Guiding aggregation dynamics in a swarm of agents via informed individuals: an analytical study

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Abstract

Self-organised aggregation, the formation of large clusters of independent agents, is an important process in swarm robotics systems since it is the prerequisite for more complex collective behaviours. Previous work on self-organised aggregation focused on the study of the individual mechanisms required to allow a swarm to form a single aggregate. In this paper, we discuss an analytical model which looks at the possibility to use the concept of informed individuals to allow the swarm to distribute on different aggregation sites according to proportions of individuals at each site arbitrarily chosen by the designer. Informed individuals are opinionated agents that selectively prefer an aggregation site and avoid to rest on the non-preferred sites. We study environments with two aggregation sites, and consider two different scenarios: one in which the informed individuals are equally distributed in numbers between the two sites; and one in which informed individuals for one type of site are three times more numerous than those on the other site. Our objective is to find out whether and for what range of model parameters the swarm distributes between the two sites according to the relative distribution of informed agents among the two sites. The analysis of the model shows that the designer capability to exploit informed individuals to control how the swarm aggregates depends on the environmental conditions. For intermediate values of the site carrying capacity, a small minority of informed individuals is able to guide the dynamics as desired by the designer. We also show that the larger the site carrying capacity the larger the total proportion of informed individuals required to lead the swarm to the desired distribution of individuals between the two sites.

Introduction

The field of swarm robotics aims at studying and designing self-organising collective behaviours for large groups of relatively simple individuals (Dorigo and Şahin, 2004). Swarm robotics takes inspiration from nature, whereby groups of social insects or other animals rely on proximate mechanisms (simple cognitive heuristics plus local interaction rules) that allow them to exhibit complex collective patterns which tend to be functional to achieve certain tasks (Camazine et al., 2001). Examples include collective migration, site selection, pattern formation, and task specialization. In biological systems, natural evolution shapes the individual rules of action underpinning the group collective response. In artificial swarms, such as swarms of robots, artificial evolution can potentially be used to mimic natural evolution in order to automate the design of individual mechanisms (see Harvey et al., 2005; Tuci and Rabérin, 2015; Tuci et al., 2018). Alternatively, the designer can program the behaviour of each individual robot, and evaluate the performance of the collective behaviour at the group (macroscopic) level (Brambilla et al., 2013). However, given the difficulties in predicting the individual actions that result in the desired self-organising collective behaviour, the designer is required to program and evaluate multiple individual controllers before finding the one that underpins the desired group level response. Thus, this approach can be time consuming and largely dependent on the designer's intuitions on what is required to move from the individual to the group level desired behaviour.

A relatively recent idea to increase the degree of controllability of artificial swarms consists in introducing a small proportion of informed individuals which can be used to bias the collective behaviours in the direction specified by the designer (Ferrante, 2013). Informed individuals are opinionated agents that tend to bias any decision making process toward their preferred option. The effect of informed individuals on the groups dynamics have been originally discussed in biological models of collective motion, where a minority of individuals determined to move in a given direction induces the rest of the swarm to opt for their direction of motion (Couzin et al., 2005; Stroeymeyt et al., 2011; Krause and Ruxton, 2011). Informed individuals have been subsequently exploited in artificial swarms mainly as a means to control the system during collective motion (Çelikkanat and Şahin, 2010; Ferrante et al., 2012, 2014). We study the effects of informed individuals in a larger spectrum of self-organised collective behaviour. In particular, in this paper we further explore the effects of informed individuals in the context of self-organised aggregation (Firat et al., 2018).

Generally speaking, in aggregation tasks, individuals have to aggregate on a common location in the environment (Garnier et al., 2005, 2008; Bayindir and Şahin, 2009; Correll and Martinoli, 2011; Gauci et al., 2014). Swarm robotics studies have shown that robot's controllers in which the individual probability to join and to leave an aggregation site depends on the number of robots perceived by an individual at the site, lead to the emergence of a single aggregate at one site among those available in the environments (Garnier et al., 2009; Campo et al., 2010). In (Halloy et al., 2007), robots controlled by similar principles influence the aggregation dynamics of cockroaches in mixed robot-animal groups. In particular, the robots programmed to preferentially rest on the lighter (rather than on the darker) shelter, induce cockroaches to behave similarly even if the animals would preferentially aggregate on the darker shelter in the absence of robots. The idea that some individuals could influence the aggregation dynamics of a group of autonomous agents, originally discussed in (Halloy et al., 2007) in the context of the robots-cockroaches interaction, has been recently explored in (Firat et al., 2018, 2019) in the context of swarm robotics systems. The authors in (Firat et al., 2018, 2019) have extended the analysis of the aggregation process in a two-site scenario as illustrated in (Garnier et al., 2009; Campo et al., 2010), to the case in which the swarm is characterised by the presence of informed individuals. In (Firat et al., 2018, 2019) the sites have distinctive features that allow the agents to discriminate between the two of them. Informed individuals are programmed to selectively avoid to rest on one of the two sites. Non-informed individuals rest with equal probabilities on both sites. These studies show that with a small proportion of informed individual it is possible to selectively drive the aggregation dynamics on a designer preferred aggregation site.

