Gamut relativity: A new computational approach to brightness and lightness perception

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This article deconstructs the conventional theory that “brightness” and “lightness” constitute perceptual dimensions corresponding to the physical dimensions of luminance and reflectance, and builds in its place the theory that brightness and lightness correspond to computationally defined “modes,” rather than dimensions, of perception. According to the theory, called gamut relativity, “blackness” and “whiteness” constitute the perceptual dimensions (forming a two-dimensional “blackness–whiteness space”) underlying achromatic color perception (black, white, and gray shades). These perceptual dimensions are postulated to be related to the neural activity levels in the ON and OFF channels of vision. The theory unifies and generalizes a number of extant concepts in the brightness and lightness literature, such as simultaneous contrast, anchoring, and scission, and quantitatively simulates several challenging perceptual phenomena, including the staircase Gelb effect and the effects of task instructions on achromatic color-matching behavior, all with a single free parameter. The theory also provides a new conception of achromatic color constancy in terms of the relative distances between points in blackness–whiteness space. The theory suggests a host of striking conclusions, the most important of which is that the perceptual dimensions of vision should be generically specified according to the computational properties of the brain, rather than in terms of “reified” physical dimensions. This new approach replaces the computational goal of estimating absolute physical quantities (“inverse optics”) with the goal of computing object properties relatively.

Keywords: brightness, lightness, dimension, physical, perceptual, computation, gamut relativity


Introduction

Conventional approaches to understanding visual perception have historically identified, with few exceptions, the dimensions of perception with the physical dimensions of the world (Gibson, 1979). Standard theories of visual surface perception, for instance, posit that brightness (“perceived luminance”) and lightness (“perceived reflectance”) constitute the perceptual counterparts to the physical dimensions of luminance (light intensity physically registered by the eye) and diffuse surface reflectance (ratio of physically incident and reflected light intensity) (Adelson, 1993, 2000; Arend & Goldstein, 1987, 1990; Arend & Spehar, 1993a, 1993b; Blakeslee, Reetz, & McCourt, 2008; Gilchrist et al., 1999; Gilchrist, 2006; Jameson & Hurvich, 1967; Kingdom, 2011; Land & McCann, 1971; Logvinenko, 2002; Schirillo, 1999a, 1999b; Schirillo, Reeves, & Arend, 1990). According to this view, brightness and lightness dimensions are defined with respect to fixed, or absolute, bright/dark and white/black poles, respectively (Gilchrist, 2007; Kingdom, 2011). It has been proposed, furthermore, that brightness and lightness constitute the dimensions underlying a two-dimensional (2D) perceptual space (Logvinenko & Maloney, 2006).

Insofar as the visual system can successfully discount the illuminant to estimate surface reflectance, observers are said to exhibit lightness constancy (Adelson, 1993, 2000; Arend & Goldstein, 1987, 1990; Arend & Spehar, 1993a, 1993b; Foster & Nascimento, 1994; Gilchrist, 2006; Kingdom, 2008; Jameson & Hurvich, 1967; Land & McCann, 1971). Insofar as the visual system can successfully estimate the intensity of light reflected to the eyes, observers are said to exhibit brightness sensitivity (Rudd & Popa, 2007; Stevens, 1957; Stevens & Stevens, 1963); that is, to exhibit some measure of sensitivity to illumination intensity. These seemingly contradictory perceptual properties are conventionally reconciled by the assumption of separate brightness and lightness dimensions (Gilchrist, 2006; Logvinenko & Maloney, 2006).

The conventional distinction between brightness and lightness can be illustrated in the staircase Gelb effect (Cataliotti & Gilchrist, 1995; Gilchrist et al., 1999; Gilchrist, 2006). The effect is typically observed using physical papers and a hidden spotlight: A piece of paper that appears black in “room illumination”...
appears white when illuminated by a hidden spotlight of high intensity in an otherwise darkened room. When a paper that appears a shade of gray in room illumination is now placed in the spotlight (adjacent to the paper appearing white), the newly introduced paper looks white, but is said to look brighter than the white of the original “black” paper—an effect that may be termed brightness escalation. Each new paper that previously appeared white now appears a shade of gray upon the addition of a paper of higher reflectance to the display. The process is repeated with additional papers of progressively higher reflectance up to a paper that appears white in room illumination. Each new paper appears progressively brighter along the putative brightness dimension, and all papers that previously appeared white appear to blacken along the putative lightness dimension. The effect can be simulated using a tissue placed in front of a computer monitor on which appear squares of progressively higher luminance against a black background. The square with the highest luminance always appears white, yet each “new” square in its turn appears brighter than the “previous” white square, and all “previous” squares blacken with the addition of each “new” square.

The conventional explanation of lightness perception in the staircase Gelb effect is framed in terms of an anchoring process applied to the putative lightness dimension (Bressan, 2006; Gilchrist et al., 1999; Gilchrist, 2006). According to this lightness anchoring theory, the region of highest luminance in a scene is always “assigned” to the white pole of the lightness axis, with all other regions assigned lightness values closer to the black pole. This is computationally accomplished by calculating the logarithms of the luminance ratios (log luminance ratios) formed between the region of highest luminance in the scene and each individual region in the scene (e.g., log_{10}[100 / 10] = 1, where the highest luminance value is the numerator and luminance is defined in units of cd/m²). Lightness anchoring theory thereby interprets the “null” log luminance ratio—formed by the highest luminance region with itself (i.e., log_{10}[100 / 100] = 0)—as appearing white. Lightness anchoring theory correctly predicts, in this respect, that subjects match each white square in the staircase Gelb series to the white (highest reflectance) Munsell chip from a set of Munsell chips, which are typically viewed under separate illumination against a white background (Cataliotti & Gilchrist, 1995; Gilchrist et al., 1999; Gilchrist, 2006).

The lightness anchoring theory does not, however, explain why each new white square appears “brighter” than previous white squares, as the theory itself posits no perceptual dimension sensitive to illumination intensity. It is generally assumed, however, that the “new” white square in the staircase Gelb effect increases in value along a putative brightness dimension, just as each “previous” white square decreases in value along the putative lightness dimension (Gilchrist, 2006; Logvinenko & Maloney, 2006). The new theory proposed in this article suggests that the absolute white point posited in the lightness anchoring theory constitutes an unnatural way of representing the perceptual property of “whiteness.” This is because a physical nothing—a zero value along the lightness dimension—is posited to represent a perceptual something; namely, an absolute white point.

The lightness anchoring theory also fails to naturally explain other key features of the staircase Gelb effect (Gilchrist et al., 1999; Gilchrist, 2006). The theory does not, in particular, easily explain gamut compression; that is, why subjects choose Munsell chips ranging only from middle gray to white, rather than black to white, when required to match each square in the final arrangement of staircase Gelb squares, despite the fact that the paper of lowest reflectance appears black in room illumination. The main attempt to explain gamut compression has been in terms of the distinction between local and global anchoring (Bressan, 2006; Gilchrist et al., 1999; Gilchrist, 2006). Global anchoring means that the region of highest luminance in the entire scene anchors the lightness values of all other regions, and local anchoring means that the highest luminance value within each illumination level anchors all the lightness values in that level. A single free parameter, \( \lambda \), controls the balance between local (\( \lambda = 0 \)) and global (\( \lambda = 1 \)) anchoring. Global anchoring is thus favored when a scene is uniformly illuminated, and local anchoring is favored when different regions of a scene are variably illuminated. In the context of the staircase Gelb effect, an intermediate value of the anchoring parameter (\( 0 < \lambda < 1 \)) has been assumed to explain gamut compression. There seems to be little computational justification for such an assumption, however, as the experimenter goes to great lengths to keep evidence of the different illumination levels hidden from the subject. The theory also has no explanation for why gamut compression is weakened or eliminated when the staircase Gelb series is viewed against a white background, an effect we term gamut decompression.

One major advantage of lightness anchoring theory, however, is that it provides a useful computational tool for solving the problem of achromatic color constancy: A key task of the visual system is to determine whether two image regions with different luminance values have the same reflectance under different illuminants, or different reflectances under the same illuminant, or some linear combination of these two extremes. Under the assumption of an identical spatial arrangement of surfaces viewed under different illuminants, local anchoring ensures that surfaces with the same reflectance values all have the same lightness values (Gilchrist et al., 1999; Gilchrist, 2006): Local anchoring thereby effectively discounts the illuminant, giving rise
to achromatic color constancy, or, as it is conventionally termed, lightness constancy.

