Reality of Affordances: A Category-Theoretic Approach

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Abstract

The concept of affordance, proposed by James J. Gibson as an opportunity for action offered by the environment to the organism, has been adopted in various fields, including psychology, neuroscience, and robotics. However, different interpretations exist as to whether it is a feature of a relation between the environment and the organism and therefore cannot exist independently of the organism, or a "resource" that exists in the environment independent of the organism's presence and is waiting to be used, or both, or neither. In this paper, we defend the position that affordances are *both* relational and resources using a category-theoretic approach. This idea is formalized by the concept of "natural transformations" in category theory, which are structure-preserving transformations between "functors" – mathematical expressions representing "seeing from a particular perspective." We propose that formalizing the realism of affordance in terms of natural transformations offers a more rigorous and lucid understanding of this concept. Furthermore, our formalization enables us to relate the reality of affordances to a broader context, especially the shift in the meaning of "reality" in modern physics. Our category-theoretic approach offers a potential solution to the problems and limitations associated with existing set theory-based frameworks for affordances, paving the way for a future theory that better accounts for the open-ended interplay between organisms and their environments.

Introduction

The concept of affordance, introduced by James J. Gibson in the field of ecological psychology, has been widely known and applied across various disciplines, including neuroscience (Jamone et al., 2018), robotics (Zech et al., 2017), and artificial life (Ogai and Ikegami, 2008). Affordances refer to the action possibilities offered by the environment to an organism, highlighting the intimate relationship between the organism and its surroundings. Despite its widespread use, a unified and consistent definition of affordance has remained elusive, leading to various interpretations and formal definitions.

One of the primary reasons for this situation stems from the apparent contradictions in Gibson's own descriptions of affordances. On one hand, Gibson asserts that affordances are relative and relational properties that exist in dependence on the organism. On the other hand, he also refers to affordances as invariant features of the environment that exist independently of the organism, waiting to be perceived and utilized. This seeming inconsistency has given rise to different theories by subsequent researchers, such as the relational theory (Chemero, 2003, 2009) and the resource theory (Reed, 1996). Consequently, most of the existing formalizations of affordance have been based on one of these interpretations, leading to difficulties in integrating them into a cohesive framework.

However, we argue that the apparent contradiction in Gibson's description is not due to inherent inconsistency in his theory but rather a lack of appropriate language to express the true nature of affordances. We propose that the mathematical tools provided by category theory can help us formalize affordances in a way that resolves these apparent contradictions and captures the essence of Gibson's original concept. Unlike set theory, which focuses on the point-like elements that compose an object, category theory characterizes an object by the relationships called "arrows" it has to and from other objects. This nature of category theory makes it particularly apt for capturing the relational nature of affordances.

In this paper, we present a category-theoretic formalization of the reality of affordances. By viewing affordances through the lens of category theory, we can naturally understand and express both the relational and resource-like aspects of affordances without falling into contradiction. Furthermore, our category-theoretic approach allows us to relate the reality of affordances to the ongoing shift towards a more dynamic, relation-oriented perspective in physics and mathematics.

The formalization of affordances using category theory has significant implications for artificial life research. By providing a rigorous and consistent framework for understanding the relationship between organisms and their environments, our approach can inform the design of more adaptive and responsive artificial agents. Moreover, by grounding the concept of affordances in the formal language of category theory, our work facilitates the integration of ecological psychology with other theoretical frameworks used in artificial life, such as dynamical systems theory, information theory, and machine learning. This integration can lead to a deeper understanding of the mechanisms underlying the emergence of adaptive behavior in both natural and artificial systems.

Theories of Affordance

Gibson's Original Description of Affordances

In his seminal work, *The Ecological Approach to Visual Perception*, Gibson (1979) introduced the concept of affordances as "what [the environment] offers the animal, what it provides or furnishes, either for good or for ill" (p. 127). While this definition is quite concise, a closer examination of Gibson's writings reveals a more complex and sometimes ambiguous picture (Chemero, 2003; Dotov et al., 2012; Segundo-Ortin and Raja, 2024).

On one hand, Gibson suggests affordances are objective properties of the environment that exist *out there* independently of any particular animal. He argues that affordances are "invariant" and always available to be perceived, regardless of the observer's needs or actions (Gibson, 1979, pp. 128-129). In an earlier article, he even more explicitly states that "[a]lthough an affordance consists of physical properties taken with reference to a certain animal it does not depend on that animal" (Gibson, 1977, pp. 69-70). This suggests a strong ecological realism in which affordances are animal-independent features of the environment.