In this paper, we discuss the results of a mathematical model that looks at aggregation dynamics in a swarm of agents with different proportions of informed and non-informed individuals. Mathematical models are quite frequently used in the study of collective behaviour in artificial swarms to avoid the time and computational costs that robotics and others agent-based models undergo to explore the effects of a wide range of experimental conditions (Brambilla et al., 2013). Mathematical model of self-organised aggregation include geometric models (Bayindir and Şahin, 2009) and Markov chains (Soysal and Şahin, 2007). To study other collective behaviours, common approaches to modelling include ordinary differential equations (Montes de Oca et al., 2011; Valentini et al., 2015), stochastic differential equations such as rate equations (Lerman and Galstyan, 2002), chemical reaction networks (Valentini et al., 2015), Fokker-Planck and Langevin equations (Hamann and Wörn, 2008), and control theory (Hsieh et al., 2008), among others. Our model uses a system of ordinary differential equations to study how informed individuals can be used in the context of aggregation to distribute the agents of a swarm between two distinctive aggregation sites (one perceived by the individuals as black and the other as white) according to two arbitrary rules specified by the designer. There are two types of informed individuals in our model: the "informed for black" individuals which rest only on the black site, and the "informed for white" individuals which rest only on the white site. Excluding the informed individuals of any type, the rest of the swarm is made of non-informed individuals, that is agents that rest on both aggregation sites with equal probabilities. Both informed and non-informed individuals leave an aggregation site with a probability given by a non-linear function of the density of individuals at the site.

Our objective is to find out whether and eventually for which parameter range the swarm distributes between the two sites according to the relative proportion of one type of informed individuals with respect to the other type, by keeping the total proportion of informed individuals as small as possible. We analyse the system for different total percentage of informed individuals in the swarm, from 0% to 100% informed individuals. For each percentage of informed individuals, we systematically vary the relative proportion of informed individuals of one type with respect to the proportion of individuals of the other type. In this paper, we report the results of two representative scenarios: one in which informed individuals are equally distributed in numbers between the two sites; and one in which the informed individuals for one type of site are three times more than the informed individuals for the other type of site. The first scenario has been chosen to represent the designer aims to induce the agents to aggregate in equal proportion on both sites. The second scenario has been arbitrarily chosen among those representative of the designer intention to induce the agents to aggregate in different proportions on each site. For each of the two scenarios illustrated in this paper, we varies the total proportion of informed individuals from 0% to 100% of the swarm population size. Moreover, we analyse the systems for different values of the site carrying capacity, that is the total number of individuals that can simultaneously rest on a site. We are interested in identifying the conditions whereby agents equally split on the two aggregation sites when both types of informed individuals are equally represented in the swarm, and the conditions whereby aggregation dynamics see agents aggregated 75% on a site and 25% on the other site, when one type of informed individuals is three time more represented than the other type. The results of this study shows that there are parameters' values for which the distribution of individuals between the two sites matches the relative proportion of one type of informed individuals with respect to the other type. In particular circumstances, the desired aggregation dynamics can be observed with a small minority of informed individuals in the swarm. In other words, the analysis of the mathematical model indicates that informed individuals are a potentially effective means to control the aggregation dynamics in swarms of autonomous agents. In section Conclusions, we discuss the significance of our results for the swarm robotics community, and we explain how we intend to use these finding in our future research works.

Methods

In this section, we describe the system of Ordinary Differential Equations (ODEs) used to investigate the effects of different proportions of two different types of informed individuals on the aggregation dynamics in a scenario with two sites, a black and a white site. We draw inspiration from another ODEs system originally discussed in (Amé et al., 2004), and subsequently extended in (Amé et al., 2006) to model the aggregation dynamics observed in cockroaches. The distinctive feature of both the above cited models is that the individual probability of leaving a site is a non-linear function of the number of individuals currently resting at that site. Our departure point is the Amé et al. (2006)'s model, where two aggregation sites, with same characteristics, are symmetrical locations for aggregation for a group of N equal type individuals. The Amé et al. (2006)'s model is the following:

$$\dot{N_i} = -N_i\lambda_i + \mu\Big(1 - \frac{N_i}{S}\Big)N_{ext}; \qquad (1)$$

