Objects are often identified by the shape of their contours. In this study, visual search tasks were used to reveal a visual dimension critical to the analysis of the shape of a boundary-defined area. Points of maximum curvature on closed paths are important for shape coding and it was shown here that target patterns are readily identified among distractors if the angle subtended by adjacent curvature maxima at the target pattern’s center differs from that created in the distractors. A search asymmetry, indicated by a difference in performance in the visual search task when the roles of target and distractor patterns are reversed, was found when the critical subtended angle was only present in one of the patterns. Performance for patterns with the same subtended angle but differing local orientation and curvature was poor, demonstrating insensitivity to differences in these local features of the patterns. These results imply that the discrimination of objects by the shape of their boundaries relies on the relative positions of their curvature maxima rather than the local properties of the boundary from which these positions are derived.

Introduction

One goal of the visual system is to parse the visual field into objects and the system is adept at extracting luminance and contrast boundaries from a scene (Field & Hayes, 2004; Li & Gilbert, 2002) with such boundaries being enhanced in their salience by a synergistic interaction between the responses of cells with adjacent and approximately colinear receptive fields (Field & Hayes, 2004; Field, Hayes, & Hess, 1993; Gilbert, Das, Ito, Kapadia, & Westheimer, 1996; Li & Gilbert, 2002). Closed paths often define the boundaries of objects and the salience of closed paths is further enhanced, facilitating the segregation of a contained figure from ground (Kovacs & Julesz, 1993; Wagemans et al., 2012). It follows that the presence of an object bounded by a salient path within an area of the visual field might be detectable without reference to the shape of that area. Discrimination between two such areas, however, would require the identification of differences in their shapes (Tan, Dickinson, & Badcock, 2013).

Difference in shape might be expected to be evident at successive levels of analysis. A local difference in curvature might differentiate two shapes, but such a difference might only be visible through scrutiny of the boundaries, particularly if overall rotation of the shape is allowed to vary, as would normally be the case. Treisman and Gelade (1980) proposed that certain features, for example the color and size of objects, could be processed in parallel across the visual field. An object uniquely distinguished by such a feature would be rapidly recognized as being present within a scene.

Experimentally, the study of visual search typically requires an observer to report the presence or absence of a target pattern among a variable number of simultaneously presented distractor patterns (Wolfe, 2001). The response time is measured, and performance on the task for a particular target/distractor pair is characterized by the rate of change of response time with the set size (SS), the total number of patterns present in each test trial. Response times do covary with the gradient of the response time versus SS graph and are known to show a number of asymmetries. For example, response times for target-absent trials are longer than target-present trials (Chun & Wolfe, 1996), although this might be attributed to the need to search
through more items when the target is absent. One particularly interesting phenomenon of visual search is an asymmetry in response time when the roles of target and distractor patterns are reversed (Treisman & Souther, 1985). Under some circumstances, pattern one is readily identified among an arbitrary number of pattern two but when the roles are reversed, pattern two is difficult to find among examples of pattern one. This asymmetry is taken as evidence that the readily identified pattern contains a visual cue that is not present in the other. When the roles are reversed, the pattern that lacks the visual cue is difficult to find among the distractor patterns, all of which contain the cue that is specific to that pattern type. This interpretation makes the assumption that all other cues are common to the patterns, which implies a hierarchy of analysis.

Treisman and Souther (1985) demonstrated asymmetry in visual search using circles and circles with intersecting lines as patterns. Circles with intersecting lines were readily identified among circles but not vice versa. Significantly, the same asymmetry was found when lines and circles with intersecting lines were paired in stimuli as target and distractor. It was concluded that the feature that allowed the circle and line pattern to be readily identified was an emergent feature of the circle–line conjugation. For a pair of patterns there are, of course, four possible outcomes: both of the patterns are readily identified, neither is, or one of the two patterns is more readily identified among examples of the other. These alternatives can be accommodated by the existence of unique cues in both, neither, or one or the other of the two patterns. When considering the whole of the search field the difficulty of the task might be attributed to the signal to noise ratio for the critical feature in the pattern playing the role as target relative to that in the distractor. If only the target has the feature then there is essentially no noise and the ratio is 1:0, but if the target is the only pattern that lacks the visual cue is difficult to find among the distractor patterns, all of which contain the cue that is specific to that pattern type. This interpretation makes the assumption that all other cues are common to the patterns, which implies a hierarchy of analysis.

The conjugation of a circle and a line, used to demonstrate asymmetry in visual search, has the potential to introduce a number of different visual cues such as orientation, size/length, intersection, and line termination (Wolfe, 2001). Kristjánsson and Tse (2001), however, demonstrated an asymmetry in visual search in two patterns defined simply by a bounding path. One shape was an ellipse and the other was a composite pattern created by joining two different semiellipses. The pattern composed of two semiellipses was created by joining one semiellipse along its major axis to another along an axis of the same length that could be the major or minor axis, depending on the length of the second axis for that element. An area enclosed by a boundary composed of two semiellipses was shown to be readily identified among ellipses (matched for area). When the roles of the target and distractor were reversed, however, the target ellipse was difficult to find among the composite patterns. In a more recent paper, Jóhannesson, Sigurdardottir, and Kristjánsson (2013) replicated this finding and also demonstrated there was no advantage for identifying the composite patterns on the ground plane in comparison to those in the sky, as might be predicted by theories of ecological optics.

The boundary of the composite pattern used by Kristjánsson and Tse (2001) contains points where the curvature of the boundary, the rate of change of orientation, is discontinuous. Tse and Albert (1998) argued that such points are used to recover threedimensional information from a silhouette. Given this, it might be expected that patterns containing such points would pop out among others that do not contain them because the points are high in information content. The results of the first experiment of the study of Kristjánsson and Tse (2001), summarized above, are consistent with this interpretation, as curvature on the ellipses is continuous and as such, different from the composite patterns. Kristjánsson and Tse (2001), however, also considered an alternative interpretation—that the rapid identification of the composite patterns was due to their being perceived as having a third dimension to their percept, in contrast to the ellipses that were perceived as flat.

The second experiment of Kristjánsson and Tse (2001) examined the possibility that rapid identification relied on a perception of three dimensionality. Rather than using the filled composite patterns, which gave a strong subjective impression of having extension in three dimensions, they used halved filled patterns and halved outlines as stimuli, both of which were perceived as flat. Kristjánsson and Tse (2001) argued that both of these modified stimuli produced results similar to those found in the first experiment. However, on inspection, the results seem much more equivocal than they claim. For the filled half patterns, the gradients of the response time (RT) versus SS graphs are not particularly asymmetric across conditions. The gradient for filled half composite patterns among half ellipses is only 50% higher than for half ellipses among half composite patterns. The results for the outline half patterns are more asymmetrical, with the gradient for the half ellipse outline among half composite outlines approaching a factor of four higher than for a half ellipse outline among half composite outlines. This suggests that perhaps the filled and outline patterns are not processed in the same manner. The filled half patterns are perceived as shapes themselves but the halved outlines are perhaps perceived as lines or as occluded shapes. Nonetheless, Kristjánsson and Tse (2001)
treated the results as similar and concluded that it was the curvature discontinuities that facilitated rapid identification of the composite stimuli, rather than the impression of three dimensionality.

A third experiment showed that incomplete outline ellipses with high aspect ratio (height:width) and low maximum curvature were more readily identified among incomplete outline ellipses with low aspect ratio and high maximum curvature than vice versa. Kristjánsson and Tse (2001) concluded that the asymmetry in response times obtained in the first experiment could not, therefore, be accounted for by differences in absolute magnitudes of curvature. It is possible, however, that these incomplete outline patterns are analyzed by mechanisms that are different from those used in the analysis of the complete outline or filled patterns, as implied by the disparity of the results reported for the second experiment. This concern might also be raised when considering the results of a fourth experiment used to examine the implications of the observation that the composite stimulus has only a single axis of mirror symmetry but the ellipse has two. The fourth experiment used S-shaped stimuli derived from the original stimuli. These stimuli incorporated the local features but not the mirror symmetry, and they showed that some asymmetry in the response times remained. The asymmetry in the results, however, was diminished for these patterns and the absolute values for the gradients were substantially higher than for the complete filled stimuli.

