

Perceived numerosity is reduced in peripheral vision

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In four experiments we investigated the perception of numerosity in the peripheral visual field. We found that the perceived numerosity of a peripheral cloud of dots was judged to be inferior to the one of a central cloud of dots, particularly when the dots were highly clustered. Blurring the stimuli accordingly to peripheral spatial frequency sensitivity did not abolish the effect and had little impact on numerosity judgments. In a dedicated control experiment we ruled out that the reduction in peripheral perceived numerosity is secondary to a reduction of perceived stimulus size. We suggest that visual crowding might be at the origin of the observed reduction in peripheral perceived numerosity, implying that numerosity could be partly estimated through the individuation of the elements populating the array.

system uses to support our numerosity judgments has been strongly debated in the last years.

One early suggestion for a possible algorithm that might support the visual estimation of large numerosities came from Allik and Tuulmets (1991). In their occupancy model, they suggested that numerosity is estimated by integrating the area occupied by the elements over the visual field, assuming that each element occupies a “virtual” area defined by an occupancy radius, rather than its physical extent. If the areas occupied by two nearby elements overlap, the total occupied area is decreased, and this property allows the model to capture the empirical finding that perceived numerosity decreases as the spacing between the elements is reduced (Ginsburg & Goldstein, 1987).

A more recent suggestion is that the ratio of high to low spatial frequency power within the stimulus area containing the elements constitutes a first stage for the estimation of both visual density and numerosity, the latter being computed exploiting some estimate of the stimulus area (Dakin, Tibber, Greenwood, Kingdom, & Morgan, 2011; Tibber, Greenwood, & Dakin, 2012). The idea of using this ratio of high to low spatial frequency power is appealing because it allows the model to provide estimates of density that are to a large extent insensitive to the size of the elements and their contrast. Intuitively, a set of larger elements would entail a larger amount of edges, as indexed by the high spatial frequency channel activity, but also a large overall occupied area, as indexed by the low spatial frequency channel activity, so that their ratio would remain invariant. Interestingly, a similar process emerged in the numerosity-coding units of Stoianov and Zorzi’s (2012) neural network model. When they trained a multilayer neural network to encode the visual content of an element pattern, they observed the

Introduction

Human observers have the ability to estimate the number of elements that appear within a given region of the visual field and to compare the numerosities of two or more groups of elements. Quite different processes seem to subtend this ability depending on the absolute number of elements that is being processed (see Feigenson, Dehaene, & Spelke, 2004; Revkin, Piazza, Izard, Cohen, & Dehaene, 2008). In particular, there is evidence suggesting that very low numerosities are perceived and represented precisely, possibly through the recognition of geometrical patterns within a limited set of options (e.g., Mandler & Shebo, 1982; Wolters, Vankempen, & Wijlhuizen, 1987). For larger numerosities, provided that time constraints prevent a direct counting of the number of elements, numerosity has to be estimated, and the process that our visual

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emergence of numerosity-sensitive units excited by center-surround units encoding edges and inhibited by units encoding the total extent of the elements.

A large debate has emerged in recent years regarding whether density and numerosity perception are to be seen as independent processes, i.e., whether a proper visual sense of number exists. One finding pointing towards the existence of a visual sense of number, that is the existence of dedicated channels for numerosity that are also spatially selective, is the fact that perceived numerosity can be adapted (Burr & Ross, 2008b; Durgin, 1995). In particular Burr and Ross (2008b) reported that the adaptation to a small number of elements can increase the perceived numerosity of a test stimulus, suggesting the existence of channels tuned to specific numerosities. At the same time, Durgin (2008) showed that local density, rather than overall numerosity is determining the amount of adaptation. Ross and Burr (2010) found a larger dependency on luminance in numerosity judgments as compared to density judgments, although this differential effect is not evident if density and numerosity judgments are obtained using the same stimuli (Tibber et al., 2012). Furthermore, it is not clear whether positing the existence of a dedicated system for numerosity perception independent from size and density estimation is necessary to explain observers' numerosity judgments when faced with stimuli changing in those two attributes (Raphael, Dillenburger, & Morgan, 2013). Recently Anobile, Cicchini, and Burr (in press) described a sharp change in the relationship between numerosity and sensitivity as dot density exceeds 0.25 dots/deg². The authors interpret their result as evidence that different and independent mechanisms tuned to numerosity and texture density can operate in parallel. Specifically, it appears that observers rely on numerosity for low dot densities, but as dot density exceeds 0.25 dots/deg², the mechanism based on texture density becomes more sensitive than the one based on numerosity and determines the observers' judgments. Finally, fMRI evidence suggests that numerosity is represented topographically in the human parietal cortex (Harvey, Klein, Petridou, & Dumoulin, 2013). Notice, however, that as long as it is assumed that the computation of numerosity is spatially selective (as in the Stoianov & Zorzi, 2012 model), the distinction between numerosity and density appears less relevant.

One common feature of the models that has been suggested for the extraction of numerosity information from visual displays is the attempt to produce estimates that capture the pattern of dependency of numerosity judgments from some of the basic attributes of the stimulus. Up to now, the predictions of the models have been tested by manipulating some of those attributes, for instance element size, luminance or polarity, within

the stimulus. In the present study we take a different approach, and instead of changing the attributes of the stimulus, we forced our observers to process the stimuli differently. In particular, we decided to investigate how numerosity is perceived in peripheral viewing.

Peripheral vision differs from central and particularly foveal vision in many aspects (Strasburger, Rentschler, & Jüttner, 2011). On one side, low-level aspects of vision such as light and contrast sensitivity (Harvey & Poppel, 1972; Pöppel & Harvey, 1973) change in the peripheral visual field in a way that is dependent on the spatial frequency of the stimulus (Hilz & Cavonius, 1974; Johnston, 1987; Rovamo, Virsu, & Nasanen, 1978). On the other side, the appearance of spatial frequency is distorted in the peripheral visual field (Thorpe Davis, Yager, & Jones, 1987), in a way that is compatible with the idea that the visual system fails to take into account the fact that peripheral channels are tuned to lower spatial frequencies (Thorpe Davis, 1990). Perceived size is also usually found to decrease in the peripheral visual field (Brown, Halpert, & Goodale, 2005; Newsome, 1972; Schneider, Ehrlich, Stein, Flaum, & Mangel, 1978), although the size of peripheral stimuli might appear to be larger under particular viewing conditions (Bedell & Johnson, 1984).

In three experiments we investigate how peripheral viewing affects perceived numerosity. In Experiment 1 we find a consistent reduction in the perceived numerosity of peripherally presented circular arrays of randomly arranged dots. In Experiment 2 we evaluate the effect of dot spacing in the periphery of the visual field. In Experiment 3 we ask whether peripheral contrast sensitivity can explain the reduction in perceived numerosity. Finally, in Experiment 4 we exclude that a possible reduction in the perceived size of the peripheral stimulus subtends the reduction of its perceived numerosity.

Experiment 1: Perceived peripheral numerosity is reduced

The aim of Experiment 1 was to assess how perceived numerosity is affected by peripheral viewing. As stimuli we used circular arrays of randomly arranged black dots over a white background. Observers compared the numerosity of a central and a peripheral array of dots presented simultaneously. In this experiment and in the following we chose to use arrays of dots scattered over an approximately equal area. Our results and conclusions thus equally apply to numerosity and density, which are not dissociated, although the task is formulated in terms of numerosity.

