Aging effects in cueing tasks as assessed by the ideal observer: Peripheral cues

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Previous aging and cueing studies suggest that automatic orienting driven by peripheral cues is preserved with aging; however, inconsistencies can be found. One issue might be the use of response times (RT) to assess cueing effects (invalid RT − valid RT), which, in many cases, may not have clear quantitative predictions. We propose an ideal observer (IO) analysis of accuracy estimating participants' internal value of cue validity, or weight, which should equal the actual cue validity. The weight measures the use of information provided by the cue and is insensitive to variations in set size and difficulty, thus potentially providing advantages to RT. Older (_n_ = 54) and younger (_n_ = 58) participants performed a yes/no detection task of a two-dimensional (2-D) Gaussian (60 ms). Square peripheral precues (150 ms) indicated likely target locations (70% valid) across two or six locations (set sizes). For cueing effects, (valid − invalid hit rates), younger participants had set-size effects (larger cueing effects for set size 6), while older participants did not. The opposite pattern was found for weights (younger: no set-size effects, older: set-size effects) due to the IO predicting larger cueing effects for larger set sizes. Comparisons to the ideal weight (cue validity) suggested that older participants used the cue information effectively with set size 2 (as or more so than younger participants), but not with set size 6. These results suggest that attentional deficits from aging in peripheral cueing tasks may only arise as difficulty increases, such as larger set sizes.

Introduction

It is well established that normal aging causes deficits in a wide spectrum of visual, cognitive, and attentional abilities. Specifically, aging effects upon vision (for reviews see Owsley, 2011; R. Sekuler & Sekuler, 2000) include visual acuity (Horswill et al., 2008; Owsley, Sekuler, & Siemsen, 1983), retinal illuminance (Weale, 1961), and motion (Bennett, Sekuler, & Sekuler, 2007; Hutchinson, Arena, Allen, & Ledgeway, 2012; Snowden & Kavanagh, 2006). Cognitive deficits include executive function (Daigneault, Braun, & Whitaker, 1992; Moscovitch & Winocur, 1995; West, 1996), memory (Craik, Anderson, Kerr, & Li, 1995; Moscovitch & Winocur, 1995; Zacks, Hasher, & Li, 2000), speed of processing (Cerella, 1985; Salthouse, 1996), and also inhibition (Hasher & Zacks, 1988), such as that shown in the antisaccade task (Butler & Zacks, 2006; Klein, Fischer, Hartnegg, Heiss, & Roth, 2000) or negative priming (Hasher, Stoltzfus, Zacks, & Rypma, 1991).

Another aspect affected by aging is visual spatial attention—for example, as shown in visual search tasks, a standard paradigm in visual attention in which participants search for a target in a field of nontargets. A standard manipulation is varying the number of items, or set size. Generally harder searches lead to findings of decreasing performance with set size (a set-size effect), whereas easier searches lead to findings of no changes in performance with set size (no set-size

effect). A still common interpretation is that set-size effects (particularly in response times [RTs]) indicate the effect of a serial attentional mechanism, whereas no set-size effects indicate a parallel search not needing attention, presented most notably by Treisman and Gelade (1980), with Feature Integration Theory, and by Duncan and Humphreys (1989) in assessing target–nontarget similarity and nontarget heterogeneity. It should be noted that several authors have presented potential issues with this interpretation (Palmer, 1995; Palmer, Verghese, & Pavel, 2000; Shaw, 1980, 1982, 1984; Shimozaki, Schoonveld, & Eckstein, 2012; Verghese, 2001; Vincent, 2011b; Vincent, Baddeley, Troscianko, & Gilchrist, 2009). Regardless of the interpretation, studies of aging in visual search typically find that older participants suffer from larger set-size effects as the task becomes more difficult (Folk & Lincourt, 1996; Foster, Behrmann, & Stussa, 1995; Harpur, Scialfa, & Thomas, 1995; Humphrey & Kramer, 1997; Madden, 2007; Madden, Pierce, & Allen, 1996; Madden & Whiting, 2004; Plude & Doussard-Roosevelt, 1989; Plude & Hoyer, 1986).

Perhaps one of the better-known demonstrations of aging effects in attention is the Useful Field of View (UFOV) test, (Ball, Beard, Roenker, Miller, & Griggs, 1988; R. Sekuler & Ball, 1986), which appears to be highly predictive of driving performance in the aged (e.g., Ball, Owsley, Sloane, Roenker, & Bruni, 1993; for a meta-analysis, see Clay et al., 2005). Current instantiations of the UFOV consist of three or four subtests, depending on the version. In Subtest 1 (Speed of Processing), participants discriminate between two objects (e.g., car/truck presented centrally). In Subtest 2 (Divided Attention) a second visual search task is added in which the participant must indicate the radial direction of a peripheral target (e.g., a car shape) in one of eight directions and one of three eccentricities. In Subtest 3 (Selective Attention), the dual task of Subtest 2 is made more difficult with the addition of a field of distracters (e.g., 47 triangles) to the visual search task. In Subtest 4 (Same/Different), the central task is made more difficult by using a same/different judgment of two items presented simultaneously. As the difficulty of the UFOV increases from Subtest 1 to 4, performance for the aged worsens relative to normal young participants (e.g., Ball et al., 1988; Edwards et al., 2006; A. B. Sekuler, Bennett, & Mamelak, 2000; R. Sekuler & Ball, 1986). These results indicate an increasing attentional deficit with age when a divided attention task is combined with an increasingly difficult visual search task.

Another predominant method for the study of visual attention is the cueing task. In a cueing task typically a participant must detect a target at two or more locations and view a cue before the stimulus display giving the likely target location (Posner, 1980). The standard result from cueing tasks is the cue validity effect, in which valid cues (cues giving the correct target location) lead to better performance than invalid cues (cues giving the incorrect target location). The cue validity effect is assumed to be the result of visual attention selecting the cued location over the uncued location(s). A common division in the study of cueing tasks is automatic attention driven by peripheral cues and voluntary attention driven by symbolic cues presented centrally (e.g., a number indicating a location). It is assumed that automatic attention leads to faster and more transient attentional (i.e., cueing) effects (e.g., Jonides, 1981; Müller & Rabbitt, 1989; reviewed by Wright & Ward, 2008), and also it has been hypothesized that automatic and voluntary attentional control exists in different parts of the brain (Corbetta & Shulman, 2002). With respect to aging effects, a common assumption is that automatic attention is retained and voluntary attention is lost with aging (see Table 1a, b for a review).[1]

This study assesses the aging effects upon the use of peripheral cues in a cueing task. As stated above, it is generally assumed that automatic attention driven by peripheral cues is resistant to aging effects (Table 1a). However, there have been counterexamples showing that peripheral cues can be less effective in the aged under certain conditions, particularly as task difficulty increases (Table 1c; Greenwood & Parasuraman, 1999; Greenwood & Parasuraman, 2004, experiment 2; Greenwood, Parasuraman, & Alexander, 1997; Whiting, Madden, & Babcock, 2007). This is particularly true of the three studies by Greenwood and colleagues, which tended to use larger set sizes than the other studies in Table 1, and also in two cases that cued more than one location (Greenwood & Parasuraman, 1999; Greenwood et al., 1997).

