Transfer of Information in Collective Decisions by Artificial Agents

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Abstract

Collective decision-making systems rely on many agents to gather, process and exchange information to arrive at a group decision. Critical to group success is the transfer of information among agents and between agents and their environment. Without information transfer, no consensus can be achieved. Yet, the role of individual rules in determining information transfer at the group level is poorly understood. With the aim to shed a light on how the decision mechanism of individuals affects information transfer in collectives, we analyze the information landscape of two decision-making strategies: one based on the majority rule and one based on the voter model. For each strategy, we consider a binary site-selection scenario and use transfer entropy to measure the flow of information in a spatial, multi-agent system. We find that information transferred among agents is dependent on the decision mechanism, increases with the time necessary to make a collective decision, and is loosely modulated by the uncertainty of the final outcome. This is the first study that compares collective decision making mechanisms through the lens of information dynamics. Although this approach is limited to simulated agents, similar approaches could in principle be used to study collective decisions in biological systems.

Introduction

It is clear that individual agents, with access to only limited information, can coordinate their behavior to make well-informed collective decisions. Examples from biology include collective decisions made by bees and ants (Franks et al., 2002), the coordinated motion of birds and fishes (Ballerini et al., 2008; Couzin et al., 2005), and the motion of individual cells during embryogenesis or regeneration (Pezzulo and Levin, 2015). In these and many other natural and artificial examples, intelligent decisions emerge in cases where individuals obey very simple rules because these simple rules are generally sufficient for agents to interact and exchange the information necessary to make an informed decision. However, the time and accuracy of group decision-making depend on the particular decision mechanism(s) of individual agents – meaning that not all rules perform equally when it comes to a particular problem to be solved at the collective level. Thus, even when performing the same collective computation, different groups may differ in the structure of information transfer among individuals. These differences and, in particular, how different flows of information among individuals can lead to the same emergent behavior are not well understood.

Information theory provides a quantitative framework for assessing how information is exchanged and processed in collectives and has been adopted before in the study of both natural (e.g., to study foraging behavior of ant colonies (Reznikova and Ryabako, 1994; Zenil et al., 2015; Meyer, 2017)) and artificial systems (e.g., to design multi-robot systems (Sperati et al., 2008, 2011)). Although intuition might suggest that information is transfered when two entities interact, quantification of the actual number of bits transferred often reveals that the predictive value of transferred information (i.e., predictive transfer (Lizier and Prokopenko, 2010)) is not a one-to-one map with the information of the communicated symbols. Consider the example of one person telling another that it will likely rain today and the other person should bring an umbrella. In areas of the world where rain is an uncommon event, this message has much predictive power as it will lead to the receiver taking an action that it would have otherwise been unlikely to take. However, in areas of the world where daily rain might be expected, the message will have little predictive power over the action of the receiver because the receiver would have likely been carrying an umbrella anyway.

To understand how behavioral interactions contribute to valuable information, it is important to be able to differentiate between "small talk" and meaningful communication. In other words, to understand how collectives make a decision requires therefore to detail how information flows among agents *as a result of executing specific mechanisms in a given context* rather then simply detailing how individuals implement these mechanisms. As we will show, even in cases where the same collective decision is made, differences in the decision mechanisms of individuals can lead to differences in the total amount of information transferred within a group.

To quantify information transfer, we use Schrieber's widely adopted measure of transfer entropy (Schreiber,

2000) – a directed measure of information exchanged between parts of a dynamical system. Transfer entropy has been used before to detect leader–follower relationships in the collective motion of zebrafish (Mwaffo et al., 2017; Butail et al., 2016), to study collectives of artificial agents (Walker et al., 2013), neural (Boedecker et al., 2012; Lizier et al., 2009) and Boolean networks (Kim et al., 2015), and to study informational patterns in computational neuroscience (Honey et al., 2007; Lizier et al., 2011). Here, we rely on transfer entropy to determine patterns in information exchange among agents implementing two self-organized collective decision-making strategies (Valentini et al., 2016b) and to determine how information transferred may depend on local-level rules.

The two decision-making strategies we implement couple a simple decision mechanism, the *majority rule* or the *voter model*, with direct modulation of signaling time to solve the best-of-n problem (Valentini et al., 2016b). In a best-of-n problem, agents must make a consensus decision for the best option among n available ones. A large number of alternative strategies have been designed to address this problem (see Valentini et al. (2017) for an extensive review) and could have been chosen for this study. Our choice of focusing on the majority rule and the voter model is motivated by the simplicity of their implemented interaction mechanisms and by the fact that their dynamics have been thoroughly investigated in previous works. These strategies have been studied by means of population models using both ordinary differential equations and chemical reaction networks (Valentini et al., 2016b), formal methods using symbolic model checking (Kouvaros and Lomuscio, 2016), as well as multi-agent simulations and experiments with real robot swarms. As a result of their simplicity, both of these strategies generalize to different problem scenarios (Valentini et al., 2016a) and have been recently extended with a number of additional mechanisms (Crosscombe et al., 2017; Strobel et al., 2018; Ebert et al., 2018). However, despite their wide reach in the collective decision-making literature, we know of no detailed empirical study of their information dynamics, and thus we use the comparison of these two strategies as a model system for inferring information transfer in collective decisions by artificial agents.

