Optimizing wavefront-guided corrections for highly aberrated eyes in the presence of registration uncertainty

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Dynamic registration uncertainty of a wavefront-guided correction with respect to underlying wavefront error (WFE) inevitably decreases retinal image quality. A partial correction may improve average retinal image quality and visual acuity in the presence of registration uncertainties. The purpose of this paper is to (a) develop an algorithm to optimize wavefront-guided correction that improves visual acuity given registration uncertainty and (b) test the hypothesis that these corrections provide improved visual performance in the presence of these uncertainties as compared to a full-magnitude correction or a correction by Guirao, Cox, and Williams (2002). A stochastic parallel gradient descent (SPGD) algorithm was used to optimize the partial-magnitude correction for three keratoconic eyes based on measured scleral contact lens movement. Given its high correlation with logMAR acuity, the retinal image quality metric log visual Strehl was used as a predictor of visual acuity. Predicted values of visual acuity with the optimized corrections were validated by regressing measured acuity loss against predicted loss. Measured loss was obtained from normal subjects viewing acuity charts that were degraded by the residual aberrations generated by the movement of the full-magnitude correction, the correction by Guirao, and optimized SPGD correction. Partial-magnitude corrections optimized with an SPGD algorithm provide at least one line improvement of average visual acuity over the full magnitude and the correction by Guirao given the registration uncertainty. This study demonstrates that it is possible to improve the average visual acuity by optimizing wavefront-guided correction in the presence of registration uncertainty.

Introduction

If registered properly with the underlying wavefront error (WFE), a wavefront-guided correction in highly aberrated eyes provides markedly better optical and visual performance than conventional corrections (Liang, Grimm, Goelz, & Bille, 1994; Marsack, Parker, & Applegate, 2008; Marsack, Parker, Niu, Pesudovs, & Applegate, 2007; Rocha, Vabre, Chateau, & Krueger, 2010; Sabesan et al., 2007; Sabesan et al., 2013). However, while the goal is to have perfect registration, the reality is that registration errors can only be minimized. In principle, a wavefront-guided correction can be placed on the eye in a fixed location (e.g., inlay, onlay, refractive surgery) or in a dynamically changing location (as with a scleral or soft contact lens). For a
wavefront-guided correction in a fixed location, the goal would be to register a full wavefront-guided correction exactly with the underlying WFE. However, in practice, there will always be some degree of registration uncertainty. In the case of a wavefront-guided contact lens correction, the registration uncertainty is dynamic. In both cases, the goal is to optimize the gain in visual performance in spite of the registration uncertainty. Here, we consider the dynamic registration problem of a correction on a scleral contact lens and note that the stochastic parallel gradient descent method can also be applied to static corrections when the likely registration uncertainty is defined.

It has long been a clinical standard to use a rigid gas-permeable contact lens to correct a highly aberrated eye (e.g., keratoconus, trauma, unsuccessful refractive surgery, etc.) because these rigid lenses provide a better optical surface and the tear lens (between the back surface of the contact lens and the anterior cornea) reduces anterior surface aberrations by index matching (Franklin & Wang, 2002). Despite the benefit, studies show that rigid lenses are not able to reduce the higher order aberrations of these eyes to a normal level (Kosaki et al., 2007; Marsack, Parker, Pesudovs, Donnelly, & Applegate, 2007; Negishi, Kumanomido, Utsumi, & Tsubota, 2007) because the index matching is not perfect (tear index $\sim 1.337$ [Craig, Simmons, Patel, & Tomlinson, 1995]; cornea index $\sim 1.376$ [Smith & Atchison, 1997]) and the distortions of the corneal back surface remain (Bühren, Kook, Yoon, & Kohnen, 2010; Chen & Yoon, 2008; Nakagawa et al., 2009; Piñero, Alió, Alesón, Escaf, & Miranda, 2009).

Consequently, wavefront techniques have been introduced into the contact lens in order to reduce higher-order aberrations (Chen, Sabesan, Ahmad, & Yoon, 2007; Marsack et al., 2008; Marsack, Parker, Niu et al., 2007; Sabesan et al., 2007; Sabesan et al., 2013). For example, wavefront-guided soft lenses made by Marsack et al. (Marsack et al., 2008; Marsack, Parker, Niu et al., 2007) and Yoon (Yoon, 2012) performed well in multiple keratoconic subjects. However, little is known about the influence of registration uncertainty, particularly as the result of contact lens movement over an extended wearing time of several hours or days (López-Gil, Castejón-Mochón, & Fernández-Sánchez, 2009; Thibos, Cheng, & Bradley, 2003). Registration uncertainty is important because movement of a wavefront-guided correction with respect to the underlying WFE will induce a change in the residual aberration structure. The resulting residual aberration structure depends on the design of the wavefront-guided correction, the underlying WFE, and the magnitude of the movement. A mis-registered wavefront-error correction can induce aberration worse than that of fitting a basic sphero-cylindrical correction (Bará, Mancebo, & Moreno-Barriuso, 2000; Guirao, Williams, & Cox, 2001; Jinabhai, Neil Charman, O’Donnell, & Radhakrishnan, 2012).