with

$$\lambda_i = \frac{\epsilon}{1+\gamma\Big(\frac{N_i}{S}\Big)^2}; \qquad N_{ext} = (N - \sum_{i=1}^{p} N_i); \qquad (2)$$

where N_i is the number of individual resting on site i, λ_i is the individual probability to leave site i, the parameter $\epsilon = 0.01s^{-1}$, the parameter $\gamma = 1667$, S is the maximum number of individuals that a site can host (i.e. the site carrying capacity), $\mu = 0.001s^{-1}$ is the rate of entering a site, N_{ext} is the number of individuals outside the sites, and $p = 2$ is the number of sites. The analysis of this model shows that the agents form a single aggregate only when each aggregation site can host more that the totality of the swarm's individuals. The model also predicts how the agents distribute in different environments varying for the number of aggregation sites and the diameter of each site bearing upon the site capacity to host individuals (see Amé et al., 2006).

We modified the system in Eq. 1 to take into account two novel features that distinctively characterised our study: that is, the differences between the two sites, one of which is perceived by the individuals as black, and the other as white, and the presence of two different types of informed individuals. With the introduction of colour differences between the two sites, the total number of individuals in a group N is given by $N = N_b + N_w + N_{ext}$, with N_b and N_w being the number of individuals resting on the black and on the white site, respectively.

Defining $\sigma = S/N$, $x_b = N_b/N$ and $x_w = N_w/N$, with $N_{ext} = N - N_b - N_w$, leads us to the following system:

$$\begin{cases} \dot{x_b} = -x_b\lambda_b + \mu\Big(1 - \frac{x_b}{\sigma}\Big)(1 - x_b - x_w) \\ \dot{x_w} = -x_w\lambda_w + \mu\Big(1 - \frac{x_w}{\sigma}\Big)(1 - x_b - x_w) \end{cases} \qquad (3)$$

with

$$\lambda_b = \frac{\epsilon}{1+\gamma\Big(\frac{x_b}{\sigma}\Big)^2}; \qquad \lambda_w = \frac{\epsilon}{1+\gamma\Big(\frac{x_w}{\sigma}\Big)^2}; \qquad (4)$$

where λ_b and λ_w refer to the probability of leaving the black and the white site, respectively. As shown in Eq. 3, the system is independent of N and depends only on the fraction of individuals on the two sites.

The distinction between informed and non-informed individuals is introduced into the system with the notation i_w (informed for white) for informed individuals that do not rest on the black site, i_b (informed for black) for informed individuals that do not rest on the white site, and ni (non-informed) for non-informed individuals, who can potentially rest on both sites. With this distinction in place, ρ_{i_b} and ρ_{i_w} are the proportion of informed individuals of type i_b and i_w, respectively. $x_b^{i_b}$ refers to the fraction of individuals on the black site that are of type i_b; $x_w^{i_w}$ refers to the fraction of individuals on the white site that are of type i_w; x_b^{ni} refers to the fraction of individuals on the black site that are of type ni; and x_w^{ni} refers to the fraction of individuals on the white site that are of type ni. The fraction of individual on the black site (x_b) and on the white site (x_w) is then written as:

$$\begin{cases} x_b = x_b^{ni} + x_b^{i_b} \\ x_w = x_w^{ni} + x_w^{i_w} \end{cases} \qquad (5)$$

since, by definition, informed individuals of type i_w never rest on the black site, and informed individuals of type i_b never rest on the white site.

Generalising Eq. 3 to the case with informed and non-informed individuals gives

$$\begin{cases} \dot{x_b^{i_b}} = -x_b^{i_b}\lambda_b + \mu\Big(1 - \frac{x_b}{\sigma}\Big)x_{ext}^{i_b} \\ \dot{x_b^{ni}} = -x_b^{ni}\lambda_b + \mu\Big(1 - \frac{x_b}{\sigma}\Big)x_{ext}^{ni} \\ \dot{x_w^{i_w}} = -x_w^{i_w}\lambda_w + \mu\Big(1 - \frac{x_w}{\sigma}\Big)x_{ext}^{i_w} \\ \dot{x_w^{ni}} = -x_w^{ni}\lambda_w + \mu\Big(1 - \frac{x_w}{\sigma}\Big)x_{ext}^{ni}, \end{cases} \qquad (6)$$

where $x_{ext}^{i_b}$, $x_{ext}^{i_w}$, and $x_{ext}^{i_{ni}}$ are the fraction of individuals of type i_b, i_w, and i_{ni} that are outside the two sites. These fractions can be expressed in the following way

$$\begin{cases} x_{ext} = 1 - x_b - x_w = 1 - x_b^{ni} - x_b^{i_b} - x_w^{ni} - x_w^{i_w} \\ x_{ext}^{i_b} = \rho_{i_b} - x_b^{i_b} \\ x_{ext}^{i_w} = \rho_{i_w} - x_w^{i_w} \\ x_{ext}^{ni} = x_{ext} - x_{ext}^{i_b} - x_{ext}^{i_w} \\ \qquad = (1 - \rho_{i_b} - \rho_{i_w}) - x_b^{ni} - x_w^{ni}. \end{cases} \qquad (7)$$