There is also the case of nonpredictive peripheral cues. More typically the cues give the likely location of the target and thus are predictive of target locations (Column P in Table 1, for predictive cues). Another common manipulation is the use of nonpredictive cues, or cues that give no information about the target location (e.g., a 50% valid cue for two locations; Column NP in Table 1, for nonpredictive cues). Despite the cues providing no information, cueing effects are commonly found, particularly with peripheral cues. This has been characterized as attentional capture and suggests the automatic (and nonfunctional) orienting of attention (e.g., Folk, Remington, & Johnston, 1992, experiment 3; Jonides, 1981, experiment 2; Theeuwes, 1991). The studies in Table 1 with nonpredictive cues have found generally similar cueing effects across the young and old participants, suggesting that they were equally affected by the nonfunctional attentional capture. Thus, the authors generally argue that the older participants had retained their automatic atten-

Authors	Set size	Validity	SOA	P	NP
Table 1a. Automatic attention preserved					
Brodeur & Enns, 1997	4	25%	133–800 ms		X
*[+]Faust & Balota, 1997, experiment 1	2	75%	100 ms, 800 ms	X	
[+]Folk & Hoyer, 1992	4	100% valid, 100% invalid	50–250 ms	X	
Greenwood & Parasuraman, 1994	2	77%	200–2000 ms	X	
Greenwood, Parasuraman, & Haxby, 1993	2	77%	200–2000 ms	X	
Hartley, Kieley, & Slabach, 1990, experiments 2 and 3	4	75%, 2 locations cued	100–400 ms	X	
Jennings, Dagenbach, Engle, & Funke, 2007	2	100%	100–400 ms	X	
Kingstone et al., 2002, experiment 2	2	80%	72 ms, 288 ms	X	
*Madden, 1992	4	40%, 100%	99 ms, 156 ms	X	
*Madden, Connelly, & Pierce, 1994	4	40%, 70%, 100%	156 ms	X	
*Madden et al., 2007	4	25%, 75%	young: 1000 ms, aged: 1500 ms	X	X
Madden, Whiting, Cabeza, & Huettel, 2004	4, 6	set size 4: 25%, 75%. Set size 6 = 16.7%, 83.3%	0 ms	X	X
[+]McCalley, Bouwhuis, & Juola, 1995	12	80% for 4 locations (20% at each cued location)	250–2000 ms	X	
McLaughlin & Murtha, 2010	1, 6, 12	set size 12: reduced effective size to 6	200 ms	X	
Robinson & Kertzman, 1990	2	82.3%	180 ms	X	
Tales, Muir, Bayer, & Snowden, 2002, experiments 3 and 4	2	50%	150 ms		X
Yamaguchi, Tsuchiya, & Kobayashi, 1995	2	80%	200–800 ms	X	
Table 1b. Voluntary attention deficits					
Brodeur & Enns, 1997	4	80%	133–800 ms	X	
[+]Folk & Hoyer, 1992	4	80%	50–250 ms	X	
Hoyer & Familiant, 1987	4	100%	250–1250 ms	X	
Iarocci, Enns, Randolph, & Burack, 2009	4	75%	130 ms, 830 ms	X	
Table 1c. Automatic attention deficits					
[+]Greenwood & Parasuraman, 1999	10, 15	80%, 1 or 3 locations cued	200 ms, 500 ms	X	
Greenwood & Parasuraman, 2004 (Exp 2)	18	77% (60%, 75% in experiment 1)	100–500 ms	X	
Greenwood, Parasuraman, & Alexander, 1997	10, 15	80% (cue: 1, 2, or 3 locations)	500 ms	X	
Whiting, Madden, & Babcock, 2007	4	25%, 75%	300 ms	X	X
Table 1d. Lack of automatic attention inhibition					
Juola, Koshino, Warner, McMickell, & Peterson, 2000	4	25%, 75%	0 ms, 157 ms	X	X
[+]Lincourt, Folk, & Hoyer, 1997	4	25%	50–300 ms		X
Pratt & Bellomo, 1999	4	25%	200 ms		X
Whiting, Madden, & Babcock, 2007, experiment 2	4	25%, 75%	300 ms	X	X

Table 1. Summary of articles on aging and cueing tasks. *Notes*: P = predictive cues, NP = nonpredictive cues. * = Aged participants had larger cueing effects. + = There were age differences in accuracy. SOA = stimulus onset asynchrony.

tional function. However, other studies have shown larger cueing effects with the aged with non-predictive cues (Table 1d; Juola, Koshino, Warner, McMickell, & Peterson, 2000; Lincourt, Folk, & Hoyer, 1997; Pratt & Bellomo, 1999; Whiting et al., 2007, experiment 2). These results seem to suggest a more detrimental attentional-capture cueing effect in the aged in which they have less ability to inhibit their attentional orienting to the peripheral cue. This pattern of a lack of inhibition in attention seems to correspond with the general loss of inhibition with aging described earlier (Butler & Zacks, 2006; Hasher et al., 1991; Hasher & Zacks, 1988; Klein et al., 2000).

Finally, there are cases in Table 1a with predictive cues in which the aged have shown larger cueing effects from the younger control participants (Table 1, *;

Faust & Balota, 1997, experiment 1; Madden, 1992; Madden, Connelly, & Pierce, 1994; Madden et al., 2007). These cueing effects have been interpreted as evidence of the retention of automatic attention. However, given that larger cueing effects in the cases of nonpredictive cues clearly seem to indicate a deficit of attentional control, there might be some concerns about this interpretation.

One potential issue in the analysis of aging effects on cueing tasks is the possible lack of clarity of how to assess performance in cueing tasks. All of the studies mentioned above, and almost all studies of aging effects in cueing tasks, measure cueing effects in terms of RTs (i.e., invalid RT − valid RT). This is generally accepted within cueing tasks, and a cueing effect in RTs, in and of itself, is a clear indication of the use of the cue with either automatic or voluntary attention. However, it is difficult to quantify how much of a cueing effect should be expected with RTs under different experimental conditions, such as different difficulties, set sizes, and cue validities.

We propose that a potential method to assess performance in cueing tasks is the Bayesian ideal observer (Barlow, 1978; Green & Swets, 1974). Aside from a number of visual domains (e.g., Burgess, Wagner, Jennings, & Barlow, 1981; Legge, Klitz, & Tjan, 1997; Liu, Knill, & Kersten, 1995; Najemnik & Geisler, 2005; Tjan, Braje, Legge, & Kersten, 1995), the ideal observer and associated models from Signal Detection Theory (SDT) have also been employed in the assessment of attentional tasks, such as visual search (Palmer, 1995; Palmer et al., 2000; Shaw, 1980, 1982, 1984; Shimozaki et al., 2012; Verghese, 2001; Vincent, 2011b; Vincent et al., 2009) and cueing tasks (Eckstein, Peterson, Pham, & Droll, 2009; Eckstein, Shimozaki, & Abbey, 2002; Kinchla, 1977; Kinchla, Chen, & Evert, 1995; Shimozaki, 2010; Shimozaki, Eckstein, & Abbey, 2003; Shimozaki et al., 2012; Vincent, 2011a; see also Yu & Dayan, 2005a, 2005b).[2] The ideal observer predicts optimal performance for a given task, and thus can function as a standard of performance. Specifically, with respect to cueing tasks, the ideal observer gives a prediction for the optimal use of the cue. Within the ideal observer, the likelihood of target presence at a location is scaled by the prior probability of target presence (i.e., as determined by the cue validity). This has been described as a weighted likelihood model (Eckstein et al., 2002; Shimozaki, 2010; Shimozaki et al., 2003), with the prior probabilities defined as weights. The weights used internally by the human observer may also be estimated and then compared to the optimal (ideal) weight, or the cue validity. Thus, this comparison assesses how well participants employ the information provided by the cue, separate from measures of sensitivity (d') and criterion bias. The observed weights might be charac-

terized as the internal estimate of the cue validity, or the expectation of signal appearance at a location.

One aspect of the ideal observer is that the cueing effect predicted from the optimal weighting will vary with the difficulty of the task. At low and high d's there is little predicted cueing effect, as performance is at floor and ceiling, respectively, and cueing effects are most prominent for moderate levels of difficulty (to be shown later). For a fixed cue validity, the predicted cueing effects will also vary with set size, generally increasing with increasing set size. Finally, the predicted cueing effects will vary with cue validity, generally increasing with increasing cue validity. Intuitively there should be no cue effects if the cue validity is nonpredictive (e.g., 50% for two possible locations), and maximal if the cue is entirely predictive (i.e., 100% valid). These aspects of cueing tasks may not have been recognized generally, particularly in research concerning older participants.