Ideally, one would simply stabilize the contact lens with each insertion and removal without any registration error with respect to the underlying WFE. Even if one could completely stabilize the contact lens on insertion, it still has to be registered perfectly with the underlying WFE over time.

Instead of eliminating all placement and registration errors, one can design a correction to be tolerant to registration errors. Six major pieces of information are needed to design such a correction: (a) the WFE of the eye of interest, (b) a starting wavefront-guided correction, (c) knowledge of the residual wavefront error at each registration error of interest, (d) a metric of retinal image quality that is highly correlated to the visual task of interest to gauge the relative merit of each possible wavefront-guided correction, (e) knowledge of the likely registration errors, and (f) a search strategy to find the wavefront-guided correction that meets the desired optical stability and gain in visual performance given the registration uncertainty.

Typically, the normalized Zernike expansion (ANSI standard Z80.28-2004) is used to represent the wavefront error of the eye, the design of the correction, and any residual WFE resulting from a registration error between the underlying WFE and its correction. One of the many advantages of the normalized Zernike expansion is each term is mathematically orthogonal to all other terms, allowing one to easily parcel out the major aberration. Unfortunately, in visual performance space, each Zernike term is not orthogonal. More specifically, each term impacts visual performance differently (Applegate, Sarver, & Khemsara, 2002; Chen, Singer, Guirao, Porter, & Williams, 2005), and terms interact to either improve or degrade visual performance (Applegate, Marsack, Ramos, & Sarver, 2003; Chen et al., 2005). To reduce the complexity when operating in visual space, a single-value image-quality metric is needed that is predictive of visual performance for the task of interest as the aberration structure is manipulated (Applegate, Ballentine, Gross, Sarver, & Sarver, 2003; Chen et al., 2005). Here, we elected to use visual acuity as the visual performance task because visual acuity is the universal clinical standard and its optimization will facilitate clinical acceptance. Therefore, the ideal image-quality metrics for use in this application are the ones that are highly correlated with and predictive of visual acuity.

Movement of a contact lens is primarily caused by the complex interaction between the lens and its surrounding ocular surface environment, particularly during blinks. Chen et al. (2007) report the standard deviation of conventional soft contact lens movements...
over 2 min as ±66 um horizontally, ±76 um vertically, and ±12° rotationally. Sabesan et al. (2013) report the standard deviation of movement for scleral lenses designed specifically for stability (measured over a brief 20 s test window) as ±54.5 um (vector movement in x and y) and a rotation of ±0.58°. Here, we report and use longer-term measurements of movement for a standard scleral lens measured at 5 min intervals over 1 hr on five different days.

Finally, we need a set of rules as to how to search the multidimensional Zernike space to find a correction that, on average, improves acuity given the lens decentration and rotation.

The problem of correcting dynamic registration errors is not new, and a review of prior techniques is warranted. Improved image quality can be achieved by moving a set of correcting lenses as is done widely in the camera industry (http://www.tamron-usa.com/ a20special/shake.asp, http://www.usa.canon.com/cusa/ consumer/standard_display/Lens_Advantage_IS) and by use of adaptive optics (Beckers, 1993; Booth, 2007; Roorda & Williams, 2001). However, unlike cameras or adaptive optics, the movement of the contact lens cannot be manipulated as we wish (at least not now in a cost-effective manner). Instead, alternate strategies need to be employed. To our knowledge, Guirao (Guirao et al., 2001; Guirao et al., 2002) was the first to search for a partial-magnitude wavefront-guided correction for a moving contact lens. Given registration errors between the correction and the underlying WFE cannot be manipulated as we wish (at least not now in a cost-effective manner). Instead, alternate strategies need to be employed. To our knowledge, Guirao (Guirao et al., 2001; Guirao et al., 2002) was the first to search for a partial-magnitude wavefront-guided correction for a moving contact lens. Given registration errors between the correction and the underlying WFE can make the residual WFE worse than no correction at all; the Guirao approach correction seeks to provide a correction that always yields some benefit given the standard deviation of the registration errors as measured by RMS WFE. In short, it attempts to minimize the deleterious effect of misregistration. At that time, researchers were just beginning to understand that RMS wavefront error was not a particularly good predictor of visual performance as measured by acuity, and to our knowledge, there was no literature documenting the actual movement of a contact lens on the eye, so it made sense to adopt a Gaussian movement assumption.