Finally, substituting Eq. 7 into Eq. 6 we obtain the following system:

$$\begin{cases} \dot{x}_b^{i_b} = -x_b^{i_b}\lambda_b + \mu\left(1-\frac{x_b}{\sigma}\right)\left(\rho_{i_b} - x_b^{i_b}\right) \\ \dot{x}_b^{ni} = -x_b^{ni}\lambda_b + \mu\left(1-\frac{x_b}{\sigma}\right)\left((1-\rho_{i_b}-\rho_{i_w}) - x_b^{ni} - x_w^{ni}\right) \\ \dot{x}_w^{i_w} = -x_w^{i_w}\lambda_w + \mu\left(1-\frac{x_w}{\sigma}\right)\left(\rho_{i_w} - x_w^{i_w}\right) \\ \dot{x}_w^{ni} = -x_w^{ni}\lambda_w + \mu\left(1-\frac{x_w}{\sigma}\right)\left((1-\rho_{i_b}-\rho_{i_w}) - x_b^{ni} - x_w^{ni}\right). \end{cases} \quad (8)$$

In the particular case when all the individuals of the group are informed (i.e. $\rho_{i_b} + \rho_{i_w} = 1$), this set of equations reduces to

$$\begin{cases} \dot{x_b} = -x_b\lambda_b + \mu\left(1-\frac{x_b}{\sigma}\right)\left(\rho_{i_b} - x_b\right) \\ \dot{x_w} = -x_w\lambda_w + \mu\left(1-\frac{x_w}{\sigma}\right)\left(\rho_{i_w} - x_w\right). \end{cases} \quad (9)$$

The set of equations illustrated in Eq. 8, is solved numerically to find equilibrium states (i.e., when $\dot{x} = 0$). Equilibrium states are studied with respect to the key parameters σ, ρ_{ib} and, ρ_{iw}. The results are discussed in next section.

Results

In this section, we show the results of our analysis, by discussing the equilibrium states of Eq. 8, for different sets of values for the parameters σ, ρ_{i_b}, and ρ_{i_w}. We remind the reader that the parameter σ is the ratio between the site carrying capacity S and the swarm size N. When $\sigma = 1$ each aggregation site can host as many individuals as the swarm size; when $\sigma < 1$, each aggregation site can host fewer individuals than the swarm size; when $\sigma > 1$, each site can host more individuals than the swarm size. ρ_{i_b} and ρ_{i_w} refer to the proportion of individuals of type i_b (informed for black) and i_w (informed for white), respectively. We also remind the reader that our objective is to find out the set of parameters for which the individuals distribute between the two sites according to the relative proportion of one type of informed individuals with respect to the other type. We are also particularly interested in finding what is the critical value of $\rho_i = \rho_{i_b} + \rho_{i_w}$ (i.e. the proportion of informed individuals) above which this objective is realized, and how this changes with respect to σ. For example, when $\rho_{i_b} = 0.3$ and $\rho_{i_w} = 0.3$ we expect 50% of the individuals on the white site and 50% of the individuals on the black site, and we would like to know how much we can decrease both ρ_{i_b} and ρ_{i_w} and still maintain this allocation.

When there are no informed individuals in the swarm ($\rho_{i_b} = 0$ and $\rho_{i_w} = 0$), our model reduces to the original (Amé et al., 2006)'s model. As in (Amé et al., 2006), we also find out that for $\sigma < 1$, the swarm equally distribute between the two sites. However, when $\sigma > 1$ the individuals are able to make a collective decision and to aggregate either on the black or on the white site.

When the entire swarm is composed of informed individuals (i.e. $\rho_{i_b} + \rho_{i_w} = 1$, see also Eq. 9), the fraction of individuals aggregated on the black site (i.e., x_b) is shown in