On the other hand, Gibson also describes affordances as being relative to the observing animal. He emphasizes that affordances must be measured relative to the animal and are unique to that animal, rather than being abstract physical properties (Gibson, 1979, pp. 127-128). This pulls against a purely objectivist conception and indicates that affordances are, in some sense, relational properties that depend on the specific capabilities and form of life of the animal.

Gibson expresses his own subtle position with the following somehow mysterious and seemingly paradoxical statement:

> "[...] an affordance is neither an objective property nor a subjective property; or it is both if you like. An affordance cuts across the dichotomy of subjective–objective and helps us to understand its inadequacy. It is equally a fact of the environment and a fact of behavior. It is both physical and psychical, yet neither. An affordance points both ways, to the environment and to the observer." (Gibson, 1979, p. 129)

This tension between affordances as objective environmental properties and as relational properties has led to differing interpretations and developments by subsequent theorists, such as what we refer to as the resource theory and the relational theory (but see also Turvey, 1992).

Resource Theory of Affordances

The resource theory, suggested by Edward Reed (1996), aligns with Gibson's emphasis on affordances as objective environmental properties. It sees affordances as real, objective "resources" in the environment that aid the goals of organisms, existing independently of any individual's perception or action.

Reed further extends the independence of affordances from individual animals, arguing that they act not only as behavioral opportunities on the timescale of the individual, but also as "selection pressures" that shape the evolution of species. On this view, affordances provide enduring structure across generations and evolutionary time, guiding the development of species' abilities and forms of life. This gives affordances an even stronger objectivity and explanatory role.

However, the resource theory faces several challenges. If affordances are purely objective environmental properties, it is not straightforward to explain how different animals can perceive different affordances in the same environment. The view appears to struggle with the animal-relativity and specificity that Gibson (and, in fact, Reed himself) also emphasizes. Furthermore, if affordances that *already* exist in the environment determine the way of life of species, as Reed suggests, it threatens to make evolution and the development of the biosphere an overly deterministic process, leaving little room for open-ended evolution and the creative role of organisms in shaping their own niches (Walsh, 2015; Withagen and van Wermeskerken, 2010; Heras-Escribano, 2020). Finally, the resource view has difficulty accommodating more complex, social, and conventional affordances, like a mailbox affording letter-posting, an example raised by Gibson (1979), which seem to depend on more than just the objective physical properties of objects.

Relational Theory of Affordances

The relational theory, developed by Chemero (2003, 2009), builds on Gibson's descriptions of affordances as animal-relative properties (see also Stoffregen, 2003). It characterizes affordances as relations between features of the environment and abilities of organisms.

This view nicely captures how different animals can perceive different affordances in the same environment, depending on their specific abilities and forms of life. It emphasizes the inherent relationality of affordances and avoids positing them as purely objective properties independent of any animal. In this way, it aligns with Gibson's point that affordances are always relative to an animal.

However, concerns have been raised that the relational theory may compromise the strong ecological realism of affordances (e.g., Costall, 1999). By conceptualizing affordances as relations that depend on the abilities of organisms, it risks rendering them too subjective or organism-dependent. Although Chemero explicitly rejects such a subjectivist interpretation, the relational view is sometimes seen as assigning affordances to the organism's arbitrary interpretation or internal representation of the environment rather than treating them as inherent to the environment (Rietveld and Kiverstein, 2014). This can make it challenging to defend the independent reality and objectivity of affordances that Gibson's ecological approach aimed to establish.

Affordances are *both* relational and resources

Rietveld and Kiverstein (2014) aim to reconcile the tensions between the relational and resource views while preserving the realism of affordances. They situate affordances in the context of the relatively stable patterns of behavior and ability that make up a "form of life." A form of life is not the capability of a particular individual, but consists of more general patterns of behaviors that can be shared among multiple individuals at various levels (species, society, group, family, etc.).

On this view, affordances are "relations between aspects of a material environment and abilities available in a form of life" (p. 335). Thus, affordances are relational in the sense that they depend on the animal's ability to make use of them. However, according to Rietveld and Kiverstein, this does not undermine the reality of affordances. First, they argue that "the existence of an affordance does not depend on the active use by any particular member of a form of life" (p. 340) in the here and now, but rather dependes on "a form of life whose members could *potentially* detect the affordance" (p. 338, emphasis in original). For example, the piano affords playing it only to those trained to do so, but even when they are not in the vicinity, there is no change in its nature that a piano player can play it. Chemero (2003, 2009) also defends the reality of affordances as relations in terms of their dependence on potential and counterfactual observers.