Here, we adopt a design strategy that capitalizes on recent studies (Ravikumar, Sarver, & Applegate, 2012) that identified single-value image-quality metrics that are highly correlated with visual acuity, measures of the actual movement of a scleral contact lens, and a multidimensional search algorithm commonly referred to as stochastic parallel gradient descent (SPGD) (Cauwenberghs, 1996; Dembo & Kailath, 1990). This method is designed to search a multidimensional space for the collection of parameters that achieve a better-than-baseline merit function. We were attracted to this technique due to its success in constructing ophthalmic adaptive optics systems (Hofer, Sredar, Queener, Li, & Porter, 2011; Vorontsov, 2002).

The purpose of this paper is to (a) apply the SPGD method to the design of partial-magnitude second to fourth radial order wavefront-guided corrections given registration uncertainty and (b) test the hypothesis that the SPGD correction provides improved visual performance in the presence of these uncertainties as compared to a full-magnitude second- to fourth-order correction or a correction based on the Guirao et al. approach.

Methods

The methods are divided into three parts: (a) the wavefront-guided correction design strategy, (b) applying the strategy to the design of a wavefront-guided contact lens correction for keratoconic eyes, and (c) experimental validation of the predicted vision improvement provided by the resulting correction design compared to the Guirao et al. approach and a full-magnitude second to fourth radial order correction.

The wavefront-guided correction design strategy

The design strategy is illustrated in Figure 1. There are four basic steps that form a closed-loop design that defines corrections that improve image quality given known registration uncertainty.

The purpose of step 1 is to collect the information to form an input parameter set for step 2. This includes (a) quantifying the wavefront error of the eye of interest...
using the normalized Zernike expansion (ANSI standard Z80.28-2004) over a large pupil diameter (e.g., a 7 mm pupil) while wearing a well-fitted trial scleral contact lens; (b) measuring the scleral contact lens movement with respect to the center of the pupil; and (c) defining a starting correction design, which is typically the full-magnitude correction for the number of Zernike terms the designer chooses.

In step 2, the input of step 1 is used to calculate the residual aberrations and associated criterion retinal image-quality metric associated with the registration errors between the correction and the underlying WFE due to the movement of the scleral contact lens. Step 3 computes the average retinal image quality for the residual aberration due to registration errors. The goal of our SPGD algorithm is to repeatedly modify all terms in the correction (see below) until the resulting correction converges to an asymptotic average value of retinal image quality that is better than that achieved using the full-magnitude wavefront-guided correction. Once the exit criterion is met, the loop is ended. In step 4, the SPGD algorithm updates the wavefront-guided correction in small steps such that after many loops the averaged retinal image quality is ultimately improved.

The SPGD algorithm

At the end of step 3 (Figure 1), if the goal of average retinal image quality did not meet the exit criterion, the algorithm will go back to step 1 and initiate another iteration by trying another correction design. Five equations were used to determine the new correction design.

\[ \Delta u_i^k = p \times r_i^k \] (1)

\[ u_{i, \text{plus}}^k = u_i^k + \Delta u_i^k \] (2)

\[ u_{i, \text{minus}}^k = u_i^k - \Delta u_i^k \] (3)

\[ \Delta \bar{f} = \frac{\bar{f}(u_{i, \text{plus}}^k) - \bar{f}(u_{i, \text{minus}}^k)}{2} \] (4)

\[ u_{i}^{k+1} = u_i^k + \Gamma \Delta \bar{f} \Delta u_i^k \] (5)

where \( u_i^k \) is the vector of Zernike coefficients, comprised of elements \( u_{i, \text{plus}}^k \) and \( u_{i, \text{minus}}^k \) that define the correction at iteration \( k \).

