Hyperdescriptions of Quantum Dynamics - A Case Study for Avian Magnetoreception

Dragana Laketić

Independent Researcher
Trondheim, Norway
dragana.laketic@gmail.com

Abstract

A previously established information theoretic approach for dynamical hierarchies is hereby revisited for the case when a higher hierarchical level, which is observable in the realm of classical physics, arises from quantum dynamics. The focus of the paper is on the description of information transfer from quantum to classical level. As an example of complex functionality emerging at a higher level from quantum dynamics in a biological system, a case study of coherent spin dynamics, which is hypothesised to be one of the mechanisms of avian magnetoreception, is presented within such an approach. The findings may be useful for better understanding of the phenomena emergent from quantum processes which occur in biological complex systems. Moreover, the lessons learned may provide valuable knowledge for nature-inspired engineering in a bottom-up fashion within the field of nanotechnology.

Introduction

Hierarchical organisation of complex systems emergent in nature (Simon, 1962; Pattee, 1973) is non-trivial. It is based on the dynamics undergone by physical entities, their mutual interactions, usually of a non-linear character, which result in the emergence of novel phenomena observable at higher levels. The emergent novelty, the more that is different (Anderson, 1972), may be observed as a property, behaviour or some spacial or temporal pattern, i.e., the nature of the emergents is rather organisational than structural. The entities observed at different hierarchical levels vary in size and the dynamics they undergo vary in speed: lower level dynamics happens at higher rates than the observed dynamics at higher levels. Further, each higher level assumes certain loss of information regarding the lower level. Attempts to capture all aspects of such hierarchies within a theoretical framework are far from trivial and they have been a subject of debate (Lenaerts et al., 2005; Gross and Lenaerts, 2002). Emergent novelty and loss of information are properties well captured by the information theoretic approach presented in (McGregor and Fernando, 2005). There, the relations between conditional entropies for the established state space descriptions at different levels are used to provide necessary conditions for the existence of dynamical hierarchies.

Hierarchies which characterise complex systems emergent in nature are phyical and, as such, amenable to the laws of physics. Therefore, if the rise of hierarchies is to be examined from the lowest physical level, the level which belongs to the realm of quantum physics must be addressed. Moreover, in recent years it has been recognised that quantum mechanical phenomena may have important roles in various biological processes such as the transfer of energy within photosynthetic complexes (Engel et al., 2007), avian magnetoreception for sensing the geomagnetic field (Ritz et al., 2000) or the motion of potassium ions through voltage gated ion channels in neural cells (Summhammer et al., 2018). However, such findings in biological systems were unexpected. While quantum properties and behaviours are usually observed under controlled lab conditions where the system is kept (nearly) isolated from the environment in order to preserve quantum coherence, the evidence found in biological systems shows that quantum processes do take place under the conditions where they are open to interactions with the environment. As such, they are an additional motivation to revisit the rise of hierarchies from the quantum level and address the quantum nature of the entities and the dynamics undergone at this lowest physical level.

Section II contains an overview of the information theoretic approach from (McGregor and Fernando, 2005). Section III addresses the challenge of this paper where information theoretic approach to dynamical hierarchies is revisited for the case of quantum dynamics. Section IV presents a case study of quantum dynamics which is hypothesised to be one of the mechanisms of avian magnetoreception and Section V summarises the presented work and looks into the future work.

Overview of the Information Theoretic Approach to Dynamical Hierarchies

The approach presented in (McGregor and Fernando, 2005) considers the relations between conditional entropies established for the state space descriptions of the system dynamics at two hierarchical levels which capture the loss of information from the lower level and the novelty brought about

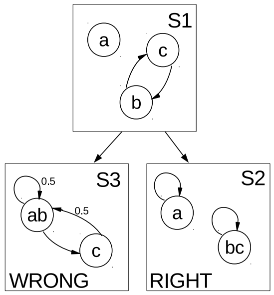

Figure 1: An illustration of the right way to hyperdescribe dynamics at a lower level, adapted from (McGregor and Fernando, 2005): the lower level dynamics given by the state space description S1 is described in two ways, S2 and S3, at a higher level. The description S3 fails to capture the emergent unit at a higher level which results from the c and b entities and their mutual interactions - the dynamics at the lower level which builds a higher emergent entity is lost in this description. Description S2 captures the emergent phenomenon well.

by the emergent higher level.