Second, such a relationship between forms of life and affordances cannot be arbitrarily defined but is constrained by the material and causal properties of the environment, or, in short, "it is not the case that everything is possible" (Rietveld and Kiverstein, 2014, p. 344). A form of life can persist as long as the environment provides opportunities to practice and live through it.

Moreover, thanks to the generality and flexibility of the concept of a form of life, this view accommodates sophisticated, social, and conventional affordances by understanding them in terms of the shared abilities, norms, and customs of sociocultural practices. Affordances like the mailbox affording letter-posting can be explained in terms of the stable practices and conventions of a form of life rather than just the physical properties of the environment or the abilities of an individual.

In summary, we can understand Rietveld and Kiverstein's view on the reality of affordances as follows: it is the lawful relationship "if there were an organism with a certain form of life in the here and now, the environment would afford certain actions to it" that is objective, public, and real. What we need next is a mathematical framework that successfully captures the nature of affordances as such, which allows us to study them more empirically.

Empirical Study on Affordance

A study by Warren (1984) on perceiving affordances for stair climbing was one of the first to empirically test and quantify the animal-relative yet objectively measurable character of affordances. He examined the boundary between the height that subjects perceived as "climbable" and "unclimbable" using a step with adjustable height. The results showed that the boundary height converged at 0.88 times the leg length for both short and tall groups. Here, the affordance of "being able to climb" is expressed as an objectively measurable and calculable ratio between the subject's leg length and the step height.

Warren's study demonstrates how affordances can be quantified in a way that captures their relational nature (being relative to the animal's bodily size) while preserving their objective reality (being measurable and expressible as an invariant ratio). This supports Rietveld and Kiverstein's claim that affordances can be "*both* relational and resources."

However, there are several limitations to Warren's approach. First, body size is only one of many factors that determine the capabilities of an animal. Other factors like strength, flexibility, skill, and even psychological factors like confidence or motivation could play a role in determining what affordances are available. Focusing solely on body dimensions may oversimplify the rich animal-environment relationship. Indeed, later experiments suggested that the climbability of a step is not simply proportional to leg length, but is determined by more complex factors such as the ratio between "stepping ability" and step height (Cesari et al., 2003). This suggests that the relevant animal-side variable is not a simple body dimension but a more complex measure of action capability (Chemero, 2009, pp. 143-145).

Second, the constant value that appears robust and universal in Warren's experiment can easily change if the experimental setting is altered even slightly, such as if the subjects wear different shoes. In real-world environments, the variability and complexity of both the animal and the environment would likely make such constant ratios rare or nonexistent. The controlled, simplified nature of the experimental setup may limit the generalizability of the findings.

These limitations highlight the challenges involved in viewing affordances as constant ratios. While Warren's study is a valuable first step, it may oversimplify the complex, multifactorial nature of the animal-environment relationship. For more complex behaviors (including socially constructed ones), it would be difficult or impossible to express affordances as simple ratios between environmental and animal-side features.

Thus, more flexible and sophisticated mathematical tools are needed to capture the full potential of the concept of affordance. In the next section, we will introduce category theory as a promising framework for formalizing affordances in a way that naturally captures their relational and objective aspects, as suggested by Rietveld and Kiverstein.

Category Theory as "Ecological" Mathematics

Category theory, a branch of mathematics that emerged in the mid-20th century, offers a unique perspective on the study of mathematical objects and their relationships. Unlike the traditional set-theoretic approach, which characterizes mathematical objects in terms of their point-like elements and then describes relationships between them as functions (mappings from one set of points to another), category theory primarily focuses on the relationships, called "arrows" (or "morphisms") between objects. As Leinster (2014, p. 9) notes, in a category, "the objects do not live in isolation": in category theory, every object is characterized by the arrows it has to and from other objects in the category and not by its constitutive elements, which are defined independently of other objects (Mac Lane, 1978). In essence, it is "arrow-first" mathematics (Hirota et al., 2023).