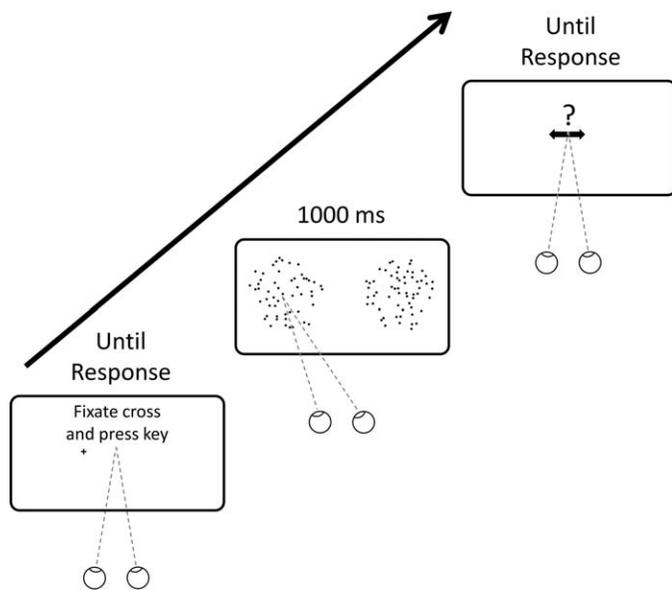


Figure 1. Procedure in Experiment 1. A black cross indicated the instructed fixation location (left). After the observer fixated on the cross and pressed a key, the two groups of points were shown for one second (center). After the two groups of points disappeared, observers indicated which group contained more dots by pressing the corresponding arrow on the keyboard (right). If observers broke fixation from the center of the group, a warning noise was given and the trial was repeated.

Methods

Observers

Eight students from the Justus-Liebig University of Giessen volunteered to participate in the study (six females, mean age = 25.2). All observers provided written informed consent in agreement with the Declaration of Helsinki. Methods and procedures were approved by the local ethics committee LEK FB06 at Giessen University (proposal number 2009-0008).

Stimuli

Two groups of dots symmetrically positioned left and right of the screen midline were used as stimuli (see Figure 1). The dots were presented in black (0.595 cd/m^2) over a white (100.1 cd/m^2) background, on a Samsung Syncmaster 1100F CRT monitor (Samsung Group, Seoul, South Korea).

The dots were arranged randomly assigning their position within a circular window whose diameter was equal to 8° of visual angle, with the constraint that the minimum center-to-center distance between any two dots had to be at least 0.281° . The dots had a radius of 0.11° of visual angle and thus were not allowed to overlap. The center of the groups of points were displaced either 6° or 9° to the left and right of the screen midline.

Procedure

At the beginning of each trial observers were asked to fixate on a cross located at the center of either the left or right group of dots (see Figure 1) and to press the keyboard spacebar as soon as they were ready. Subsequently, the two groups of dots appeared and remained visible for one second. Finally, observers were prompted to indicate which group of dots contained more elements by pressing the left or right arrow key. Gaze position on the screen was monitored online at 500 Hz with an EyeLink II system (SR Research, Mississauga, Ontario, Canada), and if the observer's gaze was not detected within 1° from the instructed fixation position, a warning white noise burst was played and the trial was repeated.

The peripheral group of dots contained either 30, 60, or 90 points, whereas the numerosity of the central group was varied following an adaptive staircase method (QUEST; Watson & Pelli, 1983). Separate staircases and thus values of Points of Subjective Equality (PSE) were obtained for each combination of peripheral numerosity (30, 60, or 90) and peripheral group eccentricity (12° or 18°). To this end, the observers' choice probabilities were fitted with a cumulative-Gaussian model using the *psignifit* toolbox version 2.5.41 for Matlab (see <http://bootstrap-software.org/psignifit/>), which implements the maximum likelihood method described by Wichmann and Hill (2001). In order to limit the differential adaptation due to the fact that the fixated group of dots always occupied the same position in retino-centric coordinates, whereas the retinal position of the peripheral group changed depending on both eccentricity and fixation side, the four combinations of the two factors' levels were tested each in a separate block of trials (whose order was counterbalanced between observers). Within a block of trials the central and peripheral stimulus always occupied the same position in retinal coordinates, thus equating the level of adaptation.

Results and discussion

The average PSEs we observed in Experiment 1 are shown in Figure 2. These values represent the numerosity of the central group of dots which observers judged to be equivalent to the numerosity of the peripheral one in the different conditions. Overall, the PSEs appear to be lower than the actual peripheral numerosity, and this effect seems to increase further in the periphery. The first impression was confirmed by the outcome a one-sample *t* test that we performed after normalizing the PSEs relative to the respective peripheral numerosity and aggregating each observer's data over all conditions, $t(7) = 4.125$, $p < 0.004$. A repeated-measure two-way ANOVA¹ with Peripheral Numerosity (30, 60, or 90) and Eccentricity (12° vs. 18°)

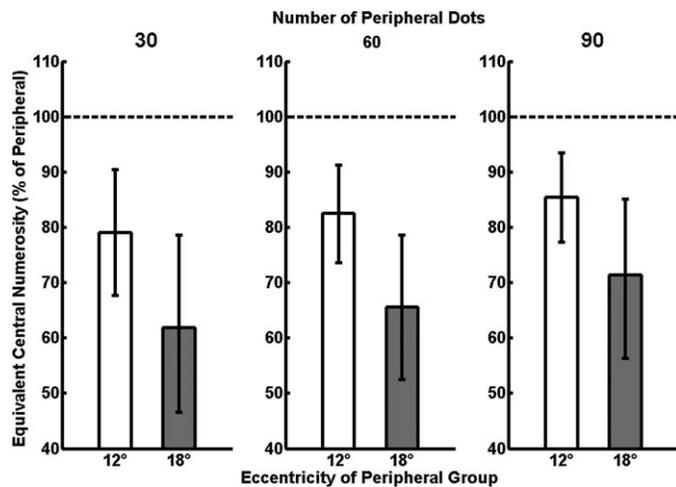


Figure 2. PSE values in Experiment 1. Results are presented in terms of the numerosity of the central group of dots, which was reported as being equivalent to the peripheral one, as a function of the eccentricity of the peripheral group and of the numerosity of the peripheral group. The values were normalized relative to the numerosity of the peripheral group. The values were in general below 100% and relatively smaller when the eccentricity of the peripheral group was 18°, indicating an eccentricity-dependent reduction of the perceived numerosity in the periphery of the visual field. Error bars represent 95% confidence intervals of the mean.

as factors performed on the normalized data evidenced that the relative PSE values decreased as the test stimulus appeared further in the periphery, Main Effect of Eccentricity: $F(1, 7) = 22.431, p < 0.002, \eta_p^2 = 0.76$. This effect was present regardless of the numerosity of the peripheral stimulus, Peripheral Numerosity \times Eccentricity interaction: $F(2, 14) = 0.447, p = 0.648, \eta_p^2 = 0.06$, which also did not have a significant effect, Main Effect of Peripheral Numerosity: $F(2, 14) = 2.545, p = 0.114, \eta_p^2 = 0.27$.

Overall, the results of Experiment 1 show clear indications that perceived numerosity is decreased in the periphery of the visual field, to an eccentricity-dependent extent. The reduction in perceived numerosity is quite conspicuous, reaching 17.7% at 12° and 33.7% at 18° and seems to extend over a quite large range of numerosities (30 to 90). Notice that this result cannot be due to some residual adaptation effects across blocks. If anything, less adaptation is expected for the peripheral location, given that the central stimulus occupied the same position in all conditions. A larger adaptation for the central location would mimic an increase in the perceived numerosity of the peripheral array. Multiple explanations for this result could be provided by the different models of numerosity perception, for instance a change of occupancy radius in the peripheral visual field (Allik & Tuulmets,

1991) or the different contrast sensitivity in the spatial frequency tuned channels (Dakin et al., 2011). In Experiment 2 we further characterize our finding by evaluating the effect of element cluster on perceived numerosity in peripheral viewing.

Experiment 2: Role of cluster

Element cluster has long been known to be a determinant of perceived numerosity (Ginsburg & Goldstein, 1987). All else (e.g., element size, contrast, array area) being equal, if the elements within a group are widely spaced, the overall perceived numerosity increases. The occupancy model by Allik and Tuulmets (1991) captured this effect by postulating a disruptive interaction between nearby elements. In Experiment 2 we test whether the peripheral reduction in perceived numerosity is dependent on element spacing. We use stimuli and an experimental procedure similar to Experiment 1; in particular, we ensure that all of our arrays cover approximately the same area independently from the degree of element cluster and numerosity.

Methods

Observers

Ten students from the Justus-Liebig University of Giessen volunteered to participate in the study (nine females, mean age = 22.8). All observers provided written informed consent in agreement with the Declaration of Helsinki. Methods and procedures were approved by the local ethics committee LEK FB06 at Giessen University (proposal number 2009-0008).