We study the information landscape of the majority rule and of the voter model by means of spatial, multi-agent simulations. We focus on the flow of information generated when agents in the collective apply a decision mechanism. To this end, we use transfer entropy to quantify the information originating from the neighbors of a focal agent applying a decision mechanism and flowing toward that same focal agent. We study transfer entropy in the majority rule and in the voter model for two problem configurations by varying the difficulty of the problem. Finally, we project the results about transfer entropy in the speed–accuracy space to analyze information transfer as a function of the performance of each strategy.

We find that the amount of information transferred differs across decision mechanisms, with the majority rule transferring more information than the voter model. Moreover, information transfer is positively correlated with the difficulty of the decision-making problem as well as with the time necessary for the collective to make a consensus decision. Information transfer is not correlated with decision accuracy (i.e., probability of choosing the best option) but is loosely modulated by the uncertainty of the collective decision – a closely related measure.

Methods

In this section, we provide details about our experimental approach. We first define the decision-making problem and the two strategies (i.e., majority rule and voter model) considered in our study. We proceed with a description of the spatial multi-agent simulations used to generate our raw data. Finally, we review the formulation of transfer entropy and then detail our approach to interpret raw data originating from the simulations and required to compute transfer entropy.

Decision-Making Problem and Strategies

As a decision-making problem, we consider a binary site-selection scenario where a collective of N agents is required to choose the best of two sites. This decision-making problem is a particular instance of the best-of-n problem (i.e., $n = 2$) and a popular benchmark to test collective decision-making strategies (Valentini et al., 2017). In our study, we make use of a site-selection scenario characterized by two options, site A and site B, with equal (or symmetric) costs, and differing qualities, $\rho_A = 1.0$ and $\rho_B \leqslant 1.0$.

This problem configuration has been used extensively to study a pair of collective decision-making strategies, one based on the majority rule (MR) and one based on the voter model (VM) (Valentini et al., 2016b). In both strategies, each agent of the collective repeatedly iterates through the same sequence of actions: explore a site i and sample its quality ρ_i; advertise (or disseminate) a preference for site i for a time that is proportional to ρ_i; and then, after this time, apply a decision mechanism (either the majority rule or the voter model) to reconsider its preference i and possibly change it in favor of a different site $j \neq i$. When using the majority rule as a decision mechanism, agents in the collective consider all preferences of their nearest neighbors and adopt the option favored by the majority of them. When using the voter model instead, agents blindly adopt the preference of another agent that is randomly chosen from those of their nearest neighbors. The duration of both the exploration and dissemination phases is stochastic and drawn at each agent state transition from an exponential distribution, cf. (Valentini et al., 2016b), with a mean duration, respectively, of 50 seconds for the exploration phase and of

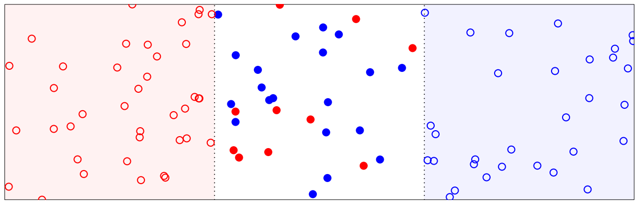


Figure 1: Illustration of the simulation environment partitioned into site A (red area), nest (white area), and site B (blue area). Symbols: filled circles represent agents in the dissemination state, empty circles represents agents in the exploration state, colors represent agent's opinion (red for site A, blue for site B).

$200 * \rho_i$ seconds for the dissemination phase.

Multi-Agent Simulations

To study the flow of information generated by the decision-making strategies described above, we used a simple, continuous-space multi-agent simulator[1] implemented in C (Valentini et al., 2014). A collective of $N = 100$ agents is confined to a closed, rectangular environment with a height of 100 units and a width of 300 units (see Figure 1). The environment itself is partitioned into three adjacent and communicating regions (100 units by 100 units). Sites A and B are located on left and right sides of the environment, respectively, and the region in the middle represents the "nest" – a central location that initially hosts the collective of agents and acts as a space for agent-to-agent communication.

The agents themselves are represented by self-propelled, massless particles that can collide with the boundaries of the environment but do not collide with each other. Agents move at a constant velocity of 20 units per second[2] and their trajectory is determined by a random walk, when moving within a certain region, and by a straight line, when moving from one region to another. Upon collision with one of the boundaries of the environment, an agent will bounce back mirroring the angle of incidence in the collision. Agents' positions are updated with a frequency of 10 Hz. Agents are capable of navigating the environment and, when necessary, moving between different regions. For example, before entering the dissemination phase, an agent moves in a straight line from one of the two sites to the nest and, once there, transits to the dissemination state to broadcast its preference to other nearby neighbors in the nest[3]. Only agents in the dissemination phase, i.e., those broadcasting their preference, can perceive each other's preference for a site and use this information during the application of the decision mechanism. For the purpose of this study, the two decision-making strategies both use a communication neighborhood represented by the agent's five nearest neighbors (see the last part of this section for a motivation of our choice) without imposing a constant communication range. Locality of interactions is obtained instead from the spatial density of agents. Simulations are terminated when a consensus decision is made by the collective.