This shift in focus in category theory (from constitution to interrelation) aligns remarkably well with Gibson's "ecological" stance, according to which, in order to understand traits and behaviors of the individual, one should not focus solely on the intrinsic properties of the individual but rather on the entire "ecology" in which it is embedded. In other words, the emphasis should be on the relationships between the organism and its environment, not one or the other of them. This stance is not limited to the animal-environment relationship but applies more generally. For example, Gibson (1979) argues that regarding the perception of light, it is not the absolute values of properties such as brightness at individual points that are fundamental, but the structured arrangement among them (called the "ambient optic array"), and, going even further, the change of such a structure over time through movement (called the "optical flow").

Despite this clear alignment between the core tenets of category theory and ecological psychology, the potential for using category theory as a mathematical framework for ecological psychology has remained largely unexplored. Instead, dynamical systems have been seen as the most promising language for the mathematical formulation of affordances (e.g., Chemero, 2009).

Dynamical systems can explicitly describe temporal changes in a system and have been widely used to model the interaction of embodied agents with their environment (e.g., Kelso, 1995). While their usefulness is unquestionable, they also have certain limitations. For example, the dynamical systems approach requires that the entire set of possible states taken by the system be specified a priori as the "phase space." Kauffman and Roli (2021) argue that we cannot appropriately formalize affordances using such a set theory-based mathematical tool. According to them, the whole set of possible usage of an object is indefinite and not "prestatable," so the emergence of new affordances and the diachronic, open-ended evolution of the biosphere are inherently not describable as a trajectory in a predetermined phase space (see also Longo et al., 2012; Montévil, 2019; Roli et al., 2022, etc.).

While they do not explicitly provide a concrete alternative to set theory (at least in the paper mentioned above) and do not mention category theory,[1] we believe that a category-theoretic approach can be a step forward, if not a complete solution, to this problem and complement the existing approaches based on dynamical systems.

Category

A category $\mathcal{C}$ consists of the following:

1. A collection of objects, denoted as $Ob(\mathcal{C})$.

2. For each pair of objects $A, B \in Ob(\mathcal{C})$, a collection of arrows (also called morphisms) from A to B, denoted as $Hom_{\mathcal{C}}(A, B)$ or $\mathcal{C}(A, B)$. This can be empty, but in general there can be multiple, sometimes infinite, arrows between objects. Thus, an arrow is (in general) not reducible to the pair of objects it connects.

   In this case, objects A, B are respectively called the "domain" and "codomain" of an arrow $f \in Hom_{\mathcal{C}}(A, B)$, and this is depicted as $f : A \to B$ or

$$A \xrightarrow{f} B \tag{1}$$

3. For each triple of objects $A, B, C \in Ob(\mathcal{C})$, a composition operation: $\circ : Hom_{\mathcal{C}}(B, C) \times Hom_{\mathcal{C}}(A, B) \to Hom_{\mathcal{C}}(A, C)$ such that given $f : A \to B$ and $g : B \to C$ we can get $g \circ f : A \to C$, depicted as follows:

$$\begin{tikzcd} & B \arrow[dr, "g"] & \\ A \arrow[ur, "f"] \arrow[rr, "g \circ f"] & & C \end{tikzcd} \tag{2}$$

   Moreover, for arrows $f : A \to B$, $g : B \to C$, and $h : C \to D$, the following "associativity condition" must hold:

$$(h \circ g) \circ f = h \circ (g \circ f) \tag{3}$$

4. For each object $A \in Ob(\mathcal{C})$, an identity arrow $id_A : A \to A$ such that for any arrow $f : A \to B$, the following "identity condition" holds:

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[1] Kauffman apparently considered that a category-theoretic approach has the same limitation at least as of 2000 (Kauffman, 2000, pp. 106-107).

$$f \circ id_A = id_B \circ f = f \tag{4}$$

This means that there is a one-to-one correspondence between the objects and the identity arrows, allowing us to identify them and say that an object *is* a special case of arrow. That is, "changing nothing" is also a kind of transformation.

