Polarimetric Radar Characteristics of Melting Hail. Part II: Practical Implications

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ABSTRACT

The results of theoretical modeling in Part I are utilized to develop practical recommendations for developing the algorithms for hail detection and determination of its size as well as attenuation correction and rainfall estimation in the presence of hail. A new algorithm for discrimination between small hail (with maximal size of less than 2.5 cm), large hail (with diameters between 2.5 and 5.0 cm), and giant hail with size exceeding 5.0 cm is proposed and implemented for applications with the S-band dual-polarization Weather Surveillance Radar-1988 Doppler (WSR-88D) systems. The fuzzy-logic algorithm is based on the combined use of radar reflectivity $Z$, differential reflectivity $Z_{DR}$, and cross-correlation coefficient $r_{hv}$. The parameters of the membership functions depend on the height of the radar resolution volume with respect to the freezing level, exploiting the size-dependent melting characteristics of hailstones. The attenuation effects in melting hail are quantified in this study, and a novel technique for polarimetric attenuation correction in the presence of hail is suggested. The use of a rainfall estimator that is based on specific differential phase $K_{DP}$ is justified on the basis of the results of theoretical simulations and comparison of actual radar retrievals at S band with gauge measurements for storms containing large hail with diameters exceeding 2.5 cm.

1. Introduction

The first part of this series (Ryzhkov et al. 2013, hereinafter Part I) provides the results of theoretical modeling of polarimetric radar characteristics of melting hail using a one-dimensional thermodynamic model of Rasmussen and Heymsfield (1987) that was generalized for arbitrary initial size distributions of ice particles at the freezing level, where melting starts. Such a model realistically reproduces vertical profiles of various radar variables in hail-bearing storms and their dependences on radar wavelength and maximal hail size. A more sophisticated 2D cloud model of The Hebrew University of Jerusalem was also used in the first part of the series to substantiate major results of theoretical simulations by a simpler 1D model. In this second part of the series, three practical issues are addressed using the results of theoretical simulations: 1) detection of hail and determination of its size, 2) attenuation correction in the presence of melting hail, and 3) radar rainfall estimation if rain is mixed with hail.

Identification of hail is an inherent part of a number of polarimetric classification algorithms suggested for research studies and operational utilization (e.g., Zrnić and Ryzhkov 1999; Vivekanandan et al. 1999; Lim et al. 2005; Heinselman and Ryzhkov 2006; Marzano et al. 2008; Park et al. 2009; Dolan and Rutledge 2009; Boodoo et al. 2009; Chandrasekar et al. 2013; Dolan et al. 2013; Al-Sakka et al. 2013). Existing polarimetric algorithms for hail detection commonly ignore the strong dependence of polarimetric variables on the height of the radar resolution volume with respect to the freezing level, however. Significant vertical variability is a consequence of the melting process, which causes polarimetric signatures of dry hail aloft to gradually transform into the ones typical for rain closer to the ground. There is also a great need to distinguish between smaller and larger hail sizes. It is hail of sufficiently large size and high density that inflicts substantial damage so that, in addition to hail detection, discrimination among hail of

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different sizes and densities is required. According to the U.S. National Weather Service (NWS) standard, hail with size exceeding 2.5 cm (1.0 in.) is considered to be high impact and dangerous (i.e., ‘severe’) and hail in excess of 5.0 cm (2.0 in.) is considered to be ‘significantly severe.’” Depue et al. (2007) suggested using the hail differential reflectivity parameter $H_{DR}$, which combines radar reflectivity $Z$ and differential reflectivity $Z_{DR}$, to discriminate between smaller- and larger-size hail, but the Depue et al. algorithm does not take into account the hail melting process. A more extensive review of the existing algorithms for hail detection and determination of its size can be found in the introduction to Part I.

In this study, a new polarimetric radar algorithm for detection of hail and determination of its size is described. The algorithm is devised for applications on S-band radars and aims at discrimination between three categories of hail size as required by the NWS. This algorithm was developed at the National Severe Storms Laboratory (NSSL) and is designed to augment the existing hydrometeor classification algorithm currently implemented on the newly deployed network of polarimetric Weather Surveillance Radar-1988 Doppler (WSR-88D) systems, which currently uses only one class category (rain mixed with hail) to designate hail without specification of its size (Park et al. 2009).

