

# Information integration in a globally coupled chaotic system

Hiroki Mori<sup>1</sup> and Masafumi Oizumi<sup>2</sup> (Equal contribution)

<sup>1</sup>Waseda University, Japan, <sup>2</sup>Araya Inc., Japan  
mori@idr.ias.sci.waseda.ac.jp, oizumi@araya.org

## Introduction

Consciousness emerges from intricate interactions between neurons in the brain. Integrated Information Theory of consciousness (IIT) hypothesizes that the amount of information integrated by the interactions within a system, called “integrated information ( $\Phi$ )”, corresponds to the level of consciousness (Tononi, 2004). Integrated information conceptually quantifies the amount of information that a system generates as a whole, above and beyond the amount of information that its parts independently generate (Tononi, 2004). To verify the hypothesis of IIT, it is important to gain theoretical insight about in which state of a dynamical system integrated information gets high or otherwise. In particular, it is intriguing to elucidate how chaos would affect the amount of integrated information. In this study, we investigate how integrated information behaves in a chaotic system by using Globally Coupled Map (GCM) (Kaneko, 1989).

GCM is one of the simplest models of a chaotic system but exhibits various activity patterns depending on the coupling strengths between the units and the bifurcation parameters of each unit. Due to the rich dynamical structure, GCM is considered to be a useful model for understanding biological information processing. The activity patterns are broadly classified into ordered phases and chaotic phases. In the ordered phases, the activities form small a number of clusters while in the chaotic phases, the activities do not form such clusters. We investigate how integrated information changes depending on the different phases. We show that integrated information takes higher values at the boundary areas between chaotic phases and ordered phases where the activities are chaotic and the coupling strength is medium. The results imply that chaos or the edge of chaos is beneficial for the system to generate high amount of information integration and potentially to generate high level of consciousness.

## Methods

### Globally Coupled Map

GCM is a general network model for investigating basic phenomena of nonlinear multi-body dynamics. The model consists of multiple logistic maps described as the following dis-

crete time equation. The units in GCM are globally coupled via the mean field activity;

$$x_{i,t+1} = (1 - \epsilon)f(x_{i,t}) + \frac{\epsilon}{N} \sum_{j=1}^N f(x_{j,t}), \quad (1)$$

$$f(x) = 1 - \alpha x^2, \quad (2)$$

where  $x_i$  is state of  $i$ -th unit  $t$  is time step,  $\alpha$  is a bifurcation parameter, and  $\epsilon$  is a parameter of coupling strength. The each unit is regarded as a logistic map in the case  $\epsilon = 0$ . The logistic map alone exhibits chaotic activities when  $\alpha$  is between 1.4 and 2.0. The second term of Equation (1) is the mean field value. The system is deterministic and thus, the entire activities of GCM are uniquely determined only by the initial value.

### Simulation procedures

We construct the GCM model consisting of 5 units. We conduct numerical experiments varying the two parameters  $\epsilon$  and  $\alpha$ . The parameter  $\epsilon$  is changed from 0.0 to 1.0 by 0.05 and the parameter  $\alpha$  is changed from 0.0 to 2.0 by 0.1. The simulations of the GCM are performed with  $10^7$  iterations for each parameter value in order to obtain an accurate estimate of integrated information.

For computing entropy and integrated information, we discretized the state of the activities  $x_{i,t+1}$ , which ranges from -1 to 1, into 5 uniform bins. We estimate the joint probability distributions of the activity states in GCM by making a histogram of the observations of each state.

### Integrated information

We use a practical measure of integrated information,  $\Phi^*$ , proposed in (Oizumi et al., 2016).  $\Phi^*$  is defined as the difference between the mutual information in the whole system and that of its parts,

$$\Phi^* = I(X_t; X_{t+1}) - I^*(X_t; X_{t+1}). \quad (3)$$

The first term  $I(X_t; X_{t+1})$  is the mutual information between the past state  $X^t$  and the present state  $X^{t+1}$ .  $I(X_t; X_{t+1})$  quantifies to what extent the past state can be