Functor

A category is *itself* a mathematical object, and thus it too is characterized by the arrows it has to/from other categories in the "category of categories" (Leinster, 2014). A functor $F : \mathcal{C} \to \mathcal{D}$ between categories $\mathcal{C}$ and $\mathcal{D}$ is such an "arrow between categories," depicted as follows:

$$\begin{tikzcd} \mathcal{C} \arrow[r, "F"] & \mathcal{D} \end{tikzcd} \tag{5}$$

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A functor F consists of:

1. A mapping $F : Ob(\mathcal{C}) \to Ob(\mathcal{D})$ that assigns to each object $A \in Ob(\mathcal{C})$ an object $F(A) \in Ob(\mathcal{D})$.
2. For each pair of objects $A, B \in Ob(\mathcal{C})$, a mapping:
   $F_{A,B} : Hom_{\mathcal{C}}(A, B) \to Hom_{\mathcal{D}}(F(A), F(B))$
   such that the following conditions hold:
   (a) For any arrow $f : A \to B$ in $\mathcal{C}$, $F(f) : F(A) \to F(B)$ in $\mathcal{D}$.
   (b) For any objects $A, B, C \in Ob(\mathcal{C})$ and arrows $f : A \to B$ and $g : B \to C$,
   $$F(g \circ f) = F(g) \circ F(f). \tag{6}$$
   (c) For any object $A \in Ob(\mathcal{C})$,
   $$F(id_A) = id_{F(A)}. \tag{7}$$

In other words, a functor is a correspondence between categories that maps objects and arrows in one category to objects and arrows in another category, respectively, in a way that preserves the structural consistency. Thus, measuring and quantifying a property of something, or mathematically modeling some phenomenon, etc., can generally be formalized as a functor.

Natural Transformation

Furthermore, since functors are also mathematical objects by themselves, in category theory, they too are characterized by the arrows between them, called *natural transformations*.

Given two functors $F, G : \mathcal{C} \to \mathcal{D}$, a natural transformation $t : F \Rightarrow G$ is depicted as follows:

$$\begin{tikzcd} \mathcal{C} \arrow[r, bend left=50, "F"{name=U}] \arrow[r, bend right=50, "G"'{name=D}] & \mathcal{D} \arrow[Rightarrow, from=U, to=D, "t"] \end{tikzcd} \tag{8}$$

It must satisfy the following conditions:

1. For each object $A \in Ob(\mathcal{C})$, an arrow $t_A : F(A) \to G(A)$ in $\mathcal{D}$. Although seemingly quite high-order and abstract, a natural transformation is in effect a collection of arrows (indexed by the objects in category $\mathcal{C}$) in category $\mathcal{D}$.
2. *Naturality condition*: for any arrow $f : A \to B$ in $\mathcal{C}$, the following diagram "commutes":

$$\begin{tikzcd} F(A) \arrow[r, "F(f)"] \arrow[d, "t_A"'] & F(B) \arrow[d, "t_B"] \\ G(A) \arrow[r, "G(f)"'] & G(B) \end{tikzcd} \tag{9}$$

A diagram is said to "commute" when the compositions of arrows along any two paths with the same start and end objects yield the same result. That is, in this case,

$$t_B \circ F(f) = G(f) \circ t_A. \tag{10}$$

As previously noted, functors can formalize observations, quantifications and modeling based on different perspectives or criteria, so a natural transformation between them captures the regular covariant structure of the change in "view" resulting from a change in "perspective" in general.

There is not always a natural transformation between functors. The existence of a natural transformation, which is a correspondence that preserves the structure of the functors, rather means that there is structural consistency among those functors.[2]

These definitions provide the foundation for understanding the basic concepts of category theory. In the next section, we will apply these concepts to formalize the notion of affordances and demonstrate how category theory can provide a rigorous and insightful framework for ecological psychology.

Category-Theoretic Formalization of the Reality of Affordances

The purpose of this section is a natural and flexible formalization, based on category theory, of the reality of affordances, described in the previous sections.

As discussed above, in the modern theory of affordance, the reality of affordances is understood in terms of the lawful correspondance between forms of life and affordances, that is, the relationship "if there were an organism with a certain form of life in the here and now, the environment would afford certain actions to it." This can be described as a counterfactual relationship: "If one had a different form of life

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[2] In particular, when two functors have two opposing natural transformations between them such that they are "inverse" to each other, those two functors are said to be "naturally equivalent" and essentially the same. Historically, it was in order to formulate this notion of "natural equivalence" that category theory was created (Eilenberg and Maclane, 1945).

than the current one, the affordances provided by the environment would change from the current ones to that one.” In other words, the objective and public nature of affordances is the covariant structure shared among multiple forms of life, including potential and counterfactual ones that do not exist in the here and now.

Furthermore, as discussed in the previous section, functors can formalize “ways of viewing” from varying perspectives or criteria, and a natural transformation between them encapsulates the covariant structure governing the dependence between changes in “perspective” and corresponding changes in “view” (see also Fong and Spivak, 2019, p. 97).