This study aims to assess the aged and young normal participants in a peripheral cueing task with predictive cues and compare their performance to the ideal observer. The analysis focused upon the use of the cue as information about signal location (through analysis of the weighting), and not upon overall performance (e.g., percent correct or d'). Both aged and young normal participants performed in a yes/no contrast discrimination of a two-dimensional (2-D) Gaussian presented for 60 ms, with equal numbers of target-present and target-absent trials; this target could appear in either two or six locations (set size). A square precue (150 ms) appeared on all trials around one of the possible target locations and indicated that the target would appear at the cued location with 70% probability (i.e., cue validity = 70%), if the target was present (Figure 1). The target was present on half the trials and absent on the other half, giving an overall proportion of 35% valid signal-present trials, 15% invalid signal-present trials, and 50% signal-absent trials.

Overview of the ideal observer

This section and Figure 2 give a brief overview of the ideal observer (IO) for this task (cued yes/no discrimination); a full description of the ideal observer appears in the Appendix. First, it is assumed that a response is generated by the observer at each location (either 2 or 6). These responses (x_1 to x_M) are assumed to be normally distributed, with a standard deviation of 1 and a mean of 0 if the signal is not present at that location, and a standard deviation of 1 and a mean of d' if the signal is present at that location. These follow the standard conventions of the equal-variance model[3] found in SDT (Green & Swets, 1974). The responses are the input variables for the IO, and the IO calculates
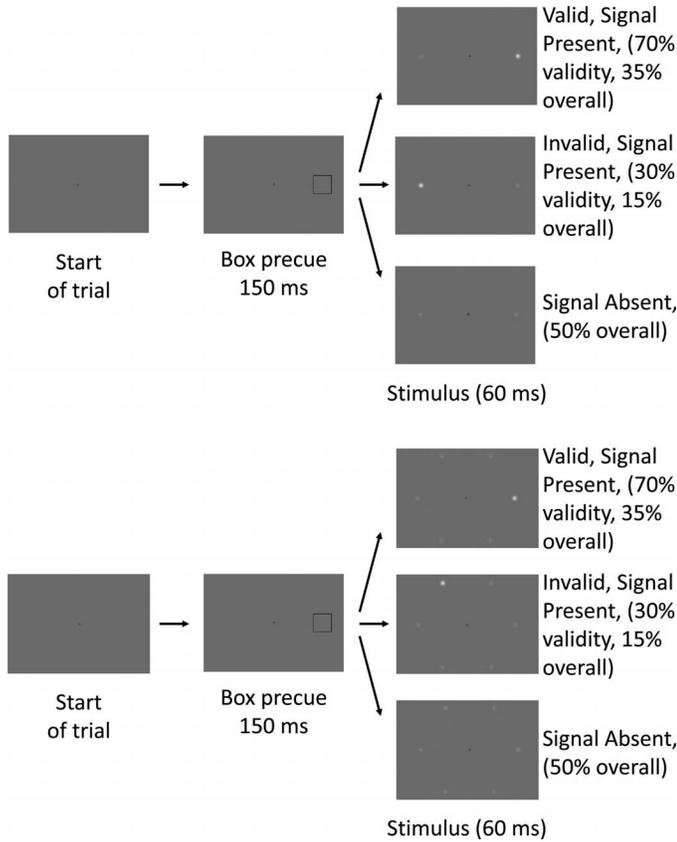
Figure 1. Stimulus conditions for the cueing task. (a) set size 2; (b) set size 6.

Figure 2. Schematic of the Bayesian ideal observer, beginning with the input to the model at the left (x_1 to x_M, observer responses for each location).

the posterior probabilities of target presence, and of target absence, given the responses at each location (p[target|all x], p[no target|all x]). The posterior probabilities are found through Bayes' theorem relating prior probability and posterior probability. The prior probabilities are comprised of two types. The first is the probability of the response at each location, given either target presence or target absence at that location (also known as the likelihood); the likelihoods are based upon the assumption of normally distributed responses at each location. The second is the probability of target presence/absence at each location, which is dependent upon the cue validity (weight) and the overall probabilities of target-present and target-absent trials. The ratio of the posterior probabilities for target presence and target absence is the output or the decision variable for the IO, and the IO compares the decision variable to the ideal criterion (crit). If the ratio is greater than or equal to the ideal criterion, the IO responds "yes" for target presence; if the ratio is below the ideal criterion, the IO responds "no." For this study, the ideal criterion is 1, and the ideal log criterion is 0 (see Appendix).

The cue validity has been described as a weight applied to the likelihood, and thus defining the ideal
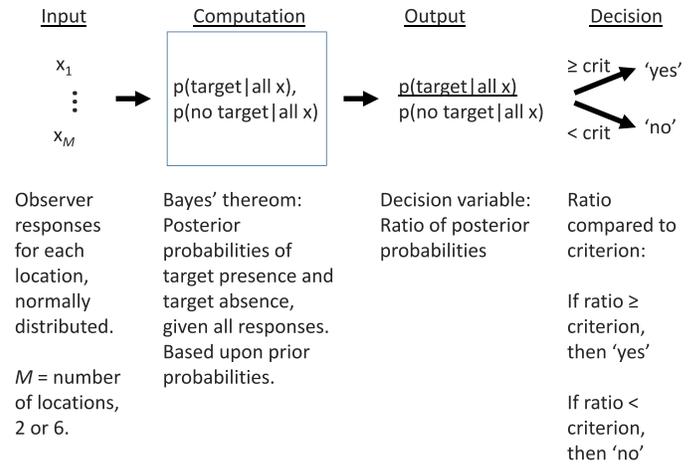
weight for the cued location as the cue validity. For this reason it also has been called a "weighted likelihood" model (e.g., Eckstein et al., 2002). When fitting the IO to human performance, the weights, criteria, and d's were allowed to vary.[4] Because of these modifications, strictly speaking this was a modified ideal observer. Except for d',[4] comparisons of the fitted human values then were made to the ideal values. For the criteria, these comparisons assessed whether human observers were biased in their responses to saying yes or no. For the weights, this comparison addressed whether the human observers could use the information provided by the cue optimally. If so, the estimated human weights would be indistinguishable from the optimal weight (i.e., the cue validity; in this study 0.70). If not, the human estimated weights would deviate from the optimal weight, either by being too low or too high. Specifically, the younger and older participants may differ in their ability to use the cue information, as indicated by their weights. This was the crucial analysis for this study. As in a standard SDT analysis, d's measured overall performance (sensitivity) in the task.

To summarize, the ideal observer fits to human performance yield three parameters, d' (overall performance), the weight (assessing use of the cue), and criterion (bias). An important aspect of the ideal observer for the cueing task is that it predicts optimal behavior. In other words, it describes what human participants should do to optimize performance, and thus, gives an objective measure of assessing the use of the cue through the analysis of the weights.

Cueing effects for this task may be characterized as the difference between the valid hit rate (correctly saying yes on target-present trials with valid cues) and the invalid hit rate (correctly saying yes on target-present trials with invalid cues). Note that the IO
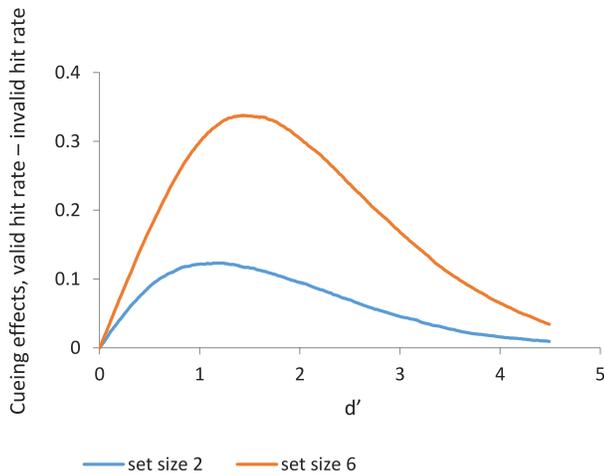
Figure 3. Ideal observer predictions of cueing effects (valid hit rate − invalid hit rate) as a function of d' with cue validity = 70%, set sizes 2 and 6, and the optimal criterion.

