

Emergence of the tradeoff law of social relationships in artificial societies driven by dual memory mechanisms

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Abstract

The trade-off between the number and closeness of friendships is one of the important features of communication systems. This distinguishes asynchronous text communication through the Internet (lightweight communications) from the face-to-face communication and social grooming of primates (elaborate communications). In this study, we modeled communication as messaging flows driven by edge and node memory mechanisms in order to investigate micro-mechanisms that realize the trade-off law and the differences between the two types of communications. Five patterns of social structures including the trade-off law emerged depending on the strengths of the memory mechanisms. This suggests how communication systems construct different social structures. These results provide insight into the design of online social networks.

Introduction

Social animals require social relationships and communications (Dunbar, 2018). Among them, primates, particularly humans, show grooming or conversation behaviors (Dunbar, 2004). In recent years, several new methods of communication have been developed for human society, such as e-mails, microblogs, social networking services (SNS), avatar chat applications, and video call communications.

In such communications, a power law is observed in the frequency distributions of message passings (Hosmann et al., 2011; Song et al., 2013; Fujihara and Miwa, 2014; Takano and Fukuda, 2017; Takano, 2018). The frequency is biased to some specific partners and frequent messaging takes place within close relationships (Takano, 2018; Tamarit et al., 2018). These relationships can be hierarchically classified into primary partners, intimate, best and good friends, friends, acquaintances, and that can be identified by name (Sutcliffe et al., 2012). In several social relationship datasets, the sizes of these groups follow a geometric series with a constant ratio ~ 3 (Dunbar, 2018). Such biased distribution can be realized via the trade-off mechanism between the number and strength of re-

lationships (Takano and Ichinose, 2018; Takano, 2018; Jiménez-Martín et al., 2020).

On the other hand, such graph structural features are connected to functional ones. Close social relationships lead to mutual cooperation (Curry et al., 2013; Harrison et al., 2011; Haan et al., 2006; Takano et al., 2016b,a), while having several weak social relationships helps in obtaining advantageous information, because novel information is often transferred to communities via such weak ties with outsiders (Granovetter, 1973; Dunbar, 2004; Onnela et al., 2007; Ellison et al., 2007; Eagle et al., 2010). Social structures show dependency on the types of communication, although the power law of messaging can be observed regardless of the types of communication. As per another reported law, such social structures are measured based on a tradeoff relationship between the number and strength of social relationships (Takano, 2018). The law can be expressed in an equation, $C = kq^a$, where k and q are the numbers and the strength of relationships respectively, and C is the cost for maintenance or construction of social relationships.

If the exponent a is small ($0 < a < 1$) strong relationships can be established easily. This can be seen in several cases of real communication data, e.g. face-to-face (F2F), telephone, and baboon grooming, several of which involve common features, strong temporal and spatial limitations and high messaging cost, classifying them as elaborate communication. Elaborate communication creates better impressions and intimate relationships (Dunbar, 2012; Vlahovic et al., 2012; Burke and Kraut, 2016); in humans, the reverse relationship is also true and elaborate communication is more commonly observed within close relationships (Burke and Kraut, 2014).

On the contrary, cases with a larger exponent ($1 < a$) often result in weak social relationships. Examples of this Twitter, e-mail, and SNS. Several of these are asynchronous communication systems, and the communication cost is low (lightweight communication).

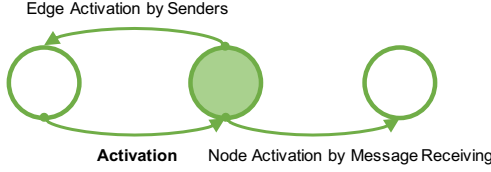


Figure 1: Two types of activation patterns

Here we study the relationship between social structures and the types of communication, using an agent-based model of artificial society considering elaborate and lightweight communication, and discuss possible relationships between social structures and the types of communication. Following the introduction of the model, we present the results, which can redisplay both the power law in communication frequency and the tradeoff law between the number and strength of relationships. Finally, we discuss on observed patterns and possible social systems.

