Debye Model Fitting for Time-Domain Modeling of Lossy Dielectrics

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Abstract

In this paper, we will show a systematic procedure for fitting a Debye model to tabulated data of dielectric constant and loss tangent at discrete frequency points. Our approach is based on generation of a guaranteed passive RC macromodel, satisfying Kronig-Kramers relations for complex permittivity. We present for the first time an RC vector fitting method.

Keywords: Materials characterization, passivity, Debye model, dielectric constant

I. Introduction

Accurate estimation of the dielectric constant and loss tangent at high frequencies is becoming increasingly important as the frequency content of the signals in printed circuit boards (PCBs) and chip packages increase. Accuracy of the design of embedded RF components and planar antennas on PCBs depend highly on the knowledge of the dielectric material properties.

The signal integrity of a high-speed link can also suffer from closed eye diagrams due to dielectric losses. Although low-loss dielectrics are available to achieve high-RF compenents and improved signal integrity, they come at a higher cost. Accurate estimation of the broadband dielectric constant and loss tangent as a function of frequency is, therefore, critical.

Many techniques are available to characterize the material properties, such as the short-pulse propagation technique based on time-domain reflectometry measurements [1], microstrip bandpass filters [2], microstrip gap or ring resonators [3]-[5] or various versions of the full sheet resonance (FSR) method [6].

These methods provide the dielectric constant and loss tangent at discrete frequency points. This is not sufficient for signal integrity analysis, which requires time-domain models or circuit models that can be simulated in SPICE, to represent the lossy dielectric.

The multi-pole Debye model is the most commonly applied lossy dielectric model in signal integrity analysis. Hence, it is critical to be able to fit a Debye model to extracted dielectric constant and loss tangent. Current methodologies, however, do not go much beyond manual fitting of the model, or using general-purpose heuristic methods such as genetic algorithms. Note that standard macromodeling or vector fitting algorithms are appropriate for RLC networks, hence do not necessarily guarantee a Debye model, which can be considered to be an RC network.

In this paper, we will show a systematic procedure for fitting a Debye model to tabulated data of dielectric constant and loss tangent at discrete frequency points. Our approach is based on generation of a guaranteed passive RC macromodel, satisfying Kronig-Kramers relations for complex permittivity. We believe the results of this paper will be very useful in generating accurate models for transmission lines, power delivery networks, and other packaging structures for signal integrity analysis.

We will present two methods for creating a Debye model. The first method is applicable when the loss tangent is approximately constant over the frequency range of interest. In this case, we can represent the complex permittivity function using an irrational analytical function.

The second method is based on a modification of the vector fitting algorithm and is applicable in case of arbitrary variations of the dielectric constant and loss tangent, hence it is very general. The result is a guaranteed passive RC model for a lossy dielectric. To the best of the author's knowledge, this is the first time that a macromodeling methodology for guaranteed RC or RL modeling is being presented. Previous macromodeling approaches do not guarantee an RC or RL model, but rather generate a general RLC model.

II. DIELECTRICS WITH APPROXIMATELY CONSTANT LOSS TANGENT

When creating a Debye model, it is useful to consider that commonly used PCB substrates such as FR-4 tend to have an approximately constant loss tangent in the frequency range of interest. For this purpose, an average loss tangent value can be used to obtain a simple broadband model for such dielectrics. A constant loss tangent however implies that the dielectric constant is frequency-dependent according to Kramers-Kronig relation. Actually, since the complex permittivity is a minimum-phase function, the dielectric constant can be exactly defined (up to a constant) for a given frequency-independent loss tangent using the equation

$$\varepsilon = as^{-\frac{2\delta}{\pi}} \quad (1)$$

where a is an arbitrary positive constant, δ is the argument of $\tan\delta$, and s is the Laplace variable [6].

Since (1) is an irrational function, it does not have any immediate circuit representation. Hence, this function cannot be integrated with other circuit elements in SPICE for time domain simulation. For this purpose, it is desirable to create a Debye model based on this function. The Debye model is given as

$$\varepsilon(s) = \varepsilon'_\infty + \sum_{i=1}^{K} \frac{a_i}{1 + s\tau_i} \quad (2)$$

where a_i and τ_i represent the strength and time constants of various relaxation processes, and s is the Laplace variable. The order of the approximation K can be chosen as high as possible as long as the extracted a_i and τ_i are all positive and real numbers. The Debye model is quite useful in time-domain simulations, as it allows the consideration of the frequency-dependent material properties using an RC type of an equivalent-circuit model in SPICE or FDTD solvers. One example is shown in Figure 1, where the Debye model in (2) has been used to model a lossy capacitor with a constant loss tangent.

