Closure to "Discussion of 'Mixed Convection From a Convectively Heated Vertical Plate to a Fluid With Internal Heat Generation'" (2011, ASME J. Heat Transfer, 33, p. 122501)

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This write-up presents a Closure to "Discussion of 'Mixed Convection From a Convectively Heated Vertical Plate to a Fluid With Internal Heat Generation,'" *Journal of Heat Transfer*, 2011, Vol. 33, 122501 and upholds the mathematical correctness of the published article.

The argument of discussion by "Asterios Pantokratoras" with respect to the correctness of the published paper [1] is totally unfounded and misleading. First, it appears that "Asterios Pantokratoras" did not understand the paper or have any idea about the convectively heated boundary layer problem with a prescribed spatially decaying exponential internal heat generation that were discussed in the paper by Makinde and Aziz [1]. The argument of Asterios Pantokratoras is mathematically erroneous, incorrect, and inaccurate with respect to the model problem described in the paper. We emphasize that the paper [1] is correct both in formulation, analysis, solution, and discussion. Below is our response;

The model local similarity equations are

$$f''' + \frac{1}{2}ff'' + \mathrm{Gr}_x\theta = 0 \tag{1}$$

$$\theta'' + \frac{1}{2}\mathrm{Pr} f\theta' + \lambda_x e^{-\eta} = 0 \tag{2}$$

with

$$f(0) = f'(0) = 0, \quad \theta'(0) = -\mathrm{Bi}[1-\theta(0)] \tag{3}$$

$$f'(\infty) = 0, \quad \theta(\infty) = 0 \tag{4}$$

where the prime symbols indicate derivatives with respect to η. Following Crepeau and Clarksean [2] and Sahin [3], the spatially decaying exponential internal heat generation within the cold fluid in contact with the right surface of the plate as prescribed in the Makinde and Aziz [1] is

$$\dot{q} = \frac{\lambda_x k \mathrm{Re}_x (T_f - T_\infty)}{x^2 e^\eta} \Rightarrow \lambda_x = \frac{\dot{q}x^2 e^\eta}{k\mathrm{Re}_x(T_f - T_\infty)} \tag{5}$$

The above prescribed expression for internal heat generation enables one to obtain the local similarity equation for energy balance in the Makinde and Aziz [1]. Equations (1)–(4) is a local similarity model problem, since some of the parameters still depend on the variable x and are solved numerically using the Runge–Kutta–Fehlberg integration method implemented on MAPLE. The solutions are displayed in the paper for various values of the local similarity parameters for $\lambda_x = 0$, 1, 5, 10 (see Tables 1 and 2 in Ref. [1]). Moreover, we emphasize in Makinde and Aziz [1] that the local similarity parameters Bi_x, Gr_x, and λ_x in the model equations (1)–(4) can become similarity parameters under certain assumptions, for instance if

$$\dot{q} = lx^{-1} \Rightarrow \lambda = \frac{l\upsilon e^\eta}{kU_\infty(T_f - T_\infty)}, \quad \text{since} \quad \mathrm{Re}_x = U_\infty x/\upsilon \tag{6}$$

Finally, both the tables and the graphical results highlighted in the Makinde and Aziz [1] satisfy the model equations (1)–(4) as well as the physics of the problem as thoroughly discussed in the paper.

References

[1] Makinde, O. D., and Aziz, P. O., 2011, "Mixed Convection From a Convectively Heated Vertical Plate to a Fluid With Internal Heat Generation," ASME J. Heat Transfer, **133**(12), p. 122501.
[2] Crepeau, J. C., and Clarksean, R., 1997, "Similarity Solutions of Natural Convection With Internal Heat Generation," ASME J. Heat Transfer, **119**(5), pp. 184–185
[3] Sahin, A. Z., 1992, "Transient Heat Conduction in Semi-Infinite Solid With Spatially Decaying Exponential Heat Generation," Int. Commun. Heat Mass Transfer, **19**(3), pp. 349–358.

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