Based on these considerations, we can hypothesize the following: *the reality of affordances can be formalized as natural transformations that represent the covariant structure between functors representing different forms of life.*

As a first step to substantiate this hypothesis and demonstrate that we can use category theory as a mathematical framework for ecological psychology, we present a simple model of the reality of affordances based on Warren’s (1984) experiment on climbability of steps, as described previously.

In this model, we consider two categories: the category $\mathcal{H}$ representing the height of steps and the category $\mathcal{C}$ representing the climbability of steps.

The objects of the category $\mathcal{H}$ are the different heights of steps, denoted as h_1, h_2, h_3, etc. The arrows in this category represent the “is lower than” relationship between step heights. For example, if h_1 is lower than h_2, there is an arrow $h_1 \to h_2$.

The category $\mathcal{C}$ consists of three objects: EC (easily climbable), PC (possibly climbable), and IC (impossible to climb). The arrows in this category represent the “is easier than” relationship between climbability degrees. For instance, there is an arrow $EC \to PC$, indicating that easily climbable is easier than possibly climbable.

These two categories belong to a kind of category called “preorder,” since for any two objects A and B, there is at most one arrow from A to B.

Two forms of life, namely, those of tall group and short group, are respectively represented by functors F_1 and F_2 from the category $\mathcal{H}$ to the category $\mathcal{C}$. These functors map the step heights to their corresponding climbability degrees for each form of life. For example, $F_1(h_1) = EC$ means that for the tall group, the step height h_1 is easily climbable.

As noted above, Warren (1984) showed that the height of the transition from climbable to unclimbable in both tall and short groups converged at about 0.88 times leg length. In addition to that, he also showed that the easiest height to climb (requiring the least energy to climb) was 0.26 times leg length for both groups. Simplifying these results, we now assume that the heights of the transition from “climbable” to “not climbable” and “easy” to “climbable” in the tall and short groups are 0.8 and 0.5 times the (average) leg length of each group, l_1 and l_2 ($l_1 > l_2$). We also assume $0 \leq h_1 < 0.5 \times l_2 \leq h_2 < 0.5 \times l_1 \leq h_3 < 0.8 \times l_2 \leq h_4 < 0.8 \times l_1 \leq h_5$. Then, two functors F_1 and F_2 are depicted by the red and blue dotted arrows in Figure 1 (the mappings between arrows are omitted).

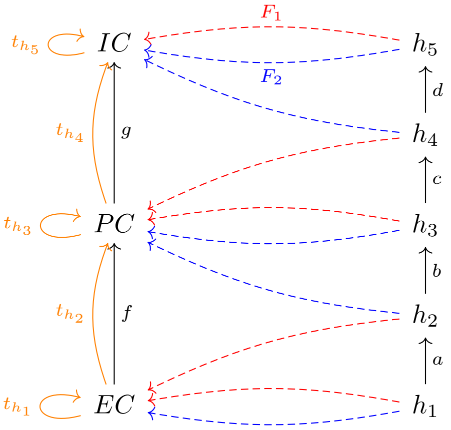

Figure 1: Diagram of the model. The category $\mathcal{H}$ (right) represents step heights, with arrows indicating the “is lower than” ($\leq$) relation. The category $\mathcal{C}$ (left) represents climbability degrees (Easy to Climb, Possible to Climb, and Impossible to Climb), with arrows indicating the “is easier than” relation. Functors F_1 (red) and F_2 (blue) map step heights to their corresponding climbability for tall and short groups, respectively. The natural transformation t (orange) captures how climbability would change if one were to transform from the form of life of the tall group to that of the short group.

A functor between two preorders is called a “monotonically increasing function.” This is one of the simplest forms of a functor and thus serves as a good starting point for modeling the perception of the environment as a functor.

The natural transformation t between the functors F_1 and F_2 captures the covariant structure of the relationship between the step heights and their climbability degrees across different forms of life. It consists of arrows $t_{h_i} : F_1(h_i) \to F_2(h_i)$ in the category $\mathcal{C}$ for each step height h_i. These arrows represent how the climbability of a step with a given height would change if one were to transform from one form of life to another.