Transfer Entropy

Central to our analysis of information transfer underlying collective decisions is the notion of *transfer entropy* introduced by Schreiber (2000). Transfer entropy, $T_{Y \to X}$, is an information-theoretic measure that quantifies the direct exchange of information from a process Y toward a process X. In its original formulation, transfer entropy is defined on time series of discrete values, but it can be extended for use with continuous-valued time series as well. We denote with x_i and y_i the discrete values of time series X and Y at time step i. Transfer entropy $T_{Y \to X}$ from process Y toward process X is given by

$$\sum_{x_{i+1}, x_i^{(k)}, y_i} p(x_{i+1}, x_i^{(k)}, y_i) \log_2 \frac{p(x_{i+1}, y_i | x_i^{(k)})}{p(x_{i+1} | x_i^{(k)}) p(y_i | x_i^{(k)})},$$

where $p(\cdot)$ gives the probability of a certain event $\cdot$ and

$$x_i^{(k)} = \{x_{i-k+1}, x_{i-k+2}, \dots, x_i\}$$

is the k-history of X at time step i. This formulation of transfer entropy is also known as *apparent* transfer entropy. A variation in which all probabilities are conditioned also on the current state of all other processes $W = \{W_1, W_2, \dots\}$ in the system is known instead as *complete* transfer entropy (Lizier et al., 2008).

In our analysis, we aim to measure the flow of information originating from the neighbors of a decision-making agent and destined to that same agent. We restrict our current analysis to the study of transfer entropy with history length $k = 1$. Therefore, the previous equation reduces to:

$$\sum_{x_{i+1}, x_i, y_i} p(x_{i+1}, x_i, y_i) \log_2 \frac{p(x_{i+1}, y_i | x_i)}{p(x_{i+1} | x_i) p(y_i | x_i)}. \quad (1)$$

Although the behavior of agents in a collective executing either of the two decision-making strategies is well defined by a memoryless, finite-state machine (Valentini et al., 2016b), longer scale interactions might still be possible due to the repeated and mutual interactions happening over time between the same agents of the collective. Our choice of $k = 1$ is motivated instead by simplicity as we aim to focus our efforts on the study of other experimental variables. We will investigate the effect of longer history lengths in future work.

[1] A video-recording of a simulation based on the voter model generated with the same simulator but different environment size can be found at `https://youtu.be/es9XXGr9Tpk`.

[2] Simulation parameters have been previously fine tuned to approximate a well-mixed interaction of agents, see (Valentini et al., 2014).

[3] A similar but specular process is implemented before an agent enters the exploration phase which happens after the application of a decision mechanism.

Measuring Information Flow from Raw Data

To compute transfer entropy and analyze the information flow during the decision-making process, we need to define two time series – X and Y – from the raw data generated by the simulations that capture the transfer of information among agents. When applying a decision-making mechanism, a focal agent may change its current preference as a result of processing information about the preferences of its neighbors. We aim to study this information-processing phase and consider therefore the preferences for a site of the focal agent and those of its neighbors.

Time series X represents the instantaneous preferences of the focal agent. Each entry $x_i \in \{0, 1\}$ gives the site preference of the focal agent (0 for site A, 1 for site B) prior to its i-th application of the decision mechanism. The next entry x_{i+1} is thus the site preference after the application of the decision mechanism and may record a change in preference of the focal agent. Time series Y represents the number of agents preferring site A in the neighborhood of the focal agent at the time of the application of the decision mechanism. Thus, the value of y_i is bounded by the neighborhood size set in the simulations (i.e., five nearest neighbors), and so $y_i \in \{0, 1, \ldots, 5\}$. The support of time series X and Y is therefore independent from the particular decision mechanism used by the agents of the collective. Specifically, the constant neighborhood size set in the simulations lets us compare the majority rule and the voter model without possible artifacts resulting from the computation of measures with time series defined over different state spaces.

For each problem configuration considered in our analysis, we performed 100 independent simulations with a collective of $N = 100$ agents randomly positioned in the nest (see next section for more details). This approach potentially leads to a set of 10^4 pairs $(X^{(i)}, Y^{(i)})$ of time series. We say potentially, however, because the length of each time series varies and is determined by a stochastic process as the number of applications of the decision mechanism by each focal agent is itself a random variable dependent on the problem configuration (see beginning of this section). This number varies anywhere from no applications to a few hundred applications. We disregard all pairs of time series whose length is less than 2 time steps as they do not yield data useful to compute transfer entropy. Rather than computing $T_{Y \to X}$ for each agent in the collective and then averaging the results, we use all remaining pairs $(X^{(i)}, Y^{(i)})$ originating from the agents within the same simulation to estimate the probabilities used in Eq. 1 and then compute transfer entropy. We repeat this process for each of the 100 simulations and then aggregate the results. All computations are performed in R using the *rinform*[4,5] package based on the *Inform*[6] C library (Moore et al., 2018).

