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Guest Editorial

Homogeneous and Isotropic Materials Failure Theory: A Critical Appraisal

1 The Challenge of Failure Theory

If there ever was a nearly ultimate field theory enigma, this is it. Failure theory for homogeneous materials has been relentlessly and aimlessly wandering around in the technical world for well over 200 years. Despite intense scientific efforts over the years, there were no historic breakthroughs. Nothing of any generality ever came together. It was hopeless, that was the unmistakable conclusion. In terms of applications, it certainly seemed destined to forever just be left as a pedestrian, blue-collar exercise. That same historic consensus remains as the common view today. All matters of generality remain as ephemeral, ambiguous.

Not accepting that dire status, there has been considerable work done on the subject in recent years. There has been substantial progress. It would be hard to imagine reaching a reasonable understanding on failure until the simplest case, that of isotropy, is under satisfactory control. That is why most of the efforts here have gone into isotropy.

There now is a complete and proven two property failure theory for homogeneous and isotropic materials [1,2]. By the term "complete," we mean the theory is completely general in going from a cohesionless, damaged material at one extreme, progressing and extending through to a perfectly ductile material at the other extreme. All states in between these two extremes must be and are of consistent and accountable physical behaviors. The complete formalism is encompassed by only two determinable failure properties, no empirical parameters are involved.

The specific obstacle impeding progress at this point is no longer the missing failure theory formulation; it is that of communications, exceedingly difficult technical communications. In going against the common, thoroughly entrenched negative view, how can anyone:

(i) be persuaded that the isotropic failure problem actually and finally has been solved, and
(ii) be convinced that the solution is nearly ready for deployment in applications and in tutorials such as books, websites, and apps.

This communications problem imposes an even more formidable barrier than did that of solving the explicit failure theory problem itself.

Acting as a palliative, many people in the practice of materials applications resolutely and routinely cling to their grossly oversimplified Mises and Tresca failure criteria. Furthermore, any attendant bandages, splints, or crutches grafted on to this prescription cannot salvage anything as far as generality is concerned.

Can the Mises-Tresca syndrome ever be overcome. If not, there may never be any progress. If it can be reversed, there could possibly be a bright future. The blue-collar group remains as an influential constituency. We will see how all of this plays out. These are interesting and crucial times.

With all this long-term confusion, it is perfectly understandable that reasonable people could give up on trying to make sense of materials failure. There are so many discordant voices and claims on all aspects of the subject. It really could seem hopeless at a certain level.

As the dedicated, committed readers and users that you are, this extremely difficult situation now relates to you. Do we recognize the critical need for a general failure solution (if it is possible) to bring forth the final resolution of this problem. More simply expressed, do we move forward or not. Is it to be blue sky opportunity ahead or will it be the continuing and probably forever lasting blue-collar state of neglect and despair. A decisive resolution is the option needed here. It could go either way but either requires proof and finality.

2 Atomic Scale Progress With Some Preliminary Guidance for Failure

Within our lifetimes, *Density Functional Theory* (Professor Walter Kohn) has revolutionized atomic physics [3]. A similar renaissance is desperately needed for materials failure. It not only seems possible, it seems quite likely. Even though materials failure ranks as one of the most difficult problems in existence, the broadly related DFT method subdued and overcame its equally difficult or even more difficult problems. So that likely success could also be done for materials failure. It should be done. Perhaps it even already has been done.

To be more specific, the failure behavior of materials is not only dictated by its atomic scale constitution but also by the distributions of all flaws and defects at all scales. All of these conditions must be taken into proper account. This does indeed seem to be an overwhelming set of complicating and restrictive conditions. Although it is not necessarily a total blockage, there could still be a reasonable approach. Just as density functional theory provided the correct direction of approach through its quantum mechanics obstacles, so too there must be a corresponding rational direction of approach for the somewhat related failure problem. It is here reported to have been discovered and then further developed.

3 Moving Ahead Strongly

The objective now is to overcome the communications issue outlined earlier and to initiate and proceed with the assimilation of this new work into the mainstream. Both of these tasks would be best done or begun through further directed and careful supporting work.