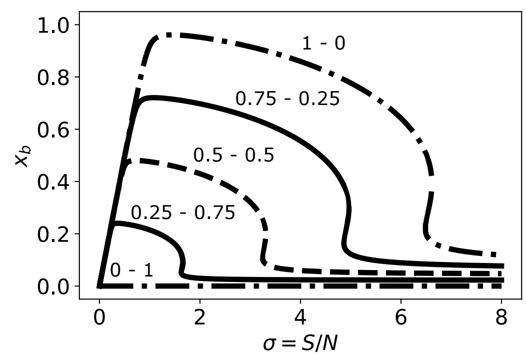
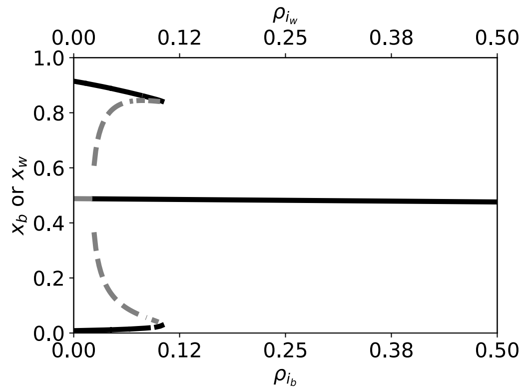


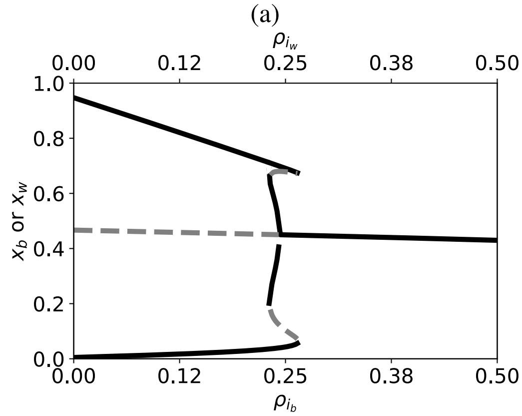
Figure 1: Graph showing the steady state for x_b when σ varies from 0 to 8 for different values of the ratio $\frac{\rho_{i_b}}{\rho_{i_w}}$ when the swarm is made of only informed individuals ($\rho_{i_b} + \rho_{i_w} = 1$). The numbers above each line indicate the fraction of informed individuals of types i_b and i_w. Dashed line: $\frac{\rho_{i_b}}{\rho_{i_w}} = 1$. Continuous lines: $\frac{\rho_{i_b}}{\rho_{i_w}} = 3$ or $1/3$. Dashed-dotted lines: $\rho_{i_b} = 0$ or $\rho_{i_w} = 0$.

Figure 1. This graph represents the steady state for x_b when σ varies from 0 to 8, and for different values of the ratio $\frac{\rho_{i_b}}{\rho_{i_w}}$.

In the low σ range, when each site is not big enough to host all the corresponding informed individuals (the black site for individuals of type i_b, and the white site for individuals of type i_w), the individuals allocate themselves to both sites until they reach the site carrying capacity. This trend does not depend on the relative ratio between $\frac{\rho_{i_b}}{\rho_{i_w}}$, therefore for low values of sigma informed agents are not able to influence the aggregation dynamics. When σ surpasses a critical value that depends on $\frac{\rho_{i_b}}{\rho_{i_w}}$, each site becomes big enough to host all the corresponding informed individuals. Steady-state dynamic for different $\frac{\rho_{i_b}}{\rho_{i_w}}$ are qualitatively different but follow a similar trend. Up to a another critical value of σ, again dependent on $\frac{\rho_{i_b}}{\rho_{i_w}}$, most individuals simply aggregate on the site they prefer. This is the regime in which informed agents have a maximal influence on the dynamics. However, above this new critical σ, individuals are no longer able to aggregate at all. This analysis reveals that, as it happened for the original model discussed in (Amé et al., 2006), and regardless of the ratio $\frac{\rho_{i_b}}{\rho_{i_w}}$, environmental parameters such as the site carrying capacity strongly influences the aggregation dynamics and that informed agents can guide self-organisation only in a limited range of this parameter. For example, when the aggregation site becomes too large, the probability to aggregate on a site, which depends on the site current density, tends to remain too low to trigger the aggregation process. In other words, the density of individuals on each site never reaches a critical value to induce the individuals to aggregate on a site. Thus, the individuals tend to disperse rather than aggregate. For each site, the transition between the two regimes illustrated above is determined by the number of informed individuals that are attracted by that particular colour: the higher the number of informed individuals of each type, the higher the size of the site required to trigger the regime change. For each ratio $\frac{\rho_{i_b}}{\rho_{i_w}}$, there exists a maximum size σ_{max} corresponding to the point of regime change. For example, when 75% of individuals are of type i_b and 25% are of type i_w, in order to have all of them aggregated on the corresponding preferred site, σ has to be smaller than 1.6 (see Figure 1, continuous lines). When $1.6 < \sigma < 4.8$, the white site becomes too large to trigger aggregation for the individuals of type i_w, while the individuals of type i_b are enough to cope with the dimension of their corresponding aggregation site. When $\sigma > 4.8$, even the black site becomes too large to trigger aggregation.



