4-PAM System Simulation for 25 Gbps Designs

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***Abstract*: This paper proposes to estimate very low PAM system BER using the Improved Importance Sampling (IIS) method. The results from both theoretical analysis and simulation show that the IIS estimation is unbiased. Based on calculation of the simulation variance and improvement ratio for IIS estimation, the IIS method is very effective for estimation of very low BER for PAM systems with intersymbol interference (ISI), jitter, crosstalk and additive noise.**

***Keywords* — 4-PAM, Jitter, BER, ISI, Signal Integrity, Simulation, Statistical Analysis, IIS Estimation, High Speed Digital.**

Introduction

IEEE 802.3 study group is exploring 4-PAM coding as an alternative to NRZ signaling to achieve higher data rate on regular FR4 substrate. There has been some literature published on 25 Gbps link level simulation utilizing traditional statistical simulation techniques but hardly any paper is devoted to extend statistical simulation applied to non-linear channels.

In high speed digital systems, the system error probability caused by ISI, jitter, crosstalk and additive noise is in very low probability region. The Monte Carlo (MC) method is commonly used in the simulation of the error probability of many digital communication systems. However, in order to estimate very low error probability, a huge number of MC samples is needed and so the MC method can be very computationally expensive. Hence, various estimation techniques such as Statistical simulation (Stateye equivalent), Quasi-Analytical (QA) method [1] and Importance Sampling (IS) techniques [2, 3] have been proposed.

Many Statistical and QA-based methods make the assumption that the channel is Linear Time Invariant (LTI) in nature and are biased. When driver/receiver non-linearity cannot be ignored, the Statistical simulation techniques may not provide accurate results and one may need to turn back to MC simulation. MC simulation can accurately predict BER but is known to be very slow because only a few simulation samples contribute to error events in the statistics. For example, if we use 1e6 simulation samples to estimate a BER of 1e-6, in average only one sample will contribute to the error event. To speed up the simulation, the IS method modifies the noise density to force the error events to happen more often. In order to keep the BER estimate unbiased, a weighting function is applied to the error count. Therefore, IS simulation is more efficient than MC.

In [2, 3] the Improved Importance Sampling (IIS) method is introduced. This paper discusses the application of the IIS method to a 4-PAM system.

The IIS Estimator

Let the input to the system be *a(t)*, the noise be *n(t)* and the output be *y(t)* as shown in Fig. 1.

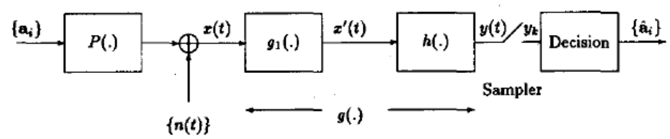

Fig 1. System model with Gating Function P(.), Memoryless function g_1(.) and filter h(.)

The input and the output are related by $y(t) = g(x(t))$, where $x(t) = a(t) + n(t)$. Let the output of the simulation be denoted by $Y_k = g(X)$, where $X = [X_k, X_{k-1,}, ..., X_{k-M+1}] = A + N$ is the input vector with M components, $A = [A_k, A_{k-1,}, ..., A_{k-M+1}]$ is an IID (Independent Identically Distributed) input data vector, and $N = [N_k, N_{k-1}, ..., N_{k-M+1}]$ is an IID noise vector. Their values are denoted by the corresponding lower case vectors ***x*, *a***, and ***n***.

For a binary signal, the a_i's are +A or –A and express the transmitted binary message. For the a_k at the *k-th* moment, we make the following hypothesis:

$H_0 \ : \ a_k = -A, \ H_1 \ : \ a_k = +A$, where A is a constant signal.

Assuming a decision threshold of *T*, if the input data and the noise are independent, the conditional error probability under H_0 can be expressed by

$$P_0 = \int_{\Omega_0} f^0(a) f_n(n) da dn \qquad (2)$$

where $f^0(a)$ is the pdf of a with the constraint a_k = –A, $f_n(n)$ is the pdf of the noise, and the decision region is $\Omega_0 = [g(a+n) \geq T]$.

Similarly, the conditional error probability under H_1 can be expressed by

$$P_1 = \int_{\Omega_1} f^1(a) f_n(n) dadn \qquad (3)$$

where $f^1(a)$ is the pdf of a with the constraint a_k = A, and the decision region is $\Omega_1 = [g(a+n) < T]$.

The error probability for the system is given by

$$P_e = P(H_0)P_0 + P(H_1)P_1$$

where $P(H_0)$ and $P(H_1)$ are the a priori probabilities for H_0 and H_1, respectively.

