How Lévy Flights Triggered by Presence of Defectors Affect Evolution of Cooperation in Spatial Games

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Abstract

Cooperation among individuals has been key to sustaining societies. However, natural selection favors defection over cooperation. Cooperation can be favored when the mobility of individuals allows cooperators to form a cluster (or group). Mobility patterns of animals sometimes follow a Lévy flight. A Lévy flight is a kind of random walk but it is composed of many small movements with a few big movements. Here, we developed an agent-based model in a square lattice where agents perform Lévy flights depending on the fraction of neighboring defectors. We focus on how the sensitivity to defectors when performing Lévy flights promotes the evolution of cooperation. Results of evolutionary simulations showed that cooperation was most promoted when the sensitivity to defectors was moderate. As the population density became larger, higher sensitivity was more beneficial for cooperation to evolve.

Introduction

Cooperative behavior is necessary to sustain human and animal societies (Rand and Nowak, 2013; Dugatkin, 1997; Clutton-Brock, 2009). However, the previous studies of evolutionary games show that cooperation is not favored by natural selection compared to defection (Nowak, 2006a,b). Therefore, it has been suggested special mechanisms are needed for cooperation to evolve (Nowak, 2006a). In the proposed mechanisms, spatial (or network) reciprocity has often been studied (Nowak and May, 1992; Santos and Pacheco, 2005). In those traditional models, individuals do not move in the spatial environment because all spaces are occupied. By contrast, mobility is a fundamental trait of animals and humans because animals forage for food and people often move when they interact. Recently, spatial reciprocity with mobility has attracted great attention (Vainstein et al., 2007; Ichinose et al., 2013; Tomassini and Antonioni, 2015).

In particular, Tomassini and Antonioni focused on a special mobility type, called a Lévy flight (Tomassini and Antonioni, 2015). A Lévy flight is a kind of random walk but it is characterized by many small movements with a few big movements. More formally, the distance of movements follows a power-law distribution. It has been shown that some animal species use Lévy flights when foraging (Viswanathan et al., 1999, 1996). When resources are randomly distributed and there is no information on their locations, a search pattern based on a Lévy flight type is optimal. Another study shows that humans also use Lévy flights (Brockmann et al., 2006).

Tomassini and Antonioni studied the evolution of cooperation in spatial games where agents perform Lévy flights (Tomassini and Antonioni, 2015). In the model, they assumed two types of conditions where Lévy flights are performed by agents: 1) Agents always perform Lévy flights, 2) agents perform Lévy flights only when more than half of their neighbors are defectors. They showed that cooperation evolved only in the latter case.

Motivated by this study, we focus on the evolution of cooperation of mobile agents that perform Lévy flights in spatial games. Tomassini and Antonioni's model was a bit extreme in the sense that they only consider both types of conditions for Lévy flights. Here, we consider a continuous range of sensitivity to the presence of defectors to identify the optimal level of Lévy flights for the evolution of cooperation in spatial games. Moreover, we reveal how the sensitivity which yields the optimal cooperation changes depending on the population density.

Model

We developed an agent-based model of the evolution of cooperation in a square lattice where the sensitivity to neighboring defectors in Lévy flights is adjusted by step functions. First, agents are randomly distributed into an $L \times L$ lattice. The density of the agents is given by ρ. Thus, the number of agents is $N = L^2\rho$. At the beginning, half of the agents are cooperators and the other half are defectors. Then, the following process is repeated until the specified number of time steps ($t_{\mathrm{end}} = 500$) is obtained.

1. One agent is randomly selected from the whole population. (This agent may be selected multiple times in one time step because we used an asymmetric update scheme.)
2. The agent plays one of four games with its neighbors and obtains the payoff. The neighboring agents also play

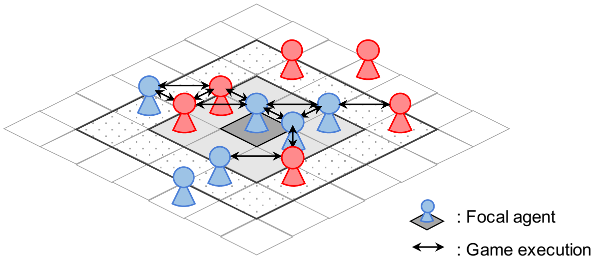

Figure 1: Game executions in Step 2.

the game with their neighbors and obtain payoffs (Fig. 1). The detail of these games is described below.