Hail produces significant attenuation of radar signal that is not well quantified (Battan 1971; Ryzhkov et al. 2007; Tabary et al. 2009; Borowska et al. 2011; Kaltenboeck and Ryzhkov 2013). No reliable methods for attenuation correction of $Z$ or $Z_{DR}$ in hail exist at the moment, as opposed to rain, where the use of differential phase $\Phi_{DP}$ is very efficient (Bringi et al. 1990, 2001). In rain, attenuation-induced biases of $Z$ and $Z_{DR}$ are linearly proportional to $\Phi_{DP}$ with coefficients of proportionality $\alpha$ and $\beta$, respectively. The factors $\alpha$ and $\beta$ are generally not constant and are affected by the variability of drop size distributions. There are two approaches to address the variability of $\alpha$ and $\beta$. One of them was suggested by Bringi et al. (2001) according to which the optimal values of $\alpha$ and $\beta$ are sought while assuming that they are constant along the whole radar ray. Another method by Gu et al. (2011) identifies “hot spots” along the propagation path (such as strong convective cells possibly containing hail) and determines the optimal values of $\alpha$ and $\beta$ within hot spots, which are treated separately from the rest of the ray. In section 3, the range of variability of the factors $\alpha$ and $\beta$ in melting hail is established on the basis of the results of the theoretical modeling in Part I as well as polarimetric radar observations at S and C band. In addition, a new method for attenuation correction in hail is suggested and tested at S band. The method is a further extension of the hotspot concept described by Gu et al. (2011), which is particularly efficient for quantification of $\alpha$ and $\beta$ within hail cells.

Although there is strong observational evidence that rainfall estimation algorithms utilizing specific differential phase $K_{DP}$ are preferable over algorithms using $Z$ and/or $Z_{DR}$ for hail-bearing storms (Balakrishnan and Zrnić 1990a; Aydin et al. 1995; Hubbert et al. 1998; Ryzhkov et al. 2005; Matrosov et al. 2013), accurate estimation of rain in the presence of hail remains a challenge. Numerous observations with operational S-band WSR-88D systems indicate that $K_{DP}$ frequently exceeds $6^{-7}$ km$^{-1}$ in the storm cores containing large/giant hail, which produce unrealistically high rain rates estimated from $K_{DP}$. In section 4, the $K_{DP}$-based approach for quantification of rain in the presence of hail is further investigated by comparing simulated vertical profiles of rain rate in melting hail and their estimates using the $R(K_{DP})$ relations at S, C, and X bands. In addition, the comparison between rain gauge accumulations and their estimates using the conventional $R(Z)$ and polarimetric $R(K_{DP})$ algorithms is performed for a storm that produced large hail (>2.5 cm) and was observed by the polarimetric WSR-88D near Wichita, Kansas (KICT), on 30 May 2012.

We emphasize that Part II (this paper) outlines the guiding principles and method for addressing the three important practical issues involving hail—1) determination of its size, 2) attenuation correction, and 3) rainfall estimation in its presence—that are driven by theoretical simulations in Part I and documented observations. Its refinement and validation is left for further studies. For example, a large-scale validation study using the Severe Hazards Analysis and Verification Experiment (SHAVE) method (Ortega et al. 2009, 2012) for the hail size discrimination algorithm (HSDA) using the whole polarimetric WSR-88D network will be described in Part III (A. Ryzhkov et al. 2013, unpublished manuscript). The National Mosaic and Multi-Sensor Quantitative Precipitation Estimation (NMQ) system (Zhang et al. 2011) will be used as a platform for validating rainfall estimation and attenuation correction in hail-bearing storms on the WSR-88D network.

2. Detection of hail and determination of its size

It is shown in Part I that the relation between $Z$ and $Z_{DR}$ strongly depends on the degree of melting and on the radar wavelength; hence, the methods for hail detection should be wavelength-specific and must take into account the height of the radar resolution volume with respect to the melting level. It is also evident that
utilizing a sole “hail differential reflectivity” parameter $H_{DR}$ (e.g., Depue et al. 2007) may not be sufficient for discrimination between small (<2.5 cm) and large (>2.5 cm) hail (Picca and Ryzhkov 2012). It is instrumental to use the cross-correlation coefficient $\rho_{hv}$ along with $Z$ and $Z_{DR}$ (Balakrishnan and Zrnić 1990b). It can be shown that the models 1 and 2 utilized in Part I correctly reproduce the decrease of $\rho_{hv}$ in melting hail as well as its wavelength dependence, but the magnitudes of such decreases are generally smaller than those experimentally observed (e.g., Ryzhkov et al. 2011). This result is likely due to the fact that secondary effects (such as roughness of hailstones or abrupt changes in the shape of the particles during collisions) that lead to possible reductions of $\rho_{hv}$ are not accounted for in the models (Mirkovic et al. 2013). Nonuniform beamfilling resulting from strong vertical gradients of radar variables in severe convective storms is another possible reason for additional reduction of $\rho_{hv}$ (Ryzhkov 2007). Picca and Ryzhkov (2012) showed that the depression of $\rho_{hv}$ above the melting layer in the area of major hail growth between $-10^\circ$ and $-20^\circ$C may indicate the presence of giant hail with sizes exceeding 5 cm that usually grows in the wet regime. Because such large hailstones reach the ground with only a relatively small decrease in their initial size, the $\rho_{hv}$ signature aloft may serve as an indicator of giant hail near the surface.