The perturbation gain, which, in combination with \( r_i^k \), determines the magnitude of the random increment or the decrement of each Zernike coefficient in the next iteration; \( r_i^k \) is a random number ranging from \(-1\) to \(1\). It is generated uniquely for each Zernike coefficient; \( \Delta J \) is the difference between average logVSX resulting from the plus correction and minus correction in the current iteration; \( \Gamma \) is the correction gain, which determines the steepness of the retinal image-quality improvement curve; \( \bar{f}(u_{i, \text{plus}}^k) \) is the average image quality across measured movements associated with the Zernike coefficients perturbed in the plus direction. \( \bar{f}(u_{i, \text{minus}}^k) \) is the average image quality across measured movements associated with the Zernike coefficients perturbed in the minus direction. \( \Delta \bar{f} \) is the average difference in image quality for the plus and minus Zernike coefficients for a given iteration \( k \). Perturbation gain \( p \) and the correction gain \( \Gamma \) are determined by a set of preliminary experiments to be \( p = 0.005 \), \( \Gamma = 200 \).

For an underlying WFE and a selected registration error, the search starts with an initial reference correction design \( u_i^k \) and a perturbation of the correction \( \Delta u_i^k \) for each Zernike coefficient as determined using Equation 1. A new set of Zernike coefficients are calculated for both a plus correction \( u_{i, \text{plus}}^k \) and a minus correction \( u_{i, \text{minus}}^k \) (from Equations 2 and 3) overlaying the pupil of interest for the underlying WFE. These two new sets of Zernike coefficients for the two perturbated correction designs \( u_{i, \text{plus}}^k \) and \( u_{i, \text{minus}}^k \) are added to the underlying WFE, defining two corresponding sets of residual error Zernike coefficients. These two sets of residual error Zernike coefficients are, in turn, used to calculate the retinal image-quality metric of interest (here, logVSX) for both the \( u_{i, \text{plus}}^k \) and \( u_{i, \text{minus}}^k \) corrections. The average retinal image quality for both the \( u_{i, \text{plus}}^k \) and \( u_{i, \text{minus}}^k \) for all registration errors of interest \( \bar{f}(u_{i, \text{plus}}^k) \) and \( \bar{f}(u_{i, \text{minus}}^k) \) are calculated and the difference taken \( \Delta \bar{f} \) as detailed in Equation 4. Finally, the Zernike coefficients \( u_i^{k+1} \) for the new reference correction are proposed by adding the product of the coefficient perturbation \( \Delta u_i^k \), the difference of average retinal image quality \( \Delta \bar{f} \), and the correction gain \( \Gamma \) to the previous iteration reference Zernike coefficients \( u_i^k \) as detailed in Equation 5. The new correction is evaluated by the exit criterion. If it meets the criterion, the loop ends; if not, the algorithm goes to the next loop and keeps updating the correction.

Applying the strategy to three keratoconic eyes

Each normal chart reader and keratoconus subject signed an IRB-approved permission to use his or her wavefront data.
The lens movement data required for step 1 was measured on keratoconic subject KC1. In order to record the lens movement, the scleral contact lens had marks at the 3, 4, 6, 9, and 12 o’clock positions. The subject was allowed to adapt to the lens for at least 10 min before recording. The lens movement was measured at 5 min intervals for 1 hr and repeated every day for five days, yielding 65 movements (13 × 5). For each movement, 10 frames of images at one frame per second (1 HZ) were taken to ensure the correction lens positions were recorded. We selected a sampling rate of 1 Hz for 10 s once we established that sampling in this manner provided statistically equivalent data to sampling at 1 Hz for 60 s. There was no restriction on when a subject could blink. At least one blink often occurred during a 10 s sampling period.

The images of the lens on the eye were recorded with a custom-designed camera system (Sarver and Associates, Carbondale, IL) and were analyzed with a custom MATLAB (MathWorks) program (UHCO Core Programming Module, Houston, TX). This analysis results in each lens movement being broken down into three components with respect to the center of the pupil: horizontal translation, vertical translation, and rotation. The recorded lens movement data are shown in Figure 2. To compare the three methods, the exact same movements were used for the full-magnitude and SPGD correction designs. The correction based on the Guirao et al. (2002) approach requires as an input the standard deviation of the movements. Guirao represents the variation in translation and rotation as a Gaussian distribution. Having measured the standard deviation in translation in both x (±0.074 mm) and y (±0.101 mm) separately, we chose to use the measured standard deviation in y (the larger of the x/y standard deviations) as representative. Translation in x and y as well as rotation are illustrated in Figure 2.

