Polarimetric Radar Characteristics of Melting Hail. Part I: Theoretical Simulations Using Spectral Microphysical Modeling

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ABSTRACT
Spectral (bin) microphysics models are used to simulate polarimetric radar variables in melting hail. Most computations are performed in a framework of a steady-state, one-dimensional column model. Vertical profiles of radar reflectivity factor $Z$, differential reflectivity $Z_{DR}$, specific differential phase $K_{DP}$, specific attenuation $A_h$, and specific differential attenuation $A_{DP}$ are modeled at S, C, and X bands for a variety of size distributions of ice particles aloft. The impact of temperature lapse rate, humidity, vertical air velocities, and ice particle density on the vertical profiles of the radar variables is also investigated. Polarimetric radar signatures of melting hail depend on the degree of melting or the height of the radar resolution volume with respect to the freezing level, which determines the relative fractions of partially and completely melted hail (i.e., rain). Simulated vertical profiles of radar variables are very sensitive to radar wavelength and the slope of the size distribution of hail aloft, which is correlated well with maximal hail size. Analysis of relative contributions of different parts of the hail/rain size spectrum to the radar variables allows explanations of a number of experimentally observed features such as large differences in $Z$ of hail at the three radar wavelengths, unusually high values of $Z_{DR}$ at C band, and relative insensitivity of the measurements at C and X bands to the presence of large hail exceeding 2.5 cm in diameter. Modeling results are consistent with S- and C-band polarimetric radar observations and are utilized in Part II for devising practical algorithms for hail detection and determination of hail size as well as attenuation correction and rainfall estimation in the presence of hail.

1. Introduction
Polarimetric weather radars reveal great potential for detection of hail and determination of its size. The combination of radar reflectivity factor $Z$ and differential reflectivity $Z_{DR}$ proves to be efficient for discrimination between pure rain and dry hail because of the more random orientation of tumbling hailstones and their lower dielectric constant relative to raindrops. Hence, low $Z_{DR}$ associated with high $Z$ is a certain indication of dry hail. Once hail starts melting, it gradually acquires polarimetric attributes of rain, and detection of melting hail mixed with rain becomes less straightforward because $Z_{DR}$ increases as hail progressively melts. Such an increase is especially pronounced at C band because of the strong effects of resonance scattering by large raindrops and smaller-sized partially melted hailstones.

It is hail of sufficiently large size and high density that inflicts substantial damage so that, in addition to hail detection, discrimination between hail of different sizes and densities is required. According to the U.S. National Weather Service standard, hail with size exceeding 2.5 cm (1.0 in.) is considered to be high impact and dangerous (i.e., “severe”), and hail in excess of 5.0 cm (2.0 in.) is considered to be “significantly severe.”
Polarimetric algorithms for hydrometeor classification, which utilize a combination of \( Z, Z_{DR} \), specific differential phase \( K_{DP} \), and the cross-correlation coefficient \( \rho_{hv} \), demonstrate good skill for hail detection at S band as limited validation studies have shown (e.g., Heinselman and Ryzhkov 2006; Depue et al. 2007). Polarimetric hail-detection algorithms that were originally developed for S band need significant modification for applications at C band, primarily because \( Z_{DR} \) of small- and medium-size melting hail at C band is greater than at S band (Ryzhkov et al. 2007; Tabary et al. 2010; Anderson et al. 2011). Wet hail is typically mixed with rain, and anomalously high \( Z_{DR} \) due to resonance scattering associated with large raindrops and small, melting hail at C band offsets the low intrinsic \( Z_{DR} \) of moderate-to-large hail. The modeling studies of Ryzhkov et al. (2009, 2011) and Kumjian et al. (2010) support this interpretation and show that \( Z_{DR} \) of melting hail is very sensitive to radar wavelength. Direct comparisons of polarimetric hail signatures observed by closely located S- and C-band radars (Borowska et al. 2011; Gu et al. 2011; Picca and Ryzhkov 2012; Kaltenboeck and Ryzhkov 2013) are generally consistent with results of these theoretical simulations. Another important feature that must be considered for hail detection using shorter wavelengths is that \( Z \) of hail at these wavelengths may be significantly lower than at S band. The corresponding difference has been termed the “hail signal” in previous studies employing dual-wavelength observations (Atlas and Ludlam 1961; Eccles and Atlas 1973; Bringer et al. 1986; Feral et al. 2003).

Validation studies of polarimetric hail-detection algorithms are rare. Notable exceptions include the works of Heinselman and Ryzhkov (2006) and Depue et al. (2007) at S band and Boodoo et al. (2009) and Tabary et al. (2009, 2010) at C band. To our knowledge, the study of Depue et al. (2007) is probably the only one in which the correlation of the maximal size of ground-truth hail observations with polarimetric signatures has been examined systematically.

Determination of hail size remains challenging. The most recent version of the hydrometeor classification algorithm (HCA) developed at the National Severe Storms Laboratory for polarimetrically upgraded Weather Surveillance Radar-1988 Doppler (WSR-88D) instruments detects “rain mixed with hail” (Park et al. 2009) and does not distinguish between large and small hail. The Colorado State University HCA distinguishes between “graupel/small hail” and “hail” without specifying the borderline size between “small hail” and “hail” (Lim et al. 2005). Depue et al. (2007) recommended using the hail differential reflectivity parameter \( H_{DR} \), which incorporates both \( Z \) and \( Z_{DR} \) (Aydin et al. 1986). Picca and Ryzhkov (2012) mention that the scheme of Depue et al. (2007) does not account for the hail melting process, which has a very strong impact on the vertical profile of \( Z_{DR} \). Kumjian et al. (2010), Picca and Ryzhkov (2011), and Kaltenboeck and Ryzhkov (2013) also suggest possible ways to identify “giant” hail with size exceeding 5 cm utilizing the pronounced reduction of \( \rho_{hv} \) and appearance of slightly negative \( Z_{DR} \) in the hail generation zone above the freezing level that indicates growth of hail in the “wet regime” for which all accreted water does not freeze on the surface of hailstones because of substantial latent heat release. Water coating on the surface of hailstones accentuates the effects of resonance scattering (Balakrishnan and Zrnić 1990b).

The methods for estimating hail size should be substantiated by retrievals from cloud models that explicitly treat the microphysics of melting hail (e.g., Khain et al. 2011; Ryzhkov et al. 2011). Melting of graupel and hail strongly affects the vertical profiles of polarimetric radar variables in convective storms. The impact of graupel/hail melting on the vertical distribution of \( Z_{DR} \) and linear depolarization ratio \( L_{DP} \) was first studied by Bringi et al. (1986), Vivekanandan et al. (1990), and Meischner et al. (1991) on the basis of the thermodynamic model of Rasmussen and Heymsfield (1987a). In this study, we use a similar thermodynamic model with several modifications regarding drop shedding and breakup, and we account for vertical air motion.

Melting hail causes significant attenuation of the radar signal that is different at orthogonal polarizations. Such attenuation can be significant even at S band. Polarimetric methods for attenuation correction are based on the measurements of differential phase \( \Phi_{DP} \) and imply knowledge of the ratios \( \alpha = A_{DP}/K_{DP} \) and \( \beta = A_{DP}/K_{DP} \), where \( A_{DP} \) is specific attenuation, \( A_{DP} \) is specific differential attenuation, and \( K_{DP} \) is specific differential attenuation. As opposed to the case of pure rain, the variability of the ratios \( \alpha \) and \( \beta \) in hail is not known, and, as a result, no reliable procedures for attenuation correction in hail exist at the moment.