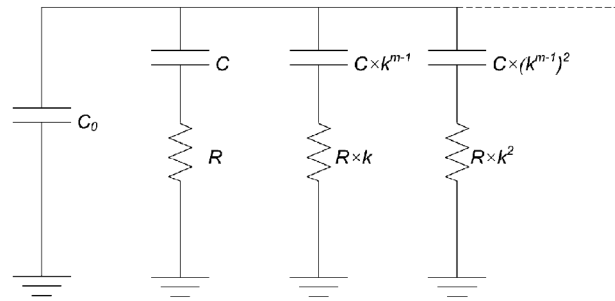

Figure 1 Debye model of a lossy capacitor with a constant loss tangent.

Based on (1), the admittance of such a network is given by

$$Y = s\frac{\varepsilon}{\varepsilon_{air}}C_{air} = \frac{C_{air}}{\varepsilon_{air}}\, a\, s^{1-2\delta/\pi} \quad (3)$$

where the subscript air refers to the permittivity or the capacitance calculated when the medium is replaced with free space. The Debye model in Figure 1 can then be considered as an approximation of this admittance.

For the case of a complex permittivity described by a constant-phase function as in (3), a simple model can be generated analytically without requiring any curve fitting [6], [7]. To find the coefficients of the Debye model, the following parameters should be provided:

- C_0 : High-frequency asymptote of the capacitance
- ω_0: Upper frequency bound for the validity of the model
- $\tan\delta$: Loss tangent
- k: Spacing factor to be chosen based on the required accuracy vs. bandwidth of the model
- N: Number of RC branches

Based on this input, the values of the circuit elements in Figure 1 can be obtained analytically following [5], [6] as

$$m = \frac{1}{1 - \frac{2\delta}{\pi}}$$

$$C = C_0(k^{m-1} - 1)$$

$$R = \frac{1}{\omega_0 C}$$

As the number of RC branches N in this network is increased, the lower frequency bound for the validity of the model decreases, which results in increased bandwidth of the model. To demonstrate the approach, consider the example with the given parameters of $C_0 = 10^{-12}$F, $\omega_0 = 10^{12}$ rad/s, and $\tan\delta = 0.02$.

Figure 2 shows the performance of the presented Debye model to represent a constant loss tangent. The spacing factor k can be increased to use less number of branches in the model. The trade-off is the increased sinusoidal variation around the desired loss tangent, decreasing the accuracy of the model. By using a small k and sufficient number of branches, perfect approximation of a constant loss tangent in the desired frequency range is possible. Note that a very accurate model for a bandwidth of approximately 8 decades was obtained using 15 branches in this example.

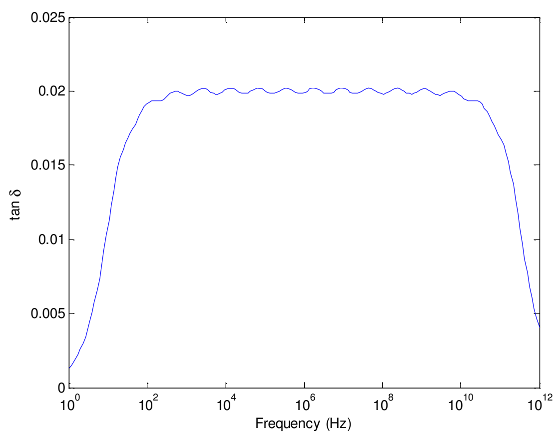

Figure 2 Loss tangent approximated using the Debye model for a material with an approximately constant loss tangent of 0.02 using 15 RC branches and $k = 5$