To test the design strategy, we applied the method to design wavefront corrections to partially corrected wavefront errors measured for three keratoconic eyes. These wavefront errors were measured over a 7 mm pupil using a COAS HD wavefront sensor (Wavefront Sciences, Albuquerque, NM) while each keratoconic eye wore a well-fitted spherical scleral contact lens (see Figures 3C, 4C, and 5C for detail). Here, for the SPGD correction, we elected to use as the starting correction a full-magnitude second to fourth radial order correction. Refer to Figures 3C, 4C, and 5C for the WFEs illustrated in Figure 2 for three different keratoconic eyes and three different correction designs (SPGD correction, full-magnitude correction, and correction based on the Guirao et al. approach) are displayed in Figures 3A (KC1), 4A (KC2), and 5A (KC3).

In step 3 (Figure 1), average logVSX across all movement samples for all five days is calculated and used in step 4. The loop is repeated until the exit criterion is reached. The exit criterion here is when the average logVSX value reaches a plateau.

### Experimental validation

#### Chart readers

Three healthy normal individuals free of systemic and ocular pathology with best-corrected distance visual acuity better than 20/20 served as chart readers. Tenets of the Declaration of Helsinki were followed; IRB-approved informed consent was obtained from the subjects after explanation of the nature and possible consequences of the study.
As described in Ravikumar et al. (2012) and repeated here for the convenience of the reader, in each chart reader, the eye with better acuity was dilated with 1% cyclopentolate hydrochloride to paralyze accommodation and best corrected via a subjective refraction through a 3 mm artificial pupil. Each chart reader’s dilated WFE was measured 10 times using a Shack-Hartmann wavefront sensor while wearing their best cycloplegic spectacle correction. Each measured WFE was fitted with a normalized Zernike expansion (ANSI-28-2004) through the 10th radial order over a 3 mm pupil. The average of the 10 measurements for each chart reader was taken as the best estimate of the test eye’s best corrected WFE over a 3 mm pupil and used for the precompensation procedure described below.

**Residual wavefront aberrations of interest**

Unlike in the Ravikumar et al. (2012) paper, in which the wavefront aberrations were selected over a range expected to cause a visual acuity change of up to five lines, the wavefront aberrations here were selected from the residual aberrations resulting from each of the three corrections for each keratoconic eye. The residual aberration data points selected were based on the percentage of their logVSX values in their specific
residual aberration series. For each of the nine residual aberration series, data points on five levels of percentage within their logVSX range (10%, 25%, 50%, 75%, and 90%) were selected. In doing so, these five data points together appropriately represent both the range and the variability of the residual aberration after applying the corrections (see Figures 3A, 4A, and 5A for enlarged data points).

**Generation of the “blurred” precompensated logMAR acuity charts**

As in previous work, the experimental approach placed the “blur” resulting from each residual aberration computationally in the target as opposed to changing the optics to induce blur (Applegate, Marsack et al., 2003; Applegate & Sarver, 1999; Applegate et al., 2002; Burton & Haig, 1984; Cheng et al., 2010; Doshi,
Acuity was measured for each subject using computationally "blurred" random letter configurations of a standard logMAR acuity chart for each of the different residual aberration after precompensating for the individual 3 mm WFE of each of the three subjects who read the charts.

The residual aberration of each chart reader was precompensated by preemphasizing (Burton & Haig, 1984; Cheng et al., 2010) the optical transfer function (OTF) of each residual aberration with each subject’s best-corrected OTF over a 3 mm pupil as defined

Figure 4. Panel A. Change in logVSX for 65 movement samples is plotted for simulated subject KC2. Second- to fourth-order SPGD correction provides better overall logVSX (green) than the other two corrections (blue, full-magnitude second- to fourth-order correction and orange, correction based on the Guirao et al. approach). The bigger symbols indicate the residual aberration structures selected for the visual acuity chart simulation study. Panel B. Change in measured logMAR for each of the selected residual aberration structures plotted against the corresponding contact lens movement sample. Fourth-order SPGD correction (green) provides better overall logMAR acuity than the other two corrections. Panel C. KC2_WFE 7 mm pupil as measured over the contact lens for Zernike terms second to 10th radial order. Black circle indicates 5 mm pupil over each of the corrections move. KC2_WFE as measured over the contact lens for all Zernike terms second to 10th radial order, KC2_4Full: second- to fourth-order full-magnitude correction, KC2_4Guirao: second- to fourth-order correction based on the Guirao et al. approach, KC2_4SPGD: second- to fourth-order SPGD correction for KC2.