The new failure theory status was briefly but succinctly stated in the third paragraph of Sec. 1 of this document. Very recent work has used the limiting case behavior (representing a cohesionless solid) to solve the previously unsolved classical problem of the "Angle of Repose" [4]. Even more recently, Ref. [5] has revealed that there are exactly three fundamental modes of failure involved with the failure of all isotropic materials. Other work for the future will show new and different derivations of the ductile/brittle transition aspect of failure. This latter area is of considerable importance as reinforcing the general theory of failure, as do Refs. [4,5].

We shall show how the simplest form of the failure theory takes an elementary form that is ideal for introductory treatments, as in undergraduate mechanics of materials texts. At the other end of the spectrum, the theory must allow further opportunities for challenging research. For example, we intend to further develop the sophisticated treatment of the bond bending and bond stretching effects at the atomic level by proceeding further along the direction presented in Ref. [6].

4 Elasticity Theory Connection is Critical and Enabling

Above all we strive to demonstrate that the present theory possesses order and organization at the highest level. If it does not have that, it would be useless. That must start right from the original and initial premise. The failure theory should represent the side-by-side development with the theory of elasticity. It must be and it is completely compatible with the fundamentals of linear elasticity theory since it represents the ultimate extension of three-dimensional elastic behavior right up to the point of exhaustion, extinction. That is what the term failure means, the extinction of the load supporting elastic capability.

Following along those lines, the pass-key to the entire failure formulation involves examining the elasticity theory and then extracting the appropriate tensor valued invariant forms that must also participate in the limiting failure process. It is a full technical partnership in mathematical synthesis between the elastic energy physical processes and the materials failure physical processes. One without the other is incomplete.

As explained in Ref. [1], the yield stress does not and should not enter into these failure considerations. It is the failure stress(es) that gives the proper coordination whether there is expected plasticity behavior or not. The 60- or 70-year time span spent in developing and verifying plasticity theory was certainly of importance for application to very ductile, polycrystalline metals. But what about the conjectured possible plasticity application to general materials failure.

The presumption (or hope) that plasticity would somehow relate to a failure methodology for all materials did not turn out to be true. It was completely irrelevant to that purpose. Some of that large expenditure of time and talent could have been better spent on failure theory and failure characterization or on many other worthy areas as well. It probably follows that the overpowering emphasis on plasticity theory for so long actually retarded any progress on the more vital theory of failure for all materials. Elasticity theory is essential, plasticity theory is not.

The transformation from elastic behavior to total, abject failure in effect represents the controlling fulcrum in the life path of the materials function. Elasticity and failure are equally important but distinctly different parts of the same larger, all covering entity. In sequential consequence, it is the failure theory that must accommodate its compatibility to that of the elasticity theory. That is the underlying basis of everything that has been done with this failure theory.

For this entire failure theory endeavor, the guidance from and connection with elasticity theory is absolutely indispensable. As developed, it completes and closes the century's long challenge posed by the absence of a credible and verified materials failure theory. Thus, quality mechanics and three-dimensional elasticity behavior as applied to isotropic materials were the specific agents for the enablement of this failure theory.

5 The Fundamental and Complete Materials Failure Discipline Is Here

The elasticity theory development, individually and in isolation, reached its apogee with the extraordinary elasticity book by Professor Love [7]. As has been seen from the forgoing account, this new failure theory is committed to continue and finalize the historic elasticity theory treatment beyond that of Love by including the tightly linked and inevitable failure occurrence.

We do have a unique opportunity to finish the development and begin the assimilation of this new and comprehensive elasticity/failure theory. The final failure discipline part is virtually complete. Not only will it provide us with an invaluable tool, it would also honor all those past but almost forgotten founders of elasticity theory. The explicit and original mathematical/physical derivation given in pp. 32–42 of Ref. [1] comprises the most meaningful account of the full failure theory origins. It is highly recommended, high concentration reading.

Rigorous failure theory for homogeneous and isotropic elastic materials will give realistic predictions of the causative and the limiting conditions of failure. Few physical science problems along with their resolutions could be of more importance than that. It would represent the fulfillment of the failure field after all the years of prolonged, agonizing, historic struggle. It would even just about complete the timeless and all inclusive discipline of the mechanics of materials.

We can make it happen.

Acknowledgment

This article is dedicated to the remembrance of my wife, Kristy Christensen. She was a very special person.

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A complete listing of references on the subject is given in Refs. [1,2]. Included here are only those works of immediate relevance to this appraisal of the current status of failure theory. These same two references also contain the necessary, detailed verifications of this new failure theory.

References

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