Optimisation of hybrid institutional incentives for cooperation in finite populations

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Introduction. Evolutionary Game Theory has been widely used to study myriad questions in diverse disciplines like Evolutionary Biology, Ecology, Physics, Sociology and Computer Science, including the mechanisms underlying the emergence and stability of cooperation (Nowak, 2006; Perc et al., 2017; Han, 2022) and how to mitigate climate and Artificial Intelligence risks (Santos and Pacheco, 2011; Góis et al., 2019; Sun et al., 2021; Han et al., 2020). Institutional incentives, either positive (reward) and negative (punishment), are among the most important mechanisms for promoting the evolution of prosocial behaviours (Sigmund et al., 2001; Van Lange et al., 2014). In institutional incentives, an external decision-maker, such as the United Nation or NATO, has a budget to interfere in the population in order to achieve a desirable outcome, for example to ensure a desired level of cooperation. The use of institutional incentives for promoting cooperation is costly so it is important to optimise the cost while, at the same time, maintaining a level of cooperation (Ostrom, 2005; Cimpeanu et al., 2021; Wang et al., 2023; Powers et al., 2016). Several theoretical models studied how to combine institutional reward and punishment for enhancing the emergence and stability of cooperation (Chen and Perc, 2014; Góis et al., 2019; Berenji et al., 2014; Hilbe and Sigmund, 2010; Sun et al., 2021). However, little attention has been given to addressing the cost optimisation of providing incentives. Sasaki et al. (Sasaki et al., 2015) looked at a rewarding policy that switches the incentive from reward to punishment when the frequency of cooperators exceeds a certain threshold. Wang et al. (Wang et al., 2019, 2023) explored the optimal incentive that not only minimises the total cost, but also guarantees a sufficient level of cooperation in both well-mixed and structured populations via optimal control theory. This work however does not take into account various stochastic effects of evolutionary dynamics such as mutation and non-deterministic behavioural update.

In this extended abstract, starting from a finite population framework in (Han and Tran-Thanh, 2018), we summarise a recent work in (Duong et al., 2023) that provides a *rigorous analysis*, supported by numerical simulations, for the problem of optimising the cost of *hybrid* incentives while sustaining a desired level of cooperation. Most relevant to this work is Duong and Han (2021) which derived analytical conditions for which a general incentive scheme can guarantee a given level of cooperation while at the same time minimising the total cost of investment. These results are highly sensitive to the intensity of selection. This work however did not study the cost-efficiency of the mixed incentive scheme, which is the focus of the present work.

Models and Methods. We consider a well-mixed, finite population of N self-regarding individuals or players, who interact with each other using one of the following cooperation dilemmas, the Donation Game (DG) and the Public Goods Game (PGG). We adopt here the finite population dynamics with the Fermi strategy update rule (Traulsen and Nowak, 2006), stating that a player X with fitness f_X adopts the strategy of another player Y with fitness f_Y with a probability given by, $P_{X,Y} = \left(1 + e^{-\beta(f_Y - f_X)}\right)^{-1}$, where β represents the intensity of selection. To reward a cooperator (respectively, punish a defector), the institution has to pay an amount θ/a (resp., θ/b) so that the cooperator's (defector's) payoff increases (decreases) by θ, where $a, b > 0$ are constants representing the efficiency ratios of providing the corresponding incentive. The population dynamics are modelled using an absorbing Markov chain consisting of $(N+1)$ states, $\{S_0, ..., S_N\}$, where S_i represents a population with i C players. S_0 and S_N are absorbing states. Let $U = \{u_{ij}\}_{i,j=1}^{N-1}$ denote the transition matrix between the $N-1$ transient states, $\{S_1, ..., S_{N-1}\}$. The entries n_{ij} of the so-called fundamental matrix $\mathcal{N} = (n_{ij})_{i,j=1}^{N-1} = (I-U)^{-1}$ of the absorbing Markov chain gives the expected number of times the population is in the state S_j if it is started in the transient state S_i (Kemeny, 1976). As a mutant can randomly occur either at S_0 or S_N, the expected number of visits at state S_i is: $\frac{1}{2}(n_{1i} + n_{N-1,i})$. Assuming that we desire to obtain at least an $\omega \in [0, 1]$ fraction of cooperation, then θ needs to satisfy the following lower bound (Han and Tran-Thanh, 2018):

$$\theta \geq \theta_0 = \frac{1}{(N-1)\beta} \log\left(\frac{\omega}{1-\omega}\right) - \delta.$$

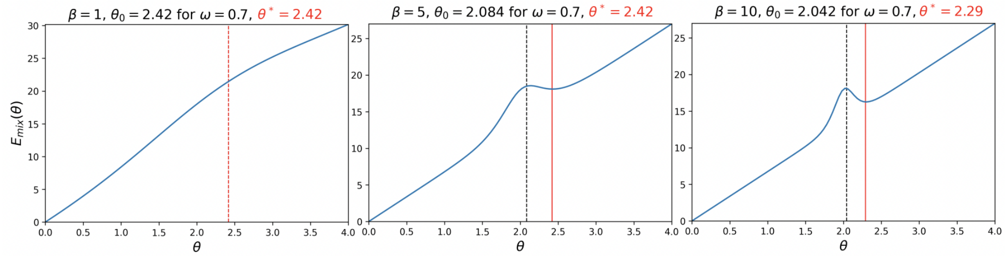

Figure 1: Finding optimal θ, denoted by θ^* (represented by the red line in the figures), that minimises E_{mix} while ensuring a minimum level of cooperation ω, where $N = 3$ for DG with $B = 2, c = 1$. The critical threshold value for the strength of selection is $\beta^* = 3.67$. The numerical results obtained are in accordance with our theoretical results: for $\beta < \beta^*$, the cost function increases, while for $\beta > \beta^*$ it is not monotonic.