3. The agent imitates the strategy of the neighbor that obtained the highest payoff within the neighborhood, including itself.
4. The agent is unsatisfied when the neighbors are defectors. If the fraction of defectors is equal to or greater than a threshold value, it performs a Lévy flight to another cell if the cell is empty. Otherwise, the agent does not move.
5. The above is repeated N times, which is regarded as one time step (t).

In Step 2, the agent and its opponent play one of four common two-person, two-strategy, symmetric games. The two strategies are cooperation (C) and defection (D). If both cooperate, they receive R; if one cooperates and the other defects, the former obtains S and the latter obtains T; if both defect, they receive P. The games are classified into the following four depending on the payoff relationships: the Harmony Game (HG); $R > T, S > P$, the Stag Hunt (SH); $R > T > P > S$, the Prisoner's Dilemma (PD); $T > R > P > S$, and the Hawk-Dove game (HD); $T > R > S > P$.

In Step 4, the condition of dissatisfaction is provided as follows. We define sensitivity $s = 1 - \frac{i}{n_{\max}}$ $(0 \leq s \leq 1)$ where $n_{\max}$ denotes the maximum number of agents in the neighborhood, that is, $n_{\max} = 8$. i denotes a threshold value for every level of sensitivity. We consider nine threshold values $i = 0, 1, ..., 8$. Then, we assume the following step functions which decide whether agents perform Lévy flights or not

$$P(s) = \begin{cases} 1 & (1 - \frac{n_{\mathrm{D}}}{n} \leq s) \\ 0 & (\text{otherwise}), \end{cases} \quad (1)$$

where n denotes the number of agents in the neighborhood and n_{D} denotes the number of defectors in the neighborhood. $P(s)$ is the probability that agents perform Lévy flights. From Eq. 1, nine step functions are obtained. When the first equation in Eq. 1 is satisfied, agents perform Lévy flights. Note that all agents have the same sensitivity.

The distance of Lévy flights is given by a power-law distribution $P(d) = d^{-\alpha}$. We fixed $d = 2$ in the simulations. Actual observations suggest that animals use $1 \leq \alpha \leq 3$.

We use $L = 50$ and $\rho = 2/3$ unless otherwise noted. For the game parameters, we fix $(R, P) = (1, 0)$ while changing $-1 \leq S \leq 1$ and $0 \leq T \leq 2$.

Result

We focus on whether and how Lévy flights promote cooperation in spatial games. Figure 2 shows the snapshots of the simulation where the sensitivity is $s = 1/2$. Here, we set $(S, T) = (-0.4, 1.4)$, thus the game is the PD. In the figure, cooperators (defectors) are shown in blue (red). First, cooperators almost go extinct but a few clusters still survive (from $t = 0$ to 50). If cooperators are clustered, they can obtain higher payoffs within the areas. Thus cooperative clusters can survive. Then, cooperators can expand their areas by moving locally and avoiding defectors based on the adaptive Lévy flights (from $t = 200$ to 500). Moreover, Lévy flights which consist of rare big movements also benefit cooperation because cooperators can inhabit new areas.

Next, we show how cooperation evolved in the whole TS plane when $s = 0, 1/2$, and 1. Figure 3 shows the average fraction of cooperators, denoted by $\bar{f}_{\mathrm{C}}$, at the final step of the simulations ($t_{\mathrm{end}} = 500$). In the right three panels, cooperation evolved when the games were the HG and the SH because cooperation between two agents (R) is most beneficial. In contrast, cooperation was hard to evolve when the games were the PD and the HD. In those two games, unilateral defection (T) is most beneficial. Moreover, defection is the dominant strategy in the PD due to $T > R$ and $P > S$. Thus, the PD resulted in the worst case for cooperation to evolve. When we compare the three results for s values, cooperation evolved in the moderate sensitivity $s = 1/2$. Here, we try to find out which s produces the optimal cooperation level. Then, we changed s with summing up $-1 \leq S \leq 1$ and $0 \leq T \leq 2$. Figure 4 shows the moderate $s = 1/2$ most promoted cooperation.