Another practical challenge is radar rainfall estimation in the presence of hail. It is generally assumed that \( K_{DP} \) is less affected by hail than is radar reflectivity \( Z \) and that the use of the \( R(K_{DP}) \) relation is preferable for the rain/hail mixture (Balakrishnan and Zrnić 1990a; Aydin et al. 1995; Bringi and Chandrasekar 2001; Ryzhkov et al. 2005). This assumption, however, should be better justified both theoretically and observationally. One of the problems is that \( K_{DP} \) may become very noisy in hail-containing areas of the storm where \( \rho_{hv} \) is reduced because of the presence of large mixed-phase hydrometeors for which the effects of resonance scattering are particularly pronounced.
This three-part series of papers is organized as follows. In the first part, both microphysical and scattering models of melting hail are described. Vertical profiles of the polarimetric radar variables are simulated and examined at three major frequency bands: S, C, and X. The sensitivity of vertical profiles to various physical factors such as 1) temperature lapse rate, 2) humidity profile, 3) strength of descending air motions (downdrafts), 4) size distributions of graupel/hail aloft, 5) density of ice particles at the freezing level, and 6) radar wavelength is investigated. Two spectral microphysical models of melting hail coupled with the polarimetric radar observation operator are utilized in this investigation, and the results of theoretical simulations at S and C bands are compared with observations in hail.

In the second part (Ryzhkov et al. 2013), the strategy for hail detection and determination of its size at different wavelengths is discussed and the algorithm for hail detection and determination of its size at S band is described. We utilize our polarimetric model of melting hail to examine attenuation effects in melting hail and the quality of the rain estimate if rain is mixed with hail. Part III (A. Ryzhkov et al. 2013, unpublished manuscript) contains results of testing and validation of the proposed algorithm using an extensive dataset of polarimetric measurements and ground observations collected in different parts of the United States with a number of polarimetric WSR-88D instruments utilizing the Severe Hazards Analysis and Verification Experiment validation method (Ortega et al. 2009).

2. Thermodynamic model description

Two thermodynamic models of melting hail are utilized in this study. Similar to earlier works of Aydin and Zhao (1990), Aydin and Giridhar (1991), and Vivekanandan et al. (1990), one of the models (model 1) makes use of the Rasmussen and Heymsfield (1987a,b) study of the physics of melting of individual hailstones. It assumes a specified distribution of graupel/hail at the freezing level and follows the change of the size distribution of partially melted ice particles/raindrops and the corresponding polarimetric radar variables as hydrometeors (totally or partially melted) reach the ground. This is a steady-state 1D model that takes into account shedding of excessive water from the surface of melting hailstones and spontaneous breakup of large raindrops but does not allow for interactions/collisions between the particles.

The second model (model 2) is the 2D nonhydrostatic mixed-phase spectral bin Hebrew University of Jerusalem Cloud Model (HUCM; e.g., Khain et al. 2004, 2011). The model contains seven classes of hydrometeors, and each class is represented by size distribution functions in 43 size bins. As opposed to model 1, this model explicitly describes both generation and melting of hail and takes into account all kinds of interaction between hydrometeors.

Model 1 is relatively simple and convenient to examine the sensitivity of the polarimetric signatures to variations of initial hail size distributions, the density of hail, and thermodynamic profiles, whereas model 2 is more sophisticated and computationally expensive. It is expected to better capture the effects of size sorting that have a strong impact on polarimetric variables.

a. Melting of individual hailstones

The 1D explicit bin microphysical model of melting hail is based on the work of Rasmussen et al. (1984) and Rasmussen and Heymsfield (1987a,b, herein RH87a,b). Initial hailstone diameters range from 0 to 40 mm in increments of 0.1 mm (400 total size “bins”). At the top of the model domain (4 km AGL, with a temperature of 0°C), an initial distribution of hailstones is prescribed. These hailstones are allowed to fall through the domain, where the temperature and humidity are prescribed on the basis of an initial sounding interpolated to the model grid (vertical resolution is 10 m) or on the basis of predefined temperature and humidity lapse rates. Any updraft or downdraft profile w may be administered. The equations for terminal velocities of melting hailstones and heat transfer equations determining the rate of melting depending on particle size (or Reynolds number) are summarized in appendixes A and B.

In our study, we perform simulations while assuming that initial density of ice particles aloft is either equal to the density of solid ice (917 kg m⁻³) or varies across the size spectrum such that the density of graupel-size particles is lower than the density of larger hailstones (Fig. 1). The latter assumption reflects the general perception that graupel is softer than large hail (e.g., Prodi 1970) and that the selected size dependence of density is consistent with the results in RH87b (their Fig. 16). In the case of high-density hail, the melted water accumulates on the surface of the hailstone and sheds after its mass exceeds a certain “shedding threshold.” If the initial density of graupel/hail is lower than 917 kg m⁻³ then melted water first soaks into the interior of the particle and starts accumulating on the surface only after all air cavities are filled up with melted water. Again, excessive water sheds after reaching a critical mass $m_{w\text{max}}$ (RH87a; Phillips et al. 2007):

$$m_{w\text{max}} = 2.68 \times 10^{-4} + 0.139(m_i + m_{ws}),$$  (1)
where \( m_i \) is mass of ice and \( m_{ws} \) is mass of retained soaked water, both expressed in kilograms. We also assume that the ratio of the mass of soaked water and the mass of ice after the particle is fully soaked is constant and equal to the mass when full soaking is attained (i.e., \( m_i \) and \( m_{ws} \) decrease proportionally before full melting occurs). This assumption is needed to ensure that the total mass of the resulting water drop does not exceed the mass of an 8-mm raindrop [or \( 2.68 \times 10^{-4} \text{kg} \) according to Eq. (1)].

The dependencies of diameters of melting hailstones and their ice cores on height for high-density and variable-density hail estimated with model 1 are illustrated in Fig. 2. The computations have been performed while assuming that the freezing level is at 4 km, temperature lapse rate is 6.5°C km\(^{-1}\), and relative humidity is 100%.

If the hailstone is spongy (i.e., originally has lower density), then the effective diameter of the ice core is defined as an equivolume diameter of a spheroid with the mass of unmelted ice and density of solid ice. For a given size of dry hailstone aloft, lower-density hail melts faster and its total diameter and effective diameter of ice core decrease more rapidly toward the ground.

Figure 2 shows that hailstones with initial diameters (i.e., diameters aloft) of less than 14 mm (for high-density hail) and 15 mm (for variable-density hail) melt completely before reaching the ground, whereas ice cores of much bigger hailstones remain large. Shedding of water from the surface of larger melting hailstones causes reduction of their mass and diameter. At each height below the freezing level, mass water fraction \( f_m \) is equal to 1 for totally melted particles and gradually decreases with size of partially melted graupel and hail (Fig. 3). Once the maximal size of a raindrop resulting from melting hail reaches 8 mm (at about 2.5–3 km below the freezing level), the \( f_m(D) \) dependence, where \( D \) is diameter, practically does not change. This is a consequence of the shedding condition in Eq. (1).

