An Optimized Search Strategy may be Induced by the Stochastic Response to Previously Visited Locations

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Abstract

In pinpoint search strategies, gradual expansion of walkers' trajectories might be efficient for the wide-range exploration when their goal locates unpredictably. However, small zigzag-shaped movements are also important to achieve the small-range exploration. An important issue is regarding rules guiding random walkers to execute such an optimized walk. Here, starting with a simple expansion model, we investigated how flexible exploration is achieved if a random walker detects whether the current position is a previously visited position and alters its directional rule based on the recent experience. The agent modifies its directional rule if the current rule disturbs recent flow of the agent's movement. We showed that our model exhibits scale-free movements so-called Lévy walks, which achieve both the local search and the global search. In addition, our model presents power-laws in respect with the first return time. These results suggest that stochastic coordination of the directional rule might produce an adaptive movement strategy.

Introduction

Animals conduct random search for goal-oriented search so as to complete an arrival to their goal such like nests or previously visited resource locations. They also adopt a movement strategy called a learning walk to obtain new spatial information when they find new resources to return there in the future (Wehner and Srinivasan, 1981, Schultheiss et al. 2013, Schultheiss and Cheng, 2013). During those movements, walkers tend to present zigzag-shaped movements and exhibit characteristic search in both cases.

Expanding search range gives a walker more chance for exploration when it searches something. Walkers can achieve such an exploration using their spatial memory or by leaving cues on the ground (Sakiyama and Gunji, 2018). Humans leave a mark on every key point when they lost their way in the mountain, for example. If a walker can detect whether it returns to previously visited locations to some extent, this strategy can be regarded as followings, as one example at the agent-level. A walker will move in the opposite direction from the previous one if the walker returns to previously visited locations. By doing so, that walker tends to expand search range as time goes on. In fact, frequent relocations to previously visited sites generate very slow diffusion (Boyer and Solis-Salas, 2014, Kazimierski et al. 2015).

On the other hand, to produce zigzag-shaped movements for pinpoint search or for acquiring information, expanding search range is not necessarily to give the walker an efficient search. The local search as well as the extent global search is inevitable for the walker to achieve such purposes, especially when it does not perfectly know its goal or for acquiring information in detail at various points (Wehner and Müller, 2010).

To this end, it is important for a random walker to conduct an efficient search where it demonstrates a complex trajectory, which including both zigzag-shaped short movements and long-straight movements. However, an issue is about rules guiding a random walker and how to execute an efficient search via decision-makings of the walker (Sakiyama & Gunji, 2013).

In this study therefore, we consider a random walker on the 1D simulation field who sometimes coordinates its directional rules while initially obeying the simple-expanding rule, i.e. it initially move in the opposite direction from previous one if it returns to previously visited positions, but as time goes on it sometimes ignore this initial rule. In proposed model, a random walker searching on a lattice environment is allowed to deposit a cue on a cell when leaving that cell so as to detect that cell has been already visited if it revisits that place. Such a mechanism can be seen in several actual living systems (Stout & Goulson 2001, Steck et al. 2009). Modification of the directional rule can occur when the directional rule disturbs the recent flow of the movements of the random walker. Thus, the walker replaces and coordinates its directional information so as to stochastically move in the opposite direction from previous one when revisiting a certain cell. We show that the walker in proposed model demonstrates a complex trajectory called a Lévy walk. The characteristics of a Lévy walk is that the probability density function of agents' steps (l) decays asymptotically as a power law:

$$P(l) \sim l^{-\mu}, \text{ where } 1 < \mu \leqq 3$$

According to several animal studies, Lévy walks can be effective methodd if the resource environment is sparse and randomly distributed (Humphries & Sims, 2014).

Materials & Methods

Space and Agents.

We set 100,000-time steps as a trial. We assume that a single agent moves in one-dimensional square lattices. The walker is set at the coordinates 5000. The field size is defined as 10,000. Periodic boundaries are assumed. The agent located at the coordinate *x* definitely chooses one cell from two cells, x-1, x+1, and update its position on each time step.

Model description.

Fixed flip walk model (FFW).