The results of a fifth experiment were particularly surprising as they showed that the asymmetry in response times remained when the curvature discontinuities were obscured by occluding circles. An infinite number of smooth paths with continuous curvature could be used to complete the composite pattern behind the occluders, so the conclusion that the curvature discontinuities are retained in the path is only one possibility, but it implies that the elliptical paths are analyzed as such by the visual system, thereby allowing for the completion of the path retaining the curvature discontinuities for analysis.

A sixth and final experiment showed that, as the magnitude of the step change in curvature at the curvature discontinuity increased, the gradient of the RT versus SS graph decreased. The changes in the patterns that modulate the step change in curvature incur concomitant changes in their gross shape. Given the discrepancies between results obtained for filled and open contours outlined above and the surprising result that the asymmetry in search performance is not diminished by occlusion of the curvature discontinuities, it seems reasonable to consider the possibility that these results are better accounted for by sensitivity to changes in global shape.

To summarize, Kristjánsson and Tse (2001) concluded that the composite pattern was readily identified among ellipses because the curvature discontinuity comprised a unique cue when the composite pattern was the target and that the ellipse was difficult to find because the ellipse did not contain a unique cue by which it could be identified when presented among the composite patterns. Previously it has been shown, however, that radial frequency (RF) patterns (Bell, Badcock, Wilson, & Wilkinson, 2007; Loffler, Wilson, & Wilkinson, 2003; Wilkinson, Wilson, & Habak, 1998)—patterns deformed from circular by a sinusoidal modulation of radius—are readily discriminated in visual search tasks, provided their frequencies of modulation differ. Specifically, patterns with three cycles of sinusoidal modulation completing 360° (RF3 patterns), are rapidly identified among RF4 patterns (Almeida, Dickinson, Maybery, Badcock, & Badcock, 2010a, 2010b, 2013, 2014; Bowden, Dickinson, Fox, & Badcock, 2015). The RF patterns used in the studies cited had approximately the same range of curvature and no curvature discontinuities and we can, therefore, presume that the patterns are not identified on the basis of the magnitude of local curvature or by the presence of discontinuities in curvature. This interpretation is supported by one experiment of Almeida et al. (2013), which showed that search tasks with elements that were patterns comprising single cycles abstracted from RF patterns were very difficult. Thus, in the current study, the conclusion that the Kristjánsson and Tse (2001) composite stimuli are identified by the existence of curvature discontinuities is revisited. We hypothesize instead that the pattern is readily identified by a component of its shape and that the equivalent shape component is absent in the ellipse.

In part, the RF pattern’s utility in the study of visual processes arises from a demonstrable integration of shape information around the pattern (Bell & Badcock, 2008; Cribb, Badcock, Maybery, & Badcock, 2016; Dickinson, Han, Bell, & Badcock, 2010; Dickinson, McGinty, Webster, & Badcock, 2012; Green, Dickinson, & Badcock, 2017a, 2017b; Hess, Wang, & Dakin, 1999; Loffler et al., 2003; Schmidtmann, Kennedy, Orbach, & Loffler, 2012; Tan, Bowden, Dickinson, & Badcock, 2015). When cycles of modulation of a particular frequency are incrementally added to a pattern (with the remainder of the pattern completed by a circular arc) the threshold for discriminating a modulated pattern from a circle decreases at a rate that cannot be accounted for by probability summation—the increase in the probability of detection of single cycles. This implies integration of information across cycles of modulation. While a recent study using RF4 patterns of fixed phase or a small set of phases has questioned whether the rate at which threshold decreases can reliably be shown to be
greater than the probability summation prediction (Baldwin, Schmidtmann, Kingdom, & Hess, 2016), thresholds for RF3 patterns, and indeed RF4 patterns, in random phase have been consistently shown to decrease almost in inverse proportion to the number of cycles of modulation present on the pattern, reflecting strong integration (Cribb et al., 2016; Dickinson et al., 2012). Furthermore, Dickinson, Bell, and Badcock (2013) have shown that patterns with different frequencies of modulation (RF3 and RF6 patterns) are perfectly discriminated; that is, they can be discriminated at their thresholds for detection of modulation. Seen in the context of the communication of information, such a result is interpreted as evidence for the existence of information lines labeled for specific qualities of the pattern (Watson & Robson, 1981), in this case perhaps related to the frequency or, equivalently, the period of modulation. Dickinson et al. (2013), however, also showed that single cycles of RF3 and RF6 modulation could not be discriminated at their threshold for detection, implying that the repetition of features on adjacent cycles of modulation is more important to discrimination performance than the modulating function itself.

Dickinson, Cribb, Riddell, and Badcock (2015) recently demonstrated that strict sinusoidal modulation of the contour is not necessary for the demonstration of integration of information. Specifically, it was shown that patterns defined by negatively or positively rectified sinusoidal modulation also displayed integration. Moreover, it was also demonstrated that patterns with the same period of modulation but different modulating functions (sinusoidal or rectified sinusoidal modulation) could not be discriminated at their thresholds for detection of modulation. This suggests that the perception of shape at threshold is not primarily concerned with the local orientation or curvature properties of the shapes. All of these results were, however, based on measurements of the minimum modulation amplitude required to detect the presence of, and discriminate between, particular frequencies of modulation at threshold. The amplitude thresholds for detection and discrimination of the patterns were only a few percent of the radius of the unmodulated circle. The results were, therefore, obtained using patterns that were always very close to circular. In the current study we employ patterns defined in the same manner but with higher amplitudes of modulation and thus, quite different from circular. Instead of using modulation detection and discrimination thresholds to infer mechanisms of shape analysis, here we use visual search to determine the properties of shapes that might allow them to be readily identified among other shapes. This allows us to test whether the rules revealed for the discrimination of patterns at threshold also apply for shapes with higher amplitudes of modulation, and also to examine if the analysis of these shapes occurs in parallel across the visual field.

**General methods**

**Observers**

Observers were recruited from the Human Vision Laboratory of The University of Western Australia. All observers gave their informed consent to participation in the study, which was approved by the Human Research Ethics Committee of The University of Western Australia and therefore complied with the tenets of the Declaration of Helsinki. Six observers participated in the study. Two, ED and KH, are authors and the remaining four, KS, AC, TM, and JP were naïve to the purpose of the study. All had normal visual acuity, assessed using a LogMAR chart.

**Apparatus**

In a dark room (<1 cd/m² ambient luminance) observers viewed a Sony G520 CRT monitor from a distance of 65.5 cm, constrained by the position of a chin rest. At this distance, each pixel of the monitor subtended 2 minutes (2') of visual angle. The screen comprised 768 × 1,024 pixels refreshed at 100 Hz. Visual stimuli were presented to the screen from the frame buffer of a Cambridge Research Systems (CRS) ViSaGe visual stimulus generator (Cambridge Research Systems, Rochester, Kent, UK). Observers made their responses using a CRS CB6 button box (Cambridge Research Systems).