Figure 2 also shows that hailstones with initial sizes between 8 and 14 mm (in the case of high-density hail) or between 9 and 15 mm (in the case of variable-density hail) end up as 8-mm raindrops, which causes an enhancement of the total concentration of 8-mm raindrops near the ground. Such an enhancement of very large drops originated from melting hail is also predicted by the simulations with HUCM (Khain et al. 2011; Ryzhkov et al. 2011). It has to be taken into account,
however, that very large raindrops with diameters of 8 mm and larger are unstable and tend to break up spontaneously. In our simulations with model 1, the loss of these large drops due to spontaneous breakup is described probabilistically using an exponential factor

\[ P_b(m, h) = \exp \left( -\frac{h_{0m} - h}{1.2H_b} \right) \],

where \( h_{0m} \) is the height at which hailstones within the original \( m \)th size bin completely melt and become 8-mm raindrops. According to Eq. (2), at the height \( h = h_{0m} - H_b \) 50% of 8-mm raindrops break up. Note that in the spectral (bin) approach we assume conservation of the concentration flux within the given size bin unless breakup occurs; that is, the product of particle concentration \( N(m, h) \) and terminal velocity \( U(m, h) \) is constant. This condition implies that the particles in separate size bins do not interact with each other. Because we use a Lagrangian approach in our simulations, both the center of a particular bin with index \( m \) (mean diameter of melting particle) and the width of the \( m \)th bin vary with height. Taking into account the breakup of 8-mm raindrops is implemented by multiplying \( N(m, h) \) by \( P_b(m, h) \) for \( h < h_{0m} \). Laboratory measurements in Kamra et al. (1991) indicate that average time of survival of an 8-mm raindrop before breakup is about 40 s. Hence, a reasonable choice of the parameter \( H_b \) in Eq. (2) is 400 m for terminal velocity of 8-mm drops of about 10 m s\(^{-1}\).

**b. Size distribution of melting hailstones and raindrops**

The advantage of model 1 is that it allows the study of the impact of the size distribution of graupel/hailstones aloft on the vertical profiles of radar variables in the most direct and straightforward way. In situ measurements of size distributions of ice particles aloft in hailstorms often reveal a biexponential type of particle spectra with different slopes for graupel and hail (Smith et al. 1976; Spahn and Smith 1976). In our simulations with model 1, we prescribe a biexponential size distribution of graupel/hail at the freezing level as

\[ N(D) = N_g \exp(-\Lambda_g D) + N_h \exp(-\Lambda_h D), \]

where subscripts \( g \) and \( h \) stand for graupel and hail, respectively. The parameters \( N_g = 8000 \text{ m}^{-3} \text{ mm}^{-1} \) and \( \Lambda_g = 1.6 \text{ mm}^{-1} \) in Eq. (3) are selected in such a way that the “graupel” part of the size spectrum yields a size distribution of raindrops at the surface that is close to the Marshall–Palmer distribution, and the corresponding values of \( Z \) and \( Z_{DR} \) at standard are 52.2 dBZ and 2.29 dB, respectively. These are in agreement with typically observed values of \( Z \) and \( Z_{DR} \) in heavy rain without hail in Oklahoma (Ryzhkov et al. 2005).

The choice of parameters \( N_h, \Lambda_h, \) and \( D_{max} \) (maximal hail size at which size distribution is truncated) is dictated by the need to match resulting size distributions of ice cores close to the surface with the observed hail size distributions reported in the literature (Ulbrich and Atlas 1982; Cheng and English 1983; Cheng et al. 1985). In this study, we present results of model simulations for four different size distributions at the freezing level \( H = 4 \text{ km} \) (shown in Fig. 4) with the following parameters characterizing distribution of hail aloft:

1) no hail aloft and at the surface \( (N_h = 0) \),
2) “small” hail, for which hail is present aloft (at \( H = 4 \text{ km} \)) but is totally melted at the surface \( (D_{max} = 14 \text{ mm}, \Lambda_h = 0.99 \text{ mm}^{-1}, \text{ and } N_h = 200\Lambda_h^{1.11} \text{ m}^{-3} \text{ mm}^{-1}) \),

![FIG. 3. Distribution of mass water fraction across size spectrum at four different heights for (a) high initial density of hail and (b) variable initial density of hail.](http://journals.ametsoc.org/jamc/article-pdf/52/12/2849/3574816/jamc-d-13-073_1.pdf)
3) “moderate” hail, with larger hail aloft with $D_{\text{max}} = 24\, \text{mm}$ so that maximal hail size at the surface is about $19\, \text{mm}$ ($L_h = 0.42\, \text{mm}^{-1}$ and $N_h = 400A_h^{4.11}\, \text{m}^{-3}\, \text{mm}^{-1}$), and

4) “large” hail, for which $D_{\text{max}} = 35\, \text{mm}$ so that the maximal size of hail at the surface is about $30\, \text{mm}$ ($L_h = 0.27\, \text{mm}^{-1}$ and $N_h = 800A_h^{4.11}\, \text{m}^{-3}\, \text{mm}^{-1}$).

Such a parameterization of hail size distributions seems reasonable for the North American high plains (e.g., Colorado and Alberta) where the cited observations of size distributions of hail at the surface have been conducted. Prevalent size distributions can be different in areas with more humid climate where smaller-size hail is usually generated.

The initial biexponential size distribution of hydrometeors at the freezing level is modified in the process of melting. As an example, size distributions of graupel/hail at $H = 4\, \text{km}$, rain and partially melted hail at $H = 0\, \text{km}$, and ice cores at $H = 0\, \text{km}$ are compared in Fig. 5. The size distribution of rain and melting hail at $H = 0\, \text{km}$ (thick solid curve) exhibits a discontinuity around a particle diameter of $8\, \text{mm}$, as a result of shedding. In addition, note the enhanced concentration of surviving $8\, \text{mm}$ drops resulting from the processes described above. Parameters $D_{\text{max}}$ and $A_h$ at the freezing level are selected in such a way that the product of the corresponding values for size distributions of ice cores at the surface is equal to $7.9$—its most likely value as reported by Ulbrich and Atlas (1982).

All raindrops in the combined rain/hail size spectrum are formed from three sources: (i) direct melting of graupel/smaller-size hail, (ii) shedding of water from the surface of larger hailstones, and (iii) spontaneous breakup of $8\, \text{mm}$ raindrops. The size distribution of raindrops from source i is determined by the assumed size distribution of graupel/small hail aloft, whereas size distributions of raindrops coming from sources ii and iii should be prescribed using the total masses of shed drops and fragments of drop breakup. An example of vertical profiles of total mass associated with ice, melted water, shed water, and breakup water is presented in Fig. 6 for the case of large-category hail with high density. Shed water and breakup water start accumulating at

![Fig. 4. Examples of graupel/hail size distribution aloft in the cases of no hail, small hail, moderate hail, and large hail for which simulations were made.](image1)

![Fig. 5. Size distributions of ice particles at $H = 4\, \text{km}$ (thin solid gray line), raindrops and melting hailstones at $H = 0\, \text{km}$ (thick solid line), and ice cores at $H = 0\, \text{km}$ (dashed line) for moderate hail with high density. Note the enhancement of $8\, \text{mm}$ drops.](image2)

![Fig. 6. Vertical profiles of mass content associated with ice (thick solid black line), melted water (solid gray line), shed water (dashed black line), and water resulting from breakup of large raindrops (dash–dotted gray line in lower-left corner) in the case of large hail with high density.](image3)
and Yau (2005) and the slope parameter $N$ total concentration of which is obtained by summing up calculated from the total mass of breakup fragments, the $L$ height parameter of melting hailstones.