Sarver, & Applegate, 2001; Marsack et al., 2004; Sarver & Applegate, 2000; Smith, Jacobs, & Chan, 1989).
in Equation 8.

\[ \text{Pre} - \text{emphasized OTF} = \frac{OTF\text{ (residual aberration)}}{OTF\text{ (3 mm)}} \]

(8)

where OTF \text{ (residual aberration)} = \text{the OTF calculated from each residual aberration of interest. OTF (3 mm)} = \text{the OTF over a 3 mm pupil with best cycloplegic spectacle correction of the viewing subject calculated from the subject's average WFE over a 3 mm pupil.}

Eleven-line logMAR acuity charts (0.7 to 0.3 logMAR) with British standard letters of 30% contrast were generated using Visual Optics Laboratory Pro-

Figure 5. Panel A. Change in logVSX for 65 movement samples is plotted for simulated subject KC3. Second- to fourth-order SPGD correction provides better overall logVSX (green) than the other two corrections (blue, full-magnitude second- to fourth-order correction and orange, correction based on the Guirao et al. approach). The bigger symbols indicate the residual aberration structures selected for the visual acuity chart simulation study. Panel B. Change in measured logMAR for each of the selected residual aberration structures plotted against the corresponding contact lens movement position. Fourth-order SPGD correction (green) provides better overall logMAR acuity than the other two corrections. Panel C. KC3_WFE 7 mm pupil as measured over the contact lens for Zernike terms second to 10th radial order. Black circle indicates 5 mm pupil over each of the corrections move. KC3_WFE as measured over the contact lens for all Zernike terms second to 10th radial order, KC3_4Full: second- to fourth-order full-magnitude correction, KC3_4Guirao: second- to fourth-order correction based on the Guirao et al. approach, KC3_4SPGD: second- to fourth-order SPGD correction for KC2.
fessional software (version 6.89, Sarver). For each of the 15 residual aberrations and one unaberrated test condition, three unique acuity charts were simulated after precompensating for the aberrations of each chart reader over a 3 mm pupil, yielding $16 \times 3 = 48$ unique logMAR simulations for each keratoconic eye. Because the residual WFEs of three keratoconus eyes were simulated, there were a total of $48 \times 3 = 144$ charts for each normal chart reader.

**Measurement of acuity**

The logMAR acuity charts were displayed on a gamma-corrected, black-and-white, high-resolution (Totoku M253i2, 1200 × 1600 pixels, 11 bit [2048]) LCD monitor. Each of the 144 charts was displayed in random order through a custom MATLAB program using Psychophysics Toolbox (Brainard, 1997).

The chart readers viewed the charts displayed on the monitor through a 3 mm artificial pupil along with their best-corrected sphero-cylindrical prescription at a distance of 12.2 feet. Chart readers started on the lowest line where all letters could be comfortably read correctly and continued until five letters were missed. Chart readers were given credit for all letters read correctly up to the fifth missed letter.

**Normalization of acuity data**

The data for each chart reader were normalized to the chart reader’s mean logMAR acuity measured on an unaberrated logMAR acuity chart such that positive values indicated a change to poorer acuity and negative values indicated a change to better acuity.

**Results**

Displayed in Table 1 are the weighting values for Zernike coefficients C3 to C14 for three keratoconic wavefront-guided corrections (KC1, KC2, and KC3) for a full-magnitude correction, a Guirao approach correction, and a SPGD correction. For the full-magnitude correction, the weighting values are all one, by definition. A full-magnitude correction is the best correction if the registration is perfect or the registration errors are so small that for the visual task of interest there is no change in retinal image quality that would impact performance. If these conditions are not true, then the residual aberrations resulting from a misregistered full-magnitude correction adversely impact image quality and resulting visual performance. The Guirao approach correction seeks to provide a correction that always yields some benefit (with respect to no correction) regardless of the amount of decen-

tration as measured by RMS WFE. The weighting function for each term depends only on that particular term and the standard deviation of the translation and rotation. The approach assumes that the user wishes to alter a full correction such that it cannot make the residual RMS WFE worse than no correction. Here, because the standard deviation of the movement is held constant, the weighting values for any given term are the same for all three test eyes. Also notice for the second radial order terms only induces prism, which impacts image location and not image quality. In contrast, the SPGD approach seeks to find the best possible average visual image quality given all movements of interest and the underlying WFE of interest. Visual image quality is objectively calculated using a metric (here, log visual Strehl ratio, logVSX) that is known to be highly correlated with visual acuity. As is true of the Guirao approach, the SPGD approach sacrifices the best possible image quality achieved with perfect registration between the underlying WFE and a full-magnitude wavefront-guided correction. Unlike the Guirao approach, the SPGD correction allows asymmetric translations and rotations (non-Gaussian) and thus actual samples of the possible registration errors. As a consequence of these design principles, all weighting functions, including those for the second radial order, can increase or decrease in magnitude or direction in order to achieve the goal of finding the wavefront-guided correction that provides the best average retinal image quality for all registration errors of interest.