A key finding that is typically taken as evidence to support the theory of brightness and lightness dimensions is that subjects instructed to match stimuli on these putative dimensions tend to correctly match the luminance or reflectance values of stimuli, respectively (Arend & Goldstein, 1987; Arend & Spehar, 1993a, 1993b; Logvinenko & Tokunaga, 2011b; Schirillo, 1999a, 1999b; Schirillo et al., 1990). In a typical lightness matching task (Arend & Spehar, 1993b), for example, subjects are instructed to adjust the luminance of a target region in a “test” display, viewed under a given illumination, to resemble a target region in a “reference” (standard) display, viewed under a different illumination, in order to “make the test and standard patches look like they were cut from the same piece of paper” (p. 447). This task instruction results in subjects adjusting the test target to have roughly the same reflectance as the reference region, effectively discounting the differential illumination across displays, resulting in approximate lightness constancy. In a typical brightness matching task (Arend & Spehar, 1993b), by contrast, subjects are instructed to adjust the luminance of the test target to “make the apparent amount of light coming from the test patch match that from the standard patch” (p. 448). Subjects receiving these instructions typically adjust the test target to have roughly the same (but typically slightly lower) luminance as the reference region. The degree of deviation from perfect luminance matching depends on the contrast polarity of the border between target and surround in the reference display: Matches made to contrast increments are consistently closer to the physical luminance of the target than matches made to contrast decrements (Arend & Goldstein, 1987; Arend & Spehar, 1993a, 1993b; Bauml, 2001; Schirillo, 1999a, 1999b; Schirillo & Shevell, 2002). This asymmetry has largely gone unexplained in the conventional theory of brightness and lightness dimensions outlined above.

In this article, we develop a new computational theory of brightness and lightness perception, termed gamut relativity, that explains the otherwise puzzling perceptual phenomena outlined above. The following section characterizes the five key postulates of the theory. Computer simulations of the perceptual phenomena described above are then provided, followed by a detailed discussion of the implications of the theory. Mathematical and simulation details are provided in Appendices A through F.

**Characterization of the new theory**

The present study makes five key theoretical contributions that depart radically from the conventional theory outlined above. First, an alternative computational specification of the dimensions underlying achromatic color space is presented. Second, the conventional notion that the range, or gamut, of perceivable achromatic colors varies between absolute poles is abandoned. The achromatic color gamut is instead defined relative to the visual system’s characterization of spatial illumination and reflectance changes in an image. Third, a new computational approach to the problem of computing achromatic surface colors independently of spatial variations in illumination (e.g., shadows) is introduced. Fourth, a new computational interpretation of brightness and lightness is provided. This interpretation suggests that brightness and lightness are more correctly characterized as computationally defined modes, rather than dimensions, of achromatic color perception. By this it is meant that brightness and lightness correspond to different representations of achromatic colors under the different “assumptions” of spatially uniform and variable illumination, respectively. Fifth, a new conception of anchoring as a scission process applied to achromatic colors is presented. Detailed descriptions of these postulates are provided below.

**Postulate I: Whiteness and blackness as perceptual dimensions**

We propose that many aspects of brightness and lightness perception, including the staircase Gelb effect, provide evidence in favor of the postulate that “blackness” and “whiteness” form the dimensions underlying achromatic color space (Figure 1) (Vladusich, 2012). As foreshadowed above, whereas the conventional brightness and lightness dimensions vary respectively from bright-to-dark and black-to-white, blackness and whiteness vary from nothing, or zero, to some arbitrary maximum value. The current theory predicts (Vladusich, 2012; Vladusich, Lu cassen, & Cornelissen, 2007) that these dimensions are related to the neural spiking rates in the visual ON and OFF channels (Kuffler, 1953; Schiller, 1992), which vary from zero to some arbitrary maximum value. The theory thus predicts that achromatic colors are composed of various proportions of blackness and whiteness—that is, as points in blackness–whiteness space—a proposal that is not new but appears to be largely forgotten (Heggelund, 1974a, 1974b, 1992).

The application of the anchoring process (Cataliotti & Gilchrist, 1995; Gilchrist et al., 1999; Gilchrist, 2006) in the context of the current theory allows us to simulate a wide range of psychophysical data outside the scope of the conventional lightness anchoring theory. To accomplish this goal, highest luminance anchoring is applied to the blackness dimension, ensuring that the
region of highest luminance in a scene is assigned a blackness value of zero.

In gamut relativity, the highest luminance value is called the blackness anchor. Unlike the lightness anchoring theory, then, the current theory suggests that the “null” log luminance ratio naturally represents an absence of blackness, rather than a fixed white point (Figure 1A). According to the theory, the property of whiteness is instead associated with the whiteness dimension. Whiteness values are computed as the log luminance ratios formed by each region and a constant luminance value, termed the whiteness anchor, which is set arbitrarily low (e.g., log[100 / 0.1] = 3, where the whiteness anchor is the denominator). Whiteness values, like blackness values, are assumed to be nonnegative. In general, variations in the blackness and whiteness coordinates can represent variations in either illumination intensity or surface reflectance, depending on the computationally defined perceptual “mode” of the visual system (see Postulate IV).

The postulate of blackness and whiteness dimensions thus leads to a new perceptual characterization of what has conventionally been treated as brightness escalation in the staircase Gelb effect. According to the new theory, the fixed whiteness anchor in the whiteness dimension leads to the conclusion that each new square of increasing luminance is represented as a whiter shade of white, rather than a brighter shade of white, as conventionally assumed (Figure 1A). This putative whiteness escalation effect corresponds to points of increasing value along the whiteness axis with the addition of each square of increasing luminance (e.g., log[10 / 0.1] = 2 becomes log[100 / 0.1] = 3). Each previous square, by comparison, is blackened slightly by the addition of each new square of higher luminance, due to blackness anchoring (e.g., log[10 / 10] = 0 becomes log[100 / 10] = 1, where the numerator is the blackness anchor). The point in blackness–whiteness space corresponding to the “previous” white square in the staircase Gelb series thus shifts, for example, from (blackness, whiteness) coordinates (0, 2) to (1, 2) (Figure 1B). The point corresponding to the “new” white square in the example above is given by the coordinates (0, 3).

Figure 1. Illustration of postulates I and II of gamut relativity. Postulates I and II relate to: (I) the existence of blackness–whiteness space; (II) the relativity of the achromatic color gamut. These postulates are illustrated through the following explanation of the staircase Gelb effect, in which a piece of “black” paper appears white when illuminated by a hidden spotlight of high intensity in an otherwise darkened room. When a “gray” paper is now placed in the spotlight, the newly introduced paper looks whiter than the white of the original “black” paper, which now appears gray. The process is repeated with additional papers, with each new paper appearing progressively whiter, and all other papers appearing progressively blacker. (A) Whitening of “new” papers: Each square of increasing luminance is anchored to the whiteness axis by application of the rule that the highest luminance square contains zero blackness (i.e., blackness anchoring). The increase in whiteness occurs because each “new” square of relatively higher luminance is computed with respect to a constant whiteness anchor. Each “new” square thus increases in whiteness but not blackness (this increase cannot be depicted in the set of white squares shown together). (B) Blackening of “previous” papers: Each “previous” white square increases in blackness with the addition of each “new” white square, as depicted by the vector shift in the direction of increasing blackness. This occurs because each “previous” white square is computed relative to each “new” blackness anchor, defined by each “new” square of relatively higher luminance. The whiteness of each “previous” square nonetheless remains constant, as the whiteness anchor remains constant. The putative line joining all blue points defines the range of perceivable achromatic colors, or gamut, corresponding to the illumination level specified by the highest luminance value. (C) Enhanced blackening of “previous” papers: Placing a background with luminance equal to the highest luminance square around all the squares results in an additional blackness component, from simultaneous contrast, being added to each “previous” white square. This component is depicted as an additional vector shift added to the shift supplied by blackness anchoring. The net result is that the achromatic color gamut becomes more negatively sloped. All these properties contribute to the simulations of lightness matching data in the staircase Gelb effect (Figure 3).
Postulate II: Relativity of the achromatic color gamut

We can generally characterize coordinates in blackness–whiteness space in terms of the following simple rule: The region of highest luminance defines the intercept, on the whiteness axis, of a straight line with negative slope, and all points of lower luminance fall on this line (e.g., gradient = [2 – 3] / [1 – 0] = −1 and intercept = 3, in the example shown in Figure 1B). Any given line is termed an achromatic color gamut, defined perceptually as the range of achromatic colors, varying from a shade of black to a shade of white, that can be seen in a region by varying the luminance of that region between the values of the whiteness and blackness anchors, keeping all other variables fixed. This definition implies that the achromatic color gamut varies with changes in illumination intensity. As illumination intensity—and hence the highest luminance value in the scene—increases, the entire achromatic color gamut is rigidly translated up the whiteness axis. The whiteness coordinate of the highest luminance region is thus taken to represent the illumination level of the associated gamut (see also Postulate V).