[4] https://CRAN.R-project.org/package=rinform.

[5] https://github.com/ELIFE-ASU/rinform.

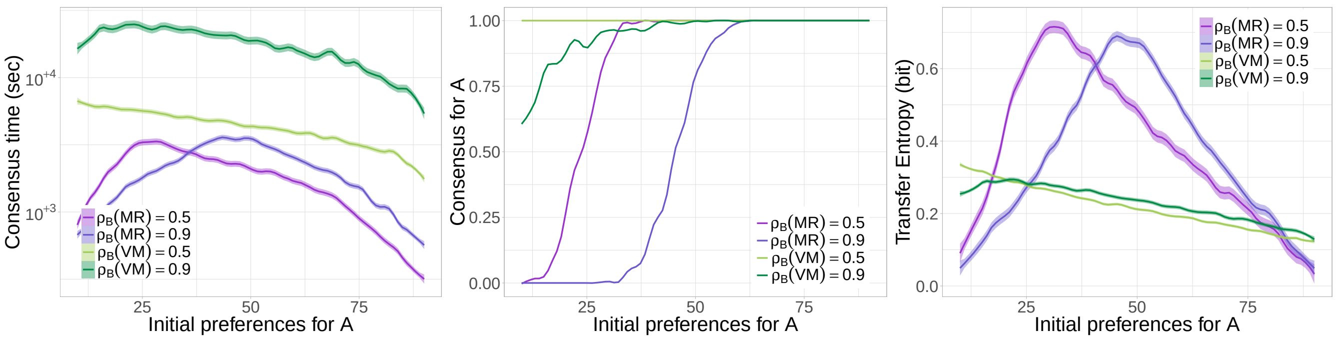
[6] https://github.com/ELIFE-ASU/inform

Experimental Results and Discussion

We consider two decision-making problems of different difficulty by varying the quality ρ_B of the worst site in the set $\{0.5, 0.9\}$. For each problem and for each decision mechanism, we study the system while varying the initial number of agents in the collective favoring the best site, i.e., site A, from a minimum of $0.1N$ to a maximum of $0.9N$, including regular intervals in between ($N = 100$). As introduced in the previous section, we performed 100 simulations for each of the above problem configurations. We let simulations run until the collective of agents reaches a unanimous consensus decision for one of the two sites (which is guaranteed as the system is finite and absorbing, as discussed by Valentini et al. (2016b)). Then we record the final collective decision and the time required to reach unanimous consensus.

We analyzed the performance of both decision-making strategies (i.e., the majority rule and the voter model) under different initial conditions, that is, different demographic distributions of sites' preferences among the agents. We study the: *consensus time*, that is, the time required by the collective of agents to reach consensus for either of the two sites; and the proportion of simulations in which the collective reached *consensus for site A* – where we define consensus as unanimous agreement among the agents for one site. This latter measure gives an estimate of the probability of the collective to choose the best option. The focus of this study was not on the relative performance of the two decision-making strategies, as such performance analyses have been studied in other work. Instead, we use these models of collective behavior to demonstrate how information dynamics and collective decision-making performance may be related.

Figure 2a shows the performance of both strategies in terms of consensus time. In agreement with previous studies (Valentini et al., 2016b), the time necessary for the collective to reach consensus generally increases with the difficulty of the problem (e.g., compare $\rho_B(\cdot) = 0.5$ with $\rho_B(\cdot) = 0.9$ in Figure 2a). The majority rule (purple lines and confidence interval) is much faster than the voter model (reported in green) with a performance gain of approximately one order of magnitude in the difficult problem configuration ($\rho_B = 0.9$). However, the majority rule and the voter model differ in terms of accuracy. Figure 2b shows the estimate of the probability of the collective to make a decision for the best site. The voter model is particularly accurate reaching consensus almost entirely on site A except for a subset of the initial conditions in the difficult problem setup. Differently, the accuracy of the majority rule is generally much lower than that of the voter model. It is well described by a sigmoid function of the initial number of agents favoring the best site. Increasing the difficulty of the decision problem shifts the critical point that determines whether the majority rule is more or less likely to yield a decision for



(a) (b) (c)

Figure 2: (a) Consensus time (logarithmic scale), (b) proportion of simulations converged on consensus for site A, and (c) transfer entropy over the initial number of agents favoring site A (i.e., $\{10, 11, \ldots, 90\}$) for a collective of $N = 100$ agents. Problem configurations: majority rule $\rho_B(MR) \in \{0.5, 0.9\}$, voter model $\rho_B(VM) \in \{0.5, 0.9\}$. Figures report the estimate of the smoothed conditional means and their confidence interval computed using LOESS regression with a span of 0.1 of the data.

the best site rather than for the inferior one.