**Stimuli**

Custom stimuli were created in Matlab version 7.04 (MathWorks, Natick, MA) installed on a PC. Each stimulus comprised a variable number of patterns defined by closed paths located near the vertices of a 7 × 7 implicit grid. The vertices of the grid were separated by 3° of visual angle (horizontally and vertically). The shapes had a nominal width and height of approximately 2° of visual angle (dimensions are provided for specific shapes as they are described). The closed paths had a Gaussian luminance contrast profile in cross-section with a maximum Weber contrast of 1 (background luminance = 45 cd/m²). Luminance calibration was performed using a CRS Optical and associated software.
Procedure

Conditions within an experiment were blocked. Each condition comprised a pairing of two patterns, where one pattern was defined as a target and a second as a distractor. In each trial of a block the observer was required to indicate whether a single target pattern was present or absent. The time from the onset of the stimulus to the response was recorded to a precision of 1 ms and the stimulus extinguished when the response was made. The number of patterns present in each trial, the set size (SS), was 1, 2, 4, 8, 16, or 32, with the target present in half of the trials. Each similar trial (e.g., a trial with four patterns and with the target absent) was repeated 10 times within a block, and five blocks were completed for each condition. Blocks of trials of different conditions were interleaved to ameliorate practice effects. Response times for each condition were collated into the similar trials, with a total of 50 instances of each similar trial, and were treated as described in the results section to determine a response time characteristic of that SS. Each experimental condition is then characterized by two graphs describing response time as a function of SS, with one describing performance when the target is present and the other when it is absent. A second pair of graphs describes performance when the role of target and distractor are reversed in a complementary condition that will be referred to as the reverse condition. The ordering of the target and distractor pairings were chosen such that in the default order, which will be referred to as the forward condition, the target would be expected to be more readily identified among the distractors than in the reverse condition. In each of two experiments, observers completed 4,800 trials (50 trials for each of six SSs in four pattern pairings in forward and reverse conditions with the targets present or absent) over 6–8 hours of testing in several shorter-duration sessions.

Despite the substantial testing time accumulated by each observer across the different conditions, the number of trials at a specific SS of a single condition was not sufficient to characterize the response time distributions with any certainty. It has been recognized that the distributions are important in constraining models of visual search (Wolfe, Palmer, & Horowitz, 2010) but the question of whether a single distribution should be preferred for all visual search tasks is not settled. Palmer, Horowitz, Torralba, and Wolfe (2011) point out that, because of the limited numbers of trials for specific SSs of a condition, the sets of response times of individual observers are usually combined prior to the estimate of a central tendency. This averaging of distributions makes the choice of the measure of central tendency difficult, but this choice influences the mapping of SS to response time. In order to inform the choice of processing procedure observer ED, who is an author, performed an additional 10,080 trials for each of two conditions. One of the conditions was a task that was insensitive to SS and the second was a task that was highly sensitive to SS. The results of these conditions are considered in Appendix A, where the choices made in the analysis of the data are considered.

The gradient of the linear graph describing response time as a function of SS is the conventional parameter used in comparing performance in visual search tasks and, after analyses of the response time distributions reported in Appendix A, we conformed to that convention in this study using the median of the response time distribution as the measure of the response time for each particular SS for each observer.

Patterns

The shapes used were circles, RF patterns with three and four cycles of modulation in 360° (RF3, RF4), negatively rectified RF patterns with three periods of modulation (RRF3), ellipses, and the composite stimulus comprised of a combination of two semi-ellipses (K&T). These patterns differ in terms of the presence or absence of features that have demonstrable or implied importance to local and global processing of shape.

Circles

The circles had a radius of 60° of visual angle. From left to right, the top row of Figure 1 shows the radius of the circle as a function of polar angle, a polar plot of the radius illustrating the pattern shape, the local orientation of circular path as a function of polar angle, and the curvature of the path as a function of polar angle. A path defining a circle has a constant curvature; therefore, a circle is devoid of both discrete points of maximum curvature and curvature discontinuities. The properties of the other pattern types are illustrated similarly on subsequent rows.

RF patterns

RF patterns are defined by a sinusoidal modulation of radius. The properties of an RF pattern are illustrated in the second row of Figure 1. The radius of an RF pattern is given by the following equation,

$$R(\theta) = R_0(1 + A \sin(\omega \theta + \phi)),$$

where $R(\theta)$ is the radius at an angle $\theta$ relative to the positive x-axis, $R_0$ is the unmodulated radius, $A$ is the amplitude of modulation, $\omega$ is the frequency of modulation, and $\phi$ is the phase of modulation of the
pattern (controlling the orientation of the pattern in the frontoparallel plane—the RF3 pattern illustrated in Figure 1 is in zero phase). An $R_0$ value of 60° was used for both RF patterns. Different integer frequencies of modulation ($\omega$) create patterns with different characteristic shapes. The RF patterns were given amplitudes of modulation, defined as a proportion of the unmodulated radius, of $1/(1 + \omega^2)$, where $\omega$ is the frequency of modulation (cycles of modulation in 360°). At these amplitudes the patterns have a curvature, a rate of change of orientation with $\theta$, of zero at the minima of the modulation of radius and therefore resemble familiar geometrical shapes with flat sides but smoothed corners (Dickinson et al., 2012). An RF4 pattern, for example, resembles a square but with rounded corners (the RF4 pattern is not depicted but contains the same local features as the RF3 and differs in periodicity of those local features). Curvature defined as the rate of change in orientation with $\theta$ will differ slightly from curvature defined as the rate of change of orientation with path length. However, curvature discontinuities require abrupt changes in orientation and therefore the curvature discontinuities are constrained to particular points defined in polar...
coordinates. Similarly, the points of maximum curvature occur at points where the tangent to the path is perpendicular to the local radius, so RF patterns are characterized by the existence of points of maximum curvature and their relative positions. RF patterns with differing frequency have differing configurations of points of maximum curvature. The curvature around the pattern is, however, continuous, so RF patterns do not contain curvature discontinuities. Because the points of maximum curvature repeat periodically around the patterns, the RF patterns also contain a shape cue defined by the relative positions of the adjacent points of maximum curvature (Dickinson et al., 2013). This cue can be described in a manner that is invariant to the orientation of the pattern, as the angle subtended at the center of the pattern by adjacent points of maximum curvature. For RF3 patterns this angle is 120° and for RF4 patterns 90°.

The Kristjánsson and Tse composite stimuli (K&T), created by joining two semiellipses, are the same in profile as those used in the majority of the experiments reported in the Kristjánsson and Tse (2001) paper. The properties of the K&T stimuli are illustrated in the third row of Figure 1. One semiellipse is, in fact, a semicircle (the top half in the example illustrated in the polar plot), and the half-length of the major axis of the semiellipse that completes the shape is the same as the radius of the circle to ensure that the contours meet. The half-length of the minor axis of the ellipse is half of this distance, producing a flattened segment. The distance used in the current experiment for the radius of the circular part of the stimulus is 66° equating to the maximum radius of the RF3 patterns. These K&T stimuli are defined by a path which has a local orientation which is a continuous function of polar angle. They do, however, contain curvature discontinuities on the path which are also points of maximum curvature. The curvature discontinuities can be seen as vertical lines in the graph of curvature versus polar angle. In the illustration the curvature discontinuities are seen at polar angles of zero and 180°. This pattern, however, differs from all of the others in that the center, as specified by the polar coordinates in which it is constructed, is not the center of gravity of the shape. We will argue that the positions of points of maximum curvature are referenced to the center of gravity of the pattern in the analysis of shape and the points of maximum curvature on these patterns subtend an angle of 156° at the center of the pattern if the center is assumed to be the center of gravity of the plane figure bounded by the pattern (Proffitt, Thomas, & O’Brien, 1983).

The ellipses used in this experiment have a major axis with a half-length equal to the radius of the circular part of the K&T pattern (see the fourth row of Figure 1). The half-length of the minor axis is the average of the radius of the circular part of the K&T pattern and the half-length of the minor axis of the second component of that pattern in order to equate the maximum and minimum widths of the patterns. Ellipses have two points of maximum curvature on their major axes but the curvature at these points is continuous. The points of maximum curvature on an ellipse subtend an angle of 180° at its center of gravity.

Rectified RF3 patterns (RRF3 patterns) were created by applying a negatively rectified modulation of radius with a frequency of 1.5 cycles per 2π radians to a circular path according to the function below,

$$R(\theta) = R_0(1 + A - \text{abs}(A \cos((\omega/2)\theta + \phi)))$$

(2)

where ω = 3. To control for the range of local orientation differences from a circle, the amplitude, A, was double that used for the RF3 patterns (the frequency of the modulation is halved resulting in the magnitude of the gradient of the function being reduced by half and so a doubling of amplitude is applied to compensate). This results in the patterns having the same histogram of local orientation difference from circular, precluding the use of orientation statistics to differentiate the patterns. So that the RRF3 patterns approximated the RF3 patterns in size, a value of 54° was used for $R_0$ for the RRF3 patterns. Rectification of the sinusoidal modulation creates curvature discontinuities at the zero crossings of the sinusoidal modulator. In contrast to the K&T patterns, these curvature discontinuities are also orientation discontinuities. These rectified RF patterns, therefore, have curvature discontinuities that transition from continuous curvature to effectively infinite curvature and then back again at their points of maximum curvature. They also contain the same shape cue as the RF3 pattern. That is, the angle between adjacent points of maximum curvature is 120°.