predicts a specific cueing effect for specific values of d', criterion, and weight. Also, the IO predicts a cueing effect without cue-induced sensitivity changes; this is entirely due to the effect of the differential weighting of information at the cued and uncued locations. Figure 3 represents the ideal observer predictions for cueing effects (valid hit rate − invalid hit rate) for the conditions of the current study as a function of sensitivity (d'), with the cue validity of 70%, set sizes of two and six, and the optimal criterion. Previously it has been shown that the ideal observer predicts a dependence of the cueing effect upon task sensitivity (also see Shimozaki et al., 2003; Shimozaki et al., 2012). For a given cue validity, predicted cueing effects are maximal for a moderate sensitivity (approximately 1.1 for set size 2 and 1.5 for set size 6 in Figure 3), with cueing effects approaching zero for low and high difficulties. Essentially these represent floor and ceiling effects, in which performance is either too poor or too good to benefit greatly from the information provided by the cue. The predicted cueing effects will also vary with set size, as shown in Figure 3, with larger predicted cueing effects with increasing set size (see also Shimozaki et al., 2012). Finally, an issue not addressed in this study is that predicted cueing effects will increase with cue validity/weight, as shown by Vincent (2011a). For example, no cueing effects are predicted when the cue is nonpredictive (e.g., 50% valid for set size 2), and maximal cueing effects are predicted for 100% valid cues.

Thus, for these reasons (the influence of sensitivity, set size, and cue validity) cueing effects in RTs may not be the most robust measure of the use of the cue, an issue that may not have been appreciated in previous studies of aging effects on cueing. From Table 1 it is apparent that a range of set sizes and validities have

been employed. Generally age differences in accuracy were not an issue in previous studies, as performance tended to be near ceiling performance for both younger and older participants to focus on RTs. However, some studies in Table 1 did have significant age effects in accuracy (indicated by "+" in Table 1; Faust & Balota, 1997; Folk & Hoyer, 1992; Lincourt et al., 1997; McCalley, Bouwhuis, & Juola, 1995). One advantage of the weighting measure is that optimally it should always equal the cue validity and essentially is insensitive to (or adjusts for) the changes in cueing effects predicted with task sensitivity and set size.

For this task with cue validity equal to 70%, a priori deviations from optimal weighting by human participants may occur from underweighting or overweighting the cue location. At the extreme, underweighting would occur if the participant performed as if the cue was not presented and did not use the cue information, and thus weighted the cue location equally to the other uncued locations (1/set size, or 0.50 with set size 2 and 0.167 with set size 6). Equivalently, this would indicate the equal distribution of attention over all locations, similar to a zoom lens model of attention (Eriksen & Yeh, 1985). The extreme for overweighting would be weighting only the cued location and ignoring the uncued locations completely; in this case, the weight would equal 1.00. This is similar to the common description of visual attention as a serial spotlight focused solely on the cued location (e.g., Posner, 1980; Treisman & Gelade, 1980).

Methods

Ethical approval was obtained through the School of Psychology at the University of Leicester in accordance with the Declaration of Helsinki. Aged and young participants performed in a series of yes/no cued contrast-discrimination tasks. Two studies were run, the first having two possible signal locations (set size = 2), the second having six possible signal locations (set size = 6). There were 54 older participants overall (M = 69.61 years, SD = 4.92, range = 60–83 years) with 28 in Study 1 (12 females) and 26 in Study 2 (10 females). There were 58 younger participants overall (M = 19.78 years, SD = 1.71, range = 18–26 years), with 30 in Study 1 (24 female) and 28 in Study 2 (24 female). The older participants were recruited from the community and paid a nominal fee, and the younger participants were students from the University of Leicester either volunteering or participating for course credit. All participants reported normal or corrected-to-normal vision and no history of any ocular or neural diseases. Also, before starting it was confirmed that participants were able to detect the cue and the stimuli (at or above

96% correct) on the computer screen. The older participants were screened for dementia by using the Mini Mental State Examination (Folstein, Folstein, & McHugh, 1975), with all participants scoring 27 (out of 30) or above.

For both studies the signal was a 2-D Gaussian blob ($\sigma = 0.25°$, eccentricity = $8.00°$) presented on a 15.6% contrast pedestal, and the cue was a dark (0.77 cd/m^2) square (length = $3°$) displayed at one of the potential stimulus locations. The fixation point was indicated by a dark (0.77 cd/m^2) cross (length = $0.5°$, width = $4.2'$).

The study was performed in a darkened room with the computer screen as the only light source and with the participants' heads on a chinrest to control viewing distance. Initially the participants' ability to detect the cue was verified. For set size 2, the cue was presented for 150 ms, either to the left or the right of a central fixation point; for set size 6, the cue was presented at one of six equidistant locations, to the left, the right, or at one of the four locations at angle of $60°$ from horizontal from the fixation point, with participants indicating the location of the cue. Performance at or above 96% correct (over 100 trials) was required before participating in the main part of the study.

It is well known that older adults are markedly less sensitive to stimulus contrast than their younger counterparts (see Owsley, 2011, for a review). Indeed, poor contrast sensitivity has been shown previously to affect older adults' performance on other visual tasks such as those involving motion perception (Allen, Hutchinson, Ledgeway & Gayle, 2010). To minimize any spurious contribution of differences in absolute contrast sensitivity between participants, we used an adaptive staircase routine to determine the signal threshold contrast for each individual participant. This allowed us to match cue visibility across participants in the main study. For both studies the task involved the yes/no discrimination of the Gaussian signal (of varying contrast) appearing in half the trials at one (of two or six locations) for 60 ms. If the signal was not present at a location, the 15.6% contrast pedestal appeared otherwise (i.e., at all nontarget locations). Prior to the stimulus, the cue appeared for 150 ms, indicating the signal location with 100% validity, if the signal appeared on that trial. The total duration of 210 ms was chosen to reduce the effects of saccades. On each trial participants received feedback on accuracy for that trial.

A three-down, one-up adaptive staircase was employed using a step size reduction of 0.5 for each reversal and a criterion of 16 reversals, yielding contrast thresholds equal to 79.4% correct performance (Levitt, 1971; Wetherill & Levitt, 1965). The staircase procedure was run four times, and the mean contrast threshold was calculated across the four staircases for the cueing task. The cueing studies outlined below were run at two visibility levels (contrasts), at threshold and one octave below threshold.[5]

In the cueing task (Figure 1), the participants judged upon signal presence, with the signal present on half the trials. The temporal parameters were the same as the staircase procedure, with the cue appearing for 150 ms and the stimulus appearing for 60 ms. The cue was 70% valid when the signal was present (and thus was valid for 35% over all trials). Each participant performed in five sessions of 100 trials (for a total of 500 trials) for each contrast, run in random order. For Study 1 (set size = 2, Figure 1a), the signal could appear either to the right or the left from central fixation. For Study 2 (set size = 6, Figure 1b), the signal could appear at locations equally distributed around the central fixation point; relative to the fixation point, these locations were to the left, the right, and the four locations at an angle of $60°$ from horizontal. The 15.6% contrast pedestal was placed in all locations not having the signal. On each trial participants received feedback on accuracy for that trial.

Stimuli were viewed on a Viewsonic P227f with a screen size of 39.1 × 29.3 cm (1280 × 960 pixels, 0.3032 mm/pixel; ViewSonic, Brea, CA) and viewing distance of 50 cm, giving a total viewing area of 42.7° × 32.7°. The mean luminance was 81.2 cd/m^2, and the Commission internationale de l'éclairage (1932) coordinates of the screen were $x = 0.284$, $y = 0.309$. The computer was calibrated with a ColorCal photometer on a Bits++ graphics board (both from Cambridge Research Systems, Cambridge, UK). The stimuli were controlled by software written in Interactive Data Language (Exelis Visual Information Solutions, Boulder, CO).