For a 4-PAM signal, we make the following hypothesis:

$$H_0 : a_k = -3A,\ H_1 : a_k = -A,$$
$$H_2 : a_k = A, \quad H_3 : a_k = +3A,$$

where A is a constant signal.

In the 4-PAM system, three thresholds need to be set. For a decision threshold of T_j, j = 0, 1, 2, if the input data and the noise are independent, the conditional error probability under H_j, j = 0, 1, 2, 3 can be expressed by

$$P_j = \int_{\Omega_j} f^j(a) f_n(n) dadn \qquad (4)$$

where $f^j(a)$ is the pdf of a with the constraint that a_k has the value under the hypothesis H_j, j = 0, 1, 2, 3, $f_n(n)$ is the pdf of the noise, and the decision regions are

$$\Omega_0 = [g(a+n) \geq T_0]$$
$$\Omega_1 = [(g(a+n) < T_0) \cup (g(a+n) \geq T_1)]$$
$$\Omega_2 = [(g(a+n) < T_1) \cup (g(a+n) \geq T_2)].$$
$$\Omega_3 = [g(a+n) < T_2]$$

The error probability for the 4-PAM system is given by

$$P_e = P(H_0)P_0 + P(H_1)P_1 + P(H_2)P_2 + P(H_3)P_3$$

where $P(H_j)$ j = 0, 1, 2, 3 are the a priori probabilities for H_j, j = 0, 1, 2, 3 respectively.

For the BER estimation, different estimators can be constructed as described below.

A. MC Estimator

Based on (4), we can construct a MC estimator as

$$\hat{P}_{MC} = \frac{1}{N}\sum_{j=1}^{N} D(.) \qquad (4)$$

where N *is* the number of simulation samples in the MC simulation and $D(t)$ is the indicator function, which equals 1 if t belongs to Ω_j, otherwise it equals 0. It is simple to show that the MC estimator is unbiased and the MC estimation variance is given by

$$\sigma_{MC}^2 = P_e(1-P_e)/N \qquad (5)$$

where P_e is the probability of error

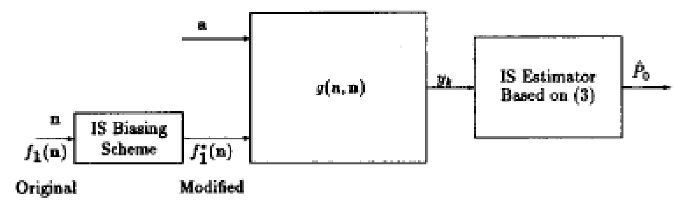

Fig. 2. IS simulation model for estimating BER

B. IS estimator

To speed up the BER simulation, the pdf of the input random variables can be modified, so that the error events occur more frequently. Thus, the number of simulation samples can be reduced. Of course, the count of error events must be properly weighted to obtain an unbiased estimation of the BER. The noise pdf can be modified by either increasing the noise variance (CIS) or translating the pdf (IIS) [3]. In the IIS method, the error probability can be written as

$$P_j = \iint_{\Omega_j} w(n) f_n^*(n) f^j(a) dadn \qquad (6)$$

where the $f_n^*(n)$ is the modified pdf for the IIS simulation and the weighting function is defined by $w(n) = \frac{f_n(n)}{f_n^*(n)}$

The IIS estimator can be constructed as

$$\hat{P}_j = \frac{1}{N_1}\sum_{j=1}^{N} w(n) \qquad (7)$$

where N_1 is the number of simulation samples in the IIS simulation. Upon direct evaluation, the IIS estimation is unbiased and the variance of the IIS estimator is given by

$$\sigma_{IS}^2 = E[(\hat{P}_j - P_j)^2]$$

$$= \frac{1}{N_1}\left[\iint_{\Omega_j} w(n)\, f_n(n)\, f^{\,j}(a)\, da\, dn - P_j^2 \right] \qquad (8)$$

In [3], an unbiased IIS Estimator has been constructed and the estimation variance has been given by using a boundary technique. Simulation speed is improved significantly using IIS.

Simulation using IIS

It is known that simulation variance is a direct measurement for simulation accuracy. In fact, the evaluation of the variance in the IIS method is quite complex. However, in [3] we have proposed a new bounding technique to solve this problem. By optimizing the variance bound the IIS can be set properly and an efficient IIS simulation can be achieved.

In Agilent EEsof simulation software SystemVue and ADS the IIS estimation technique has been implemented and can be used to efficiently estimate BER performance of very low BER systems. An example using IIS to estimate low BER is shown in Fig 3. The simulation results in Fig 4 show very good agreement between simulation and theory. At the BER=1e-8, the speed improvement IIS vs MC is great than 2e8.