According to several observational studies (e.g., Carras and Macklin 1973; Joe et al. 1976; Lesins et al. 1980; Rasmussen et al. 1984), shed drops are typically between 0.5 and 2.0 mm in diameter, with a modal diameter of about 1 mm. We parameterized the shed raindrop size distribution (DSD) as a gamma function,

$$N(D) = N_{0,sh}D^\mu \exp(-\Lambda_{sh}D), \quad (4)$$

with spectral shape parameter $\mu = 2.0$ [as in Milbrandt and Yau (2005)] and the slope parameter $\Lambda_{sh} = 2.0 \text{ mm}^{-1}$. The intercept parameter $N_{0,sh}$ is determined by using the total mass of shed water at each level. The DSD of the fragments from drop breakup is specified by using the total mass of shed water at each level. The $D$ distribution (DSD) as a gamma function,

$$N(D) = N_{0,bu} \exp(-\Lambda_{bu}D), \quad (5)$$

where $\Lambda_{bu} = 0.453 \text{ mm}^{-1}$ and the intercept parameter is calculated from the total mass of breakup fragments, the total concentration of which is obtained by summing up $N(m, h)[1 - P_b(m, h)]$ for size bins producing 8-mm raindrops [see Eq. (2)]. Size distributions of melted water, shed water, and water resulted from breakup of large raindrops at the ground in the case of large hail with high density are displayed in Fig. 7.

FIG. 7. Size distributions of melted water (thick line), shed water (dashed line), and water resulting from breakup of large raindrops (thin line) at the ground in the case of large hail with high density.

3. Scattering model and vertical profiles of polarimetric radar variables in melting hail

a. Description of the scattering model

The forward-scattering operator described in Ryzhkov et al. (2011, R11 hereinafter) is utilized to compute polarimetric radar variables from the outputs of the cloud models 1 and 2. Melting hailstones and raindrops are modeled as spongy, water-coated, or liquid oblate spheroids with aspect ratios defined as in R11. The aspect ratio of a melting particle depends on its initial size and mass water fraction $f_m$. In the case of high-density hail, the melting particle is modeled as a water-coated spheroid with an ice core having a dielectric constant equal to that of solid ice. In the case of variable-density hail, a spongy particle (before all air pockets are filled with water) is considered to be a uniformly filled spheroid with a dielectric constant that depends on volume fractions of ice, water, and air. The Maxwell Garnett mixing formula is used for the dielectric constant, in which water is treated as the matrix and ice/air are treated as inclusions (see details in R11). The melting particle is modeled as a water-coated spheroid once all air pockets are filled up and melted water starts accumulating at the surface, with the inner core treated as a uniformly filled spheroid with water matrix and ice inclusions.

Radar polarimetric variables depend on the orientation of hydrometeors. The distributions of particle orientations are not obtained from the cloud model and should be prescribed. It is assumed that melting hailstones are characterized by a Gaussian distribution of orientations with the mean orientation of their rotation axis along the vertical axis. The width of canting angle distribution $\sigma$ linearly depends on mass water fraction for particles in a particular size bin and gradually changes from 40° for dry graupel/hailstones to 10° for completely melted hydrometeors. Because $f_m$ is a function of size at any given height, $\sigma$ also varies with the size of melting hailstones.

The scattering amplitudes in two principal planes for backward and forward directions $[f^{(0)}_{a,b}]$ and $[f^{(0)}_{a,b}]$, respectively, are computed using a T-matrix code for two-layer spheroids as in Depue et al. (2007). Then, polarimetric radar variables are estimated as
\[ Z_{dr} = Z_h^r/Z_v^r, \]
\[ K_{DP} = \frac{0.18\lambda}{\pi} \int_0^{D_{\text{max}}} \text{Re}[f_{b}^{(0)} - f_{a}^{(0)}] A_5 N(D) dD, \]
\[ A_h = 8.686 \times 10^{-3} \lambda \]
\[ \times \int_0^{D_{\text{max}}} \{ \text{Im}[f_{b}^{(0)}] - A_2 \text{Im}[f_{a}^{(0)} - f_{a}^{(0)}] \} N(D) dD, \]
and
\[ A_{DP} = 8.686 \times 10^{-3} \lambda \int_0^{D_{\text{max}}} \text{Im}[f_{b}^{(0)} - f_{a}^{(0)}] A_5 N(D) dD, \]
where the subscript \( a \) stands for the scattering amplitude if the incident electric vector is parallel to the axis of rotation (smaller axis of an oblate spheroid) and subscript \( b \) is for the scattering amplitude for the orthogonal direction. \( \lambda \) is the radar wavelength, \( K_w = (e_w - 1)/(e_w + 2) \) \( (e_w \) is the dielectric constant of water), and \( A_1 - A_5 \) are angular moments defined as (Ryzhkov 2001; R11)
\[ A_1 = \frac{1}{4} (1 + r)^2, \quad A_2 = \frac{1}{4} (1 - r^2), \]
\[ A_3 = \left( \frac{3}{8} - \frac{1}{2} r + \frac{1}{8} r^4 \right)^2, \]
\[ A_4 = \left( \frac{3}{8} - \frac{1}{2} r + \frac{1}{8} r^4 \right) \left( \frac{3}{8} + \frac{1}{2} r + \frac{1}{8} r^4 \right), \quad \text{and} \]
\[ A_5 = \frac{1}{2} r (1 + r), \] (7)
where \( r = \exp(-2\sigma^2) \), with \( \sigma \) in radians. In Eq. (6), the scattering amplitudes are expressed in millimeters. \( \lambda \) is in millimeters, radar reflectivities at horizontal and vertical polarization \( Z_{dr,b} \) are in millimeters to the sixth power per meter cubed, \( K_{DP} \) is in degrees per kilometer, and \( A_h \) and \( A_{DP} \) are in decibels per kilometer.

b. Dependencies of different radar variables on size for dry and melting hail

The dependencies of different radar variables on the size of dry and high-density melting hail at S, C, and X bands for monodispersed size distributions are illustrated in Figs. 8 and 9. The values of \( Z_h, K_{DP}, A_h, \) and \( A_{DP} \) normalized by particle concentration \( N \) are computed as functions of equivolume diameter \( D \) and assuming that hydrometeors of a given size have a variety of orientations that are described by the Gaussian distribution of canting angle, with the parameter \( \sigma \) depending on mass water fraction, as specified above. The size dependencies for melting hail are presented at the surface level where smaller hailstones are completely melted (i.e., all particles with sizes of less than 8 mm are actually liquid raindrops). It is important that the size dependency of mass water fraction practically does not change in the height interval 0–1 km above ground (or 3 km below the freezing level; cf. Fig. 3) and that, therefore, the size dependencies of normalized radar variables change very little in this interval of heights.

Several important conclusions can be drawn from Figs. 8 and 9. Radar reflectivity is generally a non-monotonic function of hail size for larger hail diameters, which is the consequence of resonance scattering. Note that \( Z_h(S \text{ band}) \) is larger than \( Z_h(C \text{ band}) \) and \( Z_h(X \text{ band}) \) for dry hail with \( 1 < D < 5 \text{ cm} \). The differences between \( Z_h(S \text{ band}) \), \( Z_h(C \text{ band}) \), and \( Z_h(X \text{ band}) \) can be as high as 40 dB for certain sizes of melting hail. It should be emphasized that \( Z_h(C \text{ band}) > Z_h(S \text{ band}) \) for large raindrops and smaller melting hail with \( D < 2 \text{ cm} \), whereas the opposite is true for larger hail size (Figs. 8a,b). The positive difference \( Z_h(S \text{ band}) - Z_h(X \text{ band}) \), called the “hail signal,” was utilized for hail detection in earlier studies with dual-frequency radars (Eccles and Atlas 1973).