It is assumed that there exists a discrete state space description for each hierarchical level captured by the expression:

$$\mathcal{S} = \{s_i(t) : s_i(t) \xrightarrow{f_t} s_j(t+1)\}_{i,j=1}^{N} \quad (1)$$

where $\mathcal{S}$ stands for the state space description, $s_i(t)$ is a discrete state i of the system at a discrete time t from a finite set of N possible states, f_t a transition function which captures the probability p_{ij} of the state $s_i(t)$ transitioning to the state $s_j(t+1)$ at the next time step. The (Shannon) entropy for such system is given by:

$$H[\mathcal{S}] = -\sum_{s} P(\mathcal{S} = s)\log(P(\mathcal{S} = s)) \quad (2)$$

For a different description of the system, $\mathcal{Q}$, which is of the form as given by Eq. (1) but with the different set of probabilities which define the state transitions in consecutive discrete time steps, the conditional entropy is defined as:

$$H[\mathcal{S}|\mathcal{Q}] = -\sum_{q}\sum_{s} P(\mathcal{S} = s|\mathcal{Q} = q)\log(P(\mathcal{S} = s|\mathcal{Q} = q)) \quad (3)$$

Let us assume that a dynamical hierarchical system is described at two levels in a form as in Eq. (1) where description $\mathcal{S}_b$ is the description at a higher hierarchical level and $\mathcal{S}_a$ at a lower level. Then, according to (McGregor and Fernando, 2005):

1. There exists a *description function*, d_{ab}, which maps the states from level a to the states at level b:

$$d_{ab}(s_a(t)) = s_b(t) \quad (4)$$

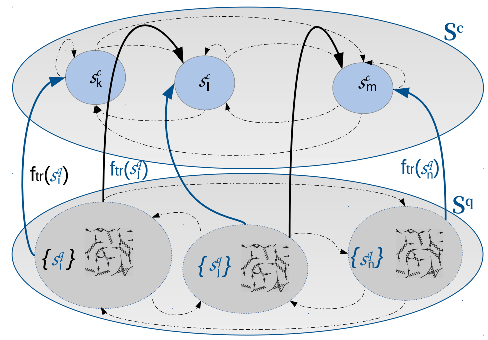

Figure 2: Schematic view of a general hierarchical system under investigation. Quantum dynamics described by S^q is observed at the classical level and described by S^c.

which captures the lower level dynamics well in a sense as illustrated by the example in Fig. 1.

2. *State dependence* is defined as:

$$r(\mathcal{S}_b, t) = 1 - \frac{H[s_b(t+1)|s_b(t)]}{H[s_b(t+1)]} \quad (5)$$

3. *Distinctness* is defined as:

$$q(\mathcal{S}_b, \mathcal{S}_a, t) = 1 - \frac{H[s_b(t+1)|s_b(t)]}{H[s_a(t+1)|s_b(t)]} \quad (6)$$

If the function given by Eq. (4) exists and if the state dependence defined by Eq. (5) and the distinctness defined by Eq. (6) are greater than 0 and less than 1, then the description $\mathcal{S}_b$ is said to hyperdescribe description $\mathcal{S}_a$. The state dependence can also be understood as a measure of how well the description at a higher level captures the novelty which arises at this level. The condition that it be less than 1 confirms that the state description $\mathcal{S}_b$ is a good description. The closer this value is to 1, the less uncertainty about the system dynamics there is if the state description $\mathcal{S}_b$ is known. Distinctness represents the measure of uncertainty about the state of the system at the higher level relative to the uncertainty about the state at the lower level provided the state at a higher level is known. Therefore, this measure captures a partial loss of information from the lower level.

From Quantum to Classical Level

The system under investigation is schematically shown in Fig. 2. At the very bottom, there are entities which obey the laws of quantum mechanics, symbolically represented by Feynman's diagrams at the level $\mathcal{S}^q$. At a higher level, the dynamics and the state transitions can be observed in the domain of classical physics, $\mathcal{S}^c$. The dynamics and the resulting state transitions are represented by the dashed arrows. Full line arrows from $\mathcal{S}^q$ to $\mathcal{S}^c$ represent the mapping from S^q to S^c, denoted as some functional dependency, $f_{tr}(*)$.

System description at the quantum level

A state of the system at the quantum level is described by a vector in a Hilbert space pertaining to the system, $\mathcal{H}_Q$, which is spanned over the orthonormal basis $\{|q_i\rangle\}$. In general, the system is in a mixed state and, therefore, best expressed by a density matrix, ρ, so that the description of the system at the quantum level, $\mathcal{S}^q$, can be written as:

$$\mathcal{S}^q = \{s_i^q(t) = \rho_i(t) : \rho_i(t) \xrightarrow{U,Env} \rho_j(t+1)\}_{i,j=1}^{N_q} \quad (7)$$

where the density matrix is assumed to be a tensor product of the density matrices of the component systems. The assumption that the system is composite stems from the fact that a higher level arises from the dynamics and interactions between the lower level entities so that a number of entities at the lower level is implicitly assumed.

The system dynamics is captured by the time evolution of the quantum system denoted by $\{U, Env\}$ over the state transition arrow. This is to mark that the state of the system may change due to some unitary transformation, U, and also due to the interactions with the environment, Env.