Predictions for the valid hit rates, invalid hit rates, and false alarm rates for the ideal observer in the cueing task first were found through Monte Carlo simulations with 50,000 trials across a range of d', criteria, and weights. Then each set of valid hit rates, invalid hit rates, and false alarm rates for each participant and for each condition of set size, contrast, and age group were converted to d', criteria and weights separately by finding the best fit (by chi-square) of the rates to the predictions from the Monte Carlo simulations (Shimozaki et al., 2003; Shimozaki, 2010; Shimozaki et al., 2012). In other words, each participant and each condition had separate values for d', criteria, and weights. As the number of input parameters (valid hit rate, invalid hit rate, false alarm rate) was equal to the number of free parameters (d', criterion, and weight), the model fits were specified exactly and chi-squares should have been small (mean $\chi^2 = 1.165$, $SE = 0.023$). This is similar to the conversion of false alarm and hit rates to d's and criteria in SDT calculations of standard yes/no tasks (e.g., Green & Swets, 1974).

	Set size 2 vs. 0				Set size 6 vs. 0			
	t	df	p	sig	t	df	p	sig
Young, threshold	−5.453	29	<0.001	*	−5.996	27	<0.001	*
Young, ob	−2.079	29	0.047	*	−1.480	27	0.151	
Old, threshold	−4.633	27	<0.001	*	−3.122	25	0.004	*
Old, ob	−2.979	27	0.006	*	−0.412	25	0.684	

Table 2. Results for log criterion, single-sample t tests against zero. *Notes*: threshold = threshold contrast, ob = octave-below threshold contrast.

Results

Analyses were performed with three-way mixed design ANOVAs, with age and set size as between-participant factors and contrast as a within-participant factor, unless stated otherwise. All main effects and interactions not mentioned in the Results were not significant.

The contrast threshold and d' results are presented first and assess overall age differences in the task and the use of the staircasing procedure. The log criterion results are presented next and assess biases in the human observers' responses. The next two sections are the most crucial for this study. The results of the cueing effects (defined as valid hit rate − the invalid hit rate) are presented, and then the results of weighting. These both assess the use of cues in the study and are complementary. The last section covers the RT results and generally assesses whether the results found in the earlier sections were due to speed/accuracy or speed/cueing (reduced cueing effects due to speeded responses) trade-offs.

Contrast thresholds

As expected, contrast thresholds as percent contrast were higher overall for the older participants (younger: mean contrast threshold = 42.83%, $SE = 1.74$; older: mean contrast threshold = 52.72%, $SE = 2.19$; $t[102.83] = 3.538$, $p = 0.001$, equal variances not assumed).

d'

For d', there was a main effect of contrast, as expected, $F(1, 108) = 131.73$, $p < 0.001$, with higher mean d's for threshold contrast (threshold: mean $d' = 2.319$, $SE = 0.077$; octave below: mean $d' = 1.605$, $SE = 0.070$). There was no main effect of age (younger: mean
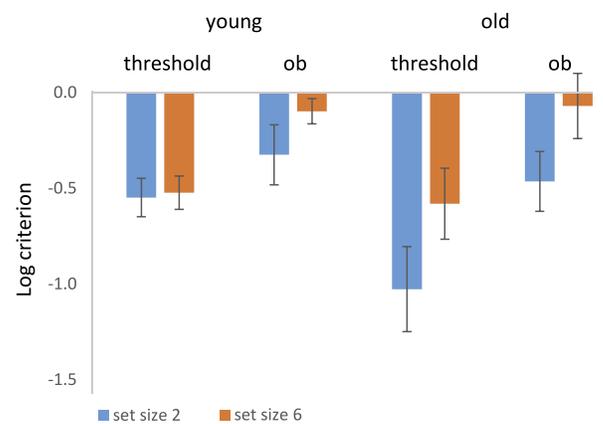

Figure 4. Log criterion. threshold = threshold contrast; ob = octave-below threshold contrast. Error bars represent standard errors of the mean.

$d' = 2.023$, $SE = 0.093$; older: mean $d' = 1.901$, $SE = 0.097$; $F[1, 108] = 0.820$, $p = 0.367$), as expected from the staircasing procedure. However, there was a significant interaction of age with set size, $F(1, 108) = 4.404$, $p = 0.038$. Two-way mixed design ANOVAs on age and contrast were performed separately for each set size. The younger participants were significantly better in the set size 2 study (younger: mean $d' = 2.146$, $SE = 0.130$; older: mean $d' = 1.743$, $SE = 0.134$; $F[1, 56] = 5.793$, $p = 0.019$), and there were no significant main effect of age for set size 6 (younger: mean $d' = 1.900$, $SE = 0.134$; older: mean $d' = 2.060$, $SE = 0.139$; $F[1, 52] = 0.569$, $p = 0.454$).[6]

The values of d' overall were somewhat higher than expected, given the staircasing procedure. As the staircase was performed before the cueing task, it appears that the higher d' values might be due to practice effects.

Log criterion

For equal numbers of signal-present and signal-absent trials, the ideal criterion for the ideal observer is 1 (see Appendix), and thus gives an ideal value of zero for the log criterion. Negative values indicate a bias for "yes" responses; positive values indicate a bias for "no" responses. For log criterion there was a main effect of contrast, $F(1, 108) = 36.57$, $p < 0.001$, with a more negative and therefore less optimal log criterion for the threshold contrast (threshold: mean log criterion = −0.669, $SE = 0.079$; octave below: mean log criterion = −0.239, $SE = 0.071$). There was also a main effect of set size, $F(1, 108) = 4.283$, $p = 0.041$, with a more negative and less optimal log criterion for set size 2 (set size 2: mean log criterion = −0.591, $SE = 0.092$; set size 6: mean log criterion = −0.318, $SE = 0.095$).
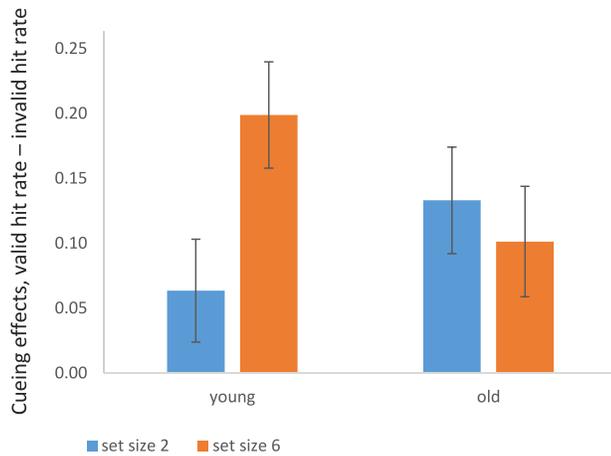
Figure 5. Cueing effects, valid hit rate − invalid hit rate, collapsed across contrast. Error bars represent standard errors of the mean.

Table 2 and Figure 4 summarize the results of log criterion. As seen in Table 2, all conditions had significantly different values from zero, except for both young and old participants with a set size of 6 and the octave-below condition. The negative values indicate a bias to say yes. This was not expected; this might have been an effect of the presence of stimuli (i.e., the pedestals) at all nontarget locations during a trial.

Cueing effects, hit rates

As described in the Introduction, cueing effects in this yes/no detection task may be quantified as the difference in the valid and invalid hit rates. There was a significant main effect of contrast, $F(1, 108) = 15.67$, $p < 0.001$, with a larger cueing effect for contrast threshold, the higher contrast (threshold: mean cueing effect, hit rates = 0.150, $SE = 0.021$; octave below: mean cueing effect, hit rates = 0.098, $SE = 0.022$).