Finally, we focus on how cooperation evolves depending on density ρ. Figure 5 shows $\bar{f}_{\mathrm{C}}$ when sensitivity s and density ρ were changed. When the sensitivity was at its highest $s = 1$, cooperation did not evolve at all. When the sensitivity was too low $s \leq 1/4$, cooperation did not evolve much. Thus, even when the density was changed, moderate sensitivities $1/4 \leq s < 7/8$ were best for cooperation to evolve.

Cooperators need to be clustered to survive. Too much sensitivity is bad because it prevents cooperators from clustering. Especially, when $s = 1$, agents (both cooperators and defectors) continue to move at all times. On the other hand, with quite low sensitivity, cooperators can form clusters but they will eventually be invaded by defectors because, with this sensitivity, cooperators are too patient with defectors. As a result, moderate sensitivity works better for cooperators to keep their clusters.

Moreover, as the density became larger, higher sensitivity promoted more cooperation. In sparse situations (low densities), cooperative clusters tend to be maintained because

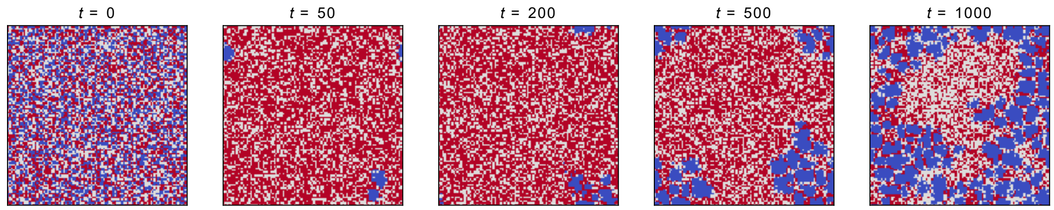

Figure 2: Snapshots of a simulation. Cooperators (Defectors) are shown in blue (red). Initially ($t = 0$), cooperators and defectors are fifty-fifty. Cooperators then almost go extinct but a few cooperative clusters survive. Finally, by utilizing Lévy flights, the cluster of cooperators can invade the sea of defectors. We used the PD game where $(R, S, T, P) = (1, -0.4, 1.4, 0)$. $L = 100$ and $\rho = 2/3$.

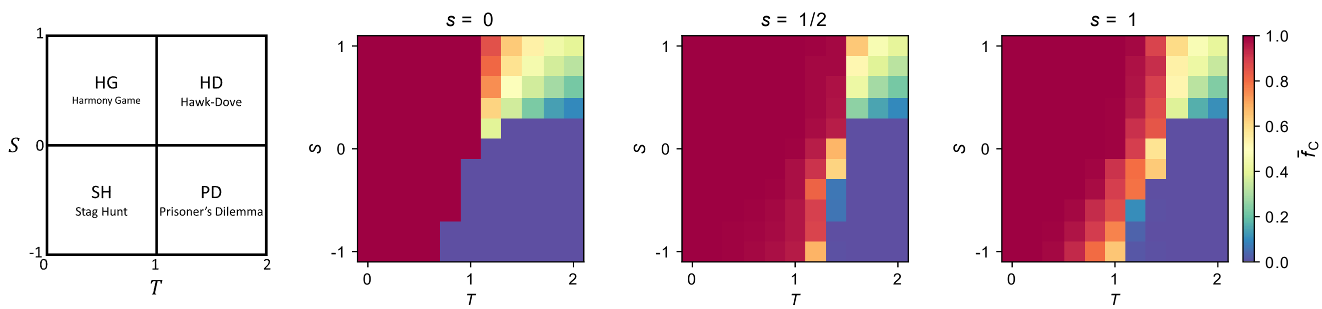

Figure 3: Fraction of cooperators $\bar{f}_{\mathrm{C}}$ in the whole TS plane when $s = 0, 1/2$, and 1. The plane is divided into the four games (HG, SH, PD, and HD) depending on the T and S values. We averaged 10 simulation runs for each data point.