System description at the classical level

The description at this level, $\mathcal{S}^c$, as in Eq. (1), is given by a finite set of possible N_c discrete states $s_i^c(t)$ observed at the classical level and a transition function f_T which describes state transitions for the successive time instances t and $t+1$:

$$\mathcal{S}^c = \{s_i^c(t) : s_i^c(t) \xrightarrow{f_T} s_j^c(t+1)\}_{i,j=1}^{N_c} \quad (8)$$

The transition function f_T is given as a set of probabilities:

$$f_T = \{p_{ij} : p_{ij} = p(s^c(t) = s_i^c, s^c(t+1) = s_j^c)\}_{i,j=1}^{N_c} \quad (9)$$

The probability that the system is in some state $s_i(t)$ at the time instance t is given by $P(s_i(t)) = \sum_j p_{ji}$ where the summation goes over all the values j for which the transition function Eq. (9) is non-zero.

Description function

Any extraction of information from a quantum system, any probing of the system by its environment, is described within the theory of quantum measurements. Therefore, the description function d_{qc}, which maps the states of the system at the quantum level to the states at the classical level, is given by an appropriate set of quantum measurement operators, $\{E_k\}$. For the open system at hand, the operator-sum formalism (Nielsen and Chuang, 2000), i.e., Kraus operators (Kraus, 1971), are best suitable. The states at the classical level are the possible values of some observable, i.e., an outcome k of the measurement obtained by an operator E_k, as shown in Fig. 3.

According to the theory of open quantum systems (Breuer and Petruccione, 2002) and the model of quantum measurements introduced by John von Neumann (Von Neumann,

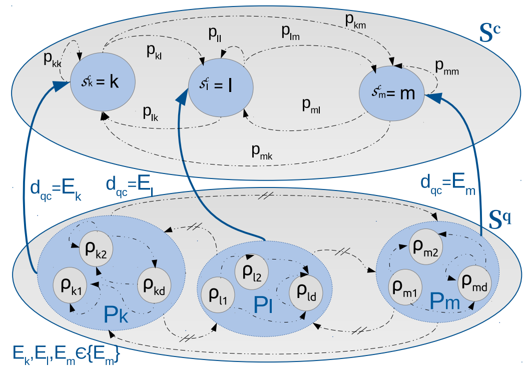

Figure 3: Schematic view of a hierarchical system under investigation where S^c hyperdescribes S^q according to the information theoretic approach to dynamical hierarchies, (McGregor and Fernando, 2005). See the text for explanation.

1955), it is sufficient to include the part of the environment which interacts with the system quantum mechanically and observe the unit evolution of such composite system. For this part of the environment, there is a Hilbert space $\mathcal{H}_E$ associated with it and assumed to be spanned over the orthonormal basis $\{|e_i\rangle\}_{i=1}^{N_E}$. The dimension of $\mathcal{H}_E$, N_E, is sufficiently large to accommodate for the quantum process under investigation, i.e. $N_E > N_Q$ (Nielsen and Chuang, 2000).

For quantum systems, the observed value depends on how the system is observed, how it is probed by the environment. The classical emergent from the quantum dynamics will be observed only as long as the coherence of the quantum system is sustained. The prolongued coherence is possible if the perturbations of the system state due to the interactions with the environment take place at the rate much higher than what it takes for the system to relax to its equilibrium state. Such conditions are known as Quantum Zeno Effect (QZE) and they were first mentioned in (Misra and Sudarshan, 1977). For the case at hand, it is needed that measurement operators $\{E_k\}$ disturb the unitary evolution of the quantum system in a way which acts as a projection operator (Itano, 2009). To account for the continuous observation time in our model, it shall be further assumed that the observation interval T consists of a number of discrete time intervals t which are much smaller than T and closely spaced in time:

$$T = nt, \quad t \lll T, \quad n \to \infty \quad (10)$$

Let us assume that the system is in the state $\rho(0)$ at the beginning of the observation and expressed by the spectral composition over its orthonormal basis:

$$\rho(0) = \sum_i \lambda_i(0)\, |q_i\rangle\langle q_i| \quad (11)$$

where $\{\lambda_i(0)\}$ are the eigenvalues of the initial density matrix. Then, after the first observation period t some value

k will be observed at the classical level as the result of the measurement operator E_k bringing the system to the new state:

$$\rho(t) = \frac{E_k \rho(0) E_k^\dagger}{tr(E_k^\dagger E_k \rho(0))} \tag{12}$$

with the probability of this outcome:

$$p_k(t) = tr(E_k^\dagger E_k \rho(0)) \tag{13}$$

The new state written as a spectral decomposition over $\mathcal{H}_Q$ with new eigenvalues $\lambda_i(t)$ per orthonormal basis vector $|q_i\rangle$:

$$\rho(t) = \sum_i \lambda_i(t) \, |q_i\rangle\langle q_i| \tag{14}$$

Further observations for $nt \in [0, T_k]$ would also lead to the change of the state of the quantum system which could be expressed in the form as in Eq. (14) with the eigenvalues $\{\lambda_i(nt)\}_{i=1}^{N_Q}$. For the observation k to be sustained during the whole observation interval T_k, it is needed that all the states of the quantum system, to which the system is brought by the originally applied operator E_k, are mapped to the same value of the observable, i.e., k:

$$p(s^c(nt) = k | s^c((n-1)t) = k) = 1, \quad \forall nt \in (0, T_k]$$
$$\rho(nt) \xrightarrow{E_k} k, \quad \forall nt \in [0, T_k] \tag{15}$$