Transfer Entropy Over Initial Conditions

Figure 2c gives a first illustration of the information flow underlying this system for both the majority rule and the voter model. For nearly all initial conditions, the majority rule lets one agent's neighbors transfer higher amounts of information than the voter model with peaks of approximately three times the information transferred using this latter mechanism. Transfer entropy $T_{Y \to X}$ is a function of the initial number of agents in the collective favoring site A, and its shape resembles that of the consensus time. For the majority rule, $T_{Y \to X}$ is maximized in the proximity of the critical point at which the collective is equally likely to choose either of the two sites. That is, the information transfer is maximal when the collective decision is characterized by the most intense competition between the two options. In the case of the voter model, information transfer behaves similarly increasing with the initial number of agents favoring the suboptimal site—an indicator of the level of competition underlying the collective decision.

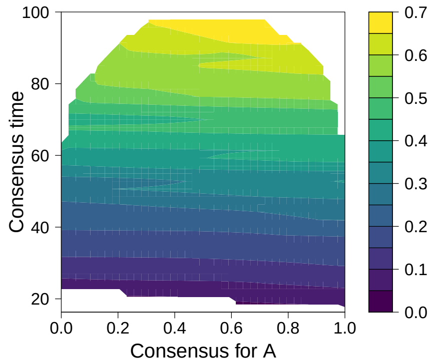
Transfer Entropy in the Speed–Accuracy Space

The results reported above seem to suggest that information transfer is an increasing function of both the time necessary to make a decision and the uncertainty of its outcome. Here, we attempt to deepen our analysis by connecting the results of transfer entropy to different measures of performance of the strategies, that is, consensus time (speed) and probability of consensus on the best site (accuracy).

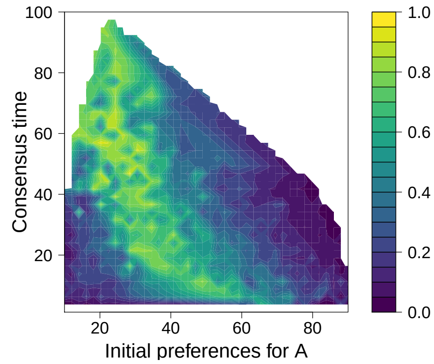
Figures 3a and 3d illustrate the information landscape in the speed–accuracy space for the majority rule and for the voter model, respectively, in the case of the difficult problem scenario with $\rho_B = 0.9$. In both figures, the scale of consensus time (i.e., the y axis) is between 0 and 100 because consensus time has been normalized by the maximum consensus time over all runs and initial conditions multiplied by the number of agents. Figures 3a and 3d show that, for both the majority rule and the voter model, the amount of transferred information increases with the time necessary for the collective to make a consensus decision. Such a monotonic relationship between information transferred and accuracy (i.e., the probability to make a collective decision for the best site), however, does not exist. Instead, information transferred is maximized at an intermediate accuracy for both the majority rule and the voter model.

For the majority rule, the observation that transfer entropy is maximized when the probability of reaching a consensus for A is approximately 0.5 is consistent with the interpretation that agents transfer more information when the outcome of the decision is maximally uncertain, and this additional transfer of information prolongs the decision and leads to a maximal consensus time at the same point. Furthermore, the lower peak value of mean transfer entropy for the voter model is consistent with the accuracy of the voter model being much higher than the majority rule (see Figure 2b) – agents can exchange less information because the decision mechanism is more accurate in general.

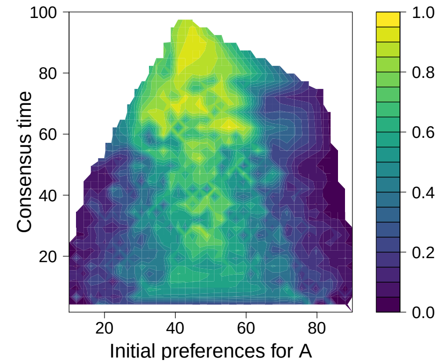
To return to the rain example from the Introduction, the eventual certainty of a consensus on a correct decision in the voter model implies that only a few decisions are due to useful information being passed from agent to agent, and all other interactions are "small talk." The decision strategy of sampling randomly from a surrounding neighborhood leads to fixation on a single opinion, the best one, in which case any further trading of opinions has no effect – agents are "preaching to the choir" or confirming their own biases. On the other hand, the blind adoption of a random sample of the environment means that although most interactions will not result in a change of opinion, some minority opinions will