Stimuli and procedure

Stimuli and procedure were similar to the ones of Experiment 1 except for the following changes:

First of all, instead of scattering randomly the points in the circular window, we varied in three levels the clustering of the dots (see Figure 3). In order to do so while still having some dots spread over the whole circular window, we first distributed 10 points imposing a minimum center-to-center distance of 2°. The subsequent points were forced to have a center-to-center distance from the nearest point within a range which determined the level of clustering: Short (0.281° to 0.682°), Medium (0.522° to 0.923°), and Large (0.803° to 1.204°) nearest point distance. Notice that this drawing algorithm bears resemblance to the *Satellite process* in Allik and Tuulmets (1991).

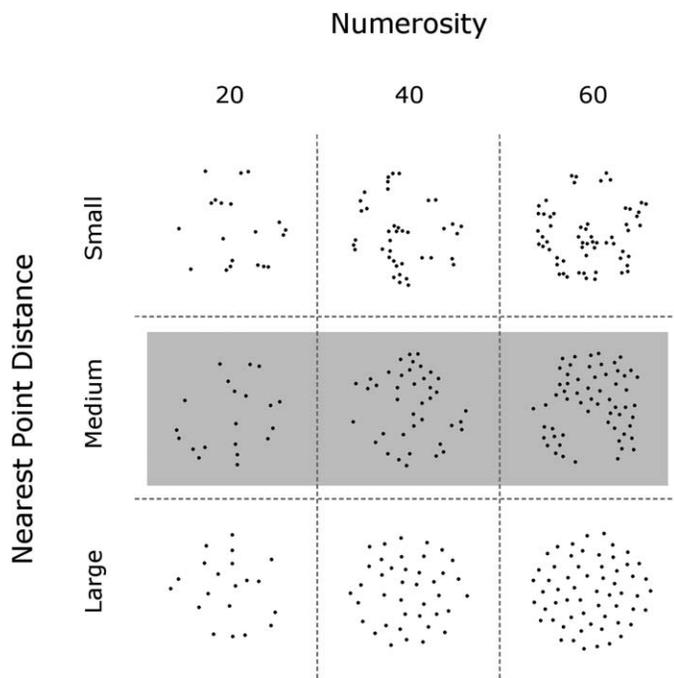


Figure 3. Peripheral stimuli used in Experiment 2. Three levels of reference numerosity and three level of nearest point distance were tested. The medium nearest point distance (indicated by the gray area) was used for the central test stimuli.

Moreover, we increased the higher level of eccentricity from 18° to 20° and we changed the possible peripheral numerosity values to 20, 40, and 60.

Results and discussion

The results of Experiment 3, expressed in terms of equivalent central numerosity, are depicted in Figure 4. Evidently, all experimental manipulations seemed to have an effect on perceived numerosity, but before addressing the whole data set, we decided to confirm the result from Experiment 1 in the most comparable subset of data. In particular, since the central group of dots in Experiment 2 had a medium dot spacing, we isolated the results obtained with peripheral medium dot spacing, averaged them across numerosities, and tested the effect of eccentricity within this subset of data. Whereas at 12° the difference between the PSEs and the reference numerosity failed to reach significance, $t(9) = 2.173$, $p = 0.058$, perceived numerosity was largely reduced at 20° , $t(9) = 5.029$, $p < 0.001$, and the PSEs also decreased reliably between 12° and 20° , $t(9) = 4.653$, $p < 0.001$. Perceived numerosity was reduced to around 90.8% at 12° eccentricity. This value is higher than the 82.3% value that we observed in Experiment 1. Most likely, this is due to the fact that the point spacing was allowed to be smaller in Experiment 1, and the results from Experiment 3 suggest that the effect of

peripheral viewing on perceived numerosity is larger when the elements cluster near to each other.

We analyzed the overall results of Experiment 2 in a three-way repeated-measure ANOVA with Peripheral Numerosity (20, 40, and 60), Nearest Point Distance (Small, Medium, Large), and Eccentricity (12° vs. 20°) as factors. This analysis revealed a significant main effect of Nearest Point Distance, $F(1.22, 11.02) = 24.826$, $p < 0.001$, $\eta_p^2 = 0.73$, and eccentricity, $F(1, 9) = 28.765$, $p < 0.001$, $\eta_p^2 = 0.76$, and a significant interaction between Nearest Point Distance and Peripheral Numerosity, $F(1.39, 12.55) = 21.317$, $p < 0.001$, $\eta_p^2 = 0.70$. All remaining main effects and interactions were not significant: main effect of Peripheral Numerosity: $F(1.16, 10.46) = 4.168$, $p = 0.063$, $\eta_p^2 = 0.32$; Nearest Point Distance \times Eccentricity interaction: $F(2, 18) = 2.484$, $p = 0.112$, $\eta_p^2 = 0.22$; Eccentricity \times Peripheral Numerosity interaction: $F(2, 18) = 1.932$, $p = 0.174$, $\eta_p^2 = 0.18$; three-way interaction: $F(4, 36) = 0.518$, $p = 0.723$, $\eta_p^2 = 0.05$.

The significant interaction between peripheral numerosity and nearest point distance can be easily explained considering how the stimuli were constructed. The spacing manipulation was only applied to the dots after the eleventh, effectively reducing the amount of cluster at lower numerosities (see Figure 3). Besides having replicated the eccentricity-dependent reduction in perceived numerosity that we had observed in Experiment 1, the results of Experiment 2 indicate that although perceived numerosity is reduced as the elements are drawn near to each other, the effect of element cluster is not dependent on eccentricity. The idea that disruptive interactions between nearby elements reduce perceived numerosity as element cluster increases is at the core of Allik and Tuulmets (1991) occupancy model of perceived numerosity, but the overall model is in general difficult to reconcile with the results of Experiment 2. This model has only one free parameter, the occupancy radius, which represents both the contribution of each element to the overall perceived numerosity and the spatial range wherein the disruptive interactions between the different elements take place. Contrary to what we observed, this model predicts that a reduction in peripheral numerosity should be associated with a reduced effect of the distance between the elements, as any reduction in perceived numerosity, all else being equal, can only be due to a reduction in the area occupied by each element.

Although the simple one parameter occupancy model proposed by Allik and Tuulmets (1991) does not seem to be powerful enough to capture the complexity of the effects that arise in peripheral numerosity perception, the slightly more complex model by Dakin and colleagues (2011) provides two possible explanations for the reduction in the perceived numerosity of peripher-

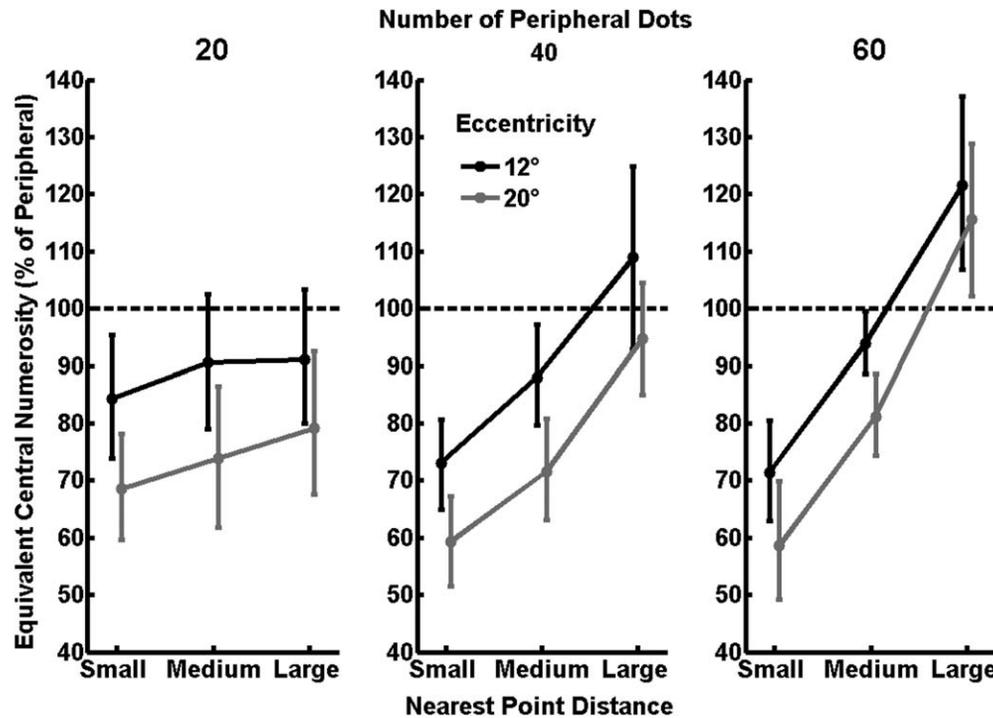


Figure 4. Results from Experiment 2. Equivalent central numerosity (expressed as a percentage of the peripheral reference numerosity) as a function of nearest point distance, reference numerosity, and eccentricity. Perceived numerosity decreased in the farther periphery and increased when the dots were more largely spaced. Central nearest point spacing was medium. Error bars are 95% between-observer confidence intervals of the mean.

ally viewed stimuli. The first explanation is in terms of peripheral contrast sensitivity and the second explanation in terms of peripheral size perception. We directly test the two explanations in Experiments 3 and 4.