The relativity of the achromatic color gamut is consistent with recent psychophysical data showing that two coordinates are required to specify achromatic colors seen under different illumination intensities (Logvinenko & Maloney, 2006; Logvinenko & Tokunaga, 2011b). Due to this illumination dependency, achromatic colors seen under different illuminants—that is, defined by different gamut lines—cannot be perfectly matched in achromatic color matching experiments (Logvinenko & Maloney, 2006; Logvinenko & Tokunaga, 2011b). The relativity postulate has no equivalent in the theory of brightness and lightness dimensions, in which achromatic color gamuts are defined absolutely as ranges between the bright/dark and black/white poles of the putative brightness and lightness dimensions, respectively. For this reason, the current theory is termed gamut relativity.

The achromatic color gamut also depends on the luminance values of the spatial background against which targets are seen (Vladusich, 2012; Vladusich et al., 2007). According to gamut relativity, this dependency is due to simultaneous contrast (Jameson & Hurvich, 1967; Wallach, 1948), which contributes additional whiteness and blackness components to the achromatic colors specified by anchoring. Each component is proportional to the log luminance ratio of a region and its background (Land & McCann, 1971; Rudd & Arrington, 2001) and is sensitive to edge polarity (Rudd, 2010; Rudd & Popa, 2007; Rudd & Zemach, 2004, 2005, 2007; Vladusich et al., 2006). Whiteness (blackness) is “induced” into a region when the luminance of the region is higher (lower) than the background, forming a contrast increment (decrement). As with the anchoring component, only positive whiteness and blackness values are allowed. The influence of simultaneous contrast is to increase (decrease) the gradient of the achromatic color gamut associated with contrast increments (decrements) (Figure 1C). Similar to the dependency of the achromatic color gamut on illumination intensity, this property of the theory ensures that achromatic colors seen against different backgrounds cannot generally be perfectly matched (Vladusich, 2012; Vladusich et al., 2007).

Postulate III: Achromatic color constancy as a correspondence problem

To solve the problem of achromatic color constancy, we apply the idea of local–global anchoring to the blackness dimension in gamut relativity: Blackness values are computed either globally with respect to the highest luminance value in the entire image (Figure 2A) or locally with respect to the highest luminance values under relatively low and high illumination levels, respectively (Figure 2B). Whiteness values continue to be computed relative to a fixed whiteness anchor. Local blackness anchoring ensures that surfaces with the same reflectance values viewed under different illuminants have the same blackness values, under the assumption of an identical spatial arrangement of surfaces viewed under different illuminants (Figure 2B).

A recent finding that plays a crucial role in the theory’s explanations of the psychophysical phenomena discussed below is that blackness values are more strongly weighted by the visual system than whiteness values (Vladusich, 2012): For equal physical changes in stimulus luminance, changes in blackness values are approximately four times greater than corresponding changes in whiteness values. This asymmetry, which is realized by applying different weights to the blackness and whiteness dimensions, is termed the blackness bias.

Due to the blackness bias, coupled with an appropriate choice of metric, the following correspondence theorem is guaranteed under local blackness anchoring (see Appendix F): Points corresponding to the same reflectance under different illumination levels lie closer to one another in blackness–whiteness space than all other points corresponding to different reflectances under different illumination levels (Figure 2B). Approximate matches between surface colors under different illumination levels are conjectured to reflect this principle: Subjects are predicted to match the blackness values of different surface colors at the expense of matching whiteness values. In highlighting the importance of the relative proximity of achromatic colors in blackness–whiteness space—at the expense of computing absolute achromatic colors irrespective of
illumination level—the correspondence theorem replaces the classical concept of lightness constancy with the concept of relative achromatic color constancy (Arend & Goldstein, 1987; Arend & Spehar, 1993a, 1993b; Gilchrist, 2006; Kingdom, 2008; Logvinenko & Maloney, 2006; Logvinenko & Tokunaga, 2011b; Schirillo, 1999a, 1999b). The correspondence theorem is also predicted to play a key role in explanations of various illusions involving real or depicted spatial illumination changes (Adelson, 1993, 2000; Gilchrist, Delman, & Jacobsen, 1983).

**Postulate IV: Brightness and lightness as perceptual modes**

A key aspect of gamut relativity is the redefinition of brightness and lightness as computationally defined modes, rather than dimensions, of vision. According to this view, the brightness mode corresponds to global anchoring ($\lambda = 1$) and the lightness mode to local anchoring ($\lambda = 0$). These modes correspond, in turn, to the respective assumptions that different image regions are uniformly ($\lambda = 1$) or variably ($\lambda = 0$) illuminated. Intermediate values of the anchoring parameter ($0 < \lambda < 1$) correspond to a continuum of perceptual modes that represent the visual system’s tendency to “hedge bets” between the brightness and lightness modes.

Various factors are predicted to modulate the balance between brightness and lightness, such as cues to the classification of edges as changes in reflectance or illumination (edge classification) (Blakeslee et al., 2008; Gilchrist et al., 1983; Logvinenko, 2002; Radonjić, Todorović, & Gilchrist, 2010), whether surface regions lie in the same depth plane (depth adjacency) (Gilchrist, 1977; Radonjić et al., 2010), the number of discriminable regions in a scene (scene articulation) (Albert, 2006; Arend & Goldstein, 1987; Arend & Spehar, 1993a, 1993b; Gilchrist & Annan, 2002; Linnell & Foster, 2002; Logvinenko, 2002; Schirillo, 1999a, 1999b; Schirillo & Shevell, 2002), and instructions influencing top-down cognitive biases (task instruc-
Postulate V: Anchoring as a scission process

The theory posits, furthermore, that anchoring can be interpreted as a process of scission (Adelson, 1993, 2000; Anderson, 2003; Anderson & Winawer, 2005, 2008; Gilchrist et al., 1983), or vector decomposition, in blackness–whiteness space (see Appendix E). This vector decomposition process splits otherwise unitary achromatic colors, computed with respect to a single global anchor, into pairs of distinct colors, which can be interpreted as surface and shadow colors. The parameter \( k \) controls the scission process: A value of \( k = 1 \) implies that no scission whatsoever occurs (brightness mode), whereas \( k = 0 \) implies that scission is complete (lightness mode). As \( k \) can take any value between these extremes, it is possible that scission is partial. The parameter \( k \) may therefore be equally well termed the anchoring or scission parameter.

A simple geometrical argument suggests that shadow colors can be represented as vectors falling directly on the blackness axis, meaning that shadows always appear a pure shade of black, with the blackness value itself depending on the difference in illumination between regions with the same reflectance and on the value of \( \lambda \) (Figure 2B). The sum of the shadow color and each decomposed surface color equals the original unitary surface color computed with respect to the global anchor (Figure 2A). This observation predicts that a type of conservation principle governs the computation of surface colors by the visual system.

We conjecture that achromatic surface colors appearing under a relatively high illumination level act as “standard” colors to which “comparison” surface colors in shadow are perceptually “assigned” by virtue of the correspondence theorem. Surfaces seen under relatively high illumination levels are thus predicted to appear in “plain view”—without the appearance of an overlaying illumination color—consistent with available psychophysical data (Anderson & Winawer, 2008; Zdravković, Economou, & Gilchrist, 2006). As surfaces in high illumination are considered the “standard” to which surface colors under lower illumination levels are perceptually assigned, the theory suggests a reason for why achromatic color space is planar. In the plane, the scission process allows points representing shadows to be displaced further away from the “standard” colors than surface colors in low illumination (Figure 2B). In one perceptual dimension, by contrast, no distinction could be made between “standard” and “comparison” surface colors, and points representing shadow and surface colors would generally lie ambiguously close to one another.