Since $E_k = \langle e_k | U | e_0 \rangle$, where $|e_0\rangle$ is the initial state of the environment, and since the quantum states are given by Eq. (14), it follows that the condition given by Eq. (15) will be satisfied if the following holds:

$$\langle e_k, q_j | e_k, q_j \rangle \neq 0, \quad \lambda_j \neq 0 \tag{16}$$

The orthonormal basis vectors $\{q_j\}$ which correspond to non-zero eigenvalues define a subspace of $\mathcal{H}_Q$. Therefore, the condition given by Eq. (16) states that for an operator E_k there is a subspace of $\mathcal{H}_Q$ from which the states are mapped to the unique value of the observable determined by the operator E_k. In Fig. 3 this subspace is denoted as P_k. In general, this condition is to hold for each operator from the set of measurement operators:

$$\forall E_k \in \{E_m\}, \quad \exists P_k = \{s^q_{j_k}\} \subset \{s^q_j\}_{j=1}^{N_Q} :$$
$$(\langle e_k, q_{j_k} | e_k, q_{j_k} \rangle \neq 0) \wedge (\forall s_j \notin P_k, \langle e_k, s_j | e_k, s_j \rangle = 0) \tag{17}$$

In Fig. 3 the subspaces $\{\mathrm{P}_k\}$ are shadowed darker in the quantum system space for each operator shown. It is also assumed that the observation is performed in a continuous time fashion as in Eq. (10) per each interval T_i when the observation is the result of the measurement operator E_i, as illustrated in Fig. 4.

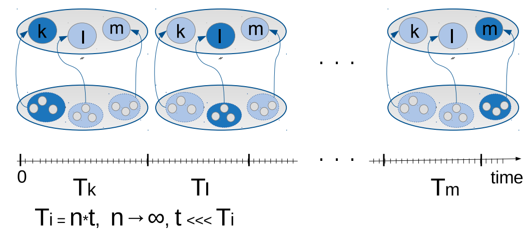

Figure 4: Illustration of the observed classical states for the description function based on the quantum measurement operators for the measurements made so as to sustain coherence per intervals T_i.

State dependence

In order to satisfy the state dependence criterion, $r < 1$, Eq. (5), the following inequality must hold:

$$H(s^c(t+1) | s^c(t)) < H(s^c(t+1)) \tag{18}$$

The conditional entropy on the left hand side is given by Eq. (3) for the two distributions S and Q of the state probabilities at two consecutive time instances. Under the assumptions made in the previous subsection about the observation of the quantum dynamics, the conditional probabilities $p_{ji} = p(s^c(t+1) = i | s^c(t) = j)$ are equal to 1 for any two consecutive instances t and $t+1$ within one observation interval T_i when the continuous measurement of the quantum dynamics maps a subspace of quantum space into the same classical observable i. Therefore, the factors in the expression for conditional entropy which correspond to such cases will be equal to 0 since $log 1 = 0$. The remaining non-zero factors stand for the transitions from states $s^c(t)$ to states $s^c(t+1)$ observed by different operators, i.e., $i \neq j$, which corresponds to the transitions from one quantum subspace P_j to its mutually orthogonal subspace P_i. Therefore, it can be written:

$$H(s^c(t+1) | s^c(t)) = -\sum_i \sum_{j, \exists j \to i} p_{ji} \cdot log_2(p_{ji}) \tag{19}$$

The right hand side of the inequality is given by Eq. (2) for the state probabilities given by Eq. (13) and is some positive non-zero value for the general case where there is more than one classical state observed. For each state of the system, the law of total probability holds:

$$p(s^c(t+1) = i) = \sum_j p_{ji} \cdot p_j \tag{20}$$

so that the unconditional entropy of the system description

S^c can be written as:

$$H(s^c(t+1)) = -\sum_i \sum_{j,\exists j \to i} (p_{ji} \cdot p_j) \cdot log_2(p_{ji} \cdot p_j) \quad (21)$$

Since the domain of the probability function is interval $[0,1]$, for each summation factor in Eq. (19) it can be written:

$$p_{ji} \cdot p_j \leq p_{ji} \quad (22)$$

Accounting for the properties of the logarithmic function (monotonically rising and negative for arguments from the interval $(0,1)$, i.e., $(x < y) \Rightarrow (log(x) < log(y))$ and $(0 < x < 1) \Rightarrow (log(x) < 0)$, it follows that:

$$H(s^c(t+1)) > -\sum_i \sum_{j,\exists j \to i} p_{ji} \cdot log_2(p_{ji}) \quad (23)$$

The right hand side of this inequality is the conditional entropy of system S^c as can be seen by comparing the last equation with Eq. (19). Therefore, it follows that the state dependence condition expressed by Eq. (18) holds.