By following the naturality condition, the natural transformation t is determined as depicted by the orange arrows in Figure 1 (Since $\mathcal{H}$ is a preorder, two arrows that share the domain and codomain are the same). Specifically,

$$\begin{aligned} t_{h_1} &= id_{EC}, \\ t_{h_2} &= f, \\ t_{h_3} &= id_{PC}, \\ t_{h_4} &= g, \\ t_{h_5} &= id_{IC} \end{aligned} \tag{11}$$

For example, the following diagram is commutative:

$$\begin{tikzcd}
F_2(h_3) = PC & F_2(h_2) = PC \arrow[l, "F_2(b)=1_{PC}"'] \\
F_1(h_3) = PC \arrow[u, "t_{h_3}=1_{PC}"] & F_1(h_2) = EC \arrow[l, "F_1(b)=f"] \arrow[u, "t_{h_2}=f"']
\end{tikzcd} \tag{12}$$

Natural transformations are called “natural” because, in many contexts, the transformations obtained by following the definitions turn out to be meaningful and useful in that context, and in some cases, they are already used and have their own names. In other words, the conditions of natural transformations, especially the naturality condition, provide exactly what’s needed for many theorems and definitions to work out smoothly.

The same is true in the current context. What natural transformation t, as defined in equation (11), represents is that if the form of life would change from that represented by F_1 to that represented by F_2, steps at height h_2 would be no longer easy to climb, steps at height h_4 would be no longer climbable, and at other heights, the climbability would remain the same. Here, the lawful covariant structure discussed in the modern theory of affordance, i.e., the relationship “if the form of life would change this way, the affordances obtained would change that way,” is “naturally” obtained by just following the definition of natural transformation. This does not depend on the specific details of the categories and functors we have defined here. Thus, although the model we now present is very elementary, the insights gained here could be applied to models of more complex forms of life.

To summarize, this simple category-theoretic model succinctly formalizes the reality of the climbability affordance in a way that captures both its relational and objective aspects in a consistent manner. On the one hand, the functors F_1 and F_2 represent the relational aspect of affordances, as they map the step heights to their corresponding climbability degrees for different forms of life, highlighting the dependence of affordances on the organism-environment relationship. On the other hand, the natural transformation t between these functors captures the objective, resource-like aspect of affordances, as it represents the covariant structure of the relationship between the environment (step heights) and the affordances (climbability degrees) that holds across different forms of life. This structure embodies the objective, resource-like nature of affordances, independent of any specific form of life. By formalizing both aspects within a single, consistent framework, this category-theoretic model provides a powerful tool for understanding the reality of affordances and their role in shaping organism-environment interactions.

From Entity to Transformation: Transitions in physics and mathematics

When addressing the reality of something, reference is often made to physics. However, even in the context of physics, the meaning of reality has changed dramatically, especially through major developments in the 20th century.

For example, the velocity of an object is relative to the choice of coordinate system or viewpoint from which it is viewed, and it is quantifiable only in dependence on that choice. Even when we are at rest on the earth, from an outside perspective, we are moving at very high speeds due to the earth’s rotation and revolution. But this dependence on perspective does not mean that it is a totally subjective quantity and nothing objective can be said about it. Rather, it is strictly calculable that “from that perspective, it would be this speed,” or more precisely, that “what appears to be this speed from this perspective would be that speed from another perspective.” Galileo Galilei discovered a method for calculating this coordinate transformation, now called the “Galilean transformation,” which he used as the basis for describing motion.

Not only velocity but almost all physical quantities depend on such a choice of coordinate system and perspective. Even so, physicists since Galilei have believed that time, as the interval between two events, remains the same in all coordinate systems. However, Einstein’s theory of relativity revealed that even time depends on the choice of the point of view (Rovelli, 2019). In other words, the problem of how physical quantities transform between different coordinate systems, known as coordinate transformation, became a fundamental problem in physics.

In the case of velocity, it may still seem to be a property of some entity existing “out there.” However, in the case of *fields* such as electromagnetic fields generated by the motion of charged particles, whether they exist or not also depends on the perspective (coordinate system) from which they are viewed. However, again, this does not mean that electromagnetic fields are arbitrarily created illusions or mere tools for the convenience of explanation; one can calculate and predict rigorously whether a field exists or not for an observer.

In other words, what is objective and real in relativity is neither a quantity measured from a particular perspective nor some entity independent of any perspective, but a structure of transformation that is established among different perspectives. In this context, existence in the classical sense, that is, “appearing to exist from any perspective,” is recognized as a special case in which its “being” does not change no matter how the relationship with the observer is changed (for more discussions, see Saigo and Taguchi, 2019).