Experiment 3: Role of peripheral contrast sensitivity

The model introduced by Dakin and colleagues (2011) rests on the assumption that numerosity is computed starting from the ratio of the output of high and low spatial frequency tuned channels (see the “Simulations” section for a direct application of the model to our stimuli). The mere fact that contrast sensitivity is reduced more for high spatial frequency stimuli in peripheral viewing as compared to low spatial frequencies (Hilz & Cavonius, 1974; Johnston, 1987; Rovamo et al., 1978) could be sufficient to explain the reduction in peripheral perceived numerosity we observed in Experiments 1 and 2. Indeed, our simulations show that the model predicts a large decrease in perceived numerosity for our stimuli when peripheral contrast sensitivity is taken into account. In Experiment 3 we test this hypothesis directly. We reason that if observers are shown a central stimulus

having the same effective contrast as a function of spatial frequency as the peripheral stimulus, their numerosity judgments should be veridical. We proceeded in four steps:

1. We decomposed the dot array images into components with different predominant spatial frequencies.
2. We measured individual detection contrast thresholds for each component image.
3. We constructed blurred images by recombining the components weighted on the basis of their central and peripheral contrast thresholds.
4. We tested numerosity judgments when observers compared blurred images in the center of the visual field and intact images in the periphery, or both central and peripheral stimuli at their respective threshold contrast level.

In Experiments 1 and 2 we showed our observers arrays of black dots on a white background, similar to those used by Allik and Tuulmets (1991) and Durgin (1995). In those stimuli the overall luminance of the display correlates with density and numerosity, providing an additional cue, which is, however, excluded by the Dakin et al. (2011) model, where the first stage of processing is constituted by an image rectification. We thus decided to opt for an array of black and white

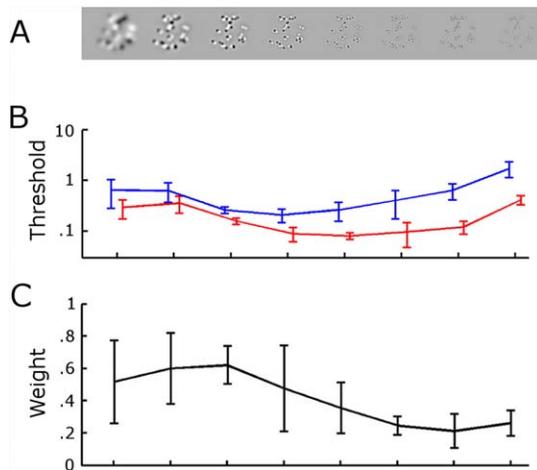


Figure 5. Stimuli and results in the detection pretest of Experiment 3. All data corresponding to one component image are aligned vertically. (A) Component images ordered from the ones with power concentrated in the lowest spatial frequency range (left) to the ones with predominantly high spatial frequency range (right). Stimulus contrast has been enhanced in the examples. (B) Detection threshold (proportion of original contrast) in both central (red) and peripheral (blue) viewing. (C) Weight (proportion of central to peripheral threshold). Error bars represent 95% confidence intervals of the mean obtained from 10,000 bootstrapped samples.

dots presented over a gray background, suppressing the total luminance cue to numerosity.

Methods

Observers

Eight students from the Justus-Liebig University of Giessen volunteered to participate in the study (seven females, mean age = 26.4). All observers provided written informed consent in agreement with the Declaration of Helsinki. Methods and procedures were approved by the local ethics committee LEK FB06 at Giessen University (proposal number 2009-0008).

Stimuli and procedure

Image decomposition: As a starting point for Experiment 3 we generated a set of images depicting arrays of dots similar to what we had done in Experiment 2. In particular, we generated 100 images with Small (0.281° to 0.682°) and Large (0.803° to 1.204°) nearest point distance for each numerosity value between 10 and 66 (the largest numerosity of elements that we could fit within the stimulus area when the spacing was large). Half of the dots were drawn in black and half of the dots were drawn in white, and after generating the images we recoded black as -1 , white as 1 , and the background as 0 .

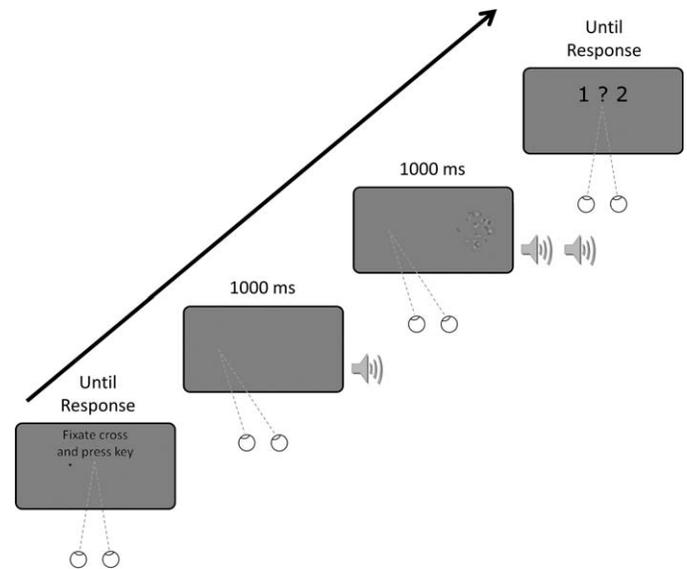


Figure 6. Trial procedure in the detection pretest. The first interval was marked by one tone and the second interval by two tones. Observers reported in which of the two intervals they saw the stimulus (an example of peripheral presentation is depicted). The contrast of the visual stimulus was reduced to near-threshold levels in the experiment.

We subsequently decomposed the images into eight component images with power concentrated at increasingly high spatial frequency (Figure 5A). The subcomponents were obtained by high-pass filtering the image and recursively subtracting the filtered image from the original one. This procedure ensured that it would be possible to re-obtain the original image by summing back the components (see Olds & Engel, 1998 for a similar approach).

The high-pass filters were obtained as the difference between the full pass filter and a Gaussian filter. We constructed seven filters with the following σ values: 0.026° , 0.054° , 0.086° , 0.130° , 0.241° , 0.347° , and 0.4864° . The eighth component contained what was left of the image after subtracting the component extracted by the seventh filter.