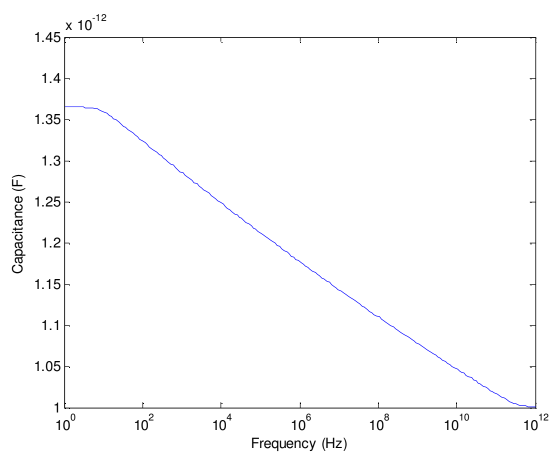

Figure 3 Capacitance obtained from the Debye model for the example of Figure 2

Figure 3 shows the variation of the capacitance with frequency. As expected, the capacitance approaches the high frequency asymptote C_0. Also at very low frequencies, the capacitance approaches the parallel connection of all the capacitors in the model.

III. Dielectrics With Arbitrary Variation of Dielectric Constant And Loss Tangent

To create a Debye model for dielectrics with arbitrary variation of the dielectric constant and the loss tangent, a curve fitting approach is needed. The standard vector-fitting algorithm [8], however, cannot be used as it cannot guarantee that the result is a passive RC type of a circuit model as in Figure 1. Even though there are various approaches for enforcing passivity or extending the vector fitting method for a guaranteed passive network realization such as [9], these methods may result in complex poles and residues (i.e., an RLC model). Hence, they cannot be used to generate a guaranteed passive RC model.

In this paper we demonstrate a modification of the vector fitting algorithm for synthesis of an RC (or RL) type of a network. In the following we outline the general steps of the vector fitting algorithm, along with the modifications necessary to generate an RC model. We call the new method, the *RC vector fitting method.*

In the standard vector fitting method, we start by specifying a set of starting poles as

$$\sigma(s)H(s) = c + \sum_{m=1}^{M} \frac{k_m}{s - \overline{p}_m} \quad (4)$$

where $\overline{p}_m$ are the starting poles and can be real or complex [8], [10]. In the RC vector fitting method, we limit the choice of the starting poles to stable real poles only. The function σ(*s*) can be represented in the rational function form:

$$\sigma(s) = 1 + \sum_{m=1}^{M} \frac{\tilde{k}_m}{s - \overline{p}_m} \quad (5)$$

which has the same poles as equation (4) but different residues. The function σ(*s*) is bounded at *s* = infinity. Multiplying equation (5) by *H*(*s*) and equating to (4),

$$c + \sum_{m=1}^{M} \frac{k_m}{s - \overline{p}_m} = \left[1 + \sum_{m=1}^{M} \frac{\tilde{k}_m}{s - \overline{p}_m}\right] H(s) \quad (6)$$

Since an initial set of poles are assumed, (6) can be reduced to the matrix equation:

$$\overline{\overline{A}}\,\overline{x} = \overline{b} \quad (7)$$

where the rows of $\overline{\overline{A}}$ correspond to the frequency samples, $\overline{x}$ contains the coefficients k_m, $\tilde{k}_m$, *c*, and $\overline{b}$

$$\begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \frac{1}{j\omega_k - \overline{p}_1} & \cdots & \cdots & \frac{1}{j\omega_k - \overline{p}_M} & \frac{-H(j\omega_k)}{j\omega_k - \overline{p}_1} & \cdots & \frac{-H(j\omega_k)}{j\omega_k - \overline{p}_M} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} c \\ k_1 \\ \vdots \\ \vdots \\ k_M \\ \tilde{k}_1 \\ \vdots \\ \tilde{k}_M \end{bmatrix} = \begin{bmatrix} H(j\omega_1) \\ H(j\omega_2) \\ \vdots \\ H(j\omega_k) \\ \vdots \\ \vdots \end{bmatrix} \quad (8)$$

contains the samples of $H(s)$. The matrix and vector elements are shown in equation (8), where ω_k is the k-th sample of frequency.

In the classical vector fitting method, (8) is solved as an overdetermined system using the least-squares solution. In the presented RC vector fitting method, we apply the least-squares solution with the additional constraint that the solution vector [i.e., the vector $\overline{x}$ in (7)] consists of positive elements only. Since the problem is convex, the solution we find is a global optimum.