In two experiments, stimuli comprising pairs of the patterns described above were chosen to examine the contributions of points of maximum curvature and curvature discontinuities to performance in visual search. Experiment 1 compares the results for the K&T and ellipse pattern pairings with pattern pairings drawn from the RF pattern derived stimuli. Experiment 2 examines pairings across the stimuli derived from ellipses and those related to RF patterns.

The patterns were presented in random phase. This particular experimental constraint was adopted because it has been shown that if RF patterns are presented in fixed phase in detection threshold tasks observers often choose to attend to particular local points on the pattern. This should not be such an issue in a search task, but we have the opportunity to test here if the search results found by Kristjánsson and Tse (2001) could be replicated using random phase patterns. The shape analysis that we propose occurs is indifferent to...
the orientation of the patterns allowing the analysis to
generalize over all pattern orientations.

**Experiment 1: Is the existence of a curvature discontinuity necessary or sufficient for the rapid identification of closed path patterns?**

### Introduction

Kristjánsson and Tse (2001) concluded that rapid identification of patterns composed of two semielipses joined along one axis (K&T patterns), among ellipses, was possible because curvature discontinuities that exist at the points at which the semielipses are joined, represent cues that are not present in the ellipses. It has recently been shown, however, that discrimination between patterns with sinusoidal modulation and patterns with negatively rectified sinusoidal modulation of the same period is not possible at the amplitude threshold for detection of modulation, despite the existence of curvature discontinuities on the rectified patterns (Dickinson et al., 2015). Moreover, Dickinson et al. (2013) showed that, while RF patterns with different frequencies of modulation could be discriminated at their thresholds for detection if they had two or more cycles of modulation applied to the patterns, patterns with only one of the cycles of modulation could not. This implies that points of maximum curvature are highly salient but also that discriminating between the frequencies of modulation in patterns with a single cycle of modulation requires much higher amplitudes, perhaps allowing the single cycles of modulation to be discriminated on the basis of the wavelength of the single cycle.

If the discrimination and detection results extrapolate to patterns with suprathreshold amplitudes of modulation, it would question whether it is the existence of a curvature discontinuity that allows the Kristjánsson and Tse patterns to be rapidly identified among ellipses. If instead the patterns are rapidly identified on the basis of a shape cue, perhaps the angle subtended at the pattern center by adjacent points of maximum curvature (Dickinson et al., 2013), then patterns that differ only in local properties such as orientation and curvature would not be easily discriminated. To summarize we hypothesize that points of maximum curvature represent salient features on closed paths, and that the angle subtended by two adjacent points of maximum curvature at the pattern center represents a shape cue by which closed paths can be rapidly discriminated. Further, we suggest that curvature discontinuities that are also curvature maxima are equivalent to curvature maxima with continuous curvature from the point of view of contributing to the hypothesized shape cue.

We predict, therefore, that suprathreshold amplitude RF patterns will be hard to find among rectified RF patterns with the same period and vice versa but, given their demonstrably global processing (Cribb et al., 2016; Dickinson et al., 2012; Green et al., 2017b; Hess et al., 1999; Loffler & Wilson, 2001; Schmidtmann et al., 2012; Tan et al., 2015), RF patterns should be rapidly identified among RF patterns with differing frequency by virtue of the shape cue. Further, RF patterns should be easy to find among circles because of the points of maximum curvature (and concomitant shape cue) they contain, but circles should be hard to find among RF patterns because the absence of these features when the circle is presented among a field of distractors containing them. Finding the circle will require scrutiny of each pattern. In short, we expect the angles on the path that define the curvature discontinuities on the rectified RF patterns to be treated equivalently to the points of maximum curvature on the conventional RF patterns. The curvature discontinuities on the Kristjánsson and Tse patterns are also points of maximum curvature because they are at the ends of the major axis of the least circular semielipse. We expect these points to also be treated by the visual system as points of maximum curvature in the analysis of two-dimensional shape. These points subtend an angle of 156° at the center of gravity of the plane figure enclosed by the pattern so we expect this angle to provide a shape cue that characterizes this pattern. The ellipses that the K&T stimuli were paired with in the study of Kristjánsson and Tse (2001) also have two points of maximum curvature situated at either end of the major axis of the ellipse. That the K&T stimuli were readily identified among ellipses but ellipses were hard to find among K&T stimuli suggests that an angle of 180° between adjacent points of maximum curvature does not represent a salient shape cue. This suggests an upper limit to the angle that represents a shape cue.

The stimuli for this experiment were composed of pairings of the patterns described in the General methods section. Four pairs of patterns were examined in this first experiment comparing the K&T and ellipse pattern pairing with pairings of RF patterns and patterns related to them. Example stimuli are shown in Figure 2. The stimuli comprised:

- A pairing of K&T patterns and ellipses: This pattern pairing allows us to verify the search asymmetry reported by Kristjánsson and Tse (2001) for these patterns (see the top row of Figure
Figure 2. Example stimuli. All of the examples above contain 16 patterns (a SS of 16). The rows of the figure represent stimuli from complementary conditions where the roles of the target and distractor are reversed from the first column to the second. Stimulus (a), for example, incorporates one K&T pattern among ellipses in the forward condition and stimulus (b) a single ellipse among K&T patterns in the reverse condition. The patterns of the pair are illustrated in the third column, a K&T stimulus above an ellipse, annotated with the positions of the curvature discontinuities (white triangles) and points of maximum curvature (black triangles).
Consistent with Kristjánsson and Tse (2001), the stimuli give the subjective impression that the K&T pattern is more readily identified among ellipses than the ellipse among K&T patterns. Similarly, in stimulus (c) an RF3 pattern is readily identified among circles, but in (d) a circle is difficult to find among RF3 patterns. In (e) an RF3 pattern is clearly evident among RF4 patterns and in (f) an RF4 pattern is similarly evident among RF3 patterns. In (g) an RF3 pattern is difficult to find among RRF3 patterns and in (h) the RRF3 pattern is difficult to find among RF3 patterns.

Both patterns contain points of maximum curvature (indicated by black triangles) but only the K&T stimulus contains curvature discontinuities (white triangles). An impression of the expected result can be obtained from inspecting the example arrays. The K&T pattern in (a) is readily identified among ellipses but the ellipse in (b) is hard to find among K&T patterns.

A pairing of RF3 patterns with circles: This pairing allows us to ensure that a comparable pattern of results may be obtained from a pair of patterns that are not differentiated by the existence of curvature discontinuities in one but not the other (see the second row of Figure 2). The RF3 pattern contains points of maximum curvature but the circle does not. Again, inspection suggests that the RF3 pattern in (c) is readily identified but the circle in (d) is hard to find.

A pairing of RF3s and RF4s (see the third row of Figure 2): The patterns of this pairing vary the number of cycles of sinusoidal modulation used to complete the pattern. Curvature around both patterns is continuous but the RF3 patterns contain three points of maximum curvature and the RF4 patterns four. As well as differing in the number of points of maximum curvature, the patterns also differ substantially in the angles subtended at the center of the patterns by adjacent points of maximum curvature. This pattern pair allows us to verify that patterns with differing frequencies of modulation are rapidly discriminated. Inspection suggests the RF3 and RF4 are readily identified in (e) and (f), respectively.