Differential reflectivity of dry hail is small because of its tumbling falling behavior and low dielectric constant. The \( Z_{DR} \) can become negative at certain resonance sizes even if the horizontal dimension of the stone is larger than the vertical dimension (Fig. 8c). Water-coated melting hailstones have higher \( Z_{DR} \) relative to dry hailstones because of the increase of effective dielectric constant and because the film of water on their surface tends to stabilize their orientation. The difference in \( Z_{DR} \) is particularly large for smaller-sized hail, which melts faster and has a higher fraction of water, as shown in Fig. 3. Mass water fraction rapidly decreases with hail size, and the thin film of water on the surface of large melting hailstones is not capable of changing their orientation significantly; thus they are still oriented more chaotically than smaller melting hailstones. As a result, smaller melting hailstones with sizes of less than 10–12 mm have relatively high \( Z_{DR} \), similar to that of large raindrops, whereas larger melting hailstones are characterized by lower \( Z_{DR} \), which is not very different from the one for dry hailstones of similar size. Notable is a strong maximum of C-band \( Z_{DR} \) for raindrops with sizes of \( \sim 6 \text{ mm} \) (Fig. 8d).

Normalized \( K_{DP} \) of dry graupel/small hail is about two orders of magnitude less than normalized \( K_{DP} \) for raindrops with similar size, and \( K_{DP} \) of hail becomes increasingly negative for hail diameters exceeding resonance sizes for each radar wavelength. This is somewhat
similar to the behavior of $Z_{DR}$ for larger hailstones (Figs. 8e,f). A vertical dotted line corresponding to $D = 8$ mm separates pure-raindrop and melting-hail parts of the size spectrum in Fig. 8.

Similar to $K_{DP}$, attenuation variables $A_h$ and $A_{DP}$ are much lower for dry graupel/hail with sizes of less than 1 cm than for raindrops of similar sizes, but they increase rapidly for larger-size hail (dry and wet) (Fig. 9). Such an increase is particularly significant for specific attenuation $A_h$, which is comparable at all three radar wavelengths for large hailstones, as opposed to raindrops and small hail for which $A_h(S \text{ band}) < A_h(C \text{ band}) < A_h(X \text{ band})$. It is known that attenuation (or extinction) of electromagnetic waves is caused by absorption and scattering of microwave radiation by atmospheric particles. The absorption is dominant for hydrometeors that are smaller than radar wavelength, whereas the “scattering loss” due to the wave energy scattered by hydrometeors in directions different from the direction of wave propagation is prevalent for larger particles such as big hailstones. Normalized $A_h$ in hail is much higher than in rain and is mainly caused by the “scattering loss,” which is heavily impacted by the effects of resonance scattering. The latter fact explains why dry hailstones may cause higher attenuation than melting hailstones for certain hail sizes. Note that specific differential attenuation exhibits oscillatory behavior and does not increase dramatically in large hail.

c. Relative contributions of different parts of the hail/rain spectrum to different radar variables

As Figs. 8 and 9 show, radar characteristics of hailstones can be very different from those of raindrops, and
the values of radar variables after integration over the whole hail/rain spectrum of sizes strongly depend on relative contributions of raindrops and hailstones of different sizes and their relative concentrations. To clarify the issue, we select the case of large, high-density hail (i.e., $D_{\text{max}} = 35 \text{ mm}$, $N_h = 0.27 \text{ mm}^{-1}$, $N_r = 800 \text{ cm}^{-3} \text{ mm}^{-1}$, and $\rho = 917 \text{ kg m}^{-3}$) and estimate separate contributions of smaller raindrops (0–4 mm), larger raindrops (4–9 mm), and four size categories of hail (9–14, 14–19, 19–25, and >25 mm) to $Z_h$, $Z_v$, $K_{DP}$, $A_h$, and $A_{DP}$. The results for S and C bands are illustrated in Figs. 10–12 for the heights of 0, 1, 2, and 3 km.

Figure 10 shows that the contribution of melting hail to integral values of $Z_h$ and $Z_v$ at S band is dominant at all heights. As a result, $Z_{DR}$ of the mixture of hail and rain remains relatively low because the intrinsic $Z_{DR}$ of hail is lower than that of rain. In contrast, the contribution of hail with sizes larger than 25 mm is significant at C band at all altitudes, whereas contributions from large raindrops and smaller-size hailstones are comparable at $H = 0$ and 1 km. Because $Z_{DR}$ of large drops at C band can be very high as a result of resonance effects, the resulting $Z_{DR}$ of a rain/hail mixture is significantly larger at C band than at S band at lower levels.

Similar plots for $K_{DP}$, $A_h$, and $A_{DP}$ are presented in Figs. 11 and 12 for S and C bands, respectively. The $K_{DP}$ is little affected by the presence of melting hail at all four heights (Figs. 11 and 12, left panels) and is almost exclusively determined by rain in the mixture with hail. Larger-size hail makes a tangible contribution to $K_{DP}$ at S band, however. On the contrary, the contribution of hail to $A_h$ is comparable with the contribution of rain at C band (especially at higher levels; see Fig. 12, middle panels) and may overwhelm the one at S band (Fig. 11), which is consistent with results of observations (e.g., Borowska et al. 2011; Picca and Ryzhkov 2012; Kaltenboeck and Ryzhkov 2013). Similar to $K_{DP}$, the bulk of $A_{DP}$ comes from large raindrops, although hail generally adds more to $A_{DP}$ than $K_{DP}$, especially at S band (Figs. 11 and 12, right panels).

The relative insensitivity of measurements at C band to large hail with sizes exceeding 25 mm was confirmed in the studies of Borowska et al. (2011) and Picca and Ryzhkov (2012), where simultaneous polarimetric radar measurements at S and C bands in hailstorms are examined. These studies show that the presence of very
large hail does not result in significant increases of $Z_h$ or $A_h$ at C band. Our simulations show that a similar conclusion holds for X band as well, although in this study we do not perform analysis at X band to the same extent as at S and C bands for two major reasons. First, attenuation in hail at X band is so overwhelming that intrinsic backscattering hail signatures at X band are almost completely masked by the effects of propagation, and reliable detection of hail and determination of its size is not feasible for larger-size hail. Second,
when examining polarimetric characteristics of hail at S and C bands, we operate with a substantial amount of well-documented polarimetric data, including the data obtained simultaneously with S-band and C-band radars, that can be used for model validation. We unfortunately do not have similar data collected simultaneously at X-band and longer-wavelength radars.

d. Vertical profiles of polarimetric radar variables in melting hail of different sizes

Vertical profiles of $Z_H$, $Z_{DR}$, $K_{DP}$, $A_h$, and $A_{DP}$ computed from the output of model 1 for the cases of large and small hail are displayed in Figs. 13 and 14. Henceforth, notation $Z_H$ is used for the radar reflectivity factor at horizontal polarization expressed in logarithmic scale; that is, $Z_H = 10 \log(Z_h)$. The set of parameters determining large and small hail in our simulations is defined in section 2b. Vertical dependencies of $Z_H$ at S and C bands for large hail are characterized by a maximum at the height about 2 km below the freezing level, where shedding of water from larger hailstones starts (Fig. 13, left panels). As melting of hail progresses, the $Z_H$ first gradually increases as a result of an increase in dielectric constant and then decreases once shedding starts.