Detection pretest: After obtaining the image components, we proceeded to measure central and peripheral detection thresholds for each observer and component in a two-interval forced choice task (Figure 6). Observers were instructed to fixate left or right of the screen midline and to hold fixation on the same spot. As in the previous experiments, if the observer broke fixation, a white noise burst sounded and the trial was repeated. One brief (100 ms) tone signaled the beginning of the first interval and a sequence of two identical tones signaled the beginning of the second interval. The component image was presented randomly during the first or second interval and the observers reported whether the stimulus was presented in the first or second interval by pressing one of two

keys. Only images with numerosity equal to 40 were used in the detection experiment and the level of spacing (small or large) was chosen randomly. The side of fixation (left vs. right) and the eccentricity of the stimulus (0° vs. 20°) were held constant in blocks of 32 trials. In order to present the component image, we reconverted the intensity unit so that -1 would map to black (0.595 cd/m^2) and 1 to white (100.1 cd/m^2) on our CRT monitor. The background was set to gray (50.3 cd/m^2). The contrast of each image was modulated by a scaling factor. In particular for each of the sixteen combinations of component (1–8) and eccentricity (central vs. peripheral), we adaptively varied the scaling factor in order to target the threshold level. Each observer underwent three 1-hr sessions and completed on average 802 trials (50.1 trials for each threshold). After collecting the data, we used the *psignifit* toolbox in order to fit a cumulative-Gaussian model to the observers' detection data and calculate the threshold (75% correct) contrast scaling level.

Numerosity experiment: The trial procedure in the numerosity experiment was substantially identical to the one in Experiments 1 and 2; i.e., the observers fixated on a cross left or right of the screen midline and maintained fixation while two groups of dots were presented, one centered on the fixation point and one centered at 20° eccentricity. At the end of each trial the observer reported whether the left or right group of dots contained more elements. As in the first two experiments, left and right fixation were tested in two separate sessions (each lasting about 30 min) in order to equate the effect of adaptation.

Based on the results of the detection pretest, we computed three levels of scaling for each observer: a blur scaling level, where the original contrast of each component was reduced by a weight calculated from the ratio of central to peripheral threshold, and two threshold scaling levels (central and peripheral), where each component was scaled to the respective threshold level. Notice that although in the latter cases the contrast of each component was reduced to threshold levels, the sum of the eight components resulted in a suprathreshold stimulus.

Three blurring conditions were tested in the numerosity experiment (Figure 7):

1. **Threshold:** The stimuli were presented at the threshold level both in the center and in the periphery.
2. **Blurred:** The blurred stimulus scaled based on the ratio of peripheral to central sensitivity was shown at fixation whereas the intact stimulus at full contrast was presented in the periphery.
3. **Original:** Intact stimuli at full contrast were shown both at fixation and in the periphery.

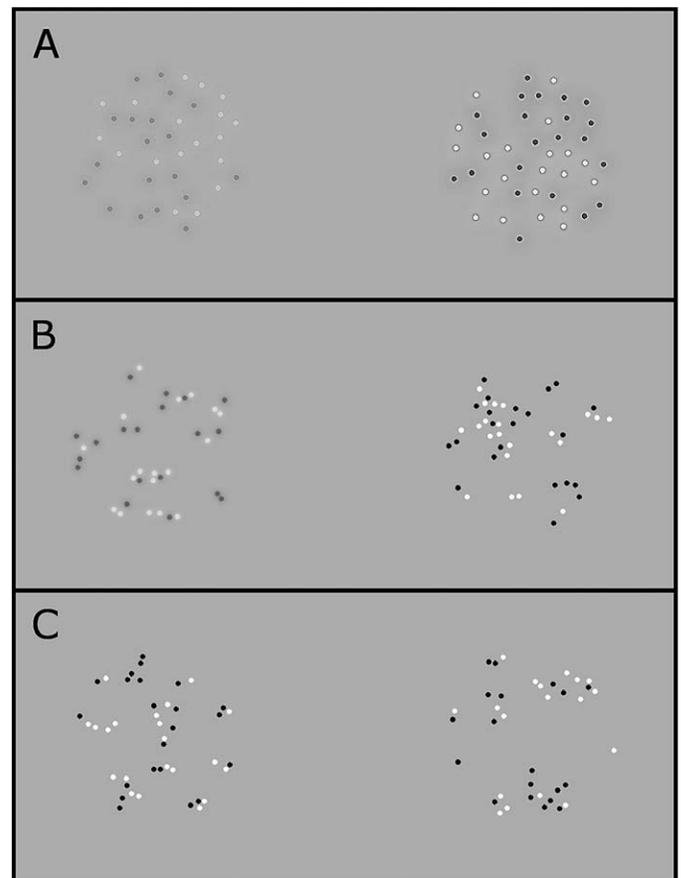


Figure 7. Stimuli used in the numerosity task of Experiment 3. For this example the images were composed averaging the weights and thresholds over all observers. (A) Threshold condition. (B) Blurred condition. (C) Original condition. All examples assume fixation on the left group. For demonstrational purposes, RGB level contrast has been increased by 50% in (A). Dot spacing is large in (A) and small in (B) and (C).

In each trial, the same level of dot spacing (small or large) was used for both the central and the peripheral stimulus. Furthermore, whereas the central stimulus numerosity varied adaptively based on the observers' responses, the peripheral stimulus numerosity could be either 37 or 43. Unlike the presentation side, which was blocked, peripheral stimulus numerosity, dot spacing and blurring were randomized across trials. Each observer provided 576 trials (equivalent to 48 trials for each combination of reference numerosity, blurring, and dot spacing).

Results and discussion

Detection pretest

The threshold scaling values for each component in central and peripheral viewing are shown in Figure 5B. The corresponding weight values computed as the ratio between central and peripheral thresholds are reported

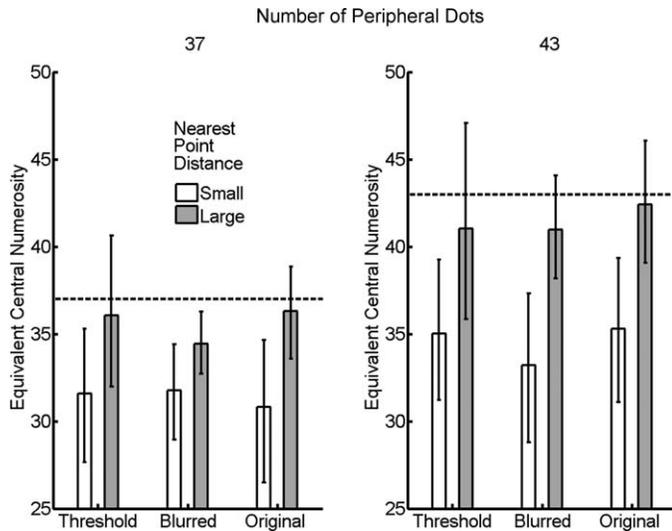


Figure 8. Results of the numerosity task of Experiment 3. Equivalent numerosity as a function of image blurring, nearest point distance and peripheral numerosity. Notice that contrary to Experiment 2 both the central and peripheral group of stimuli had the same level of point spacing and contrary to Experiments 1 and 2 the results are expressed in absolute values rather than referenced to the peripheral stimulus. Perceived numerosity reliably increased as a function of reference numerosity. The effect of peripheral viewing was more evident when the dots were tightly clustered. On the contrary, blurring the stimuli or reducing the components' contrast to the threshold levels did not affect perceived numerosity. Error bars represent 95% confidence intervals of the mean.

in Figure 5C. The threshold values in Figure 5B cannot be easily interpreted because the overall root mean square (RMS) contrast of the single components was similar but not equivalent, especially for the ones with power concentrated on the lowest spatial frequencies. Nonetheless, the ratio between the threshold expressed in the weights is compatible with the idea that contrast sensitivity reaches its maximum at lower spatial frequencies in the peripheral visual field (Hilz & Cavonius, 1974; Johnston, 1987; Rovamo et al., 1978).

Numerosity experiment

The numerosity results of Experiment 3 are depicted in Figure 8, once again expressed in terms of equivalent central numerosity. Evidently, the blurring manipulation does not seem to affect perceived numerosity in any particular way (see Raphael & Morgan, 2013 for a similar result), explicitly dismissing the hypothesis that peripheral spatial frequency sensitivity explains the reduction in perceived numerosity.