Materials and Methods

Model

In the proposed artificial society, agents message with each other and such messages strengthen relationships between them. Agents can memorize two types of messaging history (Fig. 1): That of receiving and that of the senders. The former enhances the messaging frequency of the agent, regardless of the receivers, and the latter enhances messaging frequency to the specific senders. If the agents maintain perfect memory with all senders, they will eventually communicate only with intimates. On the contrary, if they have absolutely no memory of senders, as the receive several messages, they will send significantly more messages to others in a random manner. Here, these two types of enhancement are called node activation and edge activation, respectively. Node activation indicates cascading received information to others (Yi-Qing Zhang et al., 2015), while edge activation indicates a reply for the sender (Yi-Qing Zhang et al., 2015; Paranjape et al., 2017).

The model consists of N agents, where each agent i has messaging tendency λ_{ij} to another agent j . The tendency can be expressed in an equation,

$$\lambda_{ij} = \lambda_0 + \frac{\alpha_1}{N} \sum_{j'} \sum_k \exp(-\beta_1 \Delta_{ij'k}) + \alpha_2 \sum_k \exp(-\beta_2 \Delta_{ijk}), \quad (1)$$

where Δ_{ijk} is the elapsed time from the k -th signal from j to i , λ_0 is background tendency, and the other parameters, α_1, α_2 and β_1, β_2 , are weights on each activation and forgetting coefficient, respectively. That

is, α shows the activation impact by receiving, and β shows forgetting speed.

In addition, we introduce the upper limit, L , of the number of simultaneously messages, i.e. an agents' time and cognitive capacities are finite (Dunbar, 1998; Haerter et al., 2012; Kowert et al., 2014). Along with this tendency λ_{ij} , agents choose L other agents with the way of roulette selection and send messages to them if the tendency λ_{ij} is sufficiently large ($\sum_j \lambda_{ij} > L$). Otherwise, the agent i sends a message to j by interpreting λ_{ij} in terms of probability.

Parameter Settings

We tested our model with fixed parameters $\alpha_1 = 10.0$, $\alpha_2 = 10.0$. Following this, we tested some values $\{10^{cn}\}$ ($c = 0.25, n = -12, \dots, 7$) for β_1 and β_2 , respectively. More values for α_1 and α_2 were simultaneously tested, no significant difference from the original case could be observed (see Fig. A1). Under these parameters, we iterated $T = 1000$ steps. At each step, specific agents were selected as receivers for all agents and send messages. The number of agents was $N = 1000$ and the limit was $L = 5$.

Statistical Values

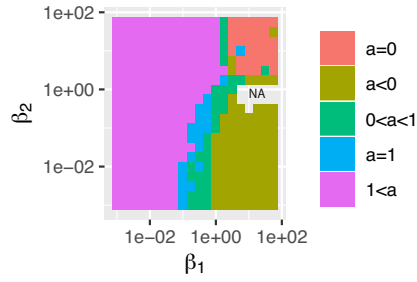
For the statistical data, we adopted the latter half of each test, $T > 500$. We confirmed that the data in the cases with $1500 \leq T \leq 2000$ did not differ from those in the case $500 \leq T \leq 1000$. Thus the results show the values in steady states. To count the relationships between agents, we adopted a threshold for the message frequency, where w_{ij} is greater than or equal to a median of w_{ij} in each trial. We also tested one more threshold, where w_{ij} is at 25 percentile or more, and the same pattern of results was confirmed.

As statistics, we computed some values based on previous work (Takano, 2018), (1) message frequency, $m_i = \sum_j w_{ij}$, (2) number of relationships or edges of agent i , k_i , and (3) mean strength of relationship of agent i , $q_i = m_i/k_i$. Exponent a was evaluated along the linear regression, $\log k = a \log q + b$ (Takano, 2018). We classified exponents with a one-sided test into three classes, $a < 0, 0 < a < 1, 1 < a$, if p-value is sufficient, ≤ 0.05 , otherwise we classified as $a = 0$ or $a = 1$.

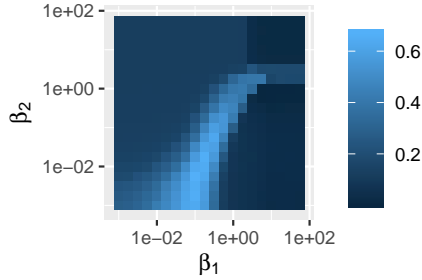
Although threshold of w_{ij} and L should be also explored these effects, in this paper, we used the above settings as an instance of the simplest settings.