Computer simulations

Simulation goals

To illustrate the quantitative properties of gamut relativity, we simulate psychophysical measurements of the staircase Gelb effect obtained by instructing subjects to match the lightness of each square to a series of Munsell chips (Cataliotti & Gilchrist, 1995; Gilchrist et al., 1999; Gilchrist, 2006). These simulations provide a cogent explanation for the gamut compression and decompression effects evident in available psychophysical data. We also simulate a classical psychophysical experiment on the influence of task instructions and scene articulation on achromatic color matching behavior (Arend & Goldstein, 1987). These simulations illustrate how task instructions and scene articulation alter the balance between the lightness mode (local anchoring, complete scission) and the brightness mode (global anchoring, zero scission), and how the blackness bias coupled with simultaneous contrast gives rise to the asymmetry in increment–decrement matching evident in the psychological data. A control simulation shows that this increment–decrement asymmetry cannot be explained under the assumption that the whiteness and blackness dimensions of gamut relativity correspond to the conventional brightness and lightness dimensions, respectively.

Methods summary

We used the theoretical parameters estimated in Vladusich (2012), together with the stimulus information provided in Cataliotti and Gilchrist (1995) and Arend and Goldstein (1987), to generate the simulation results. The simulation procedure involved calculating reference points, denoted by indices \((i, j)\), in blackness–whiteness space to which test colors, denoted by indices \((p, q)\), were matched. This was done by numerically sampling potential test luminance values within the
Simulation of lightness matching in the staircase Gelb effect

The theory is able to simulate all three key features of the staircase Gelb effect: anchoring, gamut compression, and gamut decompression (Figure 3A). As Munsell chips are typically illuminated by a light source of different intensity to the source illuminating the staircase Gelb series, the proposed dependence of the achromatic color gamut on illumination intensity means that gamuts corresponding to the staircase Gelb series and Munsell series are shifted relative to one another along the whiteness axis (Figure 3B). The theory predicts that subjects select the Munsell chip that is perceptually closest in blackness–whiteness space to each progressive shade of white in the staircase Gelb series (Figure 3A, B). Due to blackness anchoring and the blackness bias, the Munsell chip that is perceptually most similar to each white square in the staircase Gelb series is always the same white Munsell chip.

Gamut compression is also explained in terms of the blackness bias. The white background surrounding the Munsell series, on the one hand, induces a strong blackness component into each chip by means of simultaneous contrast. The whiteness component induced into each square from the farther dark background is, on the other hand, negligible. This asymmetry means that points representing the Munsell series are stretched along the blackness axis relative to points corresponding to the staircase Gelb series. As the blackness bias ensures that blackness values are far more heavily weighted than whiteness values in the simulated matching procedure, the Munsell chips that best match the staircase Gelb squares are all of relatively high reflectance (Figure 3B, C).

The theory also simulates the previously unexplained gamut decompression effect. According to the theory, the addition of a white surround induces blackness into the staircase Gelb series by means of simultaneous contrast. This manipulation weakens the asymmetry between the staircase Gelb series and the Munsell series, as both are now seen against white backgrounds, and therefore largely eliminates gamut compression (Figure 3D). The residual difference is explained by the fact that each square in the staircase Gelb series is also affected by simultaneous contrast from neighboring squares, reducing the total induced blackness signal relative to the chips in the Munsell series, each of which is completely surrounded by white.

Simulation of the effects of matching instructions and scene articulation

We first simulate the brightness matching task using simple center-surround stimuli reported in Arend and Goldstein (1987): Subjects adjusted the luminance of a “test” region, shown against a background with a certain luminance value, such that the brightness of the test region was approximately the same as that of a “reference” region, shown against a background with a different luminance value.

To simulate the brightness matching task, we assumed that subjects performed the task in the brightness mode ($\lambda = 1$). The theory simulates the finding (Arend & Goldstein, 1987) that subjects instructed to match brightness adjusted the luminance of the test region to have roughly the same luminance as the reference region when this region was a contrast increment, but adjusted the test region to have considerably lower luminance than the reference region when this region was a contrast decrement (Figure 4A through C).

This increment–decrement asymmetry can be understood by plotting the achromatic colors in blackness–whiteness space corresponding to stimuli used in the experiment (Figure 4B). Black lines indicate the gamuts that the subject is able to perceive in the target test region by varying the luminance of the region from low to high. The achromatic color of the test region is thereby physically constrained to lie on these lines, allowing only approximate matches for test stimuli in which the background luminance differs between test and reference displays. Simultaneous contrast, associated with differences in background luminance of the test region, determines the gradients and offsets of these lines (in its absence, all lines would be identical): Regions with steep and shallow negative gradients correspond to contrast increments and decrements, respectively (Vladusich, 2012). The inflection points of these functions depend on background luminance, which differs for each of nine illumination conditions. The large black dots denote the achromatic colors of the four reference regions used in the study. Achromatic color matches made to incremental targets correspond to colored regions above the black dots, whereas matches to decremental targets correspond to
Figure 3. Simulation of lightness matching in the staircase Gelb effect. (A) For each square in the staircase Gelb series, the simulation procedure selects from amongst all Munsell chips the one closest in blackness–whiteness space to the given square. Anchoring: Each square, when first presented, appears a shade of white and is matched to the Munsell chip of highest reflectance (red line). Gamut compression: When presented together, the same squares appear compressed relative to the Munsell scale (blue line). Decompression: The introduction of a white background surrounding the staircase Gelb series eliminates the compression effect (yellow line). (B) Corresponding points in blackness–whiteness space. The black points represent colors in the Munsell series. Anchoring: The Munsell series lies on a different gamut line in blackness–whiteness space due to the higher illumination intensity and the different background associated with the Munsell series relative to the staircase Gelb series. Each highest luminance square is anchored to the whiteness axis, meaning that each new square increases in whiteness but not blackness. As the point representing the highest reflectance Munsell chip is always closer to each shade of white than any other point in the Munsell series, the simulation procedure always selects this chip, despite the perceptual experience of increasing whiteness in the staircase Gelb series. An example match is indicated by the vector with the red head: Each point along the red line in (A) corresponds to a match made between each red dot in blackness–whiteness space and the perceptually closest Munsell chip. Gamut compression: The final arrangement of squares is matched to the upper half of the Munsell scale. Again, the simulation procedure selects the Munsell chip that is perceptually the closest to each square of increasing blackness in the staircase Gelb series. An example match is indicated by the vector with the blue head: Each point along the blue line in (A) corresponds to a match made between each blue dot in blackness–whiteness space and the perceptually closest Munsell chip. Decompression: The white background decompresses the staircase Gelb series due to the effect of simultaneous contrast. The simulation procedure thus selects chips along the entire Munsell scale (the relationship between matches shown in (A) and points in blackness–whiteness space is as described above.) (C) Data showing gamut compression and predictions of local and global anchoring processes in the lightness anchoring theory, reprinted from figure 6 of Gilchrist et al. (1999). The blue line shown in (A) should be compared with the data function (data corresponding to the red line in (A) is not shown). (D) Data showing gamut decompression (or insulation), reprinted from figure 12 of Gilchrist et al. (1999). The yellow line shown in (A) should be compared with these data functions. The simulation results agree well with these psychophysical measurements.
Figure 4. Simulation study of brightness and lightness matching in Arend and Goldstein (1987). We calculated the minimum perceptual distances between reference and test colors with different values of the anchoring/scission parameter $\lambda$. (A–C) Brightness matching in a simple center-surround display ($\lambda = 1$). (A) Luminance settings of the test region, plotted in terms of simulated reflectance values against the log match/reference ratio of surround luminance values. (B) Corresponding points plotted in blackness–whiteness space. Surface color gamuts seen under different illuminants correspond to different lines in blackness–whiteness space, with each line specifying its own unique set of black, gray, and white shades. The gamut specified by any given line depends on both illumination intensity and the distribution of reflectance values within the image. Each black line shows the gamut associated with the target test region as the luminance of the region is varied (computed numerically according to the equations supplied in Appendix A), and each colored dot represents a predicted luminance setting based on the simulation procedure. The large black points denote the achromatic colors of the four reference regions. The horizontal red and yellow lines denote perfect reflectance matching, and the negatively sloped red and yellow lines denote perfect luminance matching. (C) Psychophysical data from Arend and Goldstein (1987), reprinted under the “fair use” doctrine. The theory predicts the increment-decrement asymmetry very well: Increment matches to reference regions with high reflectance (yellow, green) are closer to perfect luminance matches than decrement matches to reference regions with low reflectance (red, blue), which are closer to perfect reflectance matches. (D–F) Brightness matching in a (highly articulated) Mondrian display ($\lambda = 0.5$). All matches are shifted towards the reflectance matching line, but the increment-decrement asymmetry remains intact. (G–I) Lightness matching in a Mondrian display ($\lambda = 0$). All matches fall approximately on the lines denoting perfect reflectance matching.
colored dots positioned below the black dots. The discrepancy between the colored and black dots represents the influence of simultaneous contrast, which generally precludes the possibility of perfect achromatic color matches (Vladusich, 2012).

