

# Evolution-in-Materio Computations: Hierarchies Rising from Electron Dynamics in Carbon Nanotubes

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## Abstract

Previously, Evolution-In-Materio (EIM), an unconventional computing paradigm, was addressed as a computing system which exhibits dynamical hierarchies. For different conceptual domains identified within an EIM system, a corresponding hierarchical level was defined and the state space description provided. Entropic relations established between such system descriptions show that one hyperdescribes another in an information theoretical sense. Hereby we report on those findings and revisit entropic relations between the level of material physics and the level of measurements via simulations of the addressed physical phenomenon.

## EIM Systems and Hierarchies Within Them

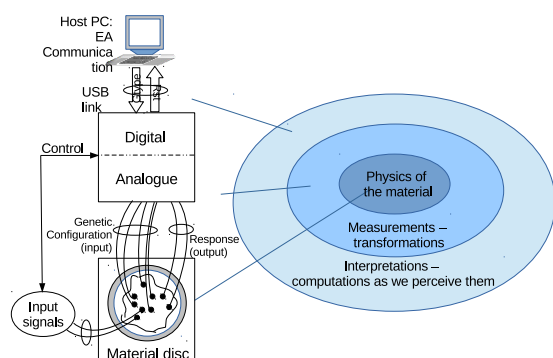


Figure 1: Conceptual domains of the EIM computing system, from (Laketić et al., 2015).

Evolution-In-Materio (EIM) (Miller et al., 2014), an unconventional computing approach, exploits physical properties of materials for computations. Materials are considered as bulks, unorganised matter, and manipulated under the guidance of an Evolutionary Algorithm (EA) towards achieving solution to a given computational problem. EA is run on a digital computer. Interaction with material physics (analogue) is realised via an interface board. For such a computing scenario different conceptual domains have been

identified as shown in Figure 1. For each domain, a system description is provided in a form of a discrete state space description, i.e., a set of system states and a transfer function, as summarised in Table 1.

The description is parameterised in the sense given by the system theory (Ashby, 1960). Since each state has a pertaining probability, the system is entropic. Also, description functions are defined which map states from a lower to a higher level in a sense described in (McGregor and Fernando, 2005).

Given such state space descriptions, the following equations were shown to hold (Laketić and Tufte, 2016):

$$H[s_b(t+1)|s_b(t)] < H[s_b(t+1)] \quad (1)$$

$$H[s_b(t+1)|s_b(t)] < H[s_a(t+1)|s_b(t)] \quad (2)$$

where indices  $a$  and  $b$  refer to a lower and higher level respectively. Such entropic relations are equivalent to *distinctness* and *state-dependence* as defined in (McGregor and Fernando, 2005) being less than 1 thereby providing a proof that recognised hierarchical levels exhibit novelty and loss of information which are prominent characteristics of dynamical hierarchies. For the level of material physics, we use carbon nanotubes (CNTs) since the physical phenomenon manipulated for achieving computations in most of our experiments within NASCENCE project (Broersma et al., 2012) was the change of CNT conductivity due to the changes in the electric field.

## From Electron Dynamics to Measured Voltages

Let us consider charge transport in a simple system as shown in Figure 2. Charge transport can be described by a set of wave vectors,  $\{\mathbf{k}_i\}$ , and occurs due to the change of a four-vector,  $\mathbf{A}$ :

$$\mathbf{A} = [\vec{A}, V/c] \quad (3)$$

where  $\vec{A}$  is a vector potential (magnetic field) and  $V/c$  a scalar, electrostatic potential. At mesoscopic level the latter corresponds to the electric field caused by the voltages

Domain	State	Transfer function	Loss of information	Novelty
Material physics	$s^1 = \{\mathbf{k}_i\}$	four vector, $\mathbf{A}$	N/A	N/A
Measurements	$s^2 = \{V_i\}$	analogue voltages on input electrodes	number of states, information on individual $\mathbf{k}$	voltage, currents (new qualitative description)
Interpretation, case-specific	$s^3 = \{s_i\}$	GA outcome	loss of voltage values	case-specific meaning assigned within a problem domain

Table 1: Overview of dynamical hierarchical levels in an EIM system

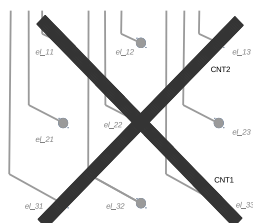


Figure 2: A schematic view of a simple CNT-based material, two CNTs, spread over a 3-by-3 electrode array for EIM.

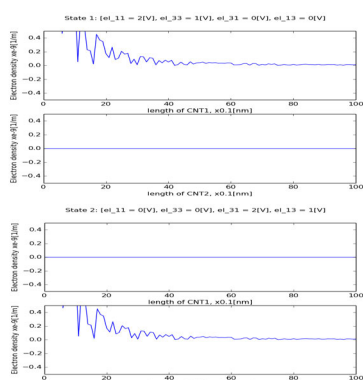


Figure 3: Two different states of material physics observed as the same state at the level of measurements.

brought to input electrodes. For simulations, a methodology as in (John et al., 2004) was used which addresses charge transport in CNT at mesoscopic level. By simulating transport for various voltages brought to electrodes  $el_{11}$ ,  $el_{33}$ ,  $el_{31}$ ,  $el_{13}$ , it can be shown that different states of electrons in the system represented by electron density along the CNTs may lead to the same potential at the position of electrode  $el_{22}$ , i.e., the same state at the level of measurements. Density of states corresponds to wave vectors  $\{\mathbf{k}_i\}$  which describe electron states at the level of material physics. In such cases, the uncertainty about the next system state is greater when it is calculated from the states at the level of material physics than when only voltages on the electrodes are used as shown in the example in Figure 3.

## Discussion and Conclusion

We have summarised the findings regarding hierarchies within EIM systems. They are of the similar nature as hierarchies present in natural, living systems, emerging from the dynamics of the units at lower levels and bringing in some novel property or behaviour at a higher level. Such hierarchies have been extensively addressed in ALife community. Information theoretic approach was used for its generality. Hierarchical organisation is important for robustness, adaptability and, above all, complexity.

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