More relevant to the current study, there was also a significant interaction of age and set size, $F(1, 108) = 4.146$, $p = 0.044$; this is shown in Figure 5

	Set size 2 vs. 0				Set size 6 vs. 0			
	t	df	p	sig	t	df	p	sig
Young, threshold	2.503	29	0.018	*	5.283	27	<0.001	*
Young, ob	0.876	29	0.388		4.558	27	<0.001	*
Old, threshold	2.979	27	0.006	*	3.573	25	0.001	*
Old, ob	1.887	27	0.070		2.616	25	0.015	*

Table 3. Results for cueing effects, hit rates (valid − invalid hit rates), single-sample t tests against zero. Notes: threshold = threshold contrast, ob = octave below threshold contrast.
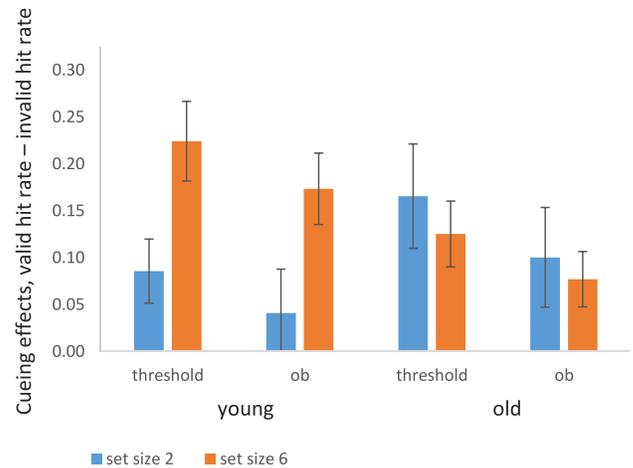

Figure 6. Cueing effects, valid hit rate − invalid hit rate. threshold = threshold contrast; ob = octave-below threshold contrast. Error bars represent standard errors of the mean.

(collapsed across contrast). As suggested by this figure, a two-way ANOVA done separately for each age group (and thus with set size and contrast as factors) found a set size effect for the younger participants, $F(1, 56) = 5.984$, $p = 0.018$, but not for the older participants, $F(1, 52) = 0.273$, $p = 0.604$. Thus, it is clear that age had an effect on the cueing effects found in this study. However, as stated in the Introduction, differences in set size will lead to different predicted cueing effects by the ideal observer; this is also true for differences in contrast or difficulty (Figure 3). Thus, an analysis of weighting might give a clearer indication of the interpretation of both the main effect of contrast and the interaction of age and set size in cueing effects.

Table 3 and Figure 6 summarize the cueing effects results for the individual (uncollapsed) conditions. As summarized in Table 3, the single-sample t tests against zero found that all conditions had a significant (nonzero) cueing effect, except for both the young and old participants in the set size 2 and the octave-below condition.

Weights

The optimal weight across all conditions was the cue validity, or 70%. Thus, for participants performing ideally, the weights should not differ from 70 and no main effects or interactions should be found.

There was a main effect of contrast, $F(1, 108) = 24.16$, $p < 0.001$, with the contrast threshold weights being larger than and closer to the optimal weighting (threshold: mean weight = 60.40, $SE = 2.86$; octave below: mean weight = 48.75, $SE = 3.10$). These results suggest that the main effect of contrast in cueing effects in hit rates was not expected by the ideal
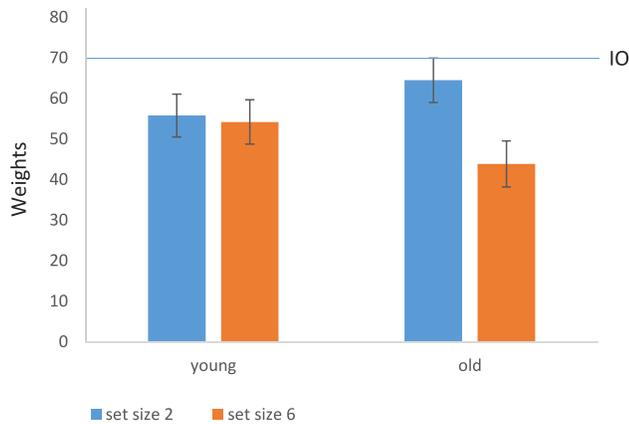
Figure 7. Weights, collapsed across contrast. IO = ideal observer weighting. Error bars represent standard errors of the mean.

observer and represented a less optimal use of the cue with the lower contrast. There was also a main effect of set size, $F(1, 108) = 4.138$, $p = 0.044$, with the set size 2 condition having the larger weights, and also closer to the optimal weighting (set size 2: mean weight = 60.13, $SE = 3.80$; set size 6: mean weight = 49.01, $SE = 3.93$). This result suggests that overall participants used the cue less effectively with the larger set size.

The interaction of age and set size (which was significant for cueing effects) was close to significance for weights, $F(1, 10) = 3.030$, $p = 0.085$. However, an inspection of this nonsignificant interaction (see Figure 7) suggests that the main effect of set size was driven mostly by the older participants. This is also suggested by the results of two-way ANOVAs (on set size and contrast) done separately for the young and old participants and the main effects of set size (young: $F[1, 56] = 0.046$, $p = 0.831$; old: $F[1, 52] = 6.650$, $p = 0.013$).

The weights for the younger participants were essentially unchanged with set size; this and the set-size effect for cueing effects reflect the expected increase of cueing effects with set size predicted by the ideal observer (see Figure 3). The opposite pattern was found for the older participants: no significant set-size effects for cueing effects and set-size effects for weights. Again, this reflects the predictions of the ideal observer (of increased cueing effects with increased set size).

Figure 8 and Table 4 give the weights' comparisons to the ideal weight and the equal weight (in which the cue information is not used). Set size 2 is shown in Figure 8a and Table 4a, and set size 6 is shown in Figure 8b and Table 4b. First, paying attention to the comparisons to the optimal weight, it can be seen that the pattern across set sizes was similar for the younger participants. Regardless of the set size, the weights did not differ significantly from the ideal
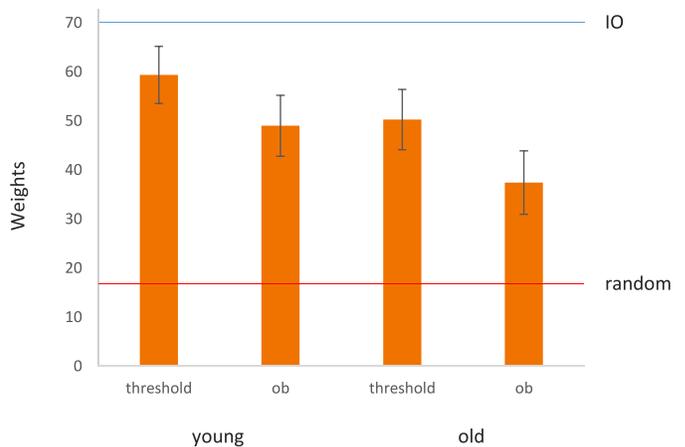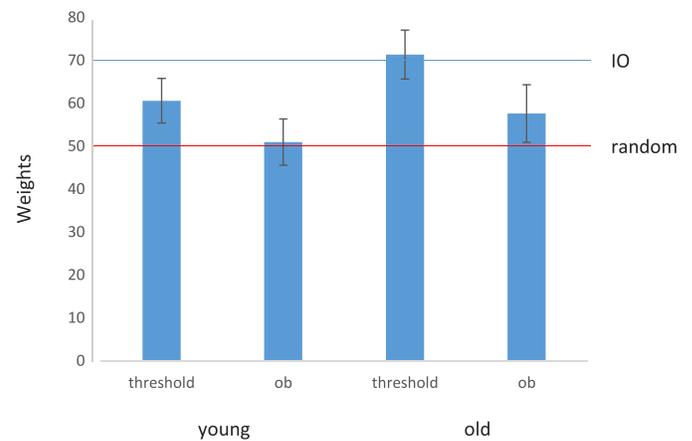
Figure 8. Weights. threshold = threshold contrast; ob = octave below threshold contrast. IO = ideal observer weighting. Error bars represent standard errors of the mean. (a) set size 2; (b) set size 6.

weight for the threshold contrast, and did differ significantly from the ideal weight for the octave-below contrast. This was expected from the results in Figure 7. The pattern across set size was different for the older participants. For set size 2 the weights did not differ significantly from the ideal weight for either contrast, whereas for set size 6 the weights did differ significantly from the ideal weight for both contrasts. These results suggest that the older participants could not use the cue information as effectively with the larger set size.