A pairing of RF3s and RRF3s: This pattern pairing examines whether the existence of curvature discontinuities (or indeed, orientation discontinuities - points of infinite curvature) is sufficient for supporting rapid identification of RRF3s among RF3 patterns (see the bottom row of Figure 2). RF3 patterns have points of maximum curvature that are coincident with curvature discontinuities, while RF3 patterns have similarly distributed points of maximum curvature that are not curvature discontinuities. Inspection suggests that both the RF3 and RRF3 patterns are hard to find in (g) and (h), respectively.

Results

For the purposes of this study, we chose to use the median as a robust measure of the central tendency of the response time distributions; this choice is defended in Appendix A. A linear function was fitted to the response time versus SS data with the intercept and gradient parameters of the line yielding SS-independent and SS-dependent measures of search performance, respectively.

A common concern with response times is their dependence on accuracy of performance. For that reason, accuracy of performance is presented first and its implications are considered later in the context of the results. Figure 3 shows the accuracy in performance of the task of reporting, whether the target was present or absent for the conditions illustrated in Figure 3.

Accuracy is close to 100% for all conditions and all SSs for target-absent conditions. For the easier tasks, such as identifying RF3 patterns among circles, the accuracy for target-present trials is also close to 100% under all circumstances. For the more difficult tasks, such as identifying circles among RF3 patterns, accuracy declines for the larger SSs. This pattern of results can be interpreted as the observers defaulting to responding “absent” after searching and failing to find a target. When the target is absent then the response is correct while when it is present but not found it is incorrect. It appears that for the harder tasks, the observers terminate their search early for larger SSs. It is, therefore, logical to assume that for the harder tasks, the response times for larger SSs might be shorter than they would have been had the observers persisted with their search. The implication of this speed/accuracy trade-off is examined by first fitting linear functions across all SSs and then to a range of SS of 1 to 8 where the accuracies are consistent with being equal.

The parameters of the fitted linear functions are, then, the gradient and the intercept of the graph relating response time to SS. The gradient shows the effect of increasing SS on the response times with shallower gradients perhaps indicating a more parallel search less influenced by an increasing number of distractor elements. The intercept represents a constant component to the response time that is independent of SS. If this SS-independent component of response time is attributed to mechanisms other than the stimulus-related decision mechanism then it should be indepen-
dent of pattern pairing. The gradients and intercepts for the reverse conditions for each of the pattern pairs are plotted against the corresponding gradients and intercepts for the forward conditions in Figure 4 below. The forward condition was defined to be easier than the reverse condition, so the data points representing the gradient data are all above the major diagonal. The data for the individual observers were fitted independently, so the error bars in Figure 4 represent the uncertainty in the means across the population of observers.

First, we consider the gradients of the fitted functions. It is clear that the search asymmetry reported by Kristjánsson and Tse (2001) is replicated. The mean of the gradients of the functions fitted to the six observers is almost four times greater in the reverse condition, identifying ellipses among K&T stimuli, than in the forward condition. A similar ratio is found for the RF3/circle pattern pairing although the gradients are shallower in both the forward and reverse conditions for that pattern pairing. Performance is symmetrical across the forward and reverse conditions for the RF3/RF4 and RF3/RRF3 pattern pairings but the gradients for the RF3/RRF3 pattern pairing are almost an order of magnitude greater than those of the RF3/RF4 pattern pairing. Table 1 presents two-tailed paired-sample t-test results demonstrating that performance in the forward and reverse conditions do not differ significantly for the RF3/RF4 and RF3/RRF3 pattern pairs but do differ for the K&T/ellipse and RF3/circle pattern pairs.

In contrast to the gradients, the intercepts were uniformly symmetrical within the pattern pairs across forward and reverse conditions. The response times at the intercepts for the RF3/RRF3 pattern pair were, however, longer for both the forward and reverse conditions compared with the other pattern pairs. This is the pattern pair that suffered the greatest degradation in accuracy for the larger SSs, so we might expect the response times for larger SSs to be shortened due to a speed/accuracy trade off and this would have the consequence of increasing the intercept at the expense of the gradient. To examine if this potential speed/accuracy trade-off might influence the pattern of results in the gradient data the response time data were fitted again using only the response times for SSs 1, 2, 4, and 8. The results of this analysis are plotted in Figure 5. Again we can see that the performance for the RF3/RF4 and RF3/RRF3 pattern pairs is approximately symmetrical across the forward and reverse conditions, while performance for the RF3/circle and K&T/ellipse pattern pairs remains highly asymmetrical, with the gradients approximately a factor of 4 larger for the reverse condition than for the forward condition. Table 2 reports the results of t-tests comparing the forward and reverse condition gradients.

The results of the t-tests are similar to those reported in Table 1, which compared gradients of functions.
fitted to response times for all SSs. Again, the gradients for the K&T/ellipse and RF3(circle) pattern pairs are asymmetrical across the forward and reverse conditions. The RF3/RF4 pattern pair gradients are, however, also shown to be asymmetrical for target-absent trials and RF3/RRF3 pattern pair gradients for target-present trials. These results are inconsistent across present and absent trials and the ratios between forward and reverse conditions are small, as can be seen in Figure 5.

While the pattern of results for the gradients is similar to those shown in Figure 4, for gradients derived from all SSs, the pattern of results for the intercepts differ. The intercepts derived from SSs 1, 2, 4, and 8 are more closely clustered, suggesting that the intercept does represent a constant component to the response time that is independent of SS.

<table>
<thead>
<tr>
<th>Pattern pair</th>
<th>Present</th>
<th>Absent</th>
</tr>
</thead>
<tbody>
<tr>
<td>K&amp;T/ellipse</td>
<td>( t(5) = 4.054, p = 0.0098^{**} )</td>
<td>( t(5) = 3.989, p = 0.0104^{*} )</td>
</tr>
<tr>
<td>RF3/circle</td>
<td>( t(5) = 4.336, p = 0.0075^{**} )</td>
<td>( t(5) = 4.013, p = 0.0102^{*} )</td>
</tr>
<tr>
<td>RF3/RF4</td>
<td>( t(5) = 1.880, p = 0.1189 )</td>
<td>( t(5) = 2.392, p = 0.0622 )</td>
</tr>
<tr>
<td>RF3/RRF3</td>
<td>( t(5) = 1.836, p = 0.1258 )</td>
<td>( t(5) = 0.252, p = 0.8114 )</td>
</tr>
</tbody>
</table>

Table 1. Results of two-tailed paired-sample t-tests comparing gradients of the linear functions fitted to response time versus SS data for the forward and reverse conditions of the four pattern pairs of Experiment 1. *\( p < 0.05 \), **\( p < 0.01 \).
Conclusions

Despite the RRF3 pattern containing curvature discontinuities that are not present in the RF3 pattern, gradients are very steep for the RF3/RRF3 pattern pair indicating that there is no cue unique to either by which they can be easily discriminated. In particular, the existence of curvature and orientation discontinuities (angles) on the RRF3 patterns does not allow them to be rapidly identified among RF3 patterns. In contrast, the very shallow gradients for the RF3/RF4 pattern pair suggest that the RF3 and RF4 both contain cues unique to each by which they are rapidly identified. Neither the RF3 nor the RF4 contain curvature discontinuities and both contain curvature maxima.

Dickinson et al. (2013) demonstrated that RF patterns with different frequencies of modulation were perfectly discriminated; that is, they could be discriminated at their thresholds for detection if they contained at least two cycles of modulation. The fact that patterns with only a single cycle of modulation were not perfectly discriminated was interpreted as evidence that the patterns were not discriminated on the basis of local properties of the modulated path. Dickinson et al. (2013), therefore, concluded that the patterns were discriminated on the basis of a shape property conferred on the patterns by the relative positions of two adjacent points of maximum curvature. Specifically it was hypothesized that the shape cue was the angle subtended at the center of the pattern by adjacent points.

Figure 5. Performance on trials for each pattern pair of Experiment 1 using only the response times for SSs of 1, 2, 4, and 8. Error bars represent the 95% confidence intervals of the means of the data across the six observers.

Table 2. Results of two-tailed paired-sample t-tests comparing gradients of the linear functions fitted to response time versus SS data for the forward and reverse conditions of the four pattern pairs of Experiment 1 using only the response times for SSs 1, 2, 4, and 8.