Results

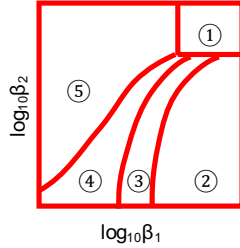
We show our results in Fig. 2a. The phase diagram describes five classes of exponent a on (β_1, β_2) , with the



(a) phase diagram of a



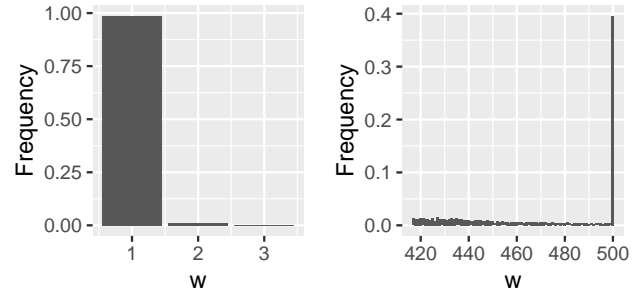
(b) Gini coefficients



(c) Five patterns

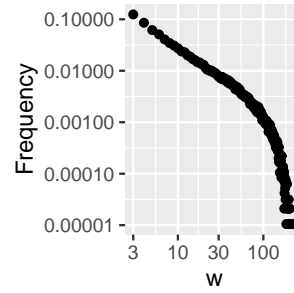
Figure 2: Results of simulation experiments. The NA (not available) area in Fig. a shows that there were no variances for agents' k and q , which were parameters of regressions for parameter a .

fixed parameter α_1, α_2 , in which both elaborate communication ($0 < a < 1$) and lightweight communication ($1 < a$) can be confirmed. We show a heatmap of the Gini coefficient based on the messaging frequency w_{ij} (Fig. 2b), and it shows that heavily biased distributions in messaging frequency are realized in the imbalanced region between forgetting coefficients β_1 and β_2 . In this mid-area, we can confirm to have moderate biased one. We classified into five sub-regions, where can be characterized by five typical patterns, shown in Fig. 2c. Among them, both features, that is, the power law in the message frequency and the tradeoff between the number and strength of the relationships can be confirmed in the sub-regions 3 and 4. We found five patterns in the message frequency distribution, as shown in Fig. 3. In those plots, we used parameters

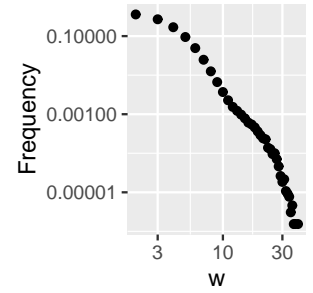


(a) Pattern 1

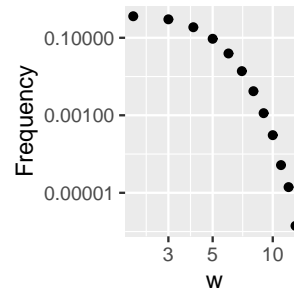
(b) Pattern 2



(c) Pattern 3



(d) Pattern 4



(e) Pattern 5

Figure 3: Typical distributions of w on each pattern. Log-log plots of cumulative distributions are shown in Fig. c-e because these are skewed.

$(56.23, 56.23)$, $(56.23, 0.001)$, $(0.316, 0.1)$, $(0.001, 0.001)$, and $(0.001, 6.23)$, for β_1 and β_2 in the five respective sub-regions. We can confirm the power law in pattern 3. The degree distributions of all patterns showed Poisson distributions, i.e. these networks were unstructured within the meaning of degree distribution.

In addition, Fig. 4 shows $(\log q_i, \log k_i)$. This indicates the relationships between the strength and number of social relationships. In these plots, we used the same parameters as in Fig. 3. The tradeoff relationship law $C = kq^a$ can be confirmed between the number and strength of social relationships in patterns 3, 4, and 5.