The same can be said about quantum mechanics, another major innovation in 20th-century physics. Philip Ball (2018) states:

> "Quantum mechanics doesn't tell us how a thing *is*, but what (with calculable probability) it *could be*, along with – and this is crucial – a logic of the relationships between those 'coulds'. If *this*, then *that*. What this means is that, to truly describe the features of quantum mechanics, as far as that is currently possible, we should replace all the conventional 'isms' with 'ifms'. For example:
>
> **Not** 'here it is a particle, there it is a wave' **but** 'if we measure things like this, the quantum object behaves in a manner we associate with particles; but if we measure it like that, it behaves as if it's a wave'
>
> **Not** 'the particle is in two states at once' **but** 'if we measure it, we will detect this state with probability X, and that state with probability Y'" (pp. 352-353)

This shift from "theory of isness" to "theory of ifness" (Ball, 2018, p. 352) does not mean that quantum mechanics has stopped talking about reality. Quite the contrary, it is the result of a thoroughgoing commitment to reality as revealed through experimental results.

In other words, rather than adhering to "existence" in the classical sense and trying to squeeze into it the "weird" phenomena that leak from it, modern physics is shifting in the direction of taking such a way of being at face value and extending the meaning of "existence" and "reality."

The category-theoretic formalization we have described in the last section can allow us to see the shift in the reality of affordances as parallel to that in modern physics (cf. Turvey, 2015), since it is the concept of natural transformations that mathematically generalizes the "transformations between different perspectives" described above.[3] Accordingly, the reality of affordances is by no means undermined by their dependence on the form of life of the observer, just as the reality of physical quantities is not undermined by their dependence on the choice of coordinate system. Instead, this dependence is embraced as a fundamental feature of the phenomena, and the focus shifts to understanding the covariant structures and patterns that govern the transformations between different perspectives.

Discussion

Since the model described in this paper is still quite rudimentary, it needs to be further extended in order to discuss affordances for a wider range of behaviors. For example, Buhrmann et al. (2013) formalize the concept of sensorimotor contingencies in a general form using the framework of dynamical systems (see also Di Paolo et al., 2017). By rethinking these formalizations in terms of category theory and connecting them to our model, we may be able to clarify the interdependence between the environment and the agent in more general and dynamic behaviors.

Also, a model has been proposed to formalize the loosened sameness between the behaviors of a soft robot called a "universal gripper" when it flexibly grasps objects of different shapes, using the concept of "equivalence of categories" (Saigo et al., 2019). It may be possible to formalize affordances more concretely as a form of sameness on the environmental side, which is complementary to this approach.

Moreover, in this paper, we formalized the reality of affordance as a natural transformation between predefined functors, but this framework does not account for the emergence of new forms of life through evolution or technical innovation. To address this, we must consider an "indeterminate natural transformation," through which a new functor is created. At present, a theory of metaphor comprehension using this concept has been proposed (Fuyama et al., 2020), which we may be able to apply and develop to explain the perception of affordances.

Category theory is useful in such extensions through connections to other frameworks. This is because it is precisely for connecting different frameworks in mathematics to each other that category theory was created.

As noted in the introduction, the formalization of affordances we have presented here has significant implications for artificial life research. By providing a rigorous and consistent framework for understanding the relationship between organisms and their environments, our approach can inform the design of more adaptive and responsive artificial agents. Moreover, by grounding the concept of affordances in the formal language of category theory, our work facilitates the integration of ecological psychology with other theoretical frameworks used in artificial life, such as dynamical systems theory, information theory, and machine learning. Such an integration can lead to a deeper understanding of the mechanisms underlying the emergence of adaptive behavior in both natural and artificial systems.

Conclusion

In this paper, we have presented a category-theoretic formalization of the reality of affordances, drawing on the insights from modern theories of affordances and the tools of category theory. This approach offers a promising avenue for advancing our understanding of the fundamental principles governing life and cognition, both natural and artificial, and for developing more sophisticated and resilient artificial life systems that can thrive in the ever-changing landscapes of the world.

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[3] In fact, category theory has been employed to formalize quantum mechanics, providing a framework for describing its seemingly strange behavior in a more natural and intuitive way (Abramsky and Coecke, 2004; Coecke and Kissinger, 2018; Heunen and Vicary, 2019).

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