For the three simulated keratoconic WFEs in our study, Figures 3, 4, and 5 display and compare (a) in panel A of each figure, logVSX as a function of 65 different movement positions (translation and rotation); (b) in panel B of each figure, the measured logMAR visual acuity resulting from reading the acuity charts computationally blurred by the residual aberrations resulting from a full-magnitude second- to fourth-order correction (4Full, for reference), the second- to fourth-order correction based on the Guirao et al. approach (4Guirao), and the second to fourth order our SPGD optimized correction (4SPGD); and (c) panel C illustrates measured wavefront error through a scleral contact lens and three corrections. Notice the coma in the wavefront error is positive (red as opposed to blue in the inferior lobe). This is commonly reported when wearing a rigid lens because the percentage of the coma originating from the posterior surface increases due to index matching of the anterior surface (Kosaki et al., 2007; Negishi et al., 2007).
As can be seen in Figure 3A, for subject KC1, over the 65 scleral contact lens movement positions sampled, the SPDG correction (green, 4SPGD correction) has better overall values of logVSX than the correction based on the Guirao et al. approach (orange, 4Guirao correction) or the full-magnitude correction (blue, 4Full correction). The bigger data points in this figure represent the corresponding residual aberration structures that were selected for visual acuity (VA) chart simulations (refer to Methods to see how these data points were selected).

To make it easier to see that the measured change in VA varies with logVSX, in Figure 3B, the normalized change in VA is plotted for each of the residual aberrations tested. For subject KC1, the measured VAs of the 4SPGD corrections were better than those of the 4Guirao and 4Full corrections.

Similar to Figure 3A, Figures 4A and 5A show that in simulated subjects KC2 and KC3 the logVSX of the SPGD correction is better than that provided by the correction based on the Guirao et al. approach and full-magnitude second- to fourth-order corrections. Similar to Figure 3B, Figures 4B and 5B show that the measured logMAR acuities for the selected residual aberration structures of the 4SPGD corrections are overall better than those of the correction based on the Guirao et al. approach and full-magnitude second- to fourth-order corrections.

To better illustrate the improvement of VA with the SPGD optimized correction over the other corrections, the average change of logMAR acuity over the three simulated keratoconic WFEs is plotted for each of the three corrections in Figure 6. As can be seen, the measured VAs are not significantly different (means within 1.5 letters) from the VAs predicted for each of the three corrections (full-magnitude second- to fourth-order correction, correction based on the Guirao et al. approach, SPGD correction) from the calculated values of logVSX ($p > 0.05$). Correction 4SPGD offered, on average, an improvement of measured VA of 0.14 logMAR (seven letters) over both corrections, 4Guirao and 4Full. The resulting visual performance for 4Full and 4Guirao were not statistically different ($p > 0.05$).

As can be seen in Figure 7, measured change in acuity and predicted change in acuity are highly correlated ($R^2 = 0.817$) with the predicted acuity being

<table>
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<th>KC1</th>
<th>KC2</th>
<th>KC3</th>
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<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.120</td>
<td>0.630</td>
<td>0.993</td>
</tr>
<tr>
<td>C4</td>
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<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.888</td>
<td>1.053</td>
<td>1.027</td>
</tr>
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<td>C5</td>
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<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.068</td>
<td>0.993</td>
<td>−5.960</td>
</tr>
<tr>
<td>C6</td>
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<td>1.000</td>
<td>1.000</td>
<td>0.928</td>
<td>0.928</td>
<td>0.928</td>
<td>10.653</td>
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</tr>
<tr>
<td>C7</td>
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<td>1.000</td>
<td>1.000</td>
<td>0.925</td>
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<td>0.925</td>
<td>1.162</td>
<td>1.655</td>
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<tr>
<td>C8</td>
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<td>1.111</td>
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<td>C9</td>
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<td>0.928</td>
<td>0.928</td>
<td>1.580</td>
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<td>0.880</td>
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<td>0.880</td>
<td>−2.305</td>
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<td>0.872</td>
<td>0.872</td>
<td>0.872</td>
<td>0.736</td>
<td>2.384</td>
<td>6.364</td>
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<tr>
<td>C12</td>
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<td>1.000</td>
<td>1.000</td>
<td>0.887</td>
<td>0.887</td>
<td>0.887</td>
<td>2.563</td>
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<td>1.249</td>
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<tr>
<td>C13</td>
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<td>1.000</td>
<td>1.000</td>
<td>0.872</td>
<td>0.872</td>
<td>0.872</td>
<td>2.521</td>
<td>1.510</td>
<td>0.603</td>
</tr>
<tr>
<td>C14</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.880</td>
<td>0.880</td>
<td>0.880</td>
<td>0.071</td>
<td>−1.018</td>
<td>−4.690</td>
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</table>