As a first step in data analysis we confirmed the replication of the basic finding of reduced perceived numerosity in peripheral vision. To this aim we isolated

the data from the original condition. Only for this analysis, we referenced the PSEs to the peripheral numerosity and averaged them across the two reference numerosities. One-sample t tests indicated that the reduction in perceived numerosity was significant when the Nearest Point Distance was small, $t(7) = 3.291$, $p < 0.013$, but not when the Nearest Point Distance was large, $t(7) = 0.407$, $p = 0.696$. This dependency of peripheral numerosity reduction on stimulus spacing was not evident in Experiment 2. We can only speculate that this has to do with the fact that the observers were comparing textures with different levels of element cluster in the center and in the periphery in Experiment 2. This is known to affect numerosity judgments also within central viewing and had a sizable effect on our results, possibly reducing the room for an additional decrease in perceived numerosity due to the peripheral viewing of tightly clustered elements. Notice that the results of Experiment 3 are even more explicitly at odds with the Allik and Tuulmets (1991) occupancy model. Within that framework, a reduction in perceived numerosity in the periphery, being the dot spacing the same, would be explained only by a reduction of the occupancy radius. This, however, would necessarily have an even stronger effect when the dots are more largely spaced and the reduction in occupancy produced by the shrinking of the radius is less counterbalanced by the reduction of the disruptive interactions between nearby elements.

As a second step in the analysis we committed the unreferenced numerosity PSEs to a three-way repeated-measure ANOVA with Nearest Point Distance (Small vs. Large), Peripheral Numerosity (37 vs. 43), and Blurring (Threshold, Blurred, or Original) as factors. This analysis revealed significant main effects of Nearest Point Distance $F(1, 7) = 6.024$, $p < 0.044$, $\eta_p^2 = 0.46$, and Peripheral Numerosity, $F(1, 7) = 42.017$, $p < 0.001$, $\eta_p^2 = 0.86$, and a significant interaction between Nearest Point Distance and Peripheral Numerosity, $F(1, 7) = 9.251$, $p < 0.019$, $\eta_p^2 = 0.57$. On the contrary, the main effect and all interactions involving blurring were not significant: main effect of Blurring: $F(2, 14) = 0.225$, $p = 0.801$, $\eta_p^2 = 0.03$; Nearest Point Distance \times Blurring interaction: $F(2, 14) = 0.437$, $p = 0.655$, $\eta_p^2 = 0.06$; Blurring \times Peripheral Numerosity interaction: $F(2, 14) = 0.35$, $p = 0.711$, $\eta_p^2 = 0.05$; three-way Interaction: $F(2, 14) = 1.226$, $p = 0.323$, $\eta_p^2 = 0.15$. Once again, the interaction between nearest point distance and peripheral numerosity might depend on the fact that the fraction of points placed following the distance rule increases as a function of total numerosity.

Notice that the highly significant effect we obtained when we increased the numerosity of the peripheral elements by just six units demonstrates that our observers responded reliably to manipulations producing relatively small changes in perceived numer-

osity. Evidently, blurring the stimuli or showing the stimuli at their detection threshold contrast did not have any remarkable effect on perceived numerosity.

Experiment 4: Peripheral size perception

The results of Experiment 3 strongly suggest that the reduction in perceived peripheral numerosity cannot be explained in terms of the differential peripheral sensitivity as a function of spatial frequency. Nonetheless, in the context of the model put forward by Dakin and colleagues (2011), there is another contributor to the estimation of numerosity beside the density estimate based on the spatial frequency content of the array: A separate mechanism provides an estimate of the array size (Raphael et al., 2013). As Tibber and colleagues (2012) acknowledge, the original suggestion that array size is estimated based on the low spatial frequency channel output is not viable because it re-introduces contrast dependence in the numerosity estimates. Regardless of what exact mechanism subtends the estimation of size, a peripheral bias in this mechanism might provide an explanation for the reduction of perceived numerosity. As we anticipated in the Introduction section, the evidence about peripheral size perception is not univocal. We thus set out to measure directly whether a misperception of the stimulus size might explain the reduction of perceived numerosity in the peripheral visual field that we observed in Experiments 1, 2, and 3.

Methods

Observers

Ten students from the Justus-Liebig University of Giessen volunteered to participate in the study (all females, mean age = 23.6). All observers provided written informed consent in agreement with the Declaration of Helsinki. Methods and procedures were approved by the local ethics committee LEK FB06 at Giessen University (proposal number 2009-0008).

Stimuli and procedure

Groups of dots similar to the ones used in Experiment 3 (black and white dots) were used as stimuli. The number of dots in the center and in the periphery was chosen randomly in the interval between 15 and 65. The dots were scattered with the only constraint that they were contained within a circular area and that the minimum interdot distance was 0.281° . The radius of the central dot array was fixed

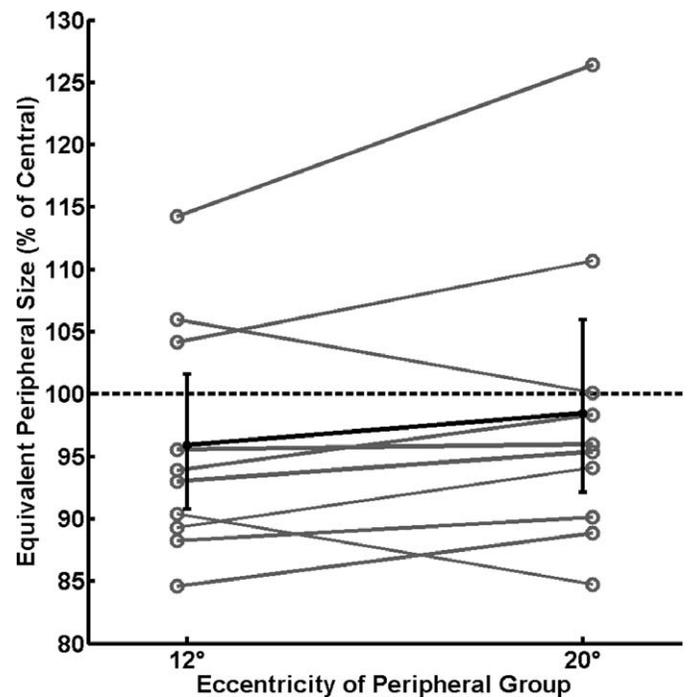


Figure 9. Size (radius) estimates in Experiment 4 expressed as percentage of the central size (8°). Observers compared the area on which dots were scattered in the central and peripheral groups. Average data are in black, whereas gray lines connect the estimates belonging to each observer. Estimates were on average veridical, but the majority of the observers (7 out of 10) reported perceiving the area of the peripheral group as larger. Error bars represent 95% confidence intervals of the mean.

throughout the experiment at 8° , similar to the previous experiments, whereas the radius of the peripheral array was allowed to vary. The trial procedure was the same as the one in Experiment 2, with the difference that observers were told explicitly to ignore the number of elements in the two arrays, which would change randomly and instead judge the size of the area where the dots were scattered. The peripheral group of dots could have an eccentricity of either 12° or 20° . Similar to the previous experiments, in order to limit the effects of differential adaptation between center and periphery, each combination of side of fixation (left or right) and eccentricity (12° or 20°) was tested in a separate block of 100 trials, meaning that each PSE was obtained by fitting a Gaussian cumulative distribution function to 200 observations.

Results and discussion

The size of PSEs obtained in Experiment 4 are depicted in Figure 9. None of the peripheral viewing conditions produced an overall effect on perceived size. The PSE values were neither statistically different from

the central array size (8°) when the peripheral stimulus was located at 12° , $t(9) = 1.388$, $p = 0.198$, nor when it was located at 20° , $t(9) = 0.403$, $p = 0.695$, from fixation. Furthermore, the PSE values did not differ between the two eccentricity conditions, $t(9) = 1.486$, $p = 0.171$. The individual data show a relatively large variability, which, given the large number of trials that were collected per condition, we do not think is simply due to measurement noise. In general, it appears that the majority of the observers (7 out of 10) reported the peripheral stimulus to appear larger than the fixated one. Previous experiments using filled shapes (Brown et al., 2005; Newsome, 1972) or lines (Schneider et al., 1978; Thompson & Fowler, 1980) generally reported reductions in the perceived size of peripheral stimuli. We can speculate that the perimeter of dot crowds is individuated in a qualitatively different way as compared to the sharp border of the objects used in the aforementioned studies. It might well be that the fuzzy edge of a dot cloud is extended when a less precise individuation of the dot positions is possible. In any case, the present result is inconsistent with an explanation of the reduction of perceived numerosity in the peripheral visual field as a consequence of a reduction of peripheral perceived stimulus size.