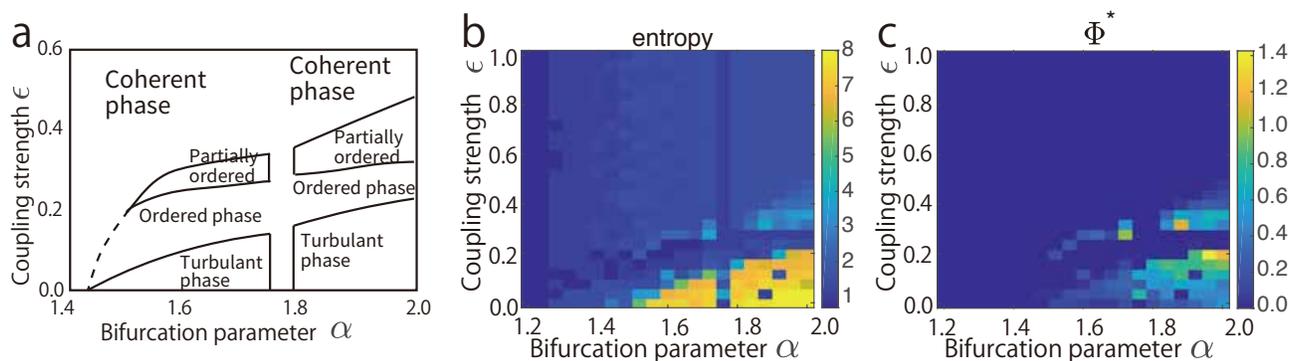


Figure 1: (a) Rough phase diagram of the GCM. (b) Entropy and (c) integrated information in the GCM as a function of the coupling strength  $\epsilon$  and the bifurcation parameter  $\alpha$ .

predicted by knowing the present state and thus, it is called “predictive information” (Bialek et al., 2001). The second term  $I^*$  quantifies the predictability of the past state under the condition where interactions between the parts are unknown.

If the parts are independent, predictability  $I$  and  $I^*$  is the same because there is no information loss ignoring interactions. However, if there are strong interactions between parts, the difference between  $I$  and  $I^*$  would get large. In this way,  $\Phi^*$  reflects how much information the system as a whole generates above and beyond its parts.

## Results

GCM is classified into four distinct phases based on activity patterns as shown in Figure 1 (a). The four phases are characterized as follows: (1) Turbulent phase: Attractors with large number of clusters, (2) Ordered phase: Attractors with small number of clusters, (3) Partially ordered phase: Attractors with large number of clusters and attractors with small number of clusters coexist depending on initial conditions. (4) Coherent phase: Almost all attractors are coherent, i.e., the activities are all synchronized. The activity of each unit is completely independent when  $\epsilon = 0$  (turbulent phase) whereas it is completely synchronized when  $\epsilon = 1.0$  (coherent phase).

Figure 1(b)(c) show the behaviors of entropy and integrated information  $\Phi^*$  as a function of the coupling strength  $\epsilon$  and the bifurcation parameter  $\alpha$ . We can see that the behaviors of entropy and integrated information are well characterized based on the phase diagram shown in Figure 1(a). The behaviors of entropy are summarized as follows: (1) In the turbulent phase, entropy is high because the activities are close to independent. (2) In the ordered phase, entropy is low because the activities are clustered. (3) In the partially ordered phase, entropy is medium. (4) In the coherent phase, entropy is low because the activities are all synchronized.

The behavior of integrated information is qualitatively different from that of entropy. Integrated information is upper bounded by entropy and thus, entropy needs to be high for integrated information being high. However, high val-

ues of entropy do not necessarily mean high values of integrated information. The behavior of integrated information is summarized as follows: (1) In the turbulent phase, if  $\epsilon$  is close to 0 and the activities are close to independent, integrated information is low. As  $\epsilon$  increases, integrated information increases and at the boundary between turbulent and ordered phase, integrated information is maximized. (2) In the ordered phase, integrated information is low because entropy is low. (3) In the partially ordered phase, integrated information is maximized when  $\epsilon$  is close to the boundary between the partially ordered phase and ordered phase. As  $\epsilon$  increases further, integrated information decreases. (4) In the coherent phase, integrated information gets to 0 because the activities are all synchronized, which is a mathematical property of  $\Phi^*$  (Oizumi et al., 2016).

## Conclusions

We investigated the behavior of integrated information in Globally Coupled Map (GCM). We found that in the boundary areas between the turbulent and the ordered phase or the ordered and the partially ordered phase, integrated information  $\Phi^*$  takes higher values compared with other areas. The activities in these areas are chaotic and moderately correlated due to the medium strength of connectivity among the units. This result implies that a non-linear chaotic system with moderate strength of connectivity may be beneficial for generating consciousness from the viewpoint of IIT.

## References

- Bialek, W., Nemenman, I., and Tishby, N. (2001). Predictability, complexity, and learning. *Neural computation*, 13(11):2409–2463.
- Kaneko, K. (1989). Chaotic but regular posi-nega switch among coded attractors by cluster size variation. *Physical Review Letters*, 63(3):219–223.
- Oizumi, M., Amari, S., Yanagawa, T., Fujii, N., and Tsuchiya, N. (2016). Measuring integrated information from the decoding perspective. *PLOS Computational Biology*, 12(1):e1004654.
- Tononi, G. (2004). An information integration theory of consciousness. *BMC Neurosci.*, 5:42.