In the case of incremental targets, the simulation procedure sets the luminance of the test region roughly equal to that of the reference region. Due to the small overall weight applied to the whiteness dimension (see Appendix A), the theory suggests that subjects place far more weight on matching blackness values. As luminance is the primary factor determining the blackness value of incremental targets (i.e., the blackness component from simultaneous contrast is zero for increments), the model sets the luminance of the test region to approximately equal the luminance of the reference region. In the case of decremental targets, the simulation procedure sets the luminance of the test region below that of the reference region. This is because the influence of simultaneous contrast in the blackness dimension is substantial. The model thus places relatively greater weight on the simultaneous contrast component of blackness, and so must set the test region to have approximately the same physical contrast as the reference region. The best approximate match therefore occurs at a substantially lower luminance value in the test region than that of the reference region.

To simulate the task of brightness matching in highly articulated Mondrian displays, we assumed that subjects performed the matching task in an intermediate mode ($\lambda = 0.5$). The theory simulates the finding that subjects instructed to match brightness in Mondrian displays tended to set the luminance of the test region much lower than in simple center-surround displays. The asymmetry between contrast increments and decrements remains similar to that observed in simple center-surround displays, consistent with the psychophysical data in Arend and Goldstein (1987) (Figure 4D through F). The effect of simultaneous contrast in the Mondrian display is to transform the straight lines occurring with center-surround displays into curvilinear functions. This occurs because as the luminance of the test region varies from low to high, the test region becomes an increment with respect to a progressively greater number of surround regions. The test region thus smoothly varies from a complete decrement (low luminance) to a complete increment (high luminance), resulting in the curvilinear functions. The achromatic color of the test region is again physically constrained to lie on these functions, generally allowing only approximate matches to be made.

We next simulated the lightness matching task in highly articulated Mondrian displays, assuming that subjects performed the task in the lightness mode ($\lambda = 0$). The theory simulates the finding that subjects instructed to match lightness performed nearly perfect reflectance matches (Figure 4G through I). Lightness matching can be geometrically understood in terms of the correspondence theorem (Figure 4H), a process that selects the unique set of achromatic colors across different gamuts corresponding to identical reflectance values under different illuminants. According to this principle, then, perfect blackness matching corresponds to veridical physical reflectance matching. We did not simulate the task of lightness matching in center-surround displays (assuming $\lambda = 0$), as psychophysical studies indicate that subjects adopt alternative strategies not based on scission into surface and illumination layers—notably, contrast matching—to perform this task (Arend & Spehar, 1993a, 1993b; Blakeslee et al., 2008).

A straightforward interpretation of brightness and lightness matching behavior is typically provided in the literature in terms of the selective matching of values in separate brightness and lightness dimensions (Arend & Goldstein, 1987; Arend & Spehar, 1993a, 1993b; Gilchrist et al., 1999; Gilchrist, 2006, 2007; Logvinenko & Maloney, 2006; Logvinenko, Petrini, & Maloney, 2008; Logvinenko & Tokunaga, 2011b; Redding & Lester, 1980; Schirillo, 1999a, 1999b; Schirillo & Shevell, 2002). As the theory of gamut relativity does not contain brightness and lightness dimensions, we instead computationally assessed the assumption that brightness and lightness matching behaviors correspond respectively to the selective matching of whiteness and blackness values rather than a change from global to local anchoring in blackness–whiteness space. We performed these control simulations by applying a metric that allowed the selective matching of either whiteness or blackness values. The results (data not shown) for the lightness matching condition were the same as those obtained in our simulation of local anchoring in gamut relativity, but the results for the brightness matching condition did not agree with the data. We found that the luminance settings for contrast decrements more closely resembled the physical luminance values of targets than did the luminance settings for contrast increments. This result did not, furthermore, depend on the value of the anchoring parameter. As the data from several studies show exactly the opposite effect (Arend & Goldstein, 1987; Arend & Spehar, 1993a, 1993b; Bauml, 2001; Schirillo, 1999a, 1999b; Schirillo & Shevell, 2002), we conclude that the matching of whiteness values, as specified in the theory of gamut relativity, cannot generally underlie brightness matching behavior. This conclusion is consistent with the results of Vladusich (2012) showing that brightness matching behavior corresponds more closely to the matching of values in the blackness dimension than in the whiteness dimension.
**Discussion**

The theory of gamut relativity explains a number of empirical observations that have not previously been explained by alternative theories of brightness or lightness (Adelson, 2000; Anderson, 2003; Anderson & Winawer, 2005, 2008; Blakeslee & McCourt, 2004; Blakeslee et al., 2008; Bressan, 2006; Gilchrist et al., 1999; Gilchrist, 2006; Grossberg & Hong, 2006; Land & McCann, 1971; Logvinenko & Maloney, 2006; Logvinenko & Tokunaga, 2011b; Rudd, 2010; Rudd & Arrington, 2001; Rudd & Zemach, 2005). The theory explains (a) whiteness escalation in the staircase Gelb effect; (b) the accompanying gamut compression effect; (c) the decompression effect that counteracts gamut compression; (d) the observation that regions under relatively high illumination intensity are generally treated by the visual system as “standard” surface colors; (e) the simultaneous sensitivity of the visual system to illumination level and surface reflectance; (f) how the visual system establishes the correspondence between achromatic surface colors under relatively high and low illumination levels; (g) the effects of task instructions on achromatic color matching behavior; (h) the asymmetry in brightness matching behavior accompanying contrast increment and decrements; and (i) the effect of scene articulation on brightness matches. This list is not, however, entirely exhaustive, and only space limitations preclude the inclusion of many additional explanations of otherwise puzzling findings in the literature, all of which provide further support for the theory.

**Some limitations of the theory**

In our simulation of the lightness matching task in Arend and Goldstein (1987), we assumed that the target test region could not become the blackness anchor. We applied this rule because had we allowed the target to become the anchor, then the theory would have incorrectly predicted poor achromatic color constancy for high reflectance reference colors (lying on the whiteness axis). As the luminance of the target test region increased above the luminance of the current blackness anchor, its blackness value would have remained locked at zero while its whiteness value increased. This would have ensured that an absolute match was possible. Such absolute matches would roughly equate the luminance values of reference and match colors, giving rise to a result that is not consistent with available data. One observation that supports our assumption is that surface regions whose luminance values change over time with respect to surrounding surface regions do not generally anchor those regions (Annan & Gilchrist, 2004). As subjects in lightness matching tasks must adjust the luminance of the test region in a continuous manner (Arend & Goldstein, 1987), it seems likely that the target region did not anchor the entire display.

In the simulations presented in this article, we defined a default constant value for the whiteness anchor. It is likely, however, that the value of the whiteness anchor generally depends on the adaptation state of the eye. Computational analysis of achromatic color matching behavior does not appear to provide a principled way to estimate the value of the whiteness anchor, as the blackness bias ensures that blackness values almost exclusively control such matching behavior. Future modeling work will focus on determining the computational utility of the whiteness anchor and the manner in which adaptation controls its value, through analysis of luminosity thresholds (Bonato & Gilchrist, 1994) and brightness magnitude judgements (Stevens, 1957; Stevens & Stevens, 1963).

In this article, we employed the Manhattan metric in order to simplify our exposition of the correspondence theorem. In a previous article simulating brightness matching in simple center-surround displays (Vladusich, 2012), we used the Euclidean metric in a version of the theory that did not incorporate highest luminance blackness anchoring. We found that the two metrics performed almost indistinguishably for all simulations conducted in this article, but the data presented in Vladusich (2012) slightly favors the Euclidean metric. It remains likely, however, that the omission of highest luminance blackness anchoring in the previous version of the theory (Vladusich, 2012) may have distorted the simulation results. We conclude that further psychophysical experiments and computational modeling are required to isolate the metric employed by the visual system.