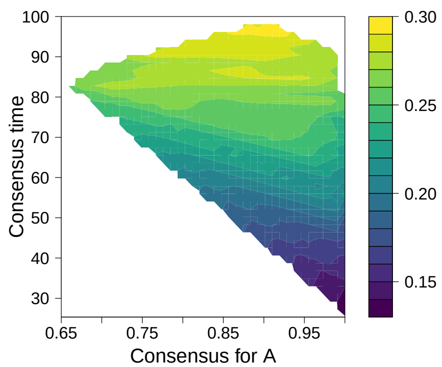
(a) MR, MTE for Speed and Accuracy



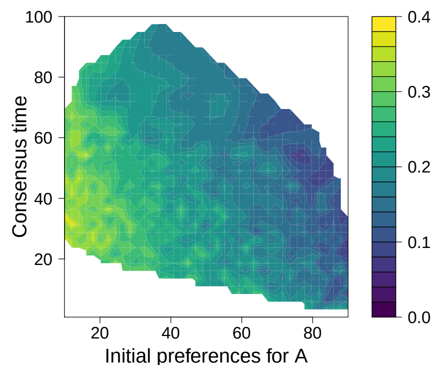
(b) MR-Simple, MTE for Speed and IC



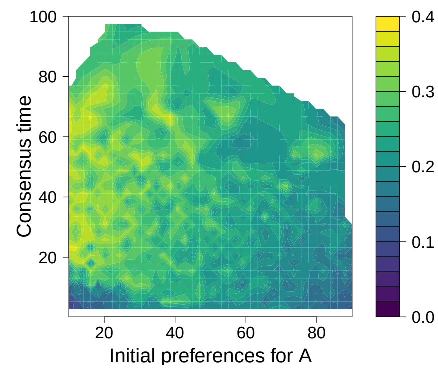
(c) MR-Hard, MTE for Speed and IC



(d) VM, MTE for Speed and Accuracy



(e) VM-Simple, MTE for Speed and IC



(f) VM-Hard, MTE for Speed and IC

Figure 3: Mean transfer entropy (MTE) (a) in the speed–accuracy space ($\rho_B = 0.9$), and in the speed–initial-condition space when (b) the decision-making problem is simple ($\rho_B = 0.5$) and (c) when it is difficult ($\rho_B = 0.9$) for agents using the majority rule. Respectively, for the voter model, mean transfer entropy (d) in the speed–accuracy space ($\rho_B = 0.9$), and in the speed–initial-condition space when (e) the decision-making problem is simple ($\rho_B = 0.5$) and (f) when it is difficult ($\rho_B = 0.9$).

persist for long periods of time before fixation purges them from the population. Thus, the low transfer of information in the voter model is concomitant with an increase in consensus time. However, in the case of the majority rule, the likelihood of an agent to change preference when encountering a randomly selected group of neighbors is high and continues to stay high especially in the case of a difficult decision. In those cases, an individual blindly following the majority of whatever group it encounters will switch rapidly from one opinion to another, and every informational transaction will have some influence on the system. However, once a system gains inertia, minority opinions do not slow the eventual consensus as much as in the voter model, and the ultimate decision occurs at a faster rate. Thus, the absolute value of mean transfer entropy seems to be an indication of convergence speed.

Finally, Figures 3b ($\rho_B = 0.5$) and 3c ($\rho_B = 0.9$) and Figures 3e ($\rho_B = 0.5$) and 3f ($\rho_B = 0.9$) show the mean transfer entropy in the speed–initial-conditions space for the majority rule and for the voter model, respectively. As in the analysis above, the consensus time is normalized to the maximum consensus time multiplied by the number of agents in the collective. The results contained in these figures confirm the positive correlation between consensus time and mean transfer entropy, and this correlation is especially strong for the majority rule. However, for both decision mechanisms, the initial conditions also seem to have modulating effect on this relationship. For the majority rule, a ridge of high transfer entropy appears to match the case when the initial preference demographics are complementary to the actual quality difference between the nest sites. That is, if initially 90% of the population have a higher opinion of a site that is in reality 90% of the quality of the other site, then the social pressure for the wrong site will perfectly balance the agent's private assessment attracting it to the correct site. However, the relationship between initial condition and mean transfer entropy in the voter model case is less clear.

Summary and Conclusions

In this paper, we attempted to shed a light upon the role of information transfer resulting from the interactions among agents using two well-studied decision-making strategies. We considered a spatial multi-agent system where agents are driven by a simple decision mechanism (i.e., the majority rule or the voter model) coupled with modulation of signaling efforts (Valentini et al., 2016b). We studied information transfer of these strategies when applied to a binary site-selection scenario for different problem difficulties. After comparing and contrasting the performance of the two strategies, we used transfer entropy (Schreiber, 2000) to measure the flow of information generated during the decision-making process. Specifically, we analyzed the information transferred to an agent from its neighbors when this agent applies a decision mechanism.