This model is the same as the deterministic walk in random environment (DWRE) where flip rate is set to 1.0 Here, flip rate represents the ratio of moving into the opposite direction when the agent returns to a cell where it has visited before. In the DWRE, a particle (an agent) moves on 1D lattice-environment where scatters are randomly distributed. The type of a scatter at a cell of the lattice changes after the *n*th visit of the particle to this cell (Bunimovich, 2000, 2004). In this paper therefore, we adopt n=1 as the scatter modification. For example, the agent selects x+1 as next position when the agent has updated its position from x to x-1 once before and returns to x. The agent updates its position randomly if it visits the current position for the first time. Detailed descriptions are follows;

if the agent visits the current position x_t for the first time,

$x_{t+1} = x_t - 1$ or $x_t + 1$ with equal probability,

else

x_{t+1} = (the opposite cell the agent selected on its last visit to x_t)

Modified flip walk model (MFW).

In this model, position update of the agent can be replaced as follows using the parameter *r*;

if the agent visits the current position x_t for the first time,

$x_{t+1} = x_t - 1$ or $x_t + 1$ with equal probability,

else

x_{t+1} = (the same cell the agent selected on its last visit to x_t) with a probability *r*,

= (the opposite cell the agent selected on its last visit to x_t) with a probability 1-*r*,

Therefore, flip rate can be 1-*r*.

In this model, flip rate is coordinated in the following manner after calculating the next position x_{t+1},

if $x_{t+1} - x_t$!= $x_t - x_{t-1}$ && *count*>=*threshold*,

$r \in [0.00, rn] = \{r \mid 0.00 \le r \le rn\}$,

Here, count is defined as follows;

count $=|\Sigma^{\text{threshold}}_{i=1} (x_{t-i+1} - x_{t-i})|$, where $|a| = \max\{a, -a\}$

Please note that this model is the same as the fixed flip walk model when the parameter *r* is fixed to 0.

Thus, the agent tends to doubt/modify the directional rule when that rule disturbs recent flow of its movement. The parameter *count* represents a degree of recent flow of the agent's movement. That value can be high if the agent moves in a certain direction for a while. For examples, the parameter *r* is replaced if $x_{t+1} - x_t$!= $x_t - x_{t-1}$, $(x_t - x_{t-1}) = 1$and $(x_{t-1} - x_{t-2}) = 1$ under the condition *threshold* = 2 since *count* = 2.

Modified flip walk -reset model (MFW-reset).

To investigate the importance of the timing that *r* and flip rate are coordinated., we set another model, named as the MFW-reset model, in which the parameter *r* is coordinated on each time step in a following manner;

$r \in [0.00, rn] = \{r \mid 0.00 \le r \le rn\}$

Parameters

Parameters are shown in Table 1. Later, we will discuss the influence of the parameter-changing.

Table 1. Parameters used for the calculation.

Parameter	Value	Description
threshold	3	threshold value for recent flow
rn	0.5	maximum value for *r*
time length	100,000	calculation time for one trial
N of trials	100	number of trials
field size	10,000	system size

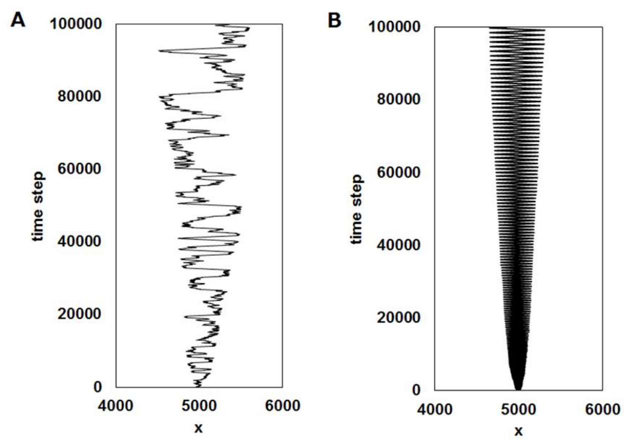

Figure 1. Examples of trajectories. A. The MFW model. B. The FFW model.

Results

Figure 1 illustrates an example of an agent trajectory in the MFW and FFW models respectively. According to this figure, the agent in the MFW model produces straight movements while walking in zigzags. On the other hand, the FFW model seems induce a pattern of ever-widening straight movement. This is because the later model forces the revisited agent to move in the opposite direction from the previous direction. As a result, straight movements monotonically increase. To investigate the dynamics of movements, we analyzed the diffusiveness of the walks. This property is one interesting feature to study to analyze search efficiency (Viswanathan et al. 2008):

$$\langle R^2 \rangle \sim t^{2H}$$

Parameter H determines the diffusive property (H > 1/2 for the super-diffusion, H = 1/2 for the diffusion). Figure 2 shows the mean-squared distance (msd) and the time step obtained from both models (msd was obtained every 1000-time steps). The results show that the MFW model shows a super-diffusive movements (H ~ 0.58, R-squared = 0.99) while the FFW model shows the normal diffusive movements (H ~ 0.49, R-squared = 0.99). For further investigation, we calculated step length distribution. Here, travel distances between two consecutive turns were defined as the step lengths.