(a)

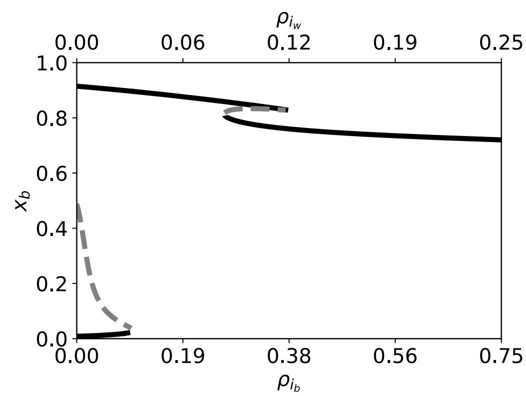


(b)

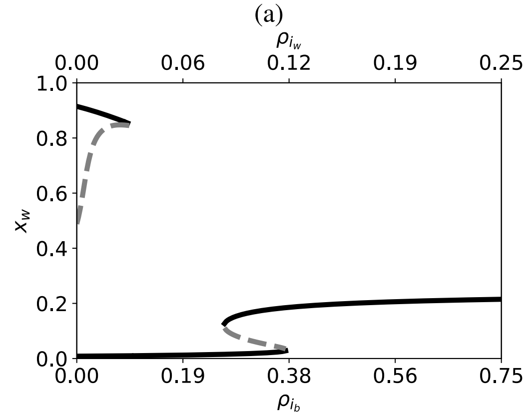
Figure 2: (a) Fraction of individuals on the black and on the white site, when $\frac{\rho_{i_b}}{\rho_{i_w}} = 1$ for (a) $\sigma = 1$ and (b) $\sigma = 2$. Black continuous lines: stable solutions. Dashed grey lines: unstable solutions.

The analysis carried out so far tell us that the most interesting regime is the one in which σ is in a range (dependent on $\frac{\rho_{i_b}}{\rho_{i_w}}$) that allows the individuals to aggregate in their respective preferred sites. In this range, we ask ourselves whether we can now have a hybrid swarm composed of informed and non-informed individuals, and whether informed individuals can still guide the dynamics in a similar way as when the swarm was only composed of informed individuals. We thus proceed by analysing the system for $\sigma = 1$ and $\sigma = 2$ for different proportions of informed individuals in the swarm, and for two different values of the ratio $\frac{\rho_{i_b}}{\rho_{i_w}}$. In all the figures that will follow, we will report stable equilibria with continuous black lines, and unstable equilibria with dashed grey lines.

Figure 2a (resp. Figure 2b) reports results with the ratio $\frac{\rho_{i_b}}{\rho_{i_w}} = 1$ for $\sigma = 1$ (resp. $\sigma = 2$), that is for each proportion of informed individuals in the swarm, 50% of them are of type i_b and 50% are of type i_w. The graph shows that the individuals aggregate on one site only (i.e. either the black or the white site), until a critical value for the total proportion of informed individuals ρ_i (about 24% of the swarm for $\sigma = 1$ and about 50% of the swarm for $\sigma = 2$), above which individuals are able to aggregate in equal numbers on both sites. Therefore, informed agents are able to guide self-organised aggregation only above a critical proportion of informed individuals, which increases with increasing σ, which therefore suggest that larger aggregation sites have again a counter-intuitive negative effect on the controllability of this self-organised behaviour.



(a)



(b)

Figure 3: (a) Fraction of individuals, when $\frac{\rho_{i_b}}{\rho_{i_w}} = 3$ and $\sigma = 1$ for (a) the black site and (b) the white site. Black continuous lines: stable solutions. Dashed grey lines: unstable solutions.

In Figure 3, the ratio $\frac{\rho_{i_b}}{\rho_{i_w}}$ is set to 3, that is for each proportion of informed individuals in the swarm, 75% of them are of type i_b and 25% are of type i_w, and $\sigma = 1$. The graphs in Figure 3a and 3b can be globally understood as follows: below a given threshold of about 10% of informed individuals of type i_b, one of two things can happen: either informed individuals of type i_b and non-informed individuals aggregate on the black site, and informed individuals of type i_w do not aggregate on any site; or informed individuals of type i_w and non-informed individuals aggregate on the white site, and informed individuals of type i_b do not aggregate on any site. Under this condition, the behaviour of informed individuals that do not aggregate on their preferred site can be explained by observing that the dimension of the site is too large relative to their number to trigger any aggregation process. Beyond 30% of informed individuals of type i_b, 75% of the swarm aggregates on the black site and 25% of the swarm aggregate on the white site. This is the regime where informed agents are able to guide self-organised dynamics.