After obtaining the coefficients by solving (8), (6) can be rewritten in the pole-zero form:

$$\frac{\prod_{m=1}^{M}(s - z_m)}{\prod_{m=1}^{M}(s - \overline{p}_m)} = \frac{\prod_{m=1}^{M}(s - \tilde{z}_m)}{\prod_{m=1}^{M}(s - \overline{p}_m)} H(s) \qquad (9)$$

where z_m and $\tilde{z}_m$ are the zeros computed by converting the pole-residue form to pole-zero form.

From equation (9), $H(s)$ can be obtained as

$$H(s) = \frac{\prod_{m=1}^{M}(s - z_m)}{\prod_{m=1}^{M}(s - \tilde{z}_m)} \qquad (10)$$

Comparing equations (10) and (6), the zeros of $\sigma(s)$ are used to replace the poles of $H(s)$. Since the starting poles $\overline{p}_m$ were real and stable, and we obtained positive residues k_m, $\tilde{k}_m$, c through the solution of (8) using non-negative least squares, the calculated poles of the function in (10) are real and stable.

This procedure is repeated until the poles converge to a constant value.

After extracting the poles of the transfer function $H(s)$, a new matrix equation can be generated and solved

$$\begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \frac{1}{j\omega_k - \overline{p}_1} & \cdots & \cdots & \frac{1}{j\omega_k - \overline{p}_M} \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} c \\ k_1 \\ \vdots \\ \vdots \\ k_M \end{bmatrix} = \begin{bmatrix} H(j\omega_1) \\ \vdots \\ H(j\omega_k) \\ \vdots \\ \end{bmatrix} \quad (11)$$

where the new coefficients c and k_m provide the constant term and the residues of $H(s)$. Note that the poles of $H(s)$ are real and stable. Similar to (8), (11) is solved using nonnegative least squares, guaranteeing a passive RC macromodel for $H(s)$.

IV. NUMERICAL EXAMPLES USING THE RC VECTOR FITTING METHOD

In this section we apply the RC vector fitting method to create a Debye model based on extracted dielectric constant and loss tangent shown in Table 1, which are taken from [6].

Freq (GHz)	1.35	2.15	2.7	3.05	6.1	10.8	13.7
ε_r	3.78	3.775	3.77	3.765	3.75	3.745	3.74
$\tan\delta$	0.0055	0.0055	0.0055	0.005	0.0065	0.0065	0.0085

Table 1 Data from [6]

First the standard vector fitting algorithm choosing four poles is run to fit the data tabulated in Table 1. Even though real and stable poles were selected as the initial four poles, the vector fitting algorithm provided complex poles as the output. Hence the result is not useful for creating a Debye model.

The RC vector fitting algorithm provided guaranteed stable and real poles as expected. Next, the residues were calculated by solving (11) using the RC vector fitting method. The global solution obtained included 3 positive residues and one residue equal to zero. Because of the observed redundancy, the RC vector fitting method was then run again with three initial poles, finally obtaining three positive residues. Hence the final model obtained was a third-order Debye model.