<table>
<thead>
<tr>
<th>Pattern pair</th>
<th>Present</th>
<th>Absent</th>
</tr>
</thead>
<tbody>
<tr>
<td>K&amp;T/ellipse</td>
<td>t(5) = 8.945, p = 0.0003***</td>
<td>t(5) = 6.492, p = 0.0013**</td>
</tr>
<tr>
<td>RF3/circle</td>
<td>t(5) = 6.152, p = 0.0017**</td>
<td>t(5) = 4.485, p = 0.0065**</td>
</tr>
<tr>
<td>RF3/RF4</td>
<td>t(5) = 1.636, p = 0.1628</td>
<td>t(5) = 3.469, p = 0.0179*</td>
</tr>
<tr>
<td>RF3/RRF3</td>
<td>t(5) = 2.697, p = 0.0429*</td>
<td>t(5) = 1.836, p = 0.1258</td>
</tr>
</tbody>
</table>

* p < 0.05, ** p < 0.01, *** p < 0.001.
points of maximum curvature. The results presented here extrapolate the significance of this result to patterns that are well above detection threshold. It would seem, therefore, that the RF3 and RRF3 patterns contain the same global shape cue and that discriminating one from the other must be done through scrutiny of the local properties of the patterns, and that this process is very slow.

The existence of the shape cue on an RF3 path also distinguishes it from a circle but then so does the existence of a point of maximum curvature. Both the K&T pattern and the ellipse exhibit points of maximum curvature but performances in the forward and reverse conditions are asymmetrical for this pattern pair. It might be argued that the RF3 patterns have fewer points of maximum curvature than the RF4 patterns, and that differing numbers of points of maximum curvature might allow the patterns to be easily discriminated. However, Dickinson et al. (2013) showed that RF patterns with differing frequency of modulation but the same number of cycles of modulation could be discriminated at their thresholds for detection, but that patterns with the same frequency of modulation but differing numbers of cycles could not. Moreover, the K&T patterns and ellipses used in the current study both have two points of maximum curvature. The K&T pattern is rapidly identified among ellipses but the ellipse is difficult to recognize among K&T patterns. That the K&T pattern can be rapidly identified among ellipses shows that the shape cue it possesses allows it to be discriminated from a pattern with the same number of points of maximum curvature, but it appears that on the ellipse, they don’t provide the shape cue that they provide on the K&T pattern and on the RF patterns. This implies an upper limit to the shape cue provided by the angle subtended by adjacent points of maximum curvature that warrants further investigation. On the basis of the findings of the current study, this upper limit must be less than 180°, the angle subtended by the two points of maximum curvature on the ellipse, but more than 156°, the angle subtended at the center of the K&T pattern.

The proposed shape cue can perhaps also account for some of the results of Kristjánsson and Tse (2001). In their Experiment 2, performance in identifying the filled half-composite and half-ellipses are small and not particularly asymmetric. By cutting the composite pattern in half, the shape cue has been changed and by cutting the ellipse in half, the shape cue has been introduced. Both, therefore, contain the postulated shape cue and that shape cue differs across the two patterns, accounting for the shallow slopes. The gradients describing performance for finding outline half-ellipses among outline half-bumps, however, are a factor of 3.5 higher than those for the converse task. One explanation for this pattern of performance is that the patterns might be amodally completed, assuming the patterns to be symmetric. A more elaborate assumption would have to be made for the S-shaped stimuli of Kristjánsson and Tse’s (2001) experiment 4 to render them amenable to a similar explanation, but in fact, the gradients reported for that experiment are quite steep. It is more likely that the paths are locally scrutinized for differences in curvature.

Perhaps the most convincing case for preferring the shape cue over the curvature discontinuity in the Kristjánsson and Tse (2001) series of experiments is the asymmetry seen in the performance in identifying the K&T stimulus among ellipses and vice versa when the curvature discontinuities and/or points of maximum curvature on the patterns are occluded by discs. An infinite number of smooth curves could be used to complete the K&T stimuli behind the occluding discs and, conversely, an infinite number of curves incorporating curvature discontinuities could be used to complete the ellipses. The stimuli would, of course, no longer be the K&T stimuli or ellipses. Completing the stimuli such that they were the K&T stimuli and genuine ellipses would require the visual system to make the assumption that the outlines should be extrapolated as elliptical curves. It is more likely that the visual system would assume a curve of least energy (Horn, 1983), which would result in points of maximum curvature behind the occluding discs. The shape cue is thereby restored. This suggestion does, of course, require testing.

**Experiment 2: A shape cue rather than a curvature discontinuity differentiates the K&T patterns from other shapes**

**Introduction**

Experiment 1 compared the results of searching for the composite stimuli of Kristjánsson and Tse (2001) among ellipses and vice versa with those obtained for pairings of patterns derived from circles, namely radial frequency (RF) patterns and rectified radial frequency (RRF) patterns. A similar asymmetry in search performance was seen for RF patterns paired with circles, but search performance was very good when searching for RF3 patterns among RF4 patterns and vice versa. In contrast, performance was poor when searching for RF3 patterns among RRF3 patterns and vice versa.

As the RRF3 patterns contained orientation discontinuities, and consequently points of infinite curvature and curvature discontinuities, while RF3...
patterns did not, it was concluded that curvature discontinuities were not responsible for rapid identification of patterns that contained them among patterns that didn’t. Furthermore, the rapid identification of RF3 patterns among RF4 patterns and vice versa, patterns with continuous curvature and very similar ranges of local curvature, suggests that a shape cue common to RF3 and RRF3 patterns but distinct across RF3 and RF4 patterns mediated performance. It was concluded that this shape cue was the angle subtended at the center of the pattern by adjacent points of maximum curvature. In drawing this conclusion it was necessary to also conclude that this angle has an upper limit that is less than the 180° subtended by the points of maximum curvature on an ellipse; otherwise, we would expect an ellipse to be readily identified among the composite patterns of Kristjánsson and Tse (2001). It might be anticipated, however, that the existence of points of maximum curvature would allow an ellipse to be rapidly identified among circles that have a constant curvature. Other pairings across the pattern types defined by modulation of a circle and manipulation of ellipses also provide informative predictions and they are examined in this experiment. The pairs considered are detailed below.

- A pairing of RF3 patterns with ellipses: This pairing has the potential to provide supporting evidence that the RF3 pattern contains a shape cue that allows for its rapid identification among patterns that do not contain the same shape cue. Both patterns contain points of maximum curvature but on the basis of the results of Experiment 1, we expect that the angle of 180° subtended by the two points of maximum curvature in the ellipse will not constitute a usable shape cue. The angle of 120° subtended by adjacent points of maximum curvature in the RF3 pattern is, however, expected to characterize its shape and allow it to be rapidly identified among ellipses.

- A pairing of ellipses with circles: Making the assumption that the ellipse is devoid of the postulated shape cue, this pairing is expected to show that the existence of points of maximum curvature on a pattern will allow it to be rapidly identified among patterns that do not contain points of maximum curvature (circles) but not vice versa.

- A pairing of the K&T pattern with a circle: This pairing is expected to show an asymmetry in performance. The K&T pattern contains points of maximum curvature, curvature discontinuities, and also the shape cue while the circle is devoid of all of these features.

- A pairing of RF3 patterns with K&T patterns: This pairing provides a strong test of whether both of these patterns contain a shape cue by which they can be discriminated, allowing each to be rapidly identified among examples of the other. The RF3 pattern has an angle subtended by adjacent points of maximum curvature, at the center of gravity of the pattern, of 120°, and the K&T pattern of 156°. The K&T pattern, of course, also contains curvature discontinuities while the RF3 pattern does not. In Experiment 1, the difficulty of identifying RRF3 patterns among RF3 patterns was taken as evidence that a curvature discontinuity does not allow the RRF3 patterns to be rapidly identified. As pointed out in the General methods section, however, the curvature discontinuities on the RRF3 patterns differ from those on the K&T patterns. The curvature discontinuities on the RRF3 patterns occur at points of infinite curvature (orientation discontinuities) so the magnitude of the discontinuity is larger for the RRF pattern but the curvature on either side of the orientation discontinuity is the same. This pattern pairing, thus allows us to rigorously test whether the curvature discontinuity on a K&T pattern is more salient than the shape cue that distinguishes an RF3 pattern.