Table 1. Weighting values for Zernike coefficients C3 to C14 for three keratoconic wavefront-guided corrections (KC1, KC2, and KC3) for a full-magnitude correction, a Guirao approach correction, and an SPGD correction.
slightly worse than measured. Different colors represent data from the three simulated keratoconic eyes and the associated corrections. The relationship between change in logVSX and change in logMAR acuity is

\[
\text{change in logMAR} = C_0 \cdot 0.3448 \cdot (\text{change in logVSX}) - 0.0486 (R^2 = 0.817)
\]

A Bland Altman analysis (Figure 8) was employed to assess agreement between the predicted and measured values of logMAR acuity to determine if the predicted logMAR acuity can reasonably be substituted for the measured acuity. Predicted acuity can be substituted for measured acuity as demonstrated by the slope of the data in the Bland Altman analysis (Figure 8), which is not significantly different from zero \((p = 0.13)\), and ±1.96 standard deviations of the difference between the predicted and measured values are within the test-retest difference in acuity that is expected in a clinical setting (±1 line or ±5 letters) (Raasch, Bailey, & Bullimore, 1998).

Discussion

Previous work by Guirao et al. (2001) and Guirao et al. (2002) documented a method for modifying a dynamically moving wavefront-guided correction such that the total RMS wavefront error would never be worse than no correction. This modification was accomplished by individually decreasing (if necessary) the magnitude of each Zernike coefficient using a set of developed formulae and the assumed standard deviation of the lens movement. While the method accomplishes the desired goal, the papers did not consider whether visual image quality or visual performance improved compared to a lens with the full wavefront guide correction or that visual performance remained relatively stable as the lens varied in registration. Instead, Guirao et al. (2001) and Guirao et al. (2002) showed, in their work, that their modification increases the area under the modulation transfer function compared to the full magnitude correction. This observation raises the question as to why the predicted loss of acuity in this experiment with a correction based on the Guirao et al. approach is essentially identical to a full-magnitude second- to fourth-order correction (see Figures 3, 4, and 5). To describe visual performance on a recognition task, such as acuity, it is important to note that the modulation transfer function is only one part of the full optical transfer function. Because phase shifts and reversals play a critical role in recognition tasks, such as letter identification, it may not be sufficient to consider only gains in the modulation transfer function without considering also the impact of the phase transfer function (Sarver & Applegate, 2004).

In addition, Thibos et al. (2004) emphasized the importance of the neural transfer function by identifying a series of metrics that they labeled visual-quality metrics. In contrast to optical-quality metrics, visual-quality metrics include in their calculation a neural transfer function, which would be expected to play a substantial role in defining performance on tasks such as visual acuity.

Here, we demonstrate a strategy for designing wavefront-guided corrections that (a) use a visual-quality metric that is highly correlated with logMAR visual acuity (i.e., the visual Strehl ratio), (b) include actual scleral contact lens movements in the modeling, and (c) search for a combination of coefficient values that improves the visual quality enough to result in a predicted gain in measured acuity.

It is important to note that the emphasis of the modeling conducted here has been on visual performance as measured by logMAR acuity. If a better metric is found for predicting acuity, it can easily be substituted in the analysis for the log visual Strehl. Likewise, if optimizing another visual task (e.g., face recognition).
recognition) is of primary interest, then the same SPGD strategy can be used once an appropriate metric for the task is identified.

**Conclusion**

SPGD optimization provides improved logMAR visual acuity in a wavefront-guided contact lens correction in the presence of lens-registration uncertainty compared to a full-magnitude correction and to the method of optimization proposed by Guirao et al. (2001) and Guirao et al. (2002). Change in logVSX can be used to predict change in logMAR acuity even in the presence of the unusual aberrations that are induced by a dynamically moving wavefront-guided correction.

*Keywords: optical design, wavefront-guided correction, stochastic parallel gradient descent, keratoconus, registration uncertainty*

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