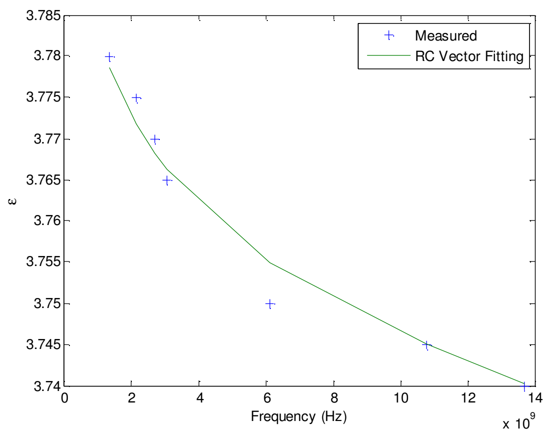

Figure 4 Fitted dielectric constant to measured data

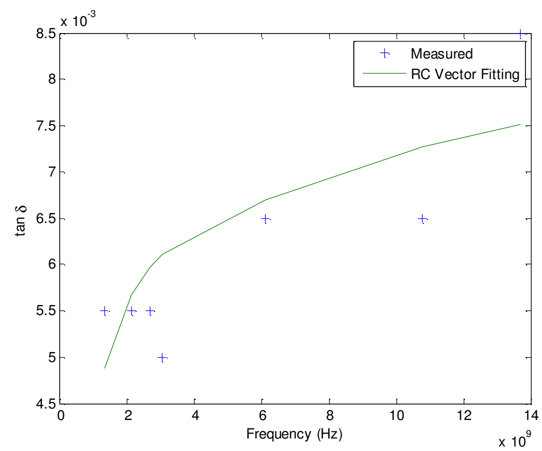

Figure 5 Fitted loss tangent to measured data

Figure 4 and Figure 5 compare the Debye model with the measured data. The frequency-dependent behavior of the complex permittivity is well captured. Note that the measured data may contain measurement errors, hence it is not expected to have an excellent match between the measurement and the Debye model. Rather, the Debye model may provide additional confidence in the extracted data, as the Debye model represents a physically plausible permittivity variation with frequency.

V. Conclusion

Using the presented approaches, causal and passive lumped models can he obtained for the capacitance and conductance of substrates either with a constant loss tangent or with an arbitrary variation of the dielectric constant and the loss tangent.

Both approaches result in a Debye model that can be implemented directly in generic SPICE solvers. We presented for the first time an RC vector fitting method that provides a global optimum solution for the poles and residues based on the convex formulation of the problem.

We believe the results of this paper will be very useful in generating accurate models for transmission lines, power delivery networks, and other packaging structures for signal integrity analysis.

References

[1] A. Deutsch, T.-M. Winkel, G. Kopcsay, C. Surovic, B. Rubin, G.Katopis, B. Chamberlin, and R. Krabbenhoft, “Extraction of ε_r and $\tan\delta$ for printed circuit board insulators up to 30 GHz using the short-pulse propagation technique,” *IEEE Trans. Adv. Packag.*, vol. 28, no. 1, pp. 4 –12, Feb. 2005.

[2] S. Yamacli, C. Ozdemir, and A. Akdagli, “A method for determining the dielectric constant of microwave PCB substrates,” *Int. J. Infrared Millim. Waves*, vol. 29, no. 2 , pp. 207–216, Feb. 2008.

[3] X. Fang, D. Linton, C. Walker, and B. Collins, “Dielectric constant characterization using a numerical method for the microstrip ring resonator,” *Microw. Opt. Technol. Lett.*, vol. 41, no. 1 , pp. 14–17, Apr. 2004.

[4] J.-M. Heinola and K. Tolsa, “Dielectric characterization of printed wiring board materials using ring resonator techniques: A comparison of calculation models,” *IEEE Trans. Dielect. Elect. Insulation*, vol. 13, no. 4 , pp. 717–726, Aug. 2006.

[5] R. K. Hoffmann, *Handbook of Microwave Integrated Circuits*. Norwood, MA: Artech House, 1987.

[6] Engin, A.E.; , "Extraction of Dielectric Constant and Loss Tangent Using New Rapid Plane Solver and Analytical Debye Modeling for Printed Circuit Boards," *Microwave Theory and Techniques, IEEE Transactions on* , vol.58, no.1, pp.211-219, Jan. 2010 doi: 10.1109/TMTT.2009.2036338

[7] R. Morrison, “RC constant-argument driving-point admittances,” IRE Trans. Circuit Theory, vol. CT-6 , no. 3 , pp. 310–317, Sep. 1959.

[8] B. Gustavsen and A. Semlyen, “Rational approximation of frequency domain responses by vector fitting,” IEEE Trans. Power Del., vol. 14, no. 3, pp. 1052–1061, Jul. 1999.

[9] de Tommasi, L.; Deschrijver, D.; Dhaene, T.; , "Single-Input-Single-Output Passive Macromodeling via Positive Fractions Vector Fitting," Signal Propagation on Interconnects, 2008. SPI 2008. 12th IEEE Workshop on , vol., no., pp.1-2, 12-15 May 2008

[10] Madhavan Swaminathan, A. Ege Engin, “*Power Integrity Modeling and Design for Semiconductors and Systems*,” Prentice Hall Press, 2007