Distinctness

For the distinctness criterion to be satisfied, $d < 1$, Eq. (6), the following inequality must hold:

$$H(s^c(t+1)|s^c(t)) < H(s^q(t+1)|s^c(t)) \quad (24)$$

For the known state at the classical level, the uncertainty about the state of the system at the quantum level is greater than the uncertainty about the system at the classical level. This condition may be understood to hold since there is an upper limit on what can be measured about the quantum system which is given by Holevo quantity χ (Holevo, 1973). For the explicit proof, we can proceed as in the previous subsection and first consider the case when the two consecutive time instances t and $t+1$ refer to the observation interval T_k where the observed values are obtained by the same operator E_k. The conditional entropy on the right hand side of Eq. (24) for such cases is equal to 0, i.e., there is no uncertainty in what the state $s^c(t+1)$ will be if the state $s^c(t)$ is known. As for the case of the state dependence, the conditional probabilities $p(s^c(t+1) = k|s^c(t) = k)$ are equal to 1 and their log value makes the corresponding summation factors equal to 0. Therefore, for the inequality given by Eq. (24) to hold, it suffices to show that:

$$H(s^q(t+1)|s^c(t)) > 0 \quad (25)$$

The left hand side represents the entropy of a quantum system conditioned on its observed state. For the case of time instances t and $t+1$ being from the same observation interval T_k, it is equal to unconditional entropy of a quantum system, given by Von Neumann entropy, $H(s^q(t+1)) = S(P_k) = \sum_{k_i} S(\rho_{k_i})$, where $\{\rho_{k_i}\}$ are the quantum states

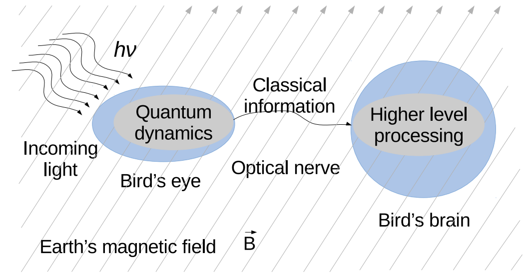

Figure 5: A schematic view of avian magnetoreception according to the model proposed in (Ritz et al., 2000).

from the subspace P_k. The Von Neumann entropy equals 0 only if the quantum system is in pure state, which is a trivial case with no interesting dynamics and is, therefore, discarded. It can be concluded that the inequality Eq. (25) holds for the case when $t, t+1 \in T_k$.

For the case when $t \in T_k$ and $t+1 \in T_l, k \neq l$, the same reasoning as for the state dependence holds with the additional condition that in this case we look at the uncertainty of quantum states dependent on the quantum level transitions. With this, it can be claimed that the inequality given by Eq. (25) holds. Finally, it can be concluded that the classical system S^c hyperdescribes the quantum system S^q.

A Case Study for Avian Magnetoreception

A model of sensing the geomagnetic field in European robin (Wiltschko and Wiltschko, 2019) which is based on the coherent spin dynamics in the cryptochrome molecules in the bird's retina was proposed in (Ritz et al., 2000). It was further confirmed in (Kerpal et al., 2019) and the underlying mechanism proven in vitro (Maeda et al., 2012). However, the experimental proof in vivo is still to be provided.

A global view on this mechanism is schematically shown in Fig. 5. The incoming light into the bird's eye creates a radical pair [1] whose spin dynamics is affected by the geomagnetic field. The spin dynamics of interest includes spin interactions among the radical pair electron spins and the nuclear spins from the surrounding molecules. The information about the geomagnetic field is encoded in the ratio of the two kinds of products of the recombination of the radical pair and is, therefore, classical. This information is assumed to be transmitted along the visual transconduction pathway towards the bird's brain where it is further processed for navigation purposes.