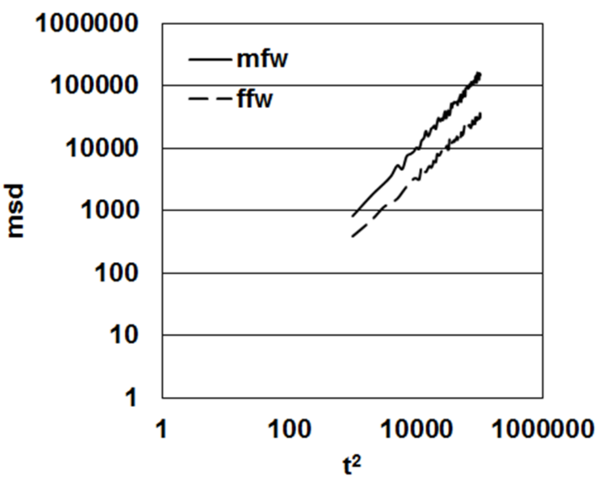

Figure 2. The diffusiveness of the MFW model (shown as a bold line) and the FFW model (shown as a broken line). Here, "msd" and "t" mean the mean squared displacement and time respectively.

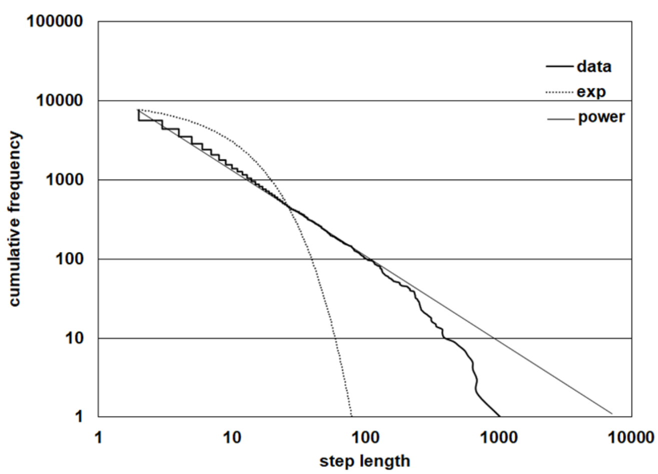

Figure 3. The relationship between step lengths and cumulative frequency of the MFW model. A bold line indicates plotted data. A broken line indicates an exponential-law distribution. A solid line indicates a power-law distribution.

Figure 3 represents the relationship between step length and its cumulative frequency obtained from the MFW model. According to Figure 3, power-law tailed distributions were achieved across some ranges (n of data from 1 trial = 7693, $\mu = 2.07$, weights of power law against exponential law = 1.00). Step lengths calculated the maximum-likelihood estimate (Edwards et al. 2007). It is obvious that the FFW model cannot produce such a distribution because step lengths are monotonic-increased after a while. Moreover, we found that the MFW model also exhibits an interesting feature in respect with the first return problem, i.e. power-law tailed distribution was achieved. Here, we calculated the time interval until the agent returns to its initial position after leaving that position. Power-law tailed distributions were achieved across some ranges (n of data from 1 trial = 100, $\mu = 2.21$, weights of power law against exponential law = 1.00). On the contrary, such a distribution disappears in the FFW model thanks to $\mu > 3$ (n of data from 1 trial = 100, $\mu = 3.18$, weights of power law against exponential law = 1.00).