**Results**

The results of Experiment 2 are summarized in Figure 6. The data were treated in the same manner as for Experiment 1. The intercepts for all of the linear functions fitted to the RT versus SS data were all tightly clustered suggesting that there was no appreciable speed accuracy trade-off for any of the pattern pairings. Although accuracies did decline somewhat for the difficult tasks, it was not to the same extent as for the RF3/RRF3 pattern pairing of Experiment 1. One of the observers, JP, was not available to participate in this experiment, so the data reported represent the performance of the remaining five observers.

In Figure 6, we can see that the pattern pairings of RF3s and ellipses, ellipses and circles, and K&T patterns and circles all produce highly asymmetric gradients for the linear function describing RT versus SS. An increase in the SS for the reverse condition of each of these pattern pairings has four times the influence on the SS-dependent component of the RTs as for the forward condition. The results for the RF3 and K&T pattern pairing are much more symmetrical with the gradients for the reverse condition 50% higher than the forward condition. Table 3 reports the results of t-tests comparing the forward and reverse condition gradients.
Conclusions

The results for the pattern pairings used in Experiment 2, which paired patterns across the pattern types derived from circles and ellipses, corroborate the conclusions of Experiment 1.

The asymmetry in the results for the RF3 and ellipse pattern pair indicate that there is a cue in the RF3 pattern that is not present in the ellipse and that the converse is not true. This result demonstrates that the shape cue is absent from the ellipse despite the fact that it contains points of maximum curvature that subtend an angle of 180° at its center.

The finding that the results for the ellipse and circle pattern pair are also asymmetric shows that patterns can be rapidly identified through the existence of points of maximum curvature among patterns that do not contain such points (circles).

That the K&T pattern is rapidly identified among circles is not surprising given that the K&T pattern is possessed of curvature discontinuities, points of maximum curvature, and the postulated shape cue, and the circle is devoid of all three properties. The two previous pattern pairings, however, point to the shape cue being most salient. The absence of the shape cue in the circle makes it hard to identify among K&T patterns.

Table 3. Results of two-tailed paired-sample t-tests comparing gradients of the linear functions fitted to response time versus SS data for the forward and reverse conditions of the four pattern pairs of Experiment 2. *p < 0.05, **p < 0.01, ***p < 0.001.
The pairing of the RF3 pattern with the K&T pattern shows that it is unlikely that it is the curvature discontinuity that facilitates rapid identification, as the gradient of the function describing RT versus SS is shallower when identifying RF3 patterns among K&T patterns than vice versa. The asymmetry for this pattern pairing is less pronounced than the other three pattern pairings with the gradient only 50% higher in the reverse condition than the forward condition, rather than the factor of four shown for the other conditions. Perhaps the 156° shape cue of the K&T pattern is somewhat less salient than the 120° shape cue of the RF3 pattern. This might be because the 156° shape cue is intrinsically weaker or because the 120° shape cue is represented three times on each RF3 pattern.

**General discussion**

The results of the two visual search task experiments reported here are consistent with the argument that the patterns examined are predominantly identified on the basis of a shape cue rather than the existence of a curvature discontinuity. This shape cue, the angle subtended at the center of the pattern by adjacent points of maximum curvature on its boundary, was first postulated on the basis of modulation thresholds for detection and discrimination of RF patterns (Dickinson et al., 2013). Patterns with different frequencies of modulation were discriminable at their thresholds for detection if two or more cycles of modulation were present, but patterns with sinusoidal and rectified sinusoidal modulation could not be discriminated at threshold if they had the same period of modulation. This implies that the shape representation is indifferent to the local curvature and orientation on the pattern. Assuming some smoothing of the boundary to mask the orientation discontinuities in the rectified RF patterns, it might be argued that local differences in curvature and orientation across the RF and rectified RF patterns were small for the almost circular patterns at threshold but the same claim could not be made for the patterns used in the visual search task here.

A hierarchy of processing in the ventral pathway of the primate visual system is dedicated to the perception of objects (Felleman & Van Essen, 1991; Ungerleider & Mishkin, 1982). Neurophysiological studies examining the activity of single neurons have revealed that cortical area V4 encodes the shape of a boundary in the population response of neurons sensitive to curvature features on the boundary (Pasupathy & Connor, 1999, 2001, 2002). It is proposed that the population response is encoded in a curvature versus polar angle space, although single neurons are often sensitive to the spatial relationships of two adjacent curvature features when referenced to the center of the pattern, an implicit representation of the proposed shape cue. A recent study has proposed a sparse coding model for V4 on the basis of a neuronal preference for high rates of curvature, which emphasizes identifying points of maximum curvature (Carlson, Rasquinha, Zhang, & Connor, 2011) above establishing the absolute magnitude of curvature. A recent study using RF3 and RF4 patterns and circles in a visual search task showed that the existence of the proposed shape cue was responsible for an enhanced N220 component in the EEG waveform and source localization of this component placed it in or around cortical area V4 (Bowden et al., 2015). Thus, it would seem that cortical area V4 might be the area responsible for encoding the identity of these simple shapes. The current study demonstrates the indifference of the shape identification mechanism to the local curvature properties of the patterns, which implies that the analysis of shape in V4 might indeed be more concerned with the positions of points of maximum curvature rather than absolute measures of curvature, a solution foreshadowed by studies inspired by information theory (Attneave, 1954). A population of neurons sensitive to the shape cue has the capacity to represent boundary conformation of simple shapes for which a center can be derived, but complex multiaxial shapes present a more demanding challenge. It has been suggested that complex patterns might be decomposed into parts, perhaps at matched concavities in their boundaries (Biederman, 1987; Hoffman & Richards, 1984), points that are rich in information (Feldman & Singh, 2005). Such complex patterns might then be recognized on the basis of the configuration of those parts. Kimia (2003) argues that the parts might be partially separable on the basis of homogeneity in surface properties (such as luminance, texture, or color) by local edge-detection mechanisms. Recent psychophysical studies have provided evidence for the integration of shape information around RF patterns, regardless of whether the patterns are circumscribed by boundaries defined by luminance increment, luminance contrast, or texture (Tan et al., 2013; Tan et al., 2015; Tan, Dickinson, & Badcock, 2016), but that contours are often ambiguous, noisy, and interrupted. Moreover, the contour properties are not necessarily an ideal representation by which the gross structure of an object might be encoded.

One representation of shape that can represent the spatial arrangement of the boundaries and that has the potential to capture the shape of an object, despite partial occlusion of the boundaries, is the medial axis (MA) representation. The MA is the locus of the centers of circles that have two or more radii tangent to the boundary of a pattern (Blum, 1973). In computer models of object recognition the shape of an object is
often represented as an abstraction of the physical MA representation of its boundaries and compared with exemplars in memory. Representation of the object shape as a hierarchical graph has the potential to render the shape representation invariant under deformations, such as bending and stretching, which allows for a general representation of the morphology of nonrigid objects.

The MAs of the shapes used in this study are very simple. For the RF patterns the MAs radiate from the center, terminating at the points of maximum curvature. The MA representations of RF3 and RF4 patterns differ, but those of the RF3 and RRF3 are identical. The angles formed by the MAs of the patterns at their centers are those of the postulated shape cue, but the RF3 and RF4 patterns are also differentiated by an abstract representation of their MA graphs, as one has a vertex with three radiations and the other has four radiations. The results of Dickinson et al. (2013), showing that RF patterns with differing frequencies of modulation but only two cycles of modulation could be discriminated at their thresholds for detection, but patterns of the same frequency with differing numbers of cycles of modulation could not, argues against an abstracted graphical representation for these simple patterns. Similarly, the finding that the shape cue is absent in the ellipse also suggests that the cue represents extension in a plane rather than simply a line.