The timeline of the changes the cryptochrome molecule undergoes is shown schematically in Fig. 6. The moment

[1] a pair of charged particles of which one is positively and another negatively charged; in this case it is a radical-ion pair

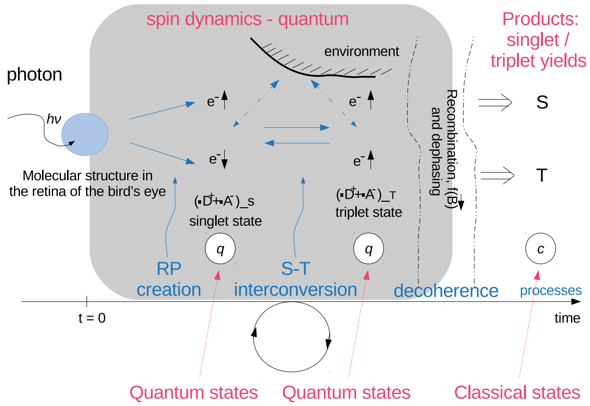

Figure 6: Radical pair creation, quantum dynamics and recombination into classical reaction yields - an overview of the processes, based on (Gauger et al., 2011; Kominis, 2009; Walters, 2014).

$t = 0$ denotes the creation of the radical pair when one of the electrons from the electron pair in the cryptochrome dyad is excited from its ground state by the incoming photon and further dislocated a few atoms away. The resulting radical pair consists of a donor, $\cdot D^+$, and acceptor, $\cdot A^-$, part of the cryptochrome dyad.

Upon the displacement, the spins of the electron pair are still of the opposite signs and the pair is in a singlet state, $|s\rangle$, the name referring to the fact that its spectrum is in the form of a single spectral line. The creation of the radical pair takes $\sim ps$, a very short time compared with its longevity estimated to be $\sim \mu s$ (Gauger et al., 2011).

The spin dynamics of the radical pair is marked by the grey shaded area in Fig, 6. Since the magnetic field is anisotropic, the two electrons experience different field gradients and, therefore, react differently with nuclear spins of the surrounding molecules. Spin interactions are hyperfine interactions and they happen in the geomagnetic field denoted $\vec{\mathbf{B}}$. The magnetic fields created by nuclear spin momenta of the surrounding molecules can be neglected in comparison to $\vec{\mathbf{B}}$. The interactions affected by the geomagnetic field are referred to as Zeeman interactions [2]. Eventually, the spin of one of the electrons may flip so that the electron pair ends up in the state known as triplet state which is characterised by the spectral line split into three lines. Therefore, for the state space based on the spectral profile, the state of the radical pair electron spins, $|e\rangle$, is one of possible four, $|e\rangle \in \{|s\rangle, |t_0\rangle, |t_-\rangle, |t_+\rangle\}$.

In Fig. 6, the two arrows of the opposite directions mark singlet-to-triplet interconversions which may go back and forth before the quantumness of the spin dynamics is lost. The decoherence of the radical pair may be due to both the recombination and the dephasing processes. The dephasing happens because of the interaction with the surrounding molecules which is, essentially, the environmental noise the radical pair experiences. Recombination occurs naturally at certain rate whereby cryptochrome molecule returns to its ground state. The reaction products or yields of this process differ based on whether they were created by recombination of the radical pair in the singlet or triplet state. In Fig. 6, these products are denoted by S and T.

Radical Pair Population Dynamics as a Quantum Measurement Process

In the bird's retina, there will be a certain amount of both kinds of reaction yields present, S and T. The dynamics of the population of these products is conveniently described by master equation which, for the case at hand, is given by the Lindblad equation (Manzano, 2020):

$$\frac{\partial}{\partial t}\rho = -\frac{i}{\hbar}[H, \rho] - \mathcal{L}(\rho) \qquad (26)$$

where the first term on the right hand side stands for the commutator between the system Hamiltonian, H, and the density matrix of the population, ρ, while the second term stands for the dissipative part of the population dynamics due to the interactions with the environment. The Hamiltonian in the first term stands for the unitary evolution of ρ. Further references to the density matrix of the quantum system will refer to the state of the three spins per radical pair: the two electron spins, one of which is dislocated, and the nearest nuclear spin of the cryptochrome molecule.

Dependent on the spin interactions included in the model, several forms of Eq. (26) have been developed (Gauger et al., 2011; Walters, 2014; Dellis and Kominis, 2012) based on the original model (Ritz et al., 2000). In (Kominis, 2009; Imamoglu and Whaley, 2015), classical yields S and T are viewed as the result of the process of quantum measurement of the radical pair electron spins by the environment. Accordingly, the Lindblad equation takes the form (Kominis, 2009; Dellis and Kominis, 2012):

$$\frac{\partial}{\partial t}\rho = -i[H_m, \rho] - \frac{(k_S + k_T)(Q_S\rho + \rho Q_S - 2Q_S\rho Q_S)}{2} \qquad (27)$$

where the first term refers to the unitary evolution of ρ due to the (magnetic) spin interactions and the second term accounts for the reaction dynamics. The first term commutator is determined by the Hamiltonian, H_m:

$$H_m = H_h + H_Z + H_{ex} \qquad (28)$$

which accounts for three contributions: hyperfine interactions between electron and nuclear spins, H_h, Zeeman interactions, H_Z, the spin exchange dynamics H_{ex}.