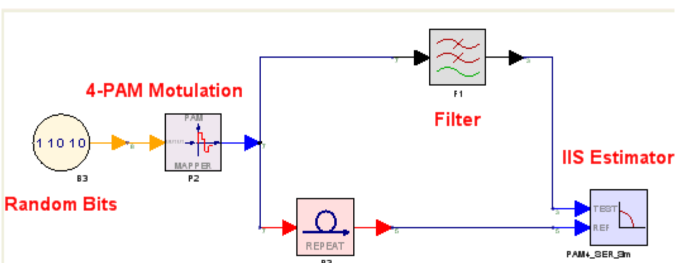

Fig. 3. Example using IIS Estimator in SystemVue

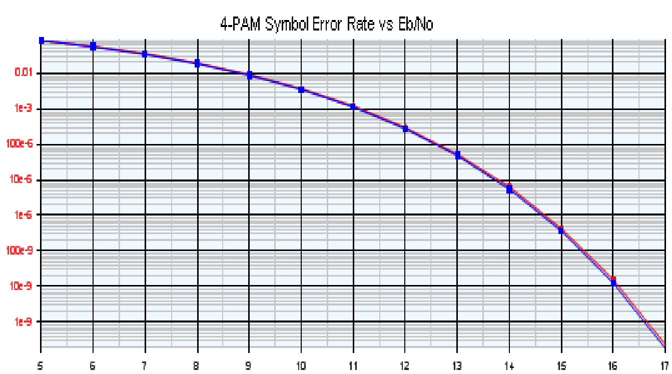

Fig. 4. The estimated BER using IIS vs Theoretical BER

SystemVue provides an environment for simulation of high speed digital systems. High speed circuits can be modelled and simulated with system level performance measurements as shown in Fig. 5 and Fig. 6. Fig. 5 shows an example design that generates a 4-PAM signal with jitter and passes it through a channel modelled with S-parameter data. The output of the channel is sent to the 89600 VSA and FlexDCA sinks for further analysis. These sinks are links to Agilent instrument software. Fig. 6 shows the eye diagram as seen in the FlexDCA display.

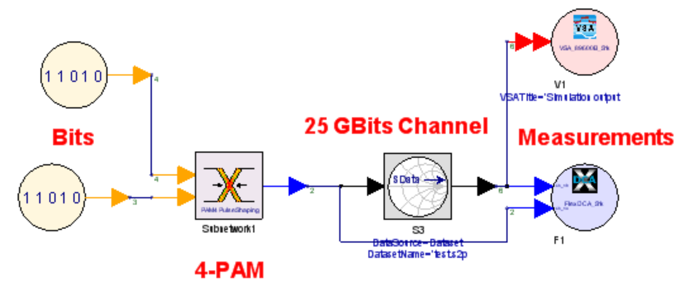

Fig. 5. 25 Gbits channel simulation in SystemVue

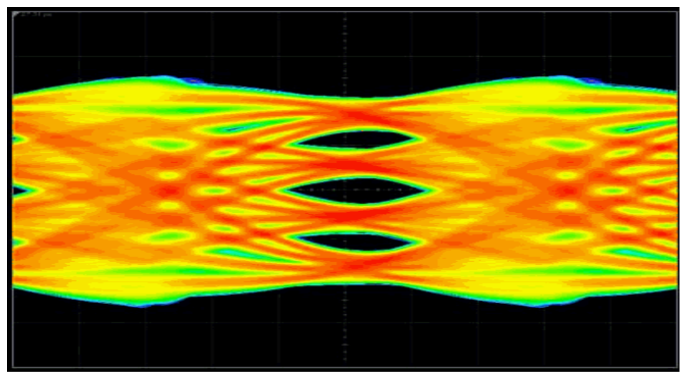

Fig. 6. 25 Gbits channel Eye Measured in SystemVue

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Conclusion and Further Work

We have shown that very low error probability in a 4-PAM system due to ISI, and Gaussian noise can be estimated by using the IIS method without making an LTI assumption. Results from analysis and simulation show that the IIS method is much better than direct MC method. This technique has been implemented in the commercially available communication system simulation package SystemVue for practical and efficient BER estimation. The IIS method can be used for M-ary signaling in linear and non-linear systems. When the noise is non-Gaussian, the IIS method can also be used for estimating the error probabilities with a desired accuracy.

Further work will be done to use the IIS estimator for very low BER simulation with high speed circuits such as 25 Gbits circuits in system level. In addition, the IIS method will be used for Signal Integrity analysis for systems with different channels, jitters, crosstalk and equalizers.

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