The patterns used in this study, however, have no concavities and it is possible that they represent elemental components of shape perception. The deformations of shape in complex objects are most likely to occur as articulations between such elements. Hung, Carlson, and Connor (2012) used an adaptive shape-sampling method to examine the selectivity of neurons in macaque infero-temporal (IT) cortex to the surface properties and MAs of multiaxial complex three-dimensional shapes. Most neurons were shown to be sensitive to both the surface properties and MA representations. Stimuli that elicited high response from single neurons, however, varied widely in shape, so it was concluded that novel objects were represented by a population of IT neurons encoding substructures of the whole, defined by both MA and surface properties. The responses of single neurons were tolerant of rotation of the stimuli around all of the Cartesian axes, which was interpreted as evidence that the response was dependent on the three-dimensional interpretation of the shape of the stimuli. The most frequently encountered substructure templates, however, had only two axes, perhaps allowing for recognition of various morphologies of a particular multiaxial shape by the same population of neurons. It might be argued that this approximates recognition of a multiaxial pattern by its abstract graphical representation.

The results of Hung et al. (2012) suggest that neurons of the IT cortex represent substructures of multiaxial shapes and imply that complex objects might be recognized on the basis of a neuronal population response. The results of this study argue that a shape cue is generated in V4 that is based on the positions of points of maximum curvature relative to the center of simple shapes that might represent the parts of more complex patterns. Such patterns might be recognized on the basis of the shapes of their parts and the morphologies of their combination.

Keywords: visual search, search asymmetry, shape, local, global

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Appendix A

The interpretation of response time distributions is contentious. It has been argued that, rather than attempting to characterize a response time by a measure of its central tendency, the shape of the distribution should always be analyzed (Heathcote, Popiel, & Mewhort, 1991). The ex-Gaussian function, a convolution of exponential and Gaussian functions, has gained some ascendancy in providing a good representation of positively skewed response time distributions (Kristjánsson & Jóhannesson, 2014; Palmer, et al., 2011). Hohle (1965) proposed that the response time could be broken down into an exponential component pertaining to the decision-making process and a Gaussian component due to other residual processes; however, there is no consensus that this is proven for any response time distributions.

In this study we are comparing distributions that result from very different search behaviors. Some of the search tasks are easy, resulting in distributions that are positively skewed and only weakly dependent upon SS, but the others are difficult and have distributions that are characterized by a serial scrutiny of each pattern of the array and, therefore, strongly influenced by SS.

Figure A1 displays response time distributions for the task of identifying RF3 patterns among RF4 patterns in target-present and target-absent trials. The times shown are for the correct responses of a single observer, ED, who performed 10,080 trials of this condition in total. The distributions for target-present trials are shown in red in the column of histograms on the left and target-absent trials are shown in blue on the right.

The response time distributions are positively skewed and become wider as the SS increases. They are all offset from a response time of zero by an amount that is similar across all SSs. It is hard to visualize the breakdown of response time distributions into exponential and Gaussian components of a convolution, so we chose to examine the response time distributions through their cumulative distribution functions. Figure A2 shows the cumulative response time distributions for the probability distributions shown in Figure A1.

Figure A1. Response time distributions for the task of identifying RF3 patterns among RF4 patterns for target-present and target-absent trials. From top to bottom, the response time distributions are for SSs of 1, 2, 4, 8, 16, and 32. Response time distributions for target-present trials are shown in red in the column of histograms on the left and target-absent trials are shown in blue in the column on the right.
representing the probability of response as a function of time. The distributions are fitted with the product of a cumulative log-Gaussian function (solid black lines) and a cumulative Gaussian function (dotted black lines). The log-Gaussian function was used in place of the exponential because the area under its curve is finite.

The products of the log-Gaussian and Gaussian-cumulative distribution functions (dashed black lines) are a very good fit to the response-time data (red and blue lines for target-present and target-absent data, respectively) and the individual log-Gaussian and Gaussian-cumulative distribution functions reveal some interesting trends in the data. The minimum response times are constrained by the log-Gaussian cumulative distribution functions (solid black lines) but for larger SSs, the Gaussian cumulative distribution

Figure A2. Cumulative distribution functions for the probability distributions shown in Figure A1. The cumulative distribution functions are shown in red for the target-present trials and in blue for target-absent trials. The fits of the product of the cumulative log-Gaussian function and the cumulative Gaussian function to these data are shown as dashed black lines. The solid black lines represent the cumulative log-Gaussian functions of the fits and the dotted black lines show the cumulative Gaussian functions.
functions (dotted black lines) approximate the data well. These observations are consistent with the Gaussian component of the response time distributions being due to the decision making process and the log-Gaussian component due to residual processes. Perhaps the cumulative distribution function of the response times can be viewed as the joint probability of a decision having been made and the capacity to respond both being satisfied.

The distributions shown in Figure A1 can be compared with those shown in Figures A3 and A4, which represent the response times for the task of identifying RF3 patterns among RRF3 patterns. Again these distributions represent 10,080 trials performed by the single observer, ED. Correct responses for target-present trials are collated in the distributions shown in Figure A3 and target-absent trials are shown in Figure A4.

Figure A3. Response time distributions for correct response to target-present trials for the task of identifying RF3 patterns among RRF3 patterns. From top to bottom, the histograms are the distributions for SSs of 1, 2, 4, 8, 16, and 32.
The response time distributions shown in Figures A3 and A4 have some interesting features that can be interpreted on the basis of the subjective experience of performing the task. For the target-present trials the distributions broaden as SS increases but the shortest response times are similar across SSs. The subjective impression of the search process is that each pattern is examined in turn and the broadening of this distribution is consistent with this observation. The target may be identified at any point in the search with the search being terminated when it is identified. If it is seen as the first item the processing time will be the same regardless of SS. Further, the distribution for a SS of two is bimodal implying that the observer recognizes the target as being present having scrutinized one or both of the patterns. In contrast, the response-time distributions for the target-absent trials are approximately Gaussian in form with the mean and standard deviation increasing with SS. This is consistent with the observer searching serially through the whole set of patterns.
patterns. Wolfe et al. (2010) modeled response-time distributions for a serial, self-terminating search with memory for rejected distractors (Horowitz, 2006) and arrived at predicted distributions that were remarkably similar to those shown here. In the model, attention was deployed to each item every 98 ms on average. For the search task described here, the observer fixated each item in turn until the target was found or until the set of distractors was exhausted. Although the question of the deployment of attention is contentious (Awh, Belopolsky, & Theeuwes, 2012; Kristjánsson, 2015) the distinction between the task for which performance is relatively independent of SS and that for which it is highly dependent here is that, for the former, the task is performed without eye movement and for the latter, fixation of each pattern is essential.

It is obvious that no single function can be used to model all of the response time distributions. In this study, our objective is to show that a shape cue that exists in a target pattern allows it to be rapidly identified among distractor patterns that do not contain the same shape cue. If the shape cue is not particular to the target, then the patterns must be scrutinized. The increase in response times with SS is very obvious for the task of identifying RF3 patterns among RRF3 patterns, but the distributions differ markedly. For target-present trials (Figure A3) the distribution for a SS of 1 is narrow and perhaps slightly skewed. For a SS of 2, it is bimodal, and for larger SSs, it becomes somewhat rectangular. The distributions for target-absent trials (Figure A4) are all approximately Gaussian. Despite this variety of distributions, it is clear that the tasks that require scrutiny of the patterns will be highly sensitive to SS and those that occur in parallel will not. In order to differentiate between these situations all we require is a robust measure of central tendency. For the tasks that require scrutiny of the patterns the median would seem to be appropriate for the skewed, bimodal, and rectangular distributions of the target-present data and the approximately Gaussian distributions of the target-absent data. For the tasks that are easy and result in a parallel search, the median value, halfway up the cumulative distribution function, appears to often be halfway up the cumulative Gaussian function that might represent the response time for the decision. For these reasons, medians have been preferred as the measure of central tendency for the response-time distributions.