Optimisation problem. The expected total cost of interference for hybrid institutional incentives is

$$E_{mix}(\theta) = \frac{\theta}{2} \sum_{j=1}^{N-1} (n_{1j} + n_{N-1,j}) \min\left(\frac{j}{a}, \frac{N-j}{b}\right).$$

In summary, we obtain the following cost-optimisation problem of institutional incentives in stochastic finite populations: $\min_{\theta \geq \theta_0} E_{mix}(\theta)$.

Main results. The main results of (Duong et al., 2023) can be summarised as follows.

1. It is always more cost efficient to use the mixed incentive approach than a separate incentive, reward or punishment,
$$E_{mix} \leq \min\{E_r, E_p\}.$$
If $\frac{b}{a} \leq \frac{1}{N-1}$, then $E_{mix}(\theta) = E_r(\theta)$. If $\frac{b}{a} \geq N-1$, then $E_{mix}(\theta) = E_p(\theta)$.

2. (Infinite population limits)
$$\lim_{N \to +\infty} \frac{E_{mix}(\theta)}{\frac{N^2\theta}{2} H_{a,b}} = \begin{cases} 1 + e^{-\beta|\theta - c|} & \text{for DG,} \\ 1 + e^{-\beta|\theta - c(1-\frac{r}{n})|} & \text{for PGG.} \end{cases}$$

3. (Weak selection limit): $\lim_{\beta \to 0} E_{mix}(\theta) = \theta N^2 H_{N,a,b}$.

4. (Strong selection limits)
$$\lim_{\beta \to +\infty} E_{mix}(\theta) = \begin{cases} \frac{N^2\theta}{2}\left(H_{N,a,b} + \frac{1}{a(N-1)}\right), & \text{for } \theta < -\delta, \\ \frac{NA}{2}\Big[2NH_{N,a,b} + \frac{1}{a(N-1)} + \frac{1}{b(N-1)} \\ \quad -\frac{\min(2/a,(N-2)/b)}{2(N-2)} - \frac{\min((N-1)/a,1/b)}{N-1}\Big], \\ \text{for } \theta = -\delta, \\ \frac{N^2\theta}{2}\left[H_{N,a,b} + \frac{1}{b(N-1)}\right] & \text{for } \theta > -\delta. \end{cases}$$

5. (Growth of the cost function) The cost function satisfies
$$\frac{N^2\theta}{2}\left(H_{N,a,b} + \frac{1}{\max(a,b)(N-1)}\right) \leq E_{mix}(\theta) \leq N(N-1)\theta\left(H_{N,a,b} + \frac{1}{\min(a,b)\lfloor\frac{(N-1)}{2}\rfloor}\right).$$

6. (Phase transition phenomena and behaviour under the threshold) There exists a threshold value β^* such that $\theta \mapsto E_{mix}(\theta)$ is non-decreasing for all $\beta \leq \beta^*$ and it is non-monotonic when $\beta > \beta^*$. As a consequence, for $\beta \leq \beta^*$
$$\min_{\theta \geq \theta_0} E_{mix}(\theta) = E_{mix}(\theta_0).$$

7. (Behaviour above the threshold value) For $\beta > \beta^*$, the number of changes of sign of $E'_{mix}(\theta)$ is at least two for all N and there exists an N_0 such that the number of changes is exactly 2 for $N \leq N_0$. As a consequence, for $N \leq N_0$, there exist $\theta_1 < \theta_2$ such that for $\beta > \beta^*$, $E_{mix}(\theta)$ is increasing when $\theta < \theta_1$, decreasing when $\theta_1 < \theta < \theta_2$, and increasing when $\theta > \theta_2$. Thus, for $N \leq N_0$,
$$\min_{\theta \geq \theta_0} E_{mix}(\theta) = \min\{E_{mix}(\theta_0), E_{mix}(\theta_2)\}.$$

Figure 1 demonstrates the behaviour of the cost function for providing incentives in different regimes of intensities of selection β, the phase transitions that occur when β is sufficiently large, as well as the accuracy of the theoretical prediction of the optimal incentive cost θ^*.

Summary and Outlook. We have summarised a recent theoretical analysis of the problem of optimising the cost of hybrid institutional incentives while guaranteeing a minimum amount of cooperation, in stochastic finite populations. In this context, institutional approaches have been widely adopted to study biological and artificial life systems (Andras et al., 2018; Jones et al., 2013; Smaldino and Lubell, 2014; Andras, 2020; Powers et al., 2016). This analysis provides new, fundamental insights into a cost-efficient design of institution-based solutions for promoting pro-social behaviours in social and artificial systems.

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