In agreement with our hypothesis, we found that the overall amount of information transferred by the two decision strategies differs considerably despite both strategies successfully solving the decision-making problem. The majority rule generally transfers more information than the voter model, sometimes up to three times the amount transferred by this latter mechanism (Figure 2c). This result may explain performance differences in the consensus time and accuracy of the final decision characterizing the two mechanisms. Although the voter model is much more accurate than the majority rule, it is also much slower (Valentini et al., 2016b). Our results, on the one hand, may suggest that this is due to the voter model transferring less information and more slowly than the majority rule. On the other hand, the fast but comparably much less accurate decisions resulting from the majority rule may be explained by a swift processing of excessive amounts of information.

In the case of the majority rule the transferred information peaks in the proximity of the critical point that divides initial conditions more likely to lead the collective to decide for the best site from those leading it to a decision for the inferior site[7]. Although this critical point is generally not the point of maximal "disorder of opinions" (i.e., equal number of agents favoring both sites), this result is similar to that of Barnett et al. (2013), which show for an Ising model that transferred information is maximized in the disordered paramagnetic phase, or that of Boedecker et al. (2012), which show for a recurrent neural network that transferred information is maximized at the edge of chaos. The critical point in our system is a function of the relative quality of the two options, i.e., problem difficulty, and represent the farthest point from a collective decision in terms of consensus time. This relation between consensus time and transferred information could provide a mechanism to infer the time required for a collective decision when only some agents of the collective are observable, for example, those within the nest.

[7] Note that the voter model is not characterized by a similar, unstable critical point, cf. (Valentini et al., 2016b).

We then deepened our analysis by projecting the transferred information first in the speed–accuracy space and then in the speed–initial-conditions space. We found that, while the overall amount of transferred information is positively correlated with the consensus time, it appears to be less dependent on the probability for the collective to make an optimal decision while being loosely modulated by the decision uncertainty (Figures 3a and 3d). Nonetheless, higher values of consensus time do not always lead to higher amounts of transferred information, but initial conditions also impact this process (see Figures 3b and 3c and Figures 3e and 3f). This result may suggest that other variables could play a driving role behind information transfer in this system.

Information is essential to collective decision making. The gathering and exchange of information performed by the agents in a collective is usually driven by rules that are very simple, yet sufficient to process information and, in doing so, let the collective make accurate and timely decisions. Collective decisions of this sort are often said to be such that, once they have been made, it is not possible to trace them back to the contribution of individual agents. This aspect of collective decision making makes understanding process dynamics particularly challenging. Even when interaction rules between agents are well known (as for the case of artificial agents and opposed to biology where rules are inferred from observations), understanding their contribution to the group-level behavior is still an open problem. Key to gaining insights into this problem is uncovering the link between group behavior and individual interactions among agents and between agents and the environment generated by a given set of rules. Characterizing the information dynamics in groups with tools such as transfer entropy can provide these initial insights and even fresh perspectives on already well-studied decision-making strategies. Moving forward with gathering enough information from natural systems to facilitate similar characterizations is an important future step for understanding naturally evolved decision-making strategies.

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References

Ballerini, M., Cabibbo, N., Candelier, R., Cavagna, A., Cisbani, E., Giardina, I., Lecomte, V., Orlandi, A., Parisi, G., Procaccini, A., Viale, M., and Zdravkovic, V. (2008). Interaction ruling animal collective behavior depends on topological rather than metric distance: Evidence from a field study. *Proceedings of the National Academy of Sciences*, 105(4):1232–1237.

Barnett, L., Lizier, J. T., Harré, M., Seth, A. K., and Bossomaier, T. (2013). Information flow in a kinetic Ising model peaks in the disordered phase. *Physical Review Letters*, 111:177203.

Boedecker, J., Obst, O., Lizier, J. T., Mayer, N. M., and Asada, M. (2012). Information processing in echo state networks at the edge of chaos. *Theory in Biosciences*, 131(3):205–213.

Butail, S., Mwaffo, V., and Porfiri, M. (2016). Model-free information-theoretic approach to infer leadership in pairs of zebrafish. *Phys. Rev. E*, 93:042411.

Couzin, I. D., Krause, J., Franks, N. R., and Levin, S. A. (2005). Effective leadership and decision-making in animal groups on the move. *Nature*, 433(7025):513–516.

Crosscombe, M., Lawry, J., Hauert, S., and Homer, M. (2017). Robust distributed decision-making in robot swarms: Exploiting a third truth state. In *2017 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pages 4326–4332. IEEE Press.

Ebert, J., Gauci, M., and Nagpal, R. (2018). Multi-feature collective decision making in robot swarms. In Dastani, M., Sukthankar, G., Andre, E., and Koenig, S., editors, *Proceedings of the 17th International Conference on Autonomous Agents and Multiagent Systems*, AAMAS '18. IFAAMAS, (in press).

Franks, N. R., Pratt, S. C., Mallon, E. B., Britton, N. F., and Sumpter, D. J. T. (2002). Information flow, opinion polling and collective intelligence in house-hunting social insects. *Philosophical Transactions of the Royal Society B: Biological Sciences*, 357(1427):1567–1583.