Mathematically, when the total proportion of informed individuals in the swarm is low, the following approximations hold:

$$\begin{cases} x_b^{i_b} \approx \rho_{i_b} \\ x_b^{ni} \approx 1 - \rho_{i_b} - \rho_{i_w} \\ x_w^{i_w} \approx 0 \\ x_w^{ni} \approx 0 \end{cases} \quad OR \quad \begin{cases} x_b^{i_b} \approx 0 \\ x_b^{ni} \approx 0 \\ x_w^{i_w} \approx \rho_{i_w} \\ x_w^{ni} \approx 1 - \rho_{i_b} - \rho_{i_w} \end{cases}. \tag{10}$$

In such a case, the swarm aggregates only on one site, with the informed individuals that prefer the other site do not join the aggregate and they do not aggregate on their preferred site. When the total proportion of informed individuals in the swarm is high, the following approximations hold:

$$\begin{cases} x_b^{i_b} \approx \rho_{i_b} \\ x_b^{ni} \approx R_b - \rho_{i_b} \\ x_w^{i_w} \approx \rho_{i_w} \\ x_w^{ni} \approx R_w - \rho_{i_w} \end{cases} \tag{11}$$

where R_b and R_w is the ratio of informed individuals of type i_b and i_w over the total number of informed individuals, respectively. These results are valid only when the critical value of σ_{max} is not reached for the specific values of R_b and R_w, as discussed previously.

Figure 4 reports results of an analysis similar to the one reported in Figure 3 but with $\sigma = 2$ instead than $\sigma = 1$, with the ratio $\frac{\rho_{i_b}}{\rho_{i_w}}$ still set to 3. The graphs in Figure 4a and 4b show that below a given threshold of about 20% of informed individuals of type i_b, the same behaviour is observed as in Figure 3a and 3b: informed individuals of type i_b and non-informed individuals aggregate on the black site, and informed individuals of type i_w do not aggregate on any site; or informed individuals of type i_w and non-informed individuals aggregate on the white site, and informed individuals of type i_b do not aggregate on any site. Beyond 20% of informed individuals of type i_b, 75% of the swarm aggregates on the black site but no individuals aggregate on the white site, since the dimension of the site is too large to trigger any aggregation process. In other words, with $\sigma = 2$, informed agents are never able to guide the aggregation dynamics. Indeed, when $\frac{\rho_{i_b}}{\rho_{i_w}} = 3$, in order to induce the individuals of type i_w to aggregate on their white site we need $\sigma < \sigma_{max} \approx 1.6$.

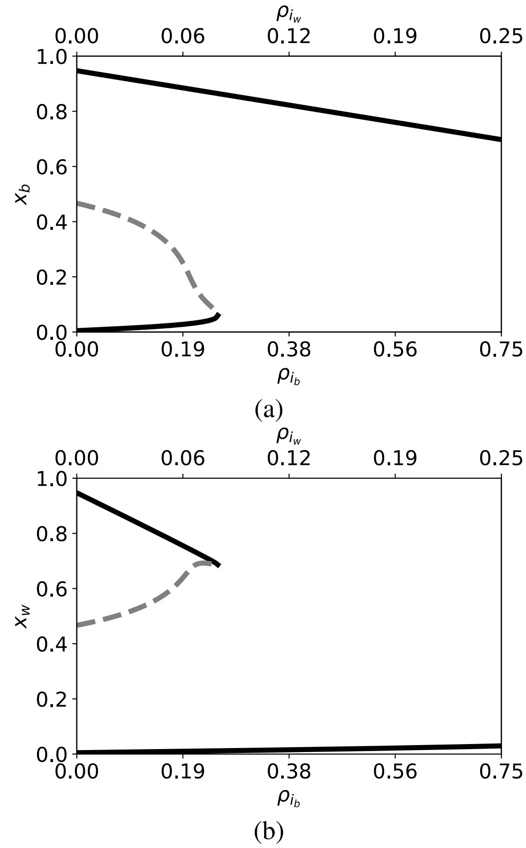


Figure 4: (a) Fraction of individuals, when $\frac{\rho_{i_b}}{\rho_{i_w}} = 3$ and $\sigma = 2$ for (a) the black site and (b) the white site. Black continuous lines: stable solutions. Dashed grey lines: unstable solutions.

To complete the discussion, analysis has been performed for values of $\sigma < 1$. As shown in Figure 5, when the fraction of informed individuals is low, the swarm behaves as predicted by (Amé et al., 2006). That is, individuals distribute equally among the two sites. The distribution of individuals then changes continuously up to the desired distribution when all the individuals are informed. Note that when the site carrying capacity is not large enough to contain the corresponding informed individuals, the amount of individuals on the site is limited by this capacity and therefore never reaches the desired fraction of individuals.