Simulations

As stated above, the results of Experiment 3 contradict the most straightforward prediction of the model introduced by Dakin and coworkers (2011; also see Tibber et al., 2012) in this context, namely that once the different spatial frequency components of two images are matched in terms of the response they elicit, they should also lead to the same estimates of numerosity. In this section we quantify the reduction in perceived numerosity for peripheral stimuli, which is predicted within the model solely based on the differential peripheral contrast sensitivity as a function of spatial frequency. The spatial frequency-dependent reduction in contrast sensitivity in the periphery acts as a bandpass filter rather than as a pure lowpass filter, i.e., the function relating spatial frequency and contrast sensitivity decrease is not monotonic (see Figure 5C). The specific filter pair that is used to derive the density estimate is chosen flexibly when the model is applied, and in principle the input to both filters in the pair could be equally attenuated in peripheral viewing. A full implementation of the model is thus necessary in order to show that it predicts a reduction of perceived numerosity in peripheral viewing. Moreover, we tested whether the decrease in perceived peripheral numerosity could be explained assuming that the visual system computes numerosity from different spatial frequency channels in the periphery and in the center of

the visual field, and specifically from the channel pairs whose outputs are most informative about numerosity.

The model is based on the assumption that perceived numerosity is computed from a metric common to the estimation of visual density, i.e., the ratio of high and low spatial frequency channel outputs, normalized by an estimate of the stimulus' extent provided by the output of the low spatial frequency channel. As a first step, we proceeded to compute the output of a series of 30 bandpass filters with different spatial frequency sensitivity (Laplacian of Gaussian filters, 30 values of σ logarithmically spaced between $32'$ and 1°) applied to images for each image type that was used in Experiment 3 (Original, Blurred, Peripheral Threshold and Central Threshold).² Specifically, we randomly drew the 50 images from the ones which were used in Experiment 3, i.e., small or large Nearest Point Distance and numerosity varying randomly between 10 and 66. We then integrated the filter outputs over each of the images and picked the combination of filters for which the ratio of the integrated filter outputs better predicted the numerosity across the 50 images. The procedure was conducted only once for the Original images, which were the same for all observers, and separately for each observer for the Blurred, Peripheral Threshold and Central Threshold images. The following filters were selected for the different image types. Original: $\sigma_{hi} = 0.63'$, $\sigma_{lo} = 19.18'$; Blurred: $\sigma_{hi} M = 1.87'$, $SD = 1.64'$ and $\sigma_{lo} M = 18.53'$, $SD = 2.17'$; Peripheral Threshold: $\sigma_{hi} M = 4.27'$, $SD = 1.64'$ and $\sigma_{lo} M = 18.03'$, $SD = 3.73'$; Central Threshold: $\sigma_{hi} M = 1.26'$, $SD = 1.97'$ and $\sigma_{lo} M = 17.34'$, $SD = 4.09'$.

At this point, we used the density estimation algorithm within the model by Dakin et al. (2011) in order to simulate the behavior of each observer during the experiment when comparing the original images with the blurred images and the peripheral threshold images with the central threshold images. Both of these comparisons should provide us with an estimate of the effect of peripheral viewing on perceived numerosity, which is predicted based on the differential contrast sensitivity.

As already noted by Tibber et al. (2012), the size-normalization stage in the numerosity perception model by Dakin et al. (2011) re-introduces a contrast dependence in the numerosity estimation, given that the size estimate is based on the low spatial frequency channel output and the normalization procedure effectively neutralizes the contribution of this channel to the estimation of numerosity. Nonetheless, our stimuli did not vary in size, reducing the need for a size normalization stage. Moreover, the results of Experiment 4 indicate that peripheral viewing does not have a straightforward effect on the perceived size of our stimuli. We thus carried out our simulated experiment using the channel output ratio as a direct estimate of

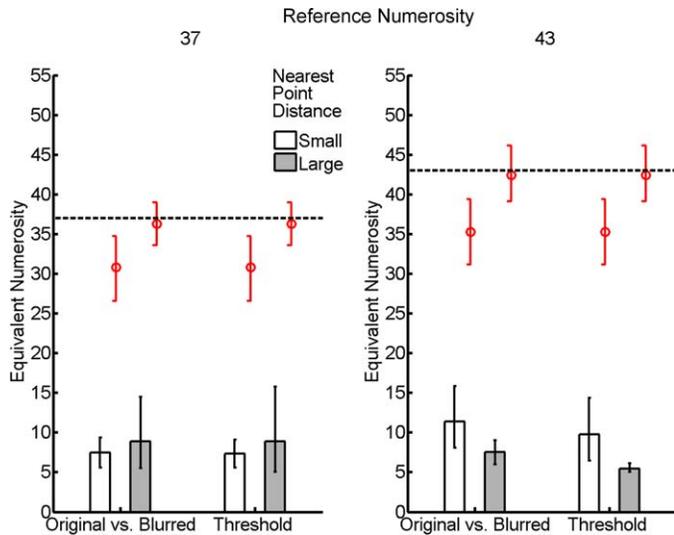


Figure 10. Simulation results. Original versus blurred indicates the numerosity of blurred images that resulted equivalent to Base N in the original images. Threshold indicates the numerosity of central threshold images that was equivalent to Base N in the peripheral threshold images. The red plots represent the corresponding observed PSEs for the original images in Experiment 3. Error bars represent 95% confidence intervals of the mean. The model predicts a disproportional reduction of perceived numerosity based on peripheral contrast sensitivity. Notice that the prediction is substantially identical in the original versus blurred and threshold comparisons, coherently with the fact that the ratio of contrast within the different spatial frequency subbands is identical.

numerosity. For each observer and condition (Original vs. Blurred and Central vs. Peripheral Threshold) we simulated 100 trials using the same adaptive staircase method that we had used in the behavioral experiments. The results of the simulation are reported in Figure 10. For comparison, we replotted the results from the original images in Experiment 3. For brevity, we only report the results we obtained using the filter pairs which we isolated from the Blurred stimuli when comparing Original and Blurred stimuli and the filter pairs from the Central Threshold stimuli when we compared the Peripheral and Central Threshold stimuli, but similar results were obtained when we used the filter pairs derived from the Original and Peripheral Threshold stimuli, respectively.

Ideally, assuming that the scaling of contrast sensitivity as a function of spatial frequency is the basis for the reduction in perceived numerosity for peripherally presented arrays of stimuli, the model should reproduce the observers' behavior when comparing the numerosity of original and blurred images and central and peripheral threshold images. This was not the case for most of the conditions. When comparing original and blurred images the model predicted extremely low PSEs, to the point that many (48.4%) of the values were

trimmed at five, as we deemed that even lower values would have been implausible. This predicted effect, amounting at least to a 79% reduction in perceived numerosity, is much more dramatic even compared to the 34% reduction in perceived numerosity which we observed at 18° eccentricity in Experiment 1 and sharply contrasts with the nonsignificant effect which physically blurring the stimuli had on perceived numerosity in Experiment 3.

For the simulations above, we compared the output of the same pair of spatial frequency channels in order to extract the numerosity information from both types of images whose numerosity was compared. However, it is possible to assume that when extracting the numerosity of central and peripheral stimuli, i.e., when comparing sets of stimuli which effectively have a different power spectrum, the visual system adaptively chooses to use the most informative pair of spatial frequency channel. Labeling channels with different spatial frequency tuning as “high” and “low” in the center and in the periphery of the visual field could in principle constitute a reason for the reduction in perceived numerosity. Indeed, Thorpe Davis (1990) suggested a similar spatial frequency channel mislabeling as an explanation for the observed increase in the perceived spatial frequency of peripherally viewed gratings.