Honey, C. J., Kötter, R., Breakspear, M., and Sporns, O. (2007). Network structure of cerebral cortex shapes functional connectivity on multiple time scales. *Proceedings of the National Academy of Sciences*, 104(24):10240–10245.

Kim, H., Davies, P., and Walker, S. I. (2015). New scaling relation for information transfer in biological networks. *Journal of The Royal Society Interface*, 12(113):20150944.

Kouvaros, P. and Lomuscio, A. (2016). Formal verification of opinion formation in swarms. In Thangarajah, J., Tuyls, K., Jonker, C., and Marsella, S., editors, *Proceedings of the 15th International Conference on Autonomous Agents and Multiagent Systems*, AAMAS '16, pages 1200–1208. IFAAMAS.

Lizier, J. T., Heinzle, J., Horstmann, A., Haynes, J.-D., and Prokopenko, M. (2011). Multivariate information-theoretic measures reveal directed information structure and task relevant changes in fmri connectivity. *Journal of Computational Neuroscience*, 30(1):85–107.

Lizier, J. T., Piraveenan, M., Pradhana, D., Prokopenko, M., and Yaeger, L. S. (2009). Functional and structural topologies in evolved neural networks. In *European Conference on Artificial Life*, volume 5777 of *LNCS*, pages 140–147. Springer.

Lizier, J. T. and Prokopenko, M. (2010). Differentiating information transfer and causal effect. *The European Physical Journal B*, 73(4):605–615.

Lizier, J. T., Prokopenko, M., and Zomaya, A. Y. (2008). Local information transfer as a spatiotemporal filter for complex systems. *Physical Review E*, 77:026110.

Meyer, B. (2017). Optimal information transfer and stochastic resonance in collective decision making. *Swarm Intelligence*, 11(2):131–154.

Moore, D. G., Valentini, G., Walker, S. I., and Levin, M. (2018). Inform: Efficient information-theoretic analysis of collective behaviors. *Frontiers in Robotics and AI*, 5:60.

Mwaffo, V., Butail, S., and Porfiri, M. (2017). Analysis of pairwise interactions in a maximum likelihood sense to identify leaders in a group. *Frontiers in Robotics and AI*, 4:35.

Pezzulo, G. and Levin, M. (2015). Re-membering the body: applications of computational neuroscience to the top-down control of regeneration of limbs and other complex organs. *Integrative biology: quantitative biosciences from nano to macro*, 7(12):1487–1517.

Reznikova, Z. and Ryabako, B. (1994). Experimental study of the ants' communication system with the application of the information theory approach. *Memorabilia Zoologica*, 48:219–236.

Schreiber, T. (2000). Measuring information transfer. *Physical Review Letters*, 85(2):461–464.

Sperati, V., Trianni, V., and Nolfi, S. (2008). Evolving coordinated group behaviours through maximisation of mean mutual information. *Swarm Intelligence*, 2(2):73–95.

Sperati, V., Trianni, V., and Nolfi, S. (2011). Self-organised path formation in a swarm of robots. *Swarm Intelligence*, 5(2):97–119.

Strobel, V., Castello Ferrer, E., and Dorigo, M. (2018). Managing byzantine robots via blockchain technology in a swarm robotics collective decision making scenario. In Dastani, M., Sukthankar, G., Andre, E., and Koenig, S., editors, *Proceedings of the 17th International Conference on Autonomous Agents and Multiagent Systems*, AAMAS '18. IFAAMAS, (in press).

Valentini, G., Brambilla, D., Hamann, H., and Dorigo, M. (2016a). Collective perception of environmental features in a robot swarm. In Dorigo, M., Birattari, M., Li, X., López-Ibáñez, M., Ohkura, K., Pinciroli, C., and Stützle, T., editors, *Swarm Intelligence*, volume 9882 of *LNCS*, pages 65–76. Springer.

Valentini, G., Ferrante, E., and Dorigo, M. (2017). The best-of-n problem in robot swarms: Formalization, state of the art, and novel perspectives. *Frontiers in Robotics and AI*, 4:9.

Valentini, G., Ferrante, E., Hamann, H., and Dorigo, M. (2016b). Collective decision with 100 Kilobots: Speed versus accuracy in binary discrimination problems. *Autonomous Agents and Multi-Agent Systems*, 30(3):553–580.

Valentini, G., Hamann, H., and Dorigo, M. (2014). Self-organized collective decision making: The weighted voter model. In Lomuscio, A., Scerri, P., Bazzan, A., and Huhns, M., editors, *Proceedings of the 13th International Conference on Autonomous Agents and Multiagent Systems*, AAMAS '14, pages 45–52. IFAAMAS.

Walker, S. I., Cisneros, L., and Davies, P. (2013). Evolutionary transitions and top-down causation. In *Proceedings of the 13th International Conference on the Simulation and Synthesis of Living Systems, ALIFE 2012*, pages 283–290.

Zenil, H., Marshall, J. A. R., and Tegnér, J. (2015). Approximations of algorithmic and structural complexity validate cognitive-behavioural experimental results.