Gamut relativity clarifies the relationship between anchoring, scission, and achromatic color constancy under conditions of spatially variable illumination. We were less concerned with the question of how the properties of “opacity” and “transparency” are assigned to surface and shadow colors, respectively (Adelson, 1993, 2000; Anderson, 2003; Anderson & Winawer, 2005, 2008; Grossberg, 1994)—or the attendant issue of depth stratification of perceptual layers (Anderson, 2003; Grossberg, 1994)—than with explaining how the properties of achromatic color constancy may arise. Future versions of the theory will, however, need to address the problem of achromatic color perception with respect to partially transmissive media (Adelson, 1993, 2000; Anderson, 2003; Anderson & Winawer, 2005, 2008).

**The computational significance of simultaneous contrast**

Gamut relativity differs from conventional approaches to achromatic color constancy, such as edge
integration theories that rely on the summation of local and remote contrast values to discount the illuminant (Land & McCann, 1971; Rudd, 2010; Rudd & Arrington, 2001; Rudd & Popa, 2007; Rudd & Zemach, 2004, 2005, 2007; Vladusich et al., 2006). Particularly notable with respect to the current study is the attempt of Rudd (2010) to model the effects of brightness and lightness instructions on achromatic color matching behavior using a version of edge integration theory. According to this theory, edge weights are differentially modulated with respect to task instructions, resulting in differential achromatic color constancy, and hence matching behavior, with different instructions. In gamut relativity, by comparison, achromatic color constancy is achieved through a blackness anchoring process that computes the blackness value of any given region under a specific illuminant with respect to the region of highest luminance under that illuminant.

According to gamut relativity, simultaneous contrast plays a modulatory role in brightness and lightness perception by enhancing achromatic color discriminability. The curvilinear functions in Figure 4, for instance, increase the perceptual distances between points representing achromatic colors under individual illuminants relative to the case of anchoring alone (Figure 1C).

This enhanced discriminability either facilitates or weakens the dependence of achromatic color on illumination level, by either increasing or decreasing the gradients $f_j$ of the gamut lines $f_j$. The correspondence theorem only holds if these gradients satisfy the constraint $|f_j| < 1$ (Appendix F). In the case of contrast increments, simultaneous contrast increases these gradients, thereby enhancing the tendency to match luminance in brightness matching tasks. In the case of contrast decrements, simultaneous contrast decreases these gradients, thereby enhancing the tendency to match reflectance. Contrast decrements therefore manifest better invariance to illumination changes than contrast increments. In the case of contrast increments, the enhanced discriminability engendered by simultaneous contrast is limited by the potential to undermine the correspondence theorem by increasing $f_j$ by too great a factor. In the case of contrast decrements, we assume that the limited coding range of neurons imposes a key constraint that prevents $f_j$ from approaching zero (implying larger blackness values).

Similar to gamut relativity, Heggelund (1974a, 1974b, 1992) suggested that achromatic color space is composed of “blackness-to-luminous” and “whiteness” dimensions. Unlike gamut relativity, however, Heggelund proposed that the “blackness-to-luminous” dimension is driven only by contrast and the “whiteness” dimension only by luminance, meaning that his model relies on simultaneous contrast within the “blackness-to-luminous” dimension to achieve achromatic color constancy. Heggelund’s theory therefore incorrectly predicts no constancy with respect to contrast increments and is also unable to explain the effects of anchoring, task instruction, and scene articulation on achromatic color matching behavior.

Predictions and scope of gamut relativity

Gamut relativity explains gamut compression in the staircase Gelb effect in terms of simultaneous contrast coupled with the blackness bias. This explanation differs from the standard explanation of gamut compression in terms of the distinction between local and global anchoring in conventional lightness anchoring theory (Gilchrist et al., 1999; Gilchrist, 2006). Unlike lightness anchoring theory, gamut relativity thus predicts that gamut compression can be eliminated by surrounding the Munsell series with a black background rather than a white background.

Gamut relativity also predicts that varying the illumination level on the Munsell series will vary the perceptual distances between each white square in the staircase Gelb and Munsell series in a psychophysically measurable manner (Logvinenko & Maloney, 2006; Logvinenko & Tokunaga, 2011b; Vladusich, 2012; Vladusich et al., 2007). The proposed computational interpretation of anchoring as scission implies, furthermore, that the visual system obeys a conservation principle in the computation of achromatic colors: Surface and illumination colors that have undergone local anchoring/scission must sum to produce a globally anchored achromatic color. Experiments manipulating the value of the anchoring/scission parameter $\lambda$ may be helpful in testing this prediction.

Gamut relativity captures, moreover, the growing recognition within the color psychophysics community that classical color matching tasks are insufficient to correctly characterize the structure of color space under conditions of variable illumination and surround reflectance (Amano & Foster, 2004; Amano, Foster, & Nascimento, 2005; Ekroll & Faul, 2009, 2012; Ekroll, Faul, Niederée, & Richter, 2002; Ekroll, Faul, & Wendt, 2011; Faul, Ekroll, & Wendt, 2008; Foster, 2003; Foster, Amano, & Nascimento, 2001a; Foster & Nascimento, 1994; Foster, Nascimento, & Amano, 2005; Foster et al., 2001b; Foster et al., 1997; Linnell & Foster, 2002; Logvinenko & Maloney, 2006; Logvinenko & Beattie, 2011; Logvinenko & Tokunaga, 2011a, 2011b; Nascimento, de Almeida, Fiadeiro, & Foster, 2005; Tokunaga & Logvinenko, 2010a, 2010b; Vladusich et al., 2006, 2007; Vladusich, 2012). Recognizing the relativity of color matches, Logvinenko and colleagues (Logvinenko & Beattie, 2011; Logvinenko & Maloney, 2006;
Logvinenko & Tokunaga, 2011a, 2011b; Tokunaga & Logvinenko, 2010a, 2010b) have recently adopted dissimilarity-rating, least-dissimilar color matching, and partial color matching techniques in the investigation of chromatic and achromatic color perception. Foster and colleagues have long advocated the notion of “relational color constancy” within the context of experiments in which subjects are required to distinguish spatial and temporal changes in reflectance and illumination (Amano & Foster, 2004; Amano et al., 2005; Foster, 2003; Foster et al., 2001a; Foster & Nascimento, 1994; Foster et al., 2005; Foster et al., 2001b; Foster et al., 1997; Linnell & Foster, 2002; Nascimento et al., 2005). In previous psychophysical studies that provided the foundation for the current theory, we had subjects rate the quality of achromatic color matches (Vladusich, 2012; Vladusich et al., 2006, 2007) or the perceptual similarity of achromatic color pairs (Vladusich et al., 2007). All these techniques move beyond the conventional paradigm that implicitly assumes the existence of absolute achromatic and chromatic color gamuts.

The relativity of black and white

Several authors have noted that achromatic colors do not appear the same under low and high illumination levels, even though subjects can reliably perform lightness matches consistent with nearly perfect achromatic color constancy between such differentially illuminated targets (Arend & Goldstein, 1990, 1987; Gilchrist, 2006; Logvinenko & Maloney, 2006; Zdravković et al., 2006). Arend and Goldstein (1990) gave the following example:

We noted in our earlier paper that, at the observers’ lightness match, the $R = 0.03$ simulated test patches were ‘better’ blacks under low illumination than under high illumination and vice versa for the whites. The same was true of the current experiment, and we still have no clear explanation for the effect. This should not, however, be interpreted as indicating only relative lightness constancy or interval-scale knowledge of surface color. The better of the two whites (or blacks) was confidently identified as white (or black), and the poorer appearance was true only for the large illuminance differences. Furthermore, when the test-patch illuminance was low the observer could find no luminance that produced a better match to the $R = 0.95$ white standard patch than the recorded match luminance (there were always ample luminance ranges available above and below the subjects’ adjustments). (p. 1936)

This quote highlights a major explanatory advantage of gamut relativity that sets it apart from alternative theories. The theory suggests, in particular, that Arend and Goldstein (1990) were incorrect in concluding that achromatic color constancy is not relative simply because subjects cannot adjust luminance to make perceptually better matches than those determined by setting reflectance equal in match and reference targets. We instead suggest, for example, that surface blacks and whites did not appear the same under different illuminants in the Arend and Goldstein (1990) study because they were not the same blacks and whites. According to the theory, a surface with given reflectance under a high illumination level is predicted to have a greater whiteness value than a region with the same reflectance under a low illumination level. Due to the correspondence theorem, however, subjects cannot do better than to set perceptual blackness values, and hence physical reflectance values, equal. The theory thereby explains the apparent paradox of why achromatic colors look different under different illuminants (Logvinenko & Maloney, 2006) even while lightness matching experiments reveal nearly perfect illumination invariance (Arend & Goldstein, 1987, 1990; Gilchrist, 2006).