[2]Zeeman interactions underlie the coupling of magnetic moments to magnetic fields; such interactions are manifested in Zeeman effect - the splitting of spectral lines due to the magnetic field

The second term of Eq. (27) refers to the dynamics of the singlet and triplet recombination processes where k_S and k_T are the recombination rates for the singlet and triplet populations respectively. According to the theory of quantum measurements, Q_S and Q_T are projection operators for the singlet and triplet yields respectively so that the following holds:

$$Q^n_{S/T} = Q_{S/T}, \quad n > 1$$
$$Q_S + Q_T = 1 \tag{29}$$

The coherent spin dynamics of the radical pair is viewed as the operation of quantum measurements of the electron spin performed by the environment where observable Q_S is measured with the rate $k = k_S + k_T$.

Further, the theory of open quantum systems allows Eq. (27) to be written in the following form:

$$\frac{\partial}{\partial t}\rho = \mathbf{M}\rho \tag{30}$$

where $\mathbf{M}$ is the matrix of dim(ρ^2). It is the eigenvalues of this matrix that reflect the regime of the population dynamics $\frac{\partial}{\partial t}\rho$. Eigenvalues are expressed by the decay rate and the pertaining oscillation frequency. For the case when $k_S \approx k_T$, the observable Q_S decays at the measurement rate. However, another regime is exhibited in the case of asymmetric recombination rates: if the radical pair spins begin in singlet state and if $k_T >> k_S$, the decay rate is such that the spin coherence is prolongued demonstrating a QZE whereby fast singlet-triplet interactions take precedence over the rapid dephasing and in that way sustain the coherence of the spin dynamics (Kominis, 2009; Dellis and Kominis, 2012).

Within the quantum measurement view, the deterministic evolution of the system's quantum state described by Eq. (27) can be expressed by the non-Hermitian operator $\mathcal{K} = H_m - ikQ_S$ related to Kraus operators. Since the observable is the result of the measurement operation, it can be expressed as the product of the measurement operator and the density matrix:

$$Q_S = \mathcal{K} \cdot \rho \tag{31}$$

Hyperdescriptions of Quantum Dynamics in RP Mechanism

The RP quantum dynamics is based on the coherent spin dynamics of the three quantum entities: 2 electrons of the RP and the nearest nucleus of the cryptochrome molecule the electrons stem from (Rieper, 2011). The state of these entities can be written in a general form as:

$$\rho = \sum_{i,j} p_{i,j} \, |e_i, n_j\rangle\langle e_i, n_j| \tag{32}$$

where $|e_i\rangle$ is the state vector of the radical pair electrons, $|n_j\rangle$ the state vector of the nuclear spin and $p_{i,j}$ the joint

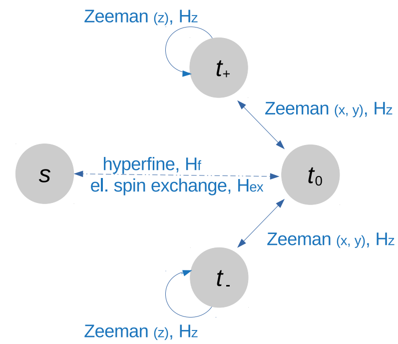

Figure 7: State transitions of RP spin states for various Hamiltonian interactions, adapted from (Poonia et al., 2015).

probability of those states. The quantum dynamics performed within the quantum level is guided by the system Hamiltonian Eq. (28). Since one electron is dislocated, it is assumed that only one electron interacts with the nuclear spin through H_h (Rieper, 2011). The spin exchange Hamiltonian, H_{ex}, is responsible for the interconversion between the singlet and triplet states of the electron pair. Figure 7 depicts the reactions between singlet and triplet states and the Hamiltonians involved. Therefore, the state space description of the system at the quantum level can be given by:

$$\mathcal{S}^q = \{s^q_i(t) = \rho_i(t) : \rho_i(t) \xrightarrow{H,\vec{\mathbf{B}}} \rho_j(t+1)\}^{N_q}_{i,j=1} \tag{33}$$

where the quantum states are expressed through the density matrix, Eq. (32). In Fig. 8 the system is schematically shown within the dynamic hierarchical view. Since the quantum triplet states are observed as classical by operator Q_T, they are shown to form a subspace P_2 which is orthogonal to P_1, a singlet state subspace.

The description function d_{qc} maps quantum states into the classical states, singlet and triplet yields, i.e., the result of the observable Q_S or $(1 - Q_S)$ as given by Eq. (29). From the theory of quantum measurements and open quantum systems, as shown in Eq. (31), these can be expressed by Kraus operators so that we can write:

$$d_{qc} = \{\mathcal{K}_1, \mathcal{K}_2\} \tag{34}$$

Description function is given by nonunitary measurement operators $\{\mathcal{K}_i\}$ given as in Eq. (31). Since the measurement operators expressed as in Eq. (34) act on mutually orthogonal subspaces, it can be said that the condition for the description function to be the good one is satisfied.