For the comparison to the equal weights, for set size 2 only the older participants' weights for the threshold contrast were significantly different. Thus, the results for set size 2 (Figures 7 and 8a) suggest that the older participants could use the cue information as or more effectively than the younger participants. For set size 6, all conditions differed

	t	*df*	*p*	sig	*t*	*df*	*p*	sig
Table 4a. Set size 2								
		vs. optimal (70)				vs. equal (50)		
Young, threshold	1.802	29	0.082		2.032	29	0.051	marg
Young, ob	3.530	29	0.001	*	0.179	29	0.859	
Old, threshold	0.238	27	0.814		3.739	27	0.001	*
Old, ob	1.851	27	0.075		1.136	27	0.266	
Table 4b. Set size 6								
		vs. optimal (70)				vs. equal (16.7)		
Young, threshold	1.828	27	0.079		7.332	27	<0.001	*
Young, ob	3.376	27	0.002	*	5.198	27	<0.001	*
Old, threshold	3.206	25	0.004	*	5.460	25	0.000	*
Old, ob	5.042	25	0.000	*	3.212	25	0.004	*

Table 4. Results for weights, single-sample *t*-tests against the optimal and the equal weights. *Notes*: threshold = threshold contrast, ob = octave-below threshold contrast. marg = marginally significant.

significantly from the equal weighting. Thus overall, these results suggest that the participants used the cue information (significant differences from the equal weight), but not optimally (significant differences from the optimal weight); the one exception was the weights for the younger participants and the threshold contrast, which were not significantly different from the optimal weighting.

Note that a priori, the weights could have been suboptimal by either being too large or too small. In these results, however, all weights were lower than the optimal weight, suggesting that attention was spread suboptimally across all locations.[7]

RTs

For overall RTs, as expected there was a significant main effect of contrast, $F(1, 108) = 5.300$, $p = 0.023$, with slower RTs for the more difficult contrast

(threshold: mean RT = 857 ms, SE = 23 ms; octave below: mean RT = 875 ms, SE = 25 ms). Also, as expected there was a significant main effect of age, $F(1, 108) = 47.36$, $p < 0.001$, with slower RTs for the older group (younger: mean RT = 702 ms, SE = 33 ms; older: mean RT = 1030 ms, SE = 34 ms).

Cueing effects in RT, Invalid RT − Valid RT

For cueing effects in RTs (invalid RT – valid RT), there was a significant main effect of contrast, $F(1, 108) = 8.306$, $p = 0.005$, with larger cueing effects at the higher contrast (threshold: mean cueing effect, RT = 109 ms, SE = 14 ms; octave below: mean cueing effect, RT = 80 ms, SE = 15 ms). This is consistent with the effect of contrast seen in the hit rate cueing effects, with the larger cueing effects at the higher (threshold) contrast. There was also a significant contrast × set size × age interaction, $F(1, 108) = 6.390$, $p = 0.013$. As summarized in Figure 9 and Table 5, this interaction was explained by a set size effect only for the older participants with the octave-below contrast, $t(53) = 2.31$, $p = 0.025$.

Table 6 summarizes the single-sample *t* test comparisons of the RT cueing effects with zero for the same values in Table 5 and Figure 9. All conditions were

Figure 9. Cueing effects, RTs, invalid RT − valid RT. threshold = threshold contrast; ob = octave below. Error bars represent standard errors of the mean.

	Set size 2 vs. Set size 6			
	t	*df*	*p*	sig
Young, threshold	−0.054	55	0.957	
Young, ob	−1.094	55	0.279	
Old, threshold	1.053	53	0.297	
Old, ob	2.31	53	0.025	*

Table 5. Results for cueing effects, RTs, paired-sample *t* tests for set-size effects. *Notes*: threshold = threshold contrast, ob = octave-below threshold contrast.

	Set size 2 vs. 0				Set size 6 vs. 0			
	t	df	p	sig	t	df	p	sig
Young, threshold	5.027	29	0.000	*	3.403	26	0.002	*
Young, ob	1.767	29	0.088		2.661	26	0.013	*
Old, threshold	3.911	27	0.001	*	3.615	26	0.001	*
Old, ob	4.749	27	0.000	*	1.305	26	0.203	

Table 6. Cueing effects, RTs, and single-sample t-tests against zero. *Notes*: threshold = threshold contrast, ob = octave-below threshold contrast.

significantly different from zero, except for the young participants in the set size 2 and octave-below contrast condition, and for the older participants in the set size 6 and octave-below contrast condition. Note that this pattern of nonsignificant cueing effects corresponds to the results for suboptimal weighting found particularly for these two conditions, and thus the suboptimal weighting was not due to a trade-off of cueing effects in RTs.

General discussion

As stated in the Introduction, the ideal observer analysis of aging effects in cueing tasks was proposed to clarify the relative lack of consistent results in previous studies. One potential issue is the use of RT measures for cueing effects, specifically invalid RT − valid RT, for which quantitative predictions for the "best" cueing effect may be difficult. Specifically, the previous studies have varied in difficulty, set size, and cue validity. The ideal observer analysis gives a specific prediction for an optimal cueing effect for specific values of difficulty, set size, and validity. It also provides a relatively straightforward measure of the use of the cue information (weight) that essentially adjusts for varying difficulty and set size and should be equal to the cue validity. Thus, this study compared performance between young and aged participants in a yes/no peripheral cueing task with a cue validity of 70% and set sizes of two and six.

Results for cueing effects, measured as the difference between the valid hit rate and the invalid hit rate, found that the younger participants had a set-size effect, with larger cueing effects for set size 6. The older participants had no set-size effects, however (Figure 5). When the use of the cue was quantified by the ideal observer analysis as the weights (the internal representation of the cue validity), the opposite pattern was found; the young participants had no significant set-size effect, while the old participants did (Figure 6). These results are a reflection of the ideal observer predictions of larger set sizes leading to larger cueing effects for a fixed cue validity (Figure 2). Finally, when comparing the weights to the optimal weight (the cue validity,

70%) and an equal weighting of location, the results suggest that for set size 2, the older participants used the cue information relatively optimally, and as or more optimally than the younger participants. However, for set size 6, the older participants' weights differed significantly from optimality. These results suggest that the older participants' ability to use the cue information was negatively affected by the larger set size.

These results were generally consistent with findings in other domains of attention. As the task difficulty increases, the aged show tendencies to be more affected by the difficulty. As reviewed in the Introduction, this is seen in visual search (Folk & Lincourt, 1996; Foster et al., 1995; Harpur, Scialfa, & Thomas, 1995; Humphrey & Kramer, 1997; Madden, 2007; Madden et al., 1996; Madden & Whiting, 2004; Plude & Doussard-Roosevelt, 1989; Plude & Hoyer, 1986) and in the UFOV test (Ball et al., 1988; Edwards et al., 2006; A. B. Sekuler et al., 2000; R. Sekuler & Ball, 1986). Also, studies indicating aging deficits with peripheral cues (Table 1c; Greenwood & Parasuraman, 1999; Greenwood & Parasuraman, 2004, experiment 2; Greenwood et al., 1997) tend to be the ones with the larger set sizes. There was also an effect of contrast on the weights that followed the general pattern of worse performance (in terms of less optimal weighting) with increasing difficulty (Figure 7). However, the effect of contrast occurred for both the young and old participants, and thus did not appear to be age-specific.

Generally cueing effects in RTs were consistent with the expected cue validity benefit, with positive values for invalid RT − valid RT, and they were also consistent with the cue validity benefit in the hit rates (with positive values for valid hit rate − invalid hit rate). Thus, there was no "speed-accuracy trade-off" of cueing effects in RTs and hit rates. Notably, the condition in which the older participants had the lowest weighting (set size 6 and octave-below contrast threshold) was also a condition in which there was no significant cueing effect in RT. This suggests that the difficulty with using the cue in this condition (indicated by the suboptimal weighting) was not due to a compensatory effect in RTs.