![Fig. 11. Relative contributions of different parts of particle size spectrum to S-band $K_{DP}$, $A_h$, and $A_{DP}$ at four height levels for the case of large, high-density hail.](image-url)
and the size of larger hailstones diminishes. In the simulations for small hail, $Z_H$ at S and C bands is 10–15 dB lower and the $Z_H$ maximum mostly disappears because of a smaller impact of shedding. The value of $Z_H$ at X band is 5–10 dBZ lower than at S and C bands in the case of large hail. A decrease in hail density at the lower end of the size spectrum as defined in Fig. 1 (variable-hail-density case) causes a reduction of $Z_H$ (except for large hail at S band), which is particularly pronounced in the case of small hail (Fig. 13, bottom-left panel). This reduction is due to the overall decrease in effective dielectric constant of spongy melting hailstones and because lower-density dry hailstones melt into smaller raindrops than do high-density hailstones of the same size.

The $Z_{DR}$ of melting hail increases with decreasing height within the first 2 km below the freezing level and then stabilizes without much change toward the ground (Fig. 13, right panels). As expected, $Z_{DR}$ near the surface is significantly lower for larger maximal hail size (large hail) than for smaller maximal hail size (small hail). For small hail, $Z_{DR}$ near the surface at C band
approaches 4 dB for high-density hail, which is more
than 1.0 dB higher than the corresponding value of $Z_{\text{DR}}$
for high-density and variable-density hail do not
differ much.

Vertical dependencies of $K_{\text{DP}}$, $A_h$, and $A_{\text{DP}}$ for large
and small hail at C band are shown in Fig. 14. The $K_{\text{DP}}$
rapidly increases with decreasing height within the first
1.5–2 km below the freezing level as smaller-size graupel/
hail transforms into raindrops. The increase slows down
below 2 km. It is interesting that both $A_h$ and $A_{\text{DP}}$ reach
their maxima at the height about 2 km below the freezing
level. This is consistent with observational evidence
that the largest attenuation/differential attenuation at
C band occurs at a certain height above the ground (e.g.,
Borowska et al. 2011; Kaltenboeck and Ryzhkov 2013).
The difference between vertical profiles of attenuation
variables for large and small hail is dramatic (especially
for $A_h$) at all three radar wavelengths, which is consistent with the general perception that larger-size hail contributes significantly to the overall extinction of electromagnetic waves.

Note that a T-matrix code for two-layer spheroids has
been used in our simulations, the results of which were
presented in Figs. 8–14. For comparison, we also per-
form computations by assuming a model of a uniformly
filled spheroid in which the effective dielectric constant
was estimated using the Maxwell Garnett mixture for-
maulas with water or ice as a matrix (R11) using the same
mass water content. Size dependencies of normalized

FIG. 13. Simulated vertical profiles of (left) $Z_H$ and (right) $Z_{\text{DR}}$ at S, C, and X bands for large hail (solid lines) and
small hail (dashed lines) of different density. The thickest lines are for S band, and the thinnest lines are for X band.
Simulations are made for $N_g = 8000 \text{m}^{-3} \text{mm}^{-1}$ and $\Lambda_e = 1.6 \text{mm}^{-1}$.
reflectivity factor and differential reflectivity for melting hail at S, C, and X bands computed three different ways are displayed in Fig. 15. It is obvious that the results of computations of radar variables for individual hail sizes using the three methods can differ significantly (as was also mentioned in R11). The difference is especially large between the two-layer model and the model with ice as matrix (green and orange curves in Fig. 15). After integration over the whole spectrum of sizes, such differences generally “wash out” and the computations performed with the T-matrix code for two-layer spheroids and the T-matrix code for uniformly filled spheroids assuming water as matrix [utilized by Jung et al. (2008)] yield close results in terms of integral values of radar variables. For example, the difference in $Z_{H}$ computed by the two methods is within 1–2 dB, and the corresponding difference in $Z_{DR}$ is usually less than 0.2 dB at S band. More detailed results of such a comparison can be found in the thesis of Ganson (2012).

e. The impact of resonance scattering on differential reflectivity in melting hail

As mentioned, $Z_{DR}$ of the mixture of hail and rain is high at C band. Further enhancement of $Z_{DR}$ can be achieved by changing the “graupel” part of the initial size spectrum. Analysis of raindrop size distributions in hail-bearing storms indicates that for $Z_H$ approaching 50 dBZ they are more consistent with smaller slopes $A_g$ of the graupel part of the ice spectrum aloft, say 1.1 mm$^{-1}$ instead of the 1.6 mm$^{-1}$ used in the previous simulations. The use of combination of $A_g = 1.1$ mm$^{-1}$ and $N_g = 1500$ m$^{-3}$ mm$^{-1}$ results in higher (up to 0.5 dB) $Z_{DR}$ near the surface. The corresponding simulated profile of $Z_{DR}$ for small hail at C band matches well the median profiles retrieved from the statistics of observations by Anderson et al. (2011) and Kaltenboeck and Ryzhkov (2013) in melting hail and are consistent with measurements of Tabary et al. (2010) (Fig. 16). In other words, the model 1 of melting hail combined with scattering computations yields realistic profiles of $Z_{DR}$ in hail at C band.

Although a simple 1D model explains the radar wavelength dependence of $Z_{DR}$ in melting hail, it falls short of reproducing anomalously high $Z_{DR}$ (up to 8 dB) that is often measured at C band. A more sophisticated 2D cloud model with spectral (bin) microphysics (HUCM) was utilized for simulating the fields of polarimetric variables in severe hailstorm in the study of R11. At each model grid cell, the masses of large hailstones with diameters exceeding 2.5 cm [large hail mass (LHM)] and smaller hailstones with diameters between 1 and 2.5 cm [small hail mass (SHM)] have been calculated for this storm. Using the computed $Z_H$ and $Z_{DR}$ and the distributions of LHM and SHM, two-dimensional frequency distributions for $Z_{H}$ and $Z_{DR}$ occurrence in grid cells that contain appreciable LHM and SHM are constructed for different height intervals for S and C bands (Fig. 17). It is obvious that in the first-kilometer layer above ground, the highest $Z_{DR}$ at C band (i.e., values approaching 8 dB) are associated with smaller hail with $Z_{H}$ between 50 and 60 dBZ. The contrast between S and C bands in terms of $Z_{DR}$ is particularly striking. As opposed to the 1D model, the HUCM more adequately...
reproduces size sorting effects in the proximity of convective updrafts, which cause additional enhancement of $Z_{DR}$. The changes in the graupel part of the size distribution of hydrometeors aloft mostly affect the resulting rain part of the size spectrum at lower levels. A more flattened graupel size distribution accentuates the contribution from larger raindrops, which causes additional increases in $Z_{DR}$. A more detailed analysis shows that the HUCM generally produces a lower slope of raindrop size distribution and more pronounced enhancement of raindrop concentration within the size interval between 5 and 9 mm containing “resonance” sizes of raindrops at C band. This can be seen, for example, from the comparison of Fig. 7 in this paper with Fig. 12 in the R11 paper. A secondary maximum of C-band $Z_{DR}$ at 6.5 dB in Fig. 16 is associated with large hail mixed with very big raindrops in the vicinity of the updraft (see also R11).