The condition for the observation (measurement) times and the sustained coherence is satisfied if the reaction rates for the two classical yields are asymmetric, i.e., $k_i >> k_j$

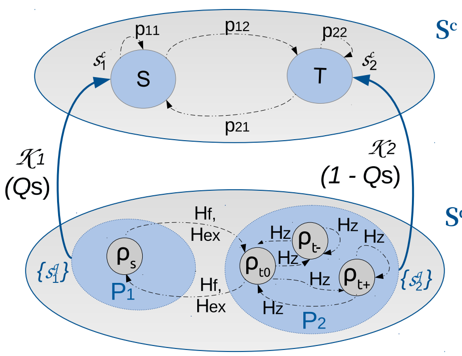

Figure 8: Radical pair quantum dynamics and its classical products viewed as a dynamical hierarchical system. See the text for explanation.

for the initial state in P_j, as described in (Kominis, 2009). Dependent on the chosen model, this condition may be extended with the conditions on the environmental noise (Gauger et al., 2011).

To complete the state space description at the classical level, beside identifying two states in which the system may be observed to be in, it is needed to determine the set of probabilities which define transitions from one state to another. At the conceptual level, we can say that the system stays in one of the observed states dependent on the ratio of the reaction rates k_S and k_T so that the probabilities p_{11} and p_{22} are functionally dependent on these values, $p_{11}, p_{22} \sim f(k_S, k_T)$. The change between two observed states is based on the change of the electron spin states which are caused by the hyperfine spin interactions also affected by the geomagnetic field. Therefore, on the conceptual level, $p_{12}, p_{21} \sim f(H_h, \vec{\mathbf{B}})$. The concrete values can be calculated based on the concrete model considered.

In conclusion, it can be said that the necessary conditions for the existence of dynamical hierarchies exist for the proposed model of avian magnetoreception (Ritz et al., 2000; Kominis, 2009; Dellis and Kominis, 2012). Within such framework, the signal carrying the information about the geomagnetic field is the emergent product of the quantum entities and their dynamics at the quantum level.

Discussion and Future Work

An information theoretic approach which provides necessary conditions for the existence of dynamical hierarchies (McGregor and Fernando, 2005) was revisited for the case when a higher hierarchical level emerges from the lower level quantum dynamics. The aspects brought about by the quantum nature of the lower level were addressed and it was found that the necessary conditions for the existence of dynamical hierarchies are dependent on the way the quantum dynamics is observed or measured which results in the right choice of the description function.

The choice of description function was related to the choice of quantum measurement operators in accordance with the theory of quantum measurements and open quantum systems. The way in which quantum information becomes available in the classical domain is determined by these operators which need to account for the effects of the environment. Moreover, if the classical emergents are to be observed, the coherence at the quantum level must be sustained for the period of observation. Such prolongued coherence can be realised only if the measurements are continuous so as to prevent the collapse of the wave functions which describe the lower level entities. This corresponds to the case of QZE. A careful investigation of quantum information flows and entropic relations between the levels requires much more space and is left for publication in the future.

The model of the quantum dynamics which is hypothesised to be inherent in avian magnetoreception in European robin was analysed as a case study for a higher level functionality emerging from lower level quantum dynamics encountered in a biological system. The focus was on the coherent spin dynamics of the biochemical RP mechanism. *Being set within dynamic hierarchical framework, the quantum processes exhibited by cryptochrome molecules in the bird's retina may be claimed to be an example of a bottom up sensing mechanism exercised within a living system.* In such case, the signal which carries the information about the geomagnetic field is the emergent product of the coherent spin dynamics at the quantum level. As such, it provides the basis for further hierarchical levels of processing the information about the geomagnetic field within higher structures of the nervous system. The information is classical and encoded as the ratio of the products of the RP coherent spin dynamics.

Therefore, a contribution of this article is also a novel view on the quantum dynamics which is assumed to be exercised within biological systems. This is in support of a bottom-up approach to building hierarchies and, in the end, complexity of patterns, functions and behaviours which characterise complex systems evolved in nature. The claims made in this paper are supported by findings from several sources - quantum information, quantum measurements, open quantum systems and quantum biology all of which are currently exciting fields of investigation.

The knowledge about the role(s) quantum processes play in complex systems evolved in nature may be applied to the challenges in man-made systems where quantum dynamics plays a significant role. The manipulation of the quantum dynamics towards the desired goal in a bottom-up fashion may find applications in the field of nanotechnology and even some less conventional computing paradigms (Laketić and Tufte, 2016). Such investigations may be extended in the future.

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