Why is achromatic color space 2D?

A significant advance afforded by gamut relativity over all alternative 2D theories of achromatic colors (Heggelund, 1974a, 1974b, 1992; Lie, 1969; Logvinenko & Maloney, 2006) is that it explains why achromatic color space must be at least 2D. This is due to the anchoring/scission process that allows achromatic colors belonging to different gamuts to be geometrically associated with one another by means of the correspondence theorem. This process can only work properly if the points corresponding to shadow colors are perceptually more distant from the “standard” surface colors than are the points representing “comparison” surface colors. The theoretical framework developed in this article shows how the visual system accomplishes this subtle computational task in terms of the geometrical relationships between points in blackness–whiteness space.

The blackness bias

Gamut relativity brings new computational meaning to the ubiquitous yet puzzling blackness bias described in many psychophysical studies (Bauml, 2001; Bowen, Pokorny, & Smith, 1989; Chubb, Landy, & Econopouly, 2004; De Weert & Spillmann, 1995; Gilchrist, 2006; Hamada, 1985; Kingdom, 1996; Magnussen &
Glad, 1975a, 1975b, 1975c; Moulden & Kingdom, 1990; Schirillo, 1999a, 1999b; Schirillo & Shevell, 2002; Shevell, 1986, 1989; Vladusich, 2012; Vladusich et al., 2006, 2007; Whittle, 1986, 1994a, 1994b; Wook Hong & Shevell, 2004). According to the correspondence theorem, the blackness bias is a necessary (though not sufficient) condition to guarantee achromatic color constancy. Unlike gamut relativity, alternative theories of achromatic color perception provide no rationale for the existence of the blackness bias and leave unexplained the role of the anchoring parameter \( k \)—an observation that is inconsistent with the effects of task instructions and scene articulation on achromatic color matching behavior in general.

The relativity of the “standard” gamut

Gamut relativity suggests that the conventional idea that highest luminance anchoring operates to generate an absolute lightness scale is not correct (Gilchrist et al., 1999; Gilchrist, 2006; Grossberg & Hong, 2006). According to gamut relativity, any given physical reflectance value can be mapped to any given point in blackness–whiteness space—depending on the reflectance values of surrounding regions, illumination intensity, and the value of the anchoring parameter \( \lambda \)—as shown in our simulations of the staircase Gelb effect. No single achromatic color gamut, moreover, provides a more correct set of estimates of physical reflectance than any other gamut: All gamuts bear equally valid relationships to physical reflectance values. The convention of assigning the gamut associated with the highest illumination level in a scene to the “standard” gamut, and gamuts in lower illumination levels to the “comparison” gamuts, for example, does not imply that the “standard” gamut represents more veridical estimates of physical reflectance values than any other gamut (Figure 5A).

The visual system could, in principle, represent the gamut in relatively lower illumination as the “standard” by reversing the convention of the assignment; that is, by defining the global blackness anchor in terms of the lowest luminance value under the lower illumination level. Under this assignment, however, achromatic colors would always be associated with negative

Figure 5. The relativity of the “standard” gamut. Vector displacements along the blackness axis define shifts between relatively high and low illumination levels. (A) Highest luminance anchoring of blackness: Let \( q \) define a relative displacement vector from high to low illumination, and let \( A \) represent the set of points corresponding to five equally (log) spaced reflectance values under high illumination intensity. \( B \) and \( B' \) represent the set of points corresponding to the same reflectance values under low illumination intensity, assuming local and global highest luminance anchoring \( (\lambda = 0 \text{ and } \lambda = 1) \), respectively. In the case of local anchoring, the points in \( A \) act as the “standard” gamut to which points in the “comparison” gamut \( B' \) become assigned by means of the correspondence theorem. (B) Lowest luminance anchoring of blackness: Let \( p \) define a relative displacement vector from low to high illumination, and let \( X \) represent the set of points corresponding to the aforementioned reflectance values under low illumination intensity. \( Y \) and \( Y' \) represent the set of points corresponding to the same reflectance values under high illumination intensity, assuming local and global lowest luminance anchoring \( (\lambda = 0 \text{ and } \lambda = 1) \), respectively. In the case of local anchoring, the points in \( X \) act as the “standard” gamut to which points in the “comparison” gamut \( Y' \) become assigned by means of the correspondence theorem. The two approaches described above are mathematically equivalent in terms of the correspondence theorem, yet the visual system’s preference for highest luminance anchoring ensures that negative whiteness and blackness values are never obtained, an observation possibly related to the biophysical constraint that spike rates in the neural ON and OFF channels of the visual system are always positive.
blackness values (Figure 5B). The visual system’s tendency to assign regions in high illumination to the “standard” gamut thus reveals a previously hidden computational expediency of highest luminance anchoring: It ensures that only positive blackness values are possible. (Positive whiteness values are also ensured if the whiteness anchor corresponds to a lower bound on registrable luminance values.) Such a proposal supports our identification of whiteness and blackness dimensions with the neural ON and OFF channels of vision, respectively. The visual system’s application of highest luminance anchoring, in this sense, reflects a key constraint imposed by biophysical limitations: Neurons can only represent whiteness and blackness through positive spike rates in the ON and OFF channels.

Gamut relativity and the computational goal of vision

Gamut relativity postulates that all achromatic color gamuts are representationally equivalent with respect to physical reflectance values, and that only the relationships between gamuts are computationally meaningful. The theory thus generically abides the extant proposal that the visual system is more concerned with the relationships between object surfaces under different illumination levels than with the representation of absolute quantities, such as luminance and reflectance, corresponding to physical dimensions (Foster & Nascimento, 1994; Gibson, 1979). Our work implies, in this sense, that the brain does not solve the “inverse optics” problem of estimating absolute physical quantities (Marr, 1982; Poggio, Torre, & Koch, 1985), consistent with recent proposals concerning the computational goal of vision (Hoffman, 2009; Mark, Marion, & Hoffman, 2010).

We suggest that conventional theories of computational vision have historically suffered from “reification” of the physical dimensions of the world (Pessoa, Thompson, & Noé, 1998); that is, the tendency to make concrete identifications between perceptual and physical dimensions where only abstract relationships exist. Reification manifests itself in the conclusion, for instance, that subjects instructed to perform lightness matches actually compute physical reflectance values, when in fact their behavior is better explained in terms of the abstract notion of the correspondence theorem defined in this article. Gamut relativity thus provides an exemplary alternative to the naive reification that has historically plagued the study of the relationship between the mind, the brain and the outside world (Gibson, 1979; Pessoa et al., 1998).

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Appendix A: Blackness–whiteness equations

Let \( i \in \{1, 2, \ldots, I\} \) index the individual regions of a Mondrian display, under the assumption that each region forms a partial border with all other regions. This assumption enforces a symmetry constraint that violates typical physical constraints of Mondrian displays (i.e., target regions do not typically abut all other regions) but is computationally expedient (i.e., no spatial structure need be built into the equations). Let the vector, \( \mathbf{r} \), represent a set of reflectance values viewed under illuminants indexed by \( j \in \{1, 2, \ldots, J\} \), with corresponding intensity values \( \ell_j \). Multiplying the reflectance vector with each scalar illuminant value, we have luminance values, \( x_j = \ell_j \mathbf{r} \), such that \( x_j = (x_i = 1, x_i = 2, \ldots, x_i = J) \), where \( x_{i,j} \) is the luminance value of the \( i \)th region under the \( j \)th illuminant.

Let \( \bar{x}_{i,j} = \prod_{n=1}^{N_{i,j}} \min(x_{i,n}, x_{n,j}) \) be the product of all luminance values smaller than, or equal to, \( x_i, j \) where \( N = I \). The whiteness value, \( \psi_i \), of the \( i \)th region under the \( j \)th illuminant is given by

\[
\psi_{i,j} = (1 - \alpha) \left[ (1 - \beta_i) \frac{x_{i,j}^N}{x_{i,j}^N} + v \log \frac{x_{i,j}}{k_{\psi}} \right]
\tag{A1}
\]

where \( k_{\psi} \) is the constant whiteness anchor, \( \alpha = 0.87 \) is the overall weight of blackness relative to whiteness, \( \beta_i \) (0 \( < \beta_i < 1 \)) is the length of the decrement border relative to the increment border formed with region \( i \), and \( v = 1.81 \) is the weight applied to the anchored luminance term.