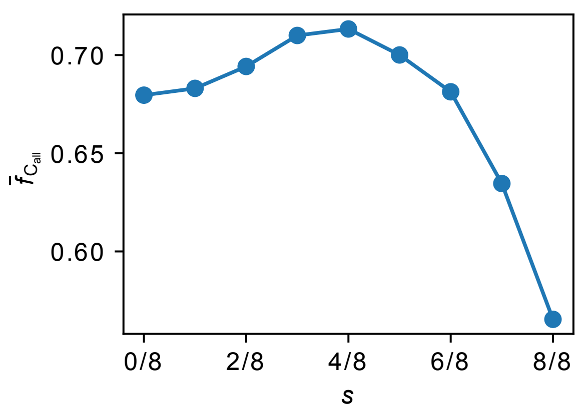

Figure 4: Fraction of cooperators $\bar{f}_{\mathrm{C_{all}}}$ as a function of sensitivity s where $\bar{f}_{\mathrm{C_{all}}}$ is obtained by averaging all $\bar{f}_{\mathrm{C}}$ in the whole parameter ranges ($0 \leq T \leq 2$ and $-1 \leq S \leq 1$). For each point on the blue line, 10 simulation runs are averaged. Cooperation was promoted the most in the moderate sensitivity $s = 1/2$. We averaged 10 simulation runs for each data point.

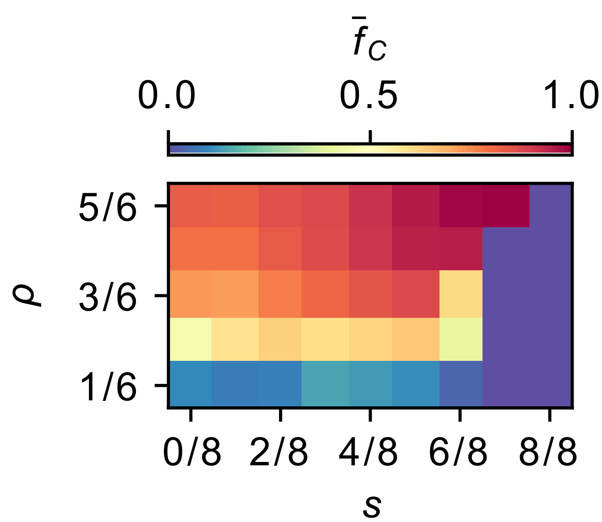

Figure 5: Fraction of cooperators $\bar{f}_{\mathrm{C}}$ as functions of densities and sensitivity. PD game with $(R, S, T, P) = (1, -0.4, 1.2, 0)$ was used. We averaged 10 simulation runs for each data point.

they are surrounded by few defectors. In contrast, in dense situations, cooperative clusters tend to be destroyed by surrounding defectors. In that case, it is better for cooperators to escape from their current positions by moving to other cells. Thus, higher sensitivity can promote cooperation in the dense situations.

Conclusion

We investigated the effect of how sensitivity to defectors when performing Lévy flights promotes the evolution of cooperation. We constructed an agent-based model where agents play games with their neighbors, update their strategies, and perform Lévy flights to move to other cells in a square lattice. Compared to the previous work, we tested various levels of sensitivity to defectors for the condition of Lévy flights and analyzed the relationship between the sensitivity and density for cooperation. The evolutionary simulations showed that cooperation was most promoted in the moderate sensitivity. Furthermore, as the density increases, higher sensitivity to defectors is better for cooperation to evolve.

We previously suggested that a uniform (not power-law) distribution of the jump distance promoted cooperation (Ichinose et al., 2013). Thus, long-range mobility may simply enhance cooperation. Comparing various distribution types for movements is necessary to understand the effect of Lévy flights on the evolution of cooperation. A future work includes the comparison between Lévy flights and jumps which follow a uniform distribution.

Acknowledgments

This work is support by HAYAO NAKAYAMA Foundation for Science & Technology and Culture .

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