We also investigated search efficiency in another way. Here, we calculated the furthest travel distance achieved by the agent until each trial finished. Thanks to its super-diffusiveness, the MFW model performs better than the FFW model (Figure 4, 1108.0±96.6 (MFW with default parameter value (th3_0.5)) vs. 630±15.1 (FFW), Mann-Whitney U-test, p<1.0E-15). For parameter investigations, we replaced the parameter *rn* and conducted same analysis. We found that the MFW model outperforms the FFW model even after parameter *rn* was replaced (Figure 4, 1087±109.6 (MFW with *rn*=0.7 (th3_0.7)) vs. FFW, Mann-Whitney U-test, p<1.0E-15, 1039±79.8 (MFW with *rn*=0.3 (th3_0.3)) vs. FFW, Mann-Whitney U-test, p<1.0E-15). In addition, we replaced the parameter *threshold* from 3 with 9 and confirmed the superiority of the MFW model (Figure 4, 778±150.6 (MFW with *threshold*=9 (th9_0.5)) vs. FFW, Mann-Whitney U-test, p<1.0E-11). As shown in Figure 4 however, the parameter *threshold* is not necessary to be large, which indicates that the agent has only to detect the last few steps to calculate its recent flow of movements (*count* value).

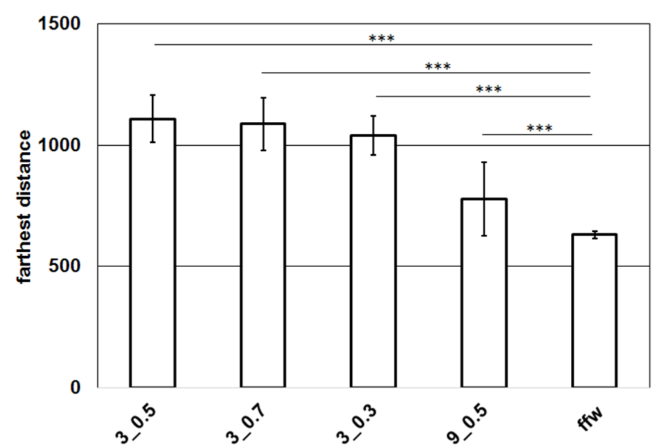

Figure 4. The averaged farthest distance of the MFW model with default parameter (th3_0.5), the MFW model with *rn*=0.7 (th3_0.7), the MFW model with *rn*=0.3 (th3_0.3), the MFW model with threshold = 9 (th9_0.5) and the FFW model

respectively. The symbol "***" indicates p<0.001. The vertical lines indicate error bars.

Finally, we would like to comment on the timing that r and flip rate are coordinated using the MFW-reset model. In the MFW model, that modification is dependent on the parameter *count* and *threshold*. We found that the ordinal MFW model again outperforms the MFW-reset model (1108.0±96.6 (MFW with default parameter value (th3_0.5)) vs. 596±661.8 (reset_0.5; *rn* is set to 0.5)), Mann-Whitney U-test, p<1.0E-15, 1108.0±126.8 (MFW with default parameter value (th3_0.5))) vs. 614±96.4 (reset_0.3), Mann-Whitney U-test, p<1.0E-15, 1108.0±96.6 (MFW with default parameter value (th3_0.5)) vs. 544±132.5 (reset_0.7), Mann-Whitney U-test, p<1.0E-15).

Discussion

Using the simulation analysis, we found that proposed model seems to allow the walker to produce small-distance movements while covering a wide range of the field. In addition, that model exhibits super-diffusiveness of the walks and a power-law tailed walks, i.e. Lévy walks (Bunimovich & Khlabystova, 2002, Bénichou et al. 2011, Humphries & Sims 2014). More interestingly, scale-free distributions are achieved also in respect with the first return time. The first return/passage problem is an important issue in many research fields including physics and biology (Redner, 2007, Bénichou et al. 2011). In this model, the walker might often revisit previously visited positions while occasionally cannot return to those positions for a long period of time. Lévy walks would compensate the balance between the local search and the global search because the agent occasionally produces straight movements.

In a scenario where the walker conducts the goal-oriented search, a scale-free walks can be efficient because it produces both zigzag-shaped movements and long straight movements. In addition, a random walker sometimes returns to the initial position, which is also important because it can search around the initial position with that place as the start-points.

In proposed model, a random walker does not always obey so-called simple-expand rule. i.e., it is not necessary to move in the opposite direction from previous one if it returns to previously visited positions. At a glance, this mechanism is contradictive because a walker in the MFW model seems to travel over wider areas compared with the FFW model. This contradiction however, points out a significant matter; the local effective strategy is not always able to be the global effective strategy.

A scale free distribution in respect with a spatial pattern can be represented based on the rule-coordination. Coordinating the directional rule will give a random walker to response in several ways to a certain event, which scenario enables the walker to present adaptive search using limited / local cues. Although the proposed models are abstract and simple models, the future work of this study can be the application of the models to the actual organisms.

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