As stated in the Introduction, a priori suboptimality for a cue validity of 70% may be expressed of an overweighting of the cued location, similar to a spotlight metaphor (e.g., Posner, 1980; Treisman & Gelade, 1980), or as an underweighting of the cued location, similar to the spread of attention suggested by a zoom lens metaphor (Eriksen & Yeh, 1985). The results, however, consistently suggest an underweighting for both young and old participants when there was suboptimal weighting.

An element of the current study was the relatively brief stimulus onset asynchrony (SOA) (150 ms). This was done to optimize cueing effects for this peripheral task (Jonides, 1981; Müller & Rabbitt, 1989; reviewed by Wright & Ward, 2008). As the aged have speed-of-processing deficits (Cerella, 1985; Salthouse, 1996), it is possible that the findings of suboptimality were due to the brief SOA. However, such an effect of SOA was not suggested with the set size 2 results, and thus would necessarily need to interact with set size. Further studies could address this concern.

Conclusions

Unlike the more standard measure of a difference in valid and invalid RTs as the cueing effect, the weight estimated from the ideal observer analysis gives a quantitative measure to compare to optimal weighting performance (i.e., the cue validity) regardless of changes in set size, cue validity, and difficulty. The analysis of weights may be interpreted as an assessment of how participants are using the information provided by the cue, and is independent from a measure of sensitivity (d') and bias. For these reasons the ideal observer analysis could clarify issues that may arise from results based upon RTs. This method was applied using a peripheral cueing task to assess potential age differences in this task. It was found that, for a small set size (two), the older participants used the cue effectively, and equally or more optimally than younger participants. However, for a larger set size (six), the older participants' weight differed from the optimal weighting, suggesting a deficit in the use of the cue information in the aged sample as the cueing task increased in difficulty (set size).

Keywords: aging, cueing, visual attention, ideal observer

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Footnotes

[1] It should be noted that aging effects upon the phenomenon of Inhibition of Return (IOR; e.g., Faust & Balota, 1997, experiment 2; Hartley & Kieley, 1995; Langley, Friesen, Saville, & Ciernia, 2011; Langley, Fuentes, Hochhalter, Brandt, & Overmier, 2001; Langley, Fuentes, Viva, & Saville, 2007) are not included in this discussion.

[2] In many of these previous studies, the focus was primarily the assessment of effects (cueing, set size) as the results of parallel models of attention instead of limited capacity attentional models (e.g., Posner, 1980; Treisman & Gelade, 1980). In this study, the ideal observer acts as a standard for the optimal use of the cue information and is relatively neutral, theoretically. It is worth noting that either a parallel model or a limited capacity model may lead to equivalent observed weights (Eckstein et al., 2002).

[3] The equal-variance assumption is ubiquitous in vision research and has been employed in previous studies on ideal observer analyses of cueing tasks (e.g., Shimozaki et al., 2012). However, it should be noted that this was a simplification of the more likely scenario in which variance scales with contrast amplitude (e.g., Dean, 1981; Tolhurst, Movshon, & Thompson, 1981).

[4] For d's this assumes that human observer performance is subject to both internal and external noise, whereas the IO has no internal noise. Studies comparing humans to the ideal observer in terms of absolute performance (i.e., d' or signal-to-noise ratio) have a long history and are sometimes known as studies of efficiency (e.g., Barlow, 1978; Burgess et al., 1981). As no external noise was added to the stimuli in this study, those comparisons could not be made.

[5] We originally presented stimuli at a half octave above contrast threshold and a half octave below contrast threshold. However, pilot studies showed that performance was at ceiling for young and older participants when stimuli were presented a half octave above contrast threshold. As such, we used stimuli presented at contrast threshold and one octave below contrast threshold in the main study.

[6] The issue of parallel versus serial attentional models was not a focus for this study. However, the lack of a main effect of set size in d' found in these results followed the prediction for a parallel attentional model.

[7] It should be noted that there was within-participant variability of the hit and false alarm rates. Estimates of the standard deviations of these rates were found with the mean rates across all conditions

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and participants using a binomial distribution assumption, where $sd = \sqrt{np(1-p)}$ with $n =$ number of trials (for a specific trial type), and $p =$ probability or rate (for a specific trial type). These SD values were: valid hit rate = 0.0287, invalid hit rate = 0.0524, false alarm rate = 0.0278.

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Appendix: The ideal observer

Generative model

The task is a cued yes/no discrimination in which the observer reports whether a target was present at any of M possible locations. Global target presence is indicated with $C = 1$, global target absence with $C = 0$, target presence at location i with $T_i = 1$, and target absence at location i with $T_i = 0$. The stimulus response at location i is denoted x_i. While not relevant to this study, this response is the cross-correlation of the stimulus with the ideal template, which in Gaussian noise is the difference between the target and nontarget templates; this linear operation is also known as the Linear Amplifier Model (e.g., Burgess et al., 1981; Green & Swets, 1974; Murray & Gold, 2004). Following the Linear Amplifier Model, it is assumed that responses at each location are drawn from normal distributions, one with the target, and one without the target. These distributions are assumed to follow the standard equal-variance assumption found in SDT (Green & Swets, 1974; $\mu_{Ti=0} = 0$, $\mu_{Ti=1} = d'$, $\sigma = 1$).

$$p(x_i|T_i = 0) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} \tag{1}$$

$$p(x_i|T_i = 1) = \frac{1}{\sqrt{2\pi}} e^{\frac{-(x_i-d')^2}{2}} \tag{2}$$

If the target is present ($C = 1$), then the probability of it being present at location i is denoted $p(T_i)$. One location is cued and has a probability $p(T_i) = \alpha$ to contain the target, where α is called the "cue validity" or "weight." All other locations have a probability $p(T_i) = (1 - \alpha)/(M - 1)$ to contain the target. Mathematically, the distribution of vector $\mathbf{T}$ conditioned on C can be expressed as follows:

$$p(\mathbf{T}|C) = \sum_i^M p(T_i)\delta(\mathbf{T} - C\mathbf{1}_i) \tag{3}$$

where $\mathbf{T}$ is a vector with 0s at locations without a target

and a 1 at the location with the target, δ is the Dirac delta function, and $\mathbf{1}_i$ is a vector of length M with a 1 at location i and 0s elsewhere.

Decision rule for the ideal observer

The ideal observer reports "target present" when $p(C = 1|\mathbf{x}) > p(C = 0|\mathbf{x})$, or when

$$\mathrm{DV} \equiv \frac{p(C = 1|\mathbf{x})}{p(C = 0|\mathbf{x})} > 1 \tag{4}$$

where DV is called the decision variable. Taking logs and applying Bayes' rule, this amounts to reporting "target present" when

$$\log \mathrm{DV} \equiv \log \frac{p(C = 1|\mathbf{x})}{p(C = 0|\mathbf{x})} = \log \frac{p(\mathbf{x}|C = 1)}{p(\mathbf{x}|C = 0)} + \log \frac{p(C = 1)}{p(C = 0)} > 0 \tag{5}$$

Note that $p(C = 1) = p(C = 0)$ in this study, and thus

$$\log \mathrm{DV} = \log \frac{p(\mathbf{x}|C = 1)}{p(\mathbf{x}|C = 0)} > 0 \tag{6}$$

With the DV expressed as the ratio in Equation 6, and if Equation 6 is described as a comparison of the log DV to an ideal log criterion, then it follows that the ideal log criterion is 0 for this task.

From the generative model, it follows that the probability of the evidence under target absence is equal to

$$p(\mathbf{x}|C = 0) = p(\mathbf{x}|\mathbf{T})p(\mathbf{T}|C = 0)$$

$$= \prod_{i=1}^{M} p(x_i|T_i = 0)$$

$$= \prod_{i=1}^{M} \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} \tag{7}$$

For the target present case we find that

$$p(\mathbf{x}|C = 1) = \sum_{j=1}^{M} p(T_j) \prod_{i \neq j} \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} \frac{1}{\sqrt{2\pi}} e^{\frac{-(x_j-d')^2}{2}} \tag{8}$$