We note five patterns respectively. In pattern 1,

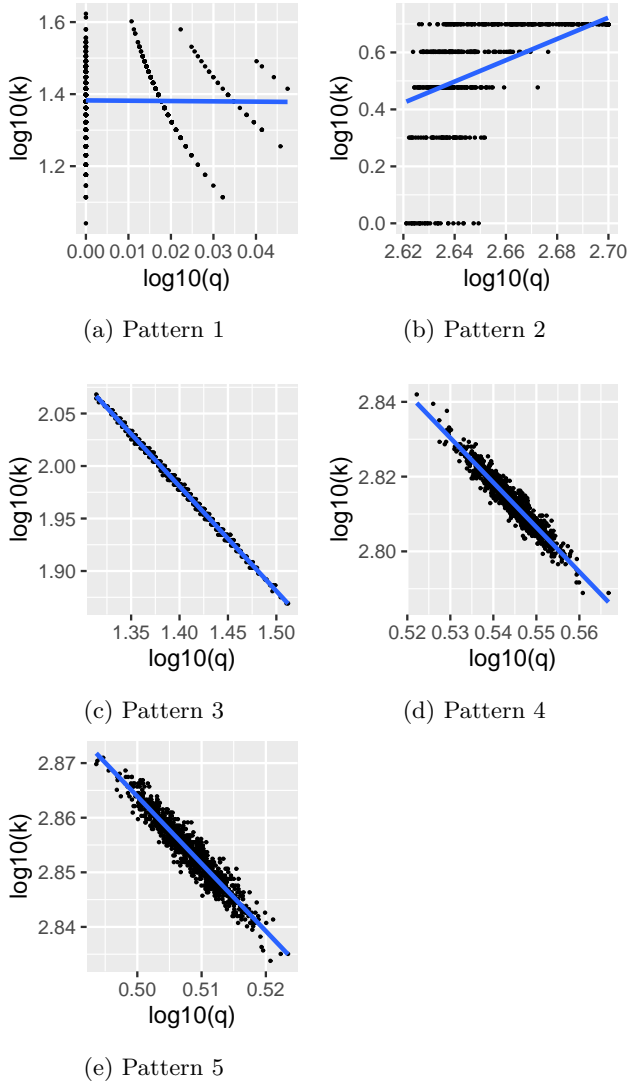


Figure 4: Scatter plots of $\log k$ and $\log q$. The blue lines are regression lines.

exponent a becomes meaningless, as can be seen in Fig. 4a. Because the forgetting coefficients β_1 and β_2 are very large, the messaging tendency λ_{ij} cannot increase and $\lambda_{ij} \sim \lambda_0$. This indicates that the messaging can simply be considered as Poisson process, and there is no specific relationship between number and strength in social communication.

In patterns 2–5, we confirmed a higher tendency (see Fig. A2b), λ_{ij} , indicating that the memories in each agent have an effect on the messaging frequency, w_{ij} , and thus on the number and strength of social relationships.

In pattern 2, the exponent is negative. Forgetting coefficient β_1 is large, while β_2 is small. This means that node activation is easily lost, while edge activation

can last longer. In the latter case, some specific edges are strongly activated and others are not, as shown in Fig. 3b; on the contrary, in principle, node activation cannot grow with limited messages. However, if an agent has a sufficient number of messages enough, the agent’s activation can be kept. In reality, the number and strength relationship shows a positive correlation, as shown in Fig. 4b.

In pattern 3 and 4, we can observe the trade-off relationships between the number and strength of social relationships, as shown in Fig. 4c and Fig. 4d. In addition, the bias in messaging frequency distribution is not very strong in Fig. 2b, Fig. 3c and Fig. 3d. The conditional difference between them is the balance between the forgetting coefficients β_1 and β_2 . Because the edges forgetting coefficient ratio, β_2/β_1 , is larger in pattern 4 than pattern 3, node activation functions better in pattern 4. This indicates that messages from certain specific agents can further enhance messages to certain other agents. On the contrary, if edge activation functions better, in pattern 3, messages from specific agents further enhance the relationship with the senders. Essentially, in pattern 3, there would be a mechanism more strongly focusing on some specific strong social relationships rather than widening the social relationships, as seen in pattern 4 (Fig. A2a).

In pattern 5, a large exponent, $1 < a$ (Fig. 4e), can be seen once more. However, the forgetting coefficient β_2 is larger than pattern 4, and this results in stronger node activation. Under such a parameter condition, the messaging frequency λ_{ij} is enhanced on average and this indicates random messaging. As a result, the message frequency distribution is exponential, as shown in Fig. 3e.

Discussion

Using an agent-based model, this study examines the relationship between social structure and types of communication. In the model, agents have two types of memory, dedicated to message receiving and senders, respectively, which can realize a variety of communication settings. In these terms, the proposed model also tests the relationship between social structure and cognitive mechanisms. It is often difficult to directly monitor cognitive processes, but using the proposed approach, cognitive mechanisms can be effectively guessed from social structure or communication features. At the same time, the cognitive mechanisms behind some specific communication tools can be investigated using communication logs, which contain information on social structures or communication features.