4. Sensitivity to temperature, humidity, and vertical motions

In addition to evaluating the dependency of polarimetric signatures of melting hail on its initial size distributions, density, and radar wavelength, we examined the sensitivity of vertical profiles of radar variables to 1) temperature lapse rate, 2) humidity profile, and 3) strength of descending air motions (downdrafts). The sensitivity analysis demonstrates that the factors 1–3 have generally weaker (but not negligible) effect on radar variables when compared with variability in size distribution, density, and radar wavelength.
A more steep temperature lapse rate speeds up the process of melting and the transition from partially melted to completely melted particles occurs at higher levels. This means that the transition from low $Z_{DR}$ at the freezing level to high $Z_{DR}$ would occur in a shallower layer. The reduction of relative humidity toward the ground produces the opposite effect: slowing down the process of melting due to evaporative cooling and increasing the depth of $Z_{DR}$ transition or decreasing vertical gradient of $Z_{DR}$. Our simulations show that changing temperature lapse rate from 6.5 to 4.5 km$^{-1}$ would lower the height of the “$Z_{DR}$ step” (i.e., the level at which $Z_{DR}$ sharply slows down its increase with decreasing height) by about 500 m. In a similar way, if relative humidity linearly decreases from 100% at the freezing level down to 60% at the surface (as in the example in RH87b), the $Z_{DR}$ step would descend by about 200 m relative to the case of 100% humidity through the whole depth of the 0-4 km layer.

Melting of hail is a major driving force for downdrafts, which may potentially produce damaging wet microbursts (Srivastava 1987). Downward air motion transports partially melted hailstones to lower height levels and, as a result, the “$Z_{DR}$ hole” [the term coined by Wakimoto and Bringi (1988)] of lower $Z_{DR}$ below the freezing level stretches farther down to the surface. We simulated vertical profiles of C-band $Z_{DR}$ for the case of small hail in the absence of downward motions and for three different profiles of downdraft velocity. Figure 18 shows that the $Z_{DR}$ contour of 1.5 dB descends by about 0.5–1 km for typical profiles of vertical velocity within strong downdrafts.

The $Z_{DR}$ hole may serve as a more reliable microburst predictor than the subsidence of the center of mass (CM) computed from a vertical profile of $Z_H$ in a traditional method for microburst prediction (Wolfson et al. 1994). This is because the height of the $Z_{DR}$ depression with respect to surrounding air is a local characteristic that depends only on the downdraft velocity, whereas CM is a function of the vertical distribution of hydrometeor concentration in a whole depth of convective cell. In other words, it is very possible that apparent CM may not descend in the presence of downdraft if additional buildup of mass occurs near the top of the cloud.

**Fig. 17.** Frequency distributions of large hail mass (shading) and small hail mass (dashed contours) on the $Z_H$–$Z_{DR}$ plane that are based on output from the HUCM at S and C bands. The title above each panel indicates the height interval (AGL) from which the distributions are computed. The environmental freezing level is at 2.5 km.
5. Conclusions

The one-dimensional thermodynamic model of melting hail by Rasmussen and Heymsfield (1987a,b) is utilized to simulate vertical profiles of polarimetric radar variables at S, C, and X bands using the polarimetric radar observation operator for a cloud model with spectral microphysics described by Ryzhkov et al. (2011) and the T-matrix code for computing the scattering amplitudes of water-coated and spongy spheroidal hydrometeors. The model realistically reproduces vertical profiles of various radar variables and their dependencies on radar wavelength and maximal size of melting hail below the freezing level. It notably explains high values of $Z_{DR}$ that are routinely observed in melting hail (particularly at C band) as opposed to dry hail. A more sophisticated spectral cloud model of The Hebrew University of Jerusalem is capable of generating anomalously high values of $Z_{DR}$ at C band and large differences between C-band and S-band $Z_{DR}$ in even better agreement with dual-wavelength polarimetric radar measurements in hail-bearing storms (Borowska et al. 2011; Kaltenboeck and Ryzhkov 2013; Picca and Ryzhkov 2012). This is attributed to the fact that the HUCM more adequately treats size sorting of raindrops and melting hailstones in the proximity of convective updrafts than does the simplified model 1, which does not explicitly address size sorting.

The 1D model predicts an enhancement in the concentration of very large raindrops originating from hail with initial sizes below 15 mm and a flattening of the raindrop spectrum at its higher end in the presence of hail. This is one of the key factors causing an increase of $Z_{DR}$ at the periphery of hail cells at all three radar wavelengths. Additional enhancement of $Z_{DR}$ (especially at C band) is attributed to intense size sorting of raindrops and smaller-size melting hailstones, processes that are well captured by the HUCM.

It is shown that specific differential phase $K_{DP}$ is least affected by the presence of hail in a mixture with rain when compared with other radar variables. This makes it an attractive parameter for quantifying the rainfall amount in the mixture. Hail contributes significantly to specific attenuation $A_h$, and this contribution increases with increasing maximal hail size so that it becomes tangible even at S band. Specific differential attenuation $A_{DP}$ is less affected by the presence of hail than is $A_h$. Its increase is more attributed to large raindrops generated from smaller-sized melting hail.

The sensitivity of the vertical profiles of radar variables to the temperature lapse rate, humidity, size distributions of graupel/hail aloft (e.g., maximal hail size), hail density, and vertical air motions was examined as part of the study. Variability of the size distribution of dry ice aloft is a primary factor affecting polarimetric signatures of the melting hail. Lower intercepts of the exponential distribution of hailstones at the freezing level [which are strongly correlated with the maximal hail size according to Ulbrich and Atlas (1982)] result in higher $Z_H$ and $A_h$ as well as lower $Z_{DR}$ beneath. Decreasing the initial density of hail causes a noticeable
reduction of $Z_H$, and decreased temperature lapse rate and relative humidity slow down the melting process and shift the $Z_{DR}$ step downward. The dependence of the depth of the $Z_{DR}$ hole on the downdraft velocity shows good promise to utilize this signature to predict microbursts driven by melting of hail.

The polarimetric model of melting hail offers a theoretical basis for development of practical algorithms for hail detection and determination of its size, attenuation correction, and rainfall estimation in the presence of hail at S, C, and X bands, which are discussed in Part II of this paper (Ryzhkov et al. 2013).

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APPENDIX A

Equations for Particle Fall Speeds

Particle velocities are determined following a simplified version of RH87a, by first computing the Best number:

$$X = \frac{8m g \rho_a \eta}{\pi \eta^2}, \quad \text{(A1)}$$

where $m$, $g$, $\rho_a$, and $\eta$ are the hailstone mass (kg), the gravitational acceleration (m s$^{-2}$), air density (kg m$^{-3}$), and dynamic viscosity of air (kg m$^{-1}$ s$^{-1}$), respectively. The Best number is then used to compute the Reynolds number $N_{Re}$ of the particle:

$$N_{Re} = 0.448 X^{0.538}; \quad X < 3.46 \times 10^8$$
$$N_{Re} = (X/0.6)^{0.5}; \quad X \geq 3.46 \times 10^8. \quad \text{(A2)}$$