Let \( \bar{x}_{i,j} = \prod_{n=1}^{N_{i,j}} \max(x_{i,n}, x_{n,j}) \) represent the product of all luminance values equal to, or greater than, \( x_i, j \). The blackness value, \( \phi_i \), of the \( i \)th region under the \( j \)th illuminant is then given by

\[
\phi_{i,j} = \alpha \left[ \beta_i \log \frac{\bar{x}_{i,j}^N}{\bar{x}_{i,j}^N} + \mu \log \frac{k_{\phi}}{x_{i,j}} \right]
\tag{A2}
\]

where \( k_{\phi} \) is the blackness anchor defined below, and \( \mu = 0.89 \) is the weight applied to the anchored luminance term. All parameter values above are estimates based on Step 2 of the brightness matching experiment reported in Vladusich (2012), which used simple center-surround stimuli. The simulation results are, however, extremely robust to significant variations in the values of the parameters, as similar results were obtained using the parameter estimates based on fitting both Step 1 and Step 2 in Vladusich (2012) simultaneously.

Appendix B: Setting the blackness and whiteness anchors

In the following, we assume \( J = 2 \), where \( \ell_1 > \ell_2 \), such that \( x_{i,1} > x_{i,2} \) for all values of \( i \). The parameters \( k_{\phi} \) and \( k_{\psi} \) perform the respective roles of whiteness and blackness anchoring in the theory. They determine the minimum and maximum luminance values at which the achromatic color \( a_{i,j} = (\phi_{i,j}, \psi_{i,j}) \) of region \( i \) under illuminant \( j \) is perceived as a pure shade of black (\( \psi_{i,j} = 0 \), \( \phi_{i,j} > 0 \)) and white (\( \psi_{i,j} > 0 \), \( \phi_{i,j} = 0 \)), respectively. The anchor \( k_{\phi = 1} \) was set equal to the highest luminance value defined by the vector \( x_{i,j} \). This anchor determined the blackness value \( \phi_{i,1} \) of region \( i \) under illuminant \( j = 1 \), excluding the influence of simultaneous contrast. In our simulations of brightness matching in the Arend and Goldstein (1987) experiment, we assumed for simplicity that \( k_{\phi = 1} \) was a constant (\( k_{\phi = 1} = 50 \text{ cd/m}^2 \)), though for two of the conditions in Arend and Goldstein (1987) the highest luminance value was greater than 50 cd/m². This simplification does not critically affect the results, but the exposition is greatly simplified. The whiteness anchor \( k_{\psi} \) was defined as a constant (\( k_{\psi} = 0.1 \text{ cd/m}^2 \)) for all simulations. Again, the simulation results do not depend critically on this setting.

Appendix C: Definition of local and global anchoring

The free parameter \( \lambda \) modifies the anchor associated to \( \ell_2 \) in the following way

\[
k_{\phi_2} = \max(x_1) \lambda \max(x_2)^{1-\lambda}
\tag{C1}
\]

where \( 0 < \lambda < 1 \). We propose that \( \lambda \) represents the weight applied by the visual system to the two extreme choices of anchor. The setting \( \lambda = 1 \) represents a global anchor for all regions indexed by \( i \) (brightness mode), whereas the setting \( \lambda = 0 \) represents local anchoring determined by the maximum luminance values in \( x_1 \) and \( x_2 \), respectively (lightness mode). For this reason, we term \( \lambda \) the anchoring parameter.

Appendix D: Log-linearity of the achromatic color gamut for a fixed background luminance

This section explains the log-linearity of the achromatic color gamut in our exposition of the theory in Figure 1. Setting the whiteness and blackness anchors...
to 1 and 100, and varying target luminance in three equal log steps (1, 10, and 100), blackness and whiteness values vary as follows: Blackness: log(100 / 1) = 2, log(100 / 10) = 1, log(100 / 100) = 0; Whiteness: log(1 / 1) = 0, log(10 / 1) = 1, log(100 / 1) = 2. It is thus clear that blackness and whiteness values respectively decrease and increase linearly with equal log luminance steps, meaning that the achromatic color gamut is log-linear. For the simultaneous contrast component, varying the luminance of a region from low to high (in equal log steps) on a background of low luminance, for example, whiteness varies as follows: log(1 / 1) = 0, log(10 / 1) = 1, log(100 / 1) = 2. Similarly, for a background of high luminance, blackness varies as follows: log(1 / 1) = 0, log(10 / 1) = 1, log(100 / 1) = 2. As the simultaneous contrast component is log-linear, the sum of anchoring and simultaneous contrast components must also be log-linear.

### Appendix E: Anchoring as a scission process

Anchoring can be interpreted as a process of perceptual scission, as follows: Define a global blackness anchor $k_\phi = \max(x_i)$, and let $\hat{\phi}_{i,2} = \min(\phi_{i,2})$ represent the blackness value of the region of lowest blackness (highest luminance) under the lower illumination level $\ell_2$, given the blackness anchor $k_\phi$. We can now reinterpret anchoring as a scission process, which behaves according to

$$\phi'_{i,2} = \phi_{i,2} - (1 - \lambda)\hat{\phi}_{i,2}$$

(E1)

where $\lambda$ is the anchoring/scission parameter. By the above transformation, the blackness values of regions under illuminants $\ell_1$ and $\ell_2$ are made equal, $\phi_{i,2} = \phi_{i,1}$, when $\lambda = 0$. The vector difference between transformed and untransformed achromatic colors, $a_{i,2} = a_{i,2} - a_{i,2}^\prime$, is interpreted as the achromatic color of the shadow overlaying the surface color, $a_{i,2}$. As in the anchoring formulation above, $\lambda = 1$ corresponds to the brightness mode and $\lambda = 0$ corresponds to the lightness mode.

### Appendix F: Proof of the correspondence theorem

The correspondence theorem states that—under the assumptions of local blackness anchoring, identical spatial distributions of reflectance values viewed under different illuminants, and application of the Manhattan metric—points in blackness–whiteness space corresponding to identical reflectance values viewed under different illumination levels are guaranteed to lie closer to one another than points corresponding to different reflectance values. Due to local blackness anchoring, this amounts to proving that points with the same blackness values defined by different gamut functions lie closer to one another than points with different blackness values. Let the functions $f_A$ and $f_{B'}$ correspond to the “standard” and “comparison” gamuts, respectively. Select an arbitrary point $q_1$ on the function $f_A$. We must then prove that the point $p_1$ on the function $f_{B'}$ is closer to point $q_1$ than another point $p_2$ on the function $f_{B'}$, given that the blackness values of points $q_1$ and $p_1$ are equal ($\phi_{q1} = \phi_{p1}$) and the blackness values of points $q_1$ and $p_2$ are unequal ($\phi_{q1} \neq \phi_{p2}$).

Let the vector $u_0$ join the points $q_1$ and $p_0$, $u_1$ join the points $p_0$ and $p_1$, and $u_2$ join the points $p_0$ and $p_2$, as shown in Figure 2B. To prove the correspondence theorem, we must prove that the inequality $||u_0 + u_1|| < ||u_0 + u_2||$ is always satisfied under the stated assumptions. We observe that $||u_0 + u_1|| - ||u_0 + u_2|| = ||u_1 - u_2||$, meaning we can restate the proof in terms of the simpler inequality $||u_1|| < ||u_2||$. Substituting $u_2 = |\phi_{p1} - \phi_{p2}|$ and $u_1 = |\psi_{p1} - \psi_{p2}|$, the ratio $|\psi_{p1} - \psi_{p2}| / |\phi_{p1} - \phi_{p2}|$ defines the gradient $f_1$ of the generic gamut function $f_j$ for any pair of points $p_1$ and $p_2$. If the function $f_j$ satisfies the constraint $|f_j| < 1$ then the correspondence theorem holds. For contrast increments, the gradient $f_j$ is characterized by $(1 - \alpha) / (1 + \alpha) = 0.13(1 + 1.81) / 0.87 = 0.42$. For contrast decrements, we have $(1 - \alpha) / (\alpha[1 + \mu]) = 0.13(0.87[1 + 0.89]) = 0.08$. The inequality $||u_1|| < ||u_2||$ is thus satisfied for both increments and decrements, and thus the theorem is proved.