Humans are continually realizing multiple types of communication and social structures via a variety of communication tools in both real and cyber

space (Takano, 2018). Therefore, there exist further possible social relationships with unknown communication tools. Because such unknown possibilities may serve as either opportunities or risks, it is useful to study them in advance using artificial societies.

As the first step to such an approach, we constructed a very simple communication model driven by two types of cognitive mechanisms, with the primary aim of explaining some existing laws behind communication. This is because unknown possibilities are better understood based on tangible experiences in known situations. As shown in the results, the proposed model shows five different patterns realized with different forgetting coefficients, β_1 and β_2 . Among them, known statistics can be observed in patterns 3 and 4. The patterns are realized under the condition in which two forgetting coefficients are moderately mixed.

The first one, pattern 3, shows the weak tradeoff between the number and strength of social relationships and is realized when the forgetting coefficient for senders, β_2 , functions less. In other words, this situation occurs when agents have a stronger memory of senders. In the real world, this pattern is seen in communication methods such as F2F, telephone, and baboon grooming (Takano, 2018). The obtained results strikingly match such real-world experiences, where people memorize partners clearly in such communications. Notably, such communication is generally preferred with some specific intimates (Dunbar, 2012; Vlahovic et al., 2012; Burke and Kraut, 2016). In the proposed model, agents enhance their messaging tendencies with some specific others via such strong social relationships. Therefore, our preference can be explained from the perspective of the cognitive mechanisms behind the communication.

On the contrary, pattern 4 shows stronger trade-off and is realized when the forgetting coefficient for senders, β_2 , functions better. This indicates that agents have weaker memory of senders but stronger ones of receiving events. In the real world, this pattern is seen in lightweight communication, like twitter, e-mail, and SNS (Takano, 2018). With such types of communication, humans often experience similar situations, owing to the large number of acquaintances we acquire in such cyber communication spaces. In these situations, information is often diffused in a random manner. Once again, such experiences can be explained from a cognitive perspective using our model. Because receiving information enhances our memory for the information and activates our tendency to inform others on average, we tend to randomly diffuse it.

Consider unknown situations. Certain unknown patterns, such as 1, 2 and 5, were observed in the proposed artificial society. In pattern 1, messaging history shows

only the independent Poisson process. This is realized under the condition of higher forgetting coefficients β_1 and β_2 . In this region, communication is very sparse and social relationships are not strong. However, if the number of agents in communities can be adjusted, the situation can be changed. We cannot mature social relationships with a large number of partners with very limited communication, but it can be done in a small community using methods such as making members more memorable. However, it may be noted that the balance between the two memory mechanisms is essential for rich social relationships.

At one extreme, the situation with a very large forgetting coefficient, β_1 can be seen in pattern 2. In this case, agents strongly memorize specific communication partners and those partners are fixed. This means that social relationships are also fixed. This type of society can be realized in a limited situation, where community is extremely small, such as immigrants (Tamarit et al., 2018). In such a situation, community members can be remembered relatively easily. This results in more lasting memory of partners and such a memory condition matches this case in our model.

At the other extreme, agents can only retain memory of receiving messages. This is pattern 5. Because there is very limited memory of partners, this can be seen as an anonymous society. A strong specific social relationship cannot be observed and this pattern can be observed in exponential messaging frequency distributions. Information transfer in this type of society is solely via diffusion and has no specific transfer routes. The networks of all patterns were unstructured within the meaning of degree distribution. This suggests that the change of social relationships due to the trade-off may not depend on network structure.

In the real world, our societies primarily look like mainly pattern 3 or 4. If this is based on our preferences and is healthy, we can guide our society to such hopeful states by considering the balance between our two types of memory. If we wish to explore new types of societies, we can also challenge such extremes. Because we already possess rich technology for developing a variety of communication tools and social relationships, the degree of freedom of design in our society has also expanded.

The power law of social relationships emerged under the conditions of costs for constructing and keeping social relationship strengths (Jiménez-Martín et al., 2020). On the proposed model, considering communication cost effects on the power law would bring more insight.