For dry hailstones,

$$u_d (m s^{-1}) = \nu N_{Re} / D, \quad \text{(A3)}$$

where $\nu$ is the kinematic viscosity of air ($= \eta/\rho_a$; m$^2$ s$^{-1}$), $D$ is the equivalent spherical diameter (m), and the density correction factor is accounted for in the definition of the Best number [Eq. (A1)]. The equation used to compute the fall speed of the equilibrium melting hailstone $u_{eq}$ is determined by $N_{Re}$ as follows. For $N_{Re} < 5000$,

$$u_{eq} = (-0.1021 + 6.116 \times 10^{-2} m^{-1/3} - 1.469 \times 10^{-4} m^{-2/3} + 1.513 \times 10^{-5} m - 5.584 \times 10^{-7} m^{-1/3})(\rho_a/\rho_a)^{0.5}, \quad \text{(A4a)}$$

which is the Brandes et al. (2002) fall speed relation for raindrops, converted so that the mass of the melting hailstone $m$ is used instead of the equivalent diameter of the raindrop. This relation lets small melting hailstones fall at the same velocity as raindrops, using an updated fall speed relation for drops [RH87a use the relation of Brandes et al. (2002)]. For larger hailstones (i.e., for $5000 \leq N_{Re} < 25000$),

$$u_{eq} = (4.80 \times 10^3 + 4.832 \times 10^6 m)(u_d/\rho_a)^{0.5}, \quad \text{(A4b)}$$

and for $N_{Re} \approx 25000$ the melting hailstone fall speed is the same as that of a dry hailstone of the same size:

$$u_{eq} = u_d, \quad \text{(A4c)}$$

Note that this assumes a drag coefficient of 0.6, or that nearly all liquid water is shed. Following RH87a, the instantaneous fall speed of the melting hailstone $u$ varies between the “dry” fall speed $u_d$ [Eq. (A3)] and the equilibrium fall speed $u_{eq}$ [Eqs. (A4)] as a linear function of the fraction of equilibrium water mass on or in the particle:

$$u = u_d + \frac{m_w (u_{eq} - u_d)}{m_{w}}. \quad \text{(A5)}$$

The equilibrium mass of water on or within a hailstone is interpreted to be the critical mass of water the hailstone can retain before the onset of shedding, given by Eq. (1) in the text.

APPENDIX B

Heat Transfer Equations

The heat transfer equations governing the physics of hail melting are dependent on $N_{Re}$, following RH87a.
Instead of solving time-dependent equations for the particle radius, however, we assume steady-state conditions and solve height-dependent equations for the ice core volume \( V_i \),

\[
\frac{dq}{dh} = \rho_i L_m \frac{dV_i}{dh},
\]

(B1)

where \( dq/dh \) is the rate of change of enthalpy with height for a hailstone, \( \rho_{\text{ice}} \) is the density of ice, \( L_m \) is the latent enthalpy of melting, and \( V_i \) is the hailstone’s volume of ice. The heat transfer equations are as follows. For \( N_{\text{Re}} < 250 \),

\[
\frac{dq}{dh} = -\frac{4\pi D}{(u-w)} [k_a(T_\infty - T_0)\overline{r}_h + L_v D_v (\rho_{v,\infty} - \rho_{v,0})\overline{r}_v],
\]

(B2a)

for \( 250 \leq N_{\text{Re}} \leq 3000 \),

\[
\frac{dq}{dh} = -\frac{2\pi D D_{k_a}(T_\infty - T_a)}{(u-w)(D - D_i)} - \frac{2\pi D}{(u-w)} [k_a(T_\infty - T_a)\overline{r}_h]
+ L_v D_v (\rho_{v,\infty} - \rho_{v,0})\overline{r}_v;
\]

(B2b)

for \( 3000 < N_{\text{Re}} < 6000 \),

\[
\frac{dq}{dh} = -\frac{2\pi D D_{k_a}(T_\infty - T_a)}{(u-w)(D - D_i)} - \frac{2\pi D}{(u-w)} [k_a(T_\infty - T_a)\overline{r}_h]
+ L_v D_v (\rho_{v,\infty} - \rho_{v,0})\overline{r}_v;
\]

(B2c)

for \( 6000 \leq N_{\text{Re}} \leq 2 \times 10^4 \),

\[
\frac{dq}{dh} = -\chi \pi D_i N_{\text{Re}}^{1/2} [N_{\text{Pr}}^{1/3} k_a(T_\infty - T_0)
+ N_{\text{Sc}}^{1/3} L_v D_v (\rho_{v,\infty} - \rho_{v,0})], \quad \chi = 0.76;
\]

and (B2d)

for \( N_{\text{Re}} > 2 \times 10^4 \),

\[
\frac{dq}{dh} = -\chi \pi D_i N_{\text{Re}}^{1/2} [N_{\text{Pr}}^{1/3} k_a(T_\infty - T_0)
+ N_{\text{Sc}}^{1/3} L_v D_v (\rho_{v,\infty} - \rho_{v,0})], \quad \chi = 0.57 + 9.0 \times 10^{-6} N_{\text{Re}}.
\]

(B2e)

The heat transfer Eqs. (B2) above are used with Eq. (B1) to determine the reduction in ice volume for each height level and hailstone size bin. This change in ice volume \( \delta V_i \) is converted to an increase in liquid water volume \( \delta V_w \) via conservation of mass:

\[
\delta V_w = (\rho_i/\rho_w) \delta V_i.
\]

(B3)

The mass of liquid meltwater is then calculated.

In the heat balance Eqs. (B2), there are several thermodynamic parameters used. Some have functional dependencies on the ambient air temperature \( T_\infty \) and are given below in mks units for convenience. Parameter \( k_a \) is the thermal conductivity of air (J m\(^{-1}\) s\(^{-1}\) K\(^{-1}\)):

\[
k_a = [2.381 + 0.0071(T_\infty - T_0)] \times 10^{-2},
\]

\( T_0 = 273.15 \) K is the reference 0°C temperature, \( \overline{r}_h \) is the thermal ventilation coefficient:

\[
\overline{r}_h = 0.78 + 0.308 N_{\text{Pr}}^{1/3} N_{\text{Re}}^{1/2},
\]

\( \overline{r}_v \) is the vapor ventilation coefficient:

\[
\overline{r}_v = 0.78 + 0.308 N_{\text{Sc}}^{1/3} N_{\text{Re}}^{1/2},
\]

\( L_v \) is the latent enthalpy of vaporization (J kg\(^{-1}\)):

\[
L_v = 2.499 \times 10^6 (T_0/T)^{0.167+3.67 \times 10^{-4} T},
\]

\( L_m \) is the latent enthalpy of melting (J kg\(^{-1}\)):

\[
L_m = 3.335 \times 10^5 [1 + 0.006(T - T_0)]
- 3.14 \times 10^{-5} (T - T_0)^2],
\]

\( D_v \) is the diffusivity of water vapor in air (m\(^2\) s\(^{-1}\)):

\[
D_v = 2.11 \times 10^{-5} (T/T_0)^{1.94} (p_0/p),
\]

\( p \) is air pressure and \( p_0 \) is surface pressure (hPa), \( \rho_{v,\infty} \) is the ambient vapor density (kg m\(^{-3}\)), \( \rho_{v,0} \) is the vapor density at temperature \( T_0 \) (kg m\(^{-3}\)), \( N_{\text{Re}} = uD/v \) is the dimensionless Reynolds number, \( N_{\text{Pr}} = v/K_a \) is the dimensionless Prandtl number, \( N_{\text{Sc}} = v/D_v \) is the dimensionless Schmidt number, \( K_a \) is the thermal diffusivity of air (m\(^2\) s\(^{-1}\)):

\[
K_a = 9.1018 \times 10^{-11} T_\infty^2 + 8.8197 \times 10^{-8} T_\infty
- 1.0654 \times 10^{-5},
\]

and \( k_w \) is the thermal conductivity of water (J m\(^{-1}\) s\(^{-1}\) K\(^{-1}\)):

\[
k_w = 0.568 \exp[0.003473(T_\infty - T_0) - 3.823
\times 10^{-5}(T_\infty - T_0)^2 + 1.087 \times 10^{-6}(T_\infty - T_0)^3].
\]

REFERENCES