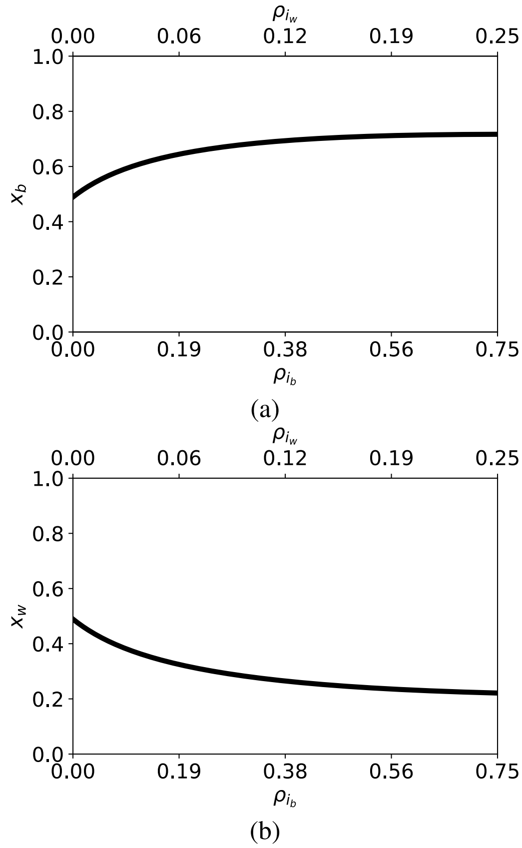


Figure 5: (a) Fraction of individuals, when $\frac{\rho_{i_b}}{\rho_{i_w}} = 3$ and $\sigma = 0.9$ for (a) the black site and (b) the white site. Black continuous lines: stable solutions.

Conclusions

In this paper, we introduced a mathematical ordinary differential equations model that is inspired by the one proposed by Amé et al. (2006). We performed an analytical study of self-organised aggregation in presence of two distinctive aggregation sites, one black and one white. We consider a swarm of agents characterised by the presence of informed individuals, that is agents that are able to recognise the colour and therefore discriminate between the two sites. Our model considers sub-populations of informed individuals, distinguishing between those that prefer the white and those that prefer the black site. Each type of informed individuals never rests on the non-preferred site. From an engineering perspective, when designing self-organised systems engaged in aggregation tasks, we would like to use informed individuals to guide the self-organised aggregation dynamics. In particular, we would like to correlate the relative proportion of one type of informed individuals with respect to the other type, with the total proportion of individuals aggregated in each site.

We analysed the equilibria of the model with respect to the site carrying capacity and to the proportion of informed individuals that prefer the white or the black site. Results show that, as in Amé et al. (2006), dynamics are strongly dependent on the environmental conditions. For intermediate values of the site carrying capacity, the informed individuals are able to guide the dynamics. And within this range, the critical mass of informed individuals needed to guide the dynamics is positively correlated with the site carrying capacity, meaning that larger sites make the collective dynamics more difficult to be guided by informed individuals. Finally, to perform a non-even allocation among the two sites, the range of the carrying capacity parameter that allows informed individuals to guide these dynamics is even more narrow compared to the case of even allocation.

This paper has based its analysis on a seminal and important model of self-organised aggregation which was derived after experiments performed with real cockroaches (see Amé et al., 2006). However, experimental results we performed in (Firat et al., 2019) have already given us insight that, by having more control on the microscopic self-organised model of aggregation, it is possible to have informed individuals guiding the dynamics in a wider range of environmental conditions. In future work, we would like to focus our efforts in two directions. First, we would like to propose a macroscopic ODE model that more closely capture the microscopic design method discussed in (Firat et al., 2019) rather than the behaviour of natural cockroaches. Both in (see Amé et al., 2006) and in (Firat et al., 2019), the individual prpobability of leaving a site is a non-linear function of the density of individuals at a site. However, we believe that the specific non-linear dependency can be tuned in a way to make dynamics less dependent on environmental conditions when informed individuals are introduced. Secondly, the almost totality of self-organised models of aggregation in swarm of agents are engineered in order to amplify the effect of positive feedback, which is the prime mechanism responsible for aggregation. This is because the focus of all these studies is in achieving a single aggregate. Our approach to the problem differs from previous research works since we aim to use informed individuals to distribute the swarm on two or more aggregation sites (rather than concentrate it on a single site) according to the relative proportions of different types of informed individuals present in the swarm. Building upon the encouraging results of this macroscopic model, we are currently working at a new microscopic model that is able to regulate the effect of positive feedback in a way that is different between informed and non-informed individuals. This work aims to design individual controllers for swarm of robots that allow the designer to chose between inducing the swarm to aggregation on one site, and allocation of individuals to different sites in proportion to the relative frequency of informed robots present in the swarm.

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