This provides insight into the design of online social network services. For example, we can provide cyberspace where people realize physical-world-like com-

munication which facilitates the construction of close relationships by reinforcing the memory of senders through the system’s user interface, such as with recommendations of communication partners. These on-line close relationships will help people with limited social resources in the physical world (such as sexual minorities and victims of sexual abuse and bullying) to support their social relationships in the physical world (McKenna and Bargh, 1998; Green-Hamann et al., 2011; Kowert et al., 2014; Takano and Tsunoda, 2019). On the other hand, this study suggests that if a service intends to function as a social media platform for sharing news it should facilitate users’ activations by receiving information.

In the near future, we may live in a world where virtual meets reality. People would be changed their communications and social relationships by the world. The artificial society model and the insight of this paper might be needed for keeping human nature in these worlds because this can provides how people keep comfortable/useful social relationships.

Here, although human communication was the primary point of discussion, it must be noted that in the near future, our world will have more agents apart from humans, such as robots, cars and other smart devices equipped with artificial intelligence. In the connected world, realized as the Internet of Things, we will be faced with more opportunities and challenges to explore further. The model proposed in this study can be used to effectively research such possible connected worlds. While new agents may or may not possess many characteristics that differ from ours, such possibilities should be thoroughly tested, in advance for the safety and robustness of the system as a whole. At the same time, a sufficient understanding of them can enable effective applications that contribute to our collective well-being. Thus, an artificial society built on existing data and our experiences will show us much about the fruitful possibilities we are facing.

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Appendixes

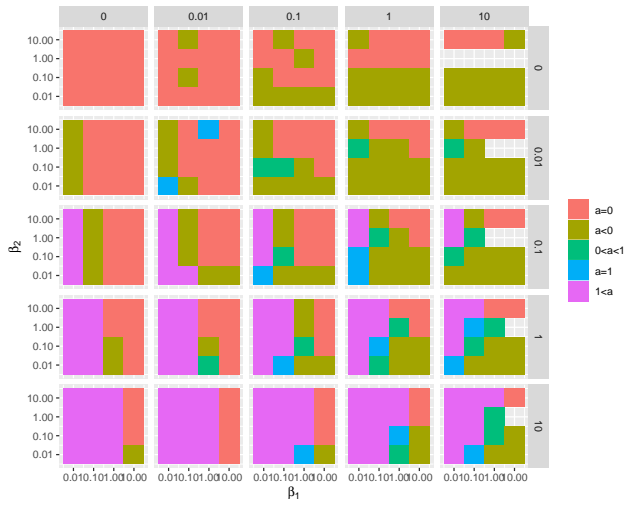
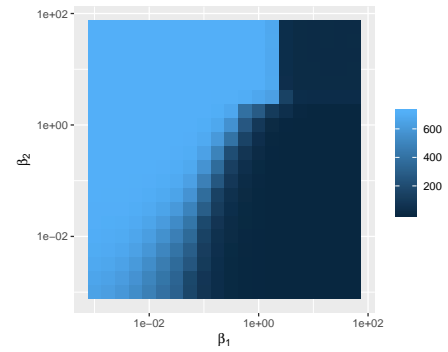
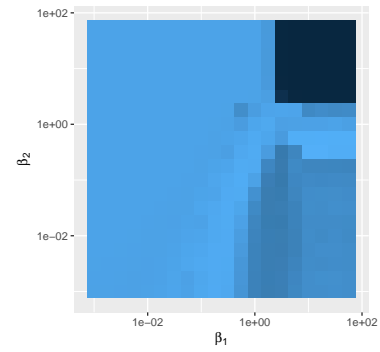


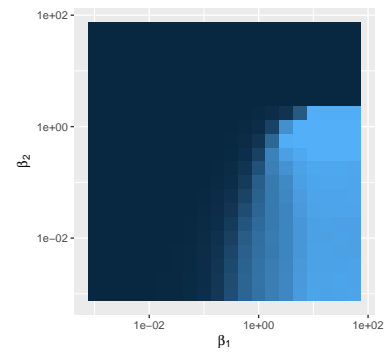
Fig. A1: Linear dependencies of α_1 and α_2 on the phase diagram for a (Fig. 2a). Fixed values were used about α_1 and α_2 .



(a) $\langle k \rangle$



(b) $\langle m \rangle$



(c) $\langle q \rangle$

Fig. A2: Heatmaps of the statistical values.