In order to test this hypothesis, we simulated a set of experiments using, for each observer, the most appropriate pair of filters separately for the central and for the peripheral stimuli. More specifically, we tested the model using the pairs of filters from the Original (corresponding to central viewing) and Blurred (corresponding to peripheral viewing) stimuli. Coherently with the fact that peripheral visual sensitivity is relatively larger for low spatial frequencies, the high-frequency channel in the pair from the Original/central stimuli ($\sigma_{hi} = 0.63'$, $\sigma_{lo} = 19.18'$) was tuned to higher spatial frequency than one from the blurred images (mean $\sigma_{hi} = 1.87'$, mean $\sigma_{lo} = 18.53'$). We compared the density/numerosity estimates between Original and Blurred stimuli, which would correspond to the effective input to the observer while seeing Original stimuli both in the center and in the periphery (i.e., in the Original condition in Experiment 3), and two Original and two Blurred stimuli, which in turn would correspond to the effective input to the observer viewing an Original stimulus in the periphery and a Blurred stimulus in the center (i.e., the Blurred condition in Experiment 3, as the logic behind the blurring was to equalize the effective contrast in the center and in the periphery). The results of this simulation are presented in Figure 11. In this case the PSEs were on average larger than the base numerosities, forcing us to trim the values extrapolated beyond the highest numerosity in our stimuli, i.e., 100 (60.7%

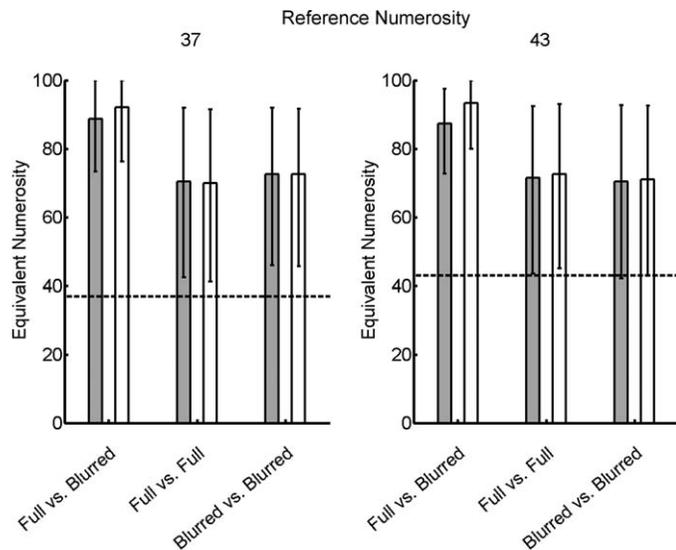


Figure 11. Results of simulation using different filter pairs for central and peripheral vision. The central and peripheral filter pairs were selected so as to best decode blurred and original stimuli, respectively. In contrast with the observed behavior in Experiment 3, under these conditions the model predicts an increase in the perceived numerosity of peripherally viewed stimuli. Error bars represent 95% confidence intervals of the mean.

of values). This result goes against an explanation of the peripheral reduction in perceived numerosity as a result of using channels tuned to different spatial frequencies in the center and periphery of the visual field within the framework of the model introduced by Dakin and colleagues (2011). If anything, using the channels whose output ratio most strongly correlates with stimulus density based on the relative spatial frequency response in the center and periphery of the visual field, the model predicts that the perceived numerosity in the peripheral visual field should increase.

General discussion

In a series of four experiments we investigated how the numerosity (Experiments 1, 2, and 3) and size (Experiment 4) of groups of dots are perceived in the peripheral visual field. Using different numerosities, eccentricities, and luminance polarity patterns we consistently found that the perceived numerosity is reduced in the periphery, whilst the area over which the dots are scattered appears unchanged. In Experiment 3, by using images of dot arrays manipulated so as to provide equivalent contrast response in the center and in the periphery for the different spatial frequency sub-bands, we exclude that the reduction in perceived

numerosity simply arises from the differential contrast sensitivity as a function of spatial frequency in the center and in the periphery of the visual field, a factor which could have explained our results in the framework of the recent model by Dakin and colleagues (2011).

To our knowledge, our study is the first to address the estimation of relatively large numerosities in the peripheral visual field, but previous studies have investigated the enumeration of peripherally presented arrays of elements within the subitizing range (Palomares, Smith, Pitts, & Carter, 2011; Parth & Rentschler, 1984). The main aim in those studies was to investigate the accuracy of enumeration in the periphery of the visual field as a function of stimulus arrangement. Parth and Rentschler (1984) were interested in the ability of observers to exactly enumerate the elements in their stimuli, so they only considered their results in terms of the accuracy drop with peripheral viewing. Moreover, they used linear arrays of dots and found that array size largely determined the observers' judgments under those conditions. Our findings are broadly consistent with the results by Palomares and colleagues (2011) that as soon as the test numerosity exceeded the subitizing range (around three elements in their observations), the observers' numerosity estimates were lower in the periphery. The authors also found that the reduction in perceived peripheral numerosity was largely abolished as soon as the elements were scaled (in terms of wavelength and size) according to cortical magnification. The stimuli they used differed to a large extent from ours, making a direct comparison unviable. On one side we used arrays of dots whereas they used Gabor patches. It makes sense to assume that the efficacy of narrow-band stimuli such as Gabor gratings is largely conditioned by the contrast sensitivity at their spatial frequency, and the shift in sensitivity towards lower spatial frequencies in the peripheral visual field needs to be compensated. Since our elements were discs, which have a broader spectrum profile, spatial frequency sensitivity should be less of an issue in our study. Indeed the results of Experiment 3, where blurring had little impact on perceived numerosity, indicate that increasing the relative power of the stimuli at lower spatial frequencies does not affect the perceived numerosity of arrays of dots dramatically. Moreover, upscaling the size of the elements within our stimuli would cause them to overlap, likely reducing the overall perceived numerosity. Even more importantly, a major difference between our paradigm and the one employed by Palomares and colleagues (2011) is the fact that they used few largely spaced elements encircling the fixation point, whereas we used a larger number of elements confined within a relatively small area. Indeed, we believe that interactions between nearby dots when

viewed peripherally might be the reason why we perceive them as reduced in number.

After dismissing perceived size and peripheral blurring as possible explanations for the reduced perceived numerosity in peripheral viewing, we would like to suggest crowding as a possible factor limiting the accessibility of the individual elements when viewed peripherally. Crowding is a general attribute of peripheral vision (for reviews see Pelli & Tillman, 2008; Strasburger et al., 2011). It is defined as an impairment in the visual system's ability to recover the features and the position of peripheral targets in the presence of nearby elements (e.g., He, Cavanagh, & Intriligator, 1996; Petrov & Popple, 2007). Generally, crowding has been evidenced in flanker tasks, where a peripheral target has to be identified and flanking distractors have to be discarded. In these tasks features of the distractors might be merged with those of the target but the presence of the target itself is still perceived. However, in a numerosity estimation task like the one we used, all of the elements are task relevant and have the same features. In this context multiple nearby elements might be merged in a single percept, resulting in a global reduction of perceived numerosity. This reasoning is coherent with the assumption that observers rely at least partially on the individuation of the elements whose numerosity they are asked to estimate (Burr & Ross, 2008a). Our results provide at least some anecdotal evidence consistent with the crowding explanation. The results from Experiment 3 indicate that the effect of peripheral viewing is stronger or possibly limited to the case when the elements are tightly clustered. It has long been known that the effect of crowding increases more than linearly as the distance between the interacting elements decreases (e.g., Eriksen & Eriksen, 1974). Distributing the elements evenly among the available area is bound to reduce the overall level of crowding.

In summary, the results of our study indicate that the perceived numerosity of ensembles of visual elements presented in the periphery of the visual field is reduced as compared to central viewing. The effect cannot be due to blurring in peripheral vision, nor is it a consequence of a misperception of peripheral stimulus size. We conclude that visual crowding possibly reduces the number of the elements to which the numerosity estimation system has access.

Keywords: numerosity, peripheral vision, crowding, size perception

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Footnotes

¹Throughout the text, a Greenhouse-Geisser correction was applied to the ANOVA degrees of freedom whenever a significant violation of the sphericity assumption was detected.

²The original model features an early rectification stage. We found that applying this rectification to our filtered images introduced sharp edges at the border between the locations occupied by dark and bright points. This in turn had the counter-intuitive effect of increasing the informativity of the high spatial frequency channel output limitedly to the case of the blurred and threshold images. We thus decided to avoid the rectification stage and applied the LOG filters after coding luminance between 0 and 1.

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