Theoretical simulations in Part I show that $Z$ and $Z_{DR}$ of melting hail are very sensitive to the slope parameter $\Lambda_h$ of the initial size distribution of hail aloft. Because $\Lambda_h$ is strongly correlated with maximal hail size $D_{max}$ (Ulbrich and Atlas 1982), the sensitivity of $Z$ and $Z_{DR}$ to $\Lambda_h$ can be utilized for indirect estimation of $D_{max}$, or at least for discrimination between different hail size categories.

In this study, we suggest a fuzzy-logic scheme to distinguish between three categories of hail size: small hail (diameter $D < 2.5$ cm), large hail ($2.5 < D < 5.0$ cm), and giant hail ($D > 5.0$ cm) on the basis of $Z$, $Z_{DR}$, and $\rho_{hv}$, with the parameters of the membership functions depending on the relative height of the center of the radar resolution volume with respect to the level of zero wet-bulb temperature, where melting starts. At the moment, HSDA is proposed for utilization at S band only, although a similar type of the algorithm can be developed for C band (and eventually for X band) by taking into consideration higher values of $Z_{DR}$ and lower values of $\rho_{hv}$ for the same size categories of hail because of the more pronounced effects of resonance scattering (Part I; Kaltenboeck and Ryzhkov 2013; Al-Sakka et al. 2013). The choice of the 2.5- and 5.0-cm diameters of hailstones to delineate the three classes of hail is dictated by the requirement of the NWS to discriminate hail that is smaller and larger than 2.5 cm (about 1 in., which is considered to be “severe”) and by apparent changes in polarimetric signatures following the transition between large and giant hail such as a significant drop in $\rho_{hv}$ aloft and the appearance of slightly negative $Z_{DR}$ associated with S-band resonance at about 5 cm in dry hail according to Part I (their Figs. 8c,d). In addition, hailstones with diameters in excess of about 5 cm (2 in.) are considered to be “significantly severe” by the NWS.

The parameters of the membership functions of HSDA are determined on the basis of the output of the model studies in Ryzhkov et al. (2009) and Part I as well as radar observations. The results of two recent observational studies in central Oklahoma by Picca and Ryzhkov (2012) and Kaltenboeck and Ryzhkov (2013) are used to justify the choice of the membership functions for the three categories of hail. Picca and Ryzhkov (2012) focus on the analysis of one extreme hail event occurring in Oklahoma City on 16 May 2010 with abundant ground reports of large and giant hail. The study of Kaltenboeck and Ryzhkov (2013) examines mean vertical profiles of $Z$, $Z_{DR}$, and $\rho_{hv}$ in the major hail cores of six storms, two of which produced giant hail. The data have been collected by the S-band KOUN WSR-88D and C-band University of Oklahoma Polarimetric Radar for Innovations in Meteorology and Engineering (OU-PRIME; Palmer et al. 2011).

The classification algorithm uses six sets of membership functions corresponding to six height intervals with respect to the melting level (i.e., where the wet-bulb temperature $T_w$ is equal to zero and melting of hail commences). A trapezoidal shape of the membership functions is selected that is similar to the existing operational Next Generation Weather Radar (NEXRAD) hydrometeor classification algorithm (HCA; Park et al. 2009). The stratification of height intervals implies knowledge of the vertical profile of $T_w$, which can be obtained either from observed soundings or from the output of numerical weather prediction models such as the Rapid Update Cycle (RUC) or High-Resolution Rapid Refresh RUC (HRRR), similar to the winter classification algorithm described in Schuur et al. (2012). Separate membership functions are utilized in the six height intervals:

1) $H > H(T_w = -25^\circ)C$,
2) $H(T_w = 0^\circ)C < H < H(T_w = -25^\circ)C$,
3) $H(T_w = 0^\circ)C - 1\,\text{km} < H < H(T_w = 0^\circ)C$,
4) $H(T_w = 0^\circ)C - 2\,\text{km} < H < H(T_w = 0^\circ)C - 1\,\text{km}$,
5) $H(T_w = 0^\circ)C - 3\,\text{km} < H < H(T_w = 0^\circ)C - 2\,\text{km}$, and
6) $H < H(T_w = 0^\circ)C - 3\,\text{km}$. 

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The membership functions for these intervals are depicted in Fig. 1. At $H > H(T_w = -25^\circ C)$ (interval 1), $Z$ is the major discriminator between the three categories of hail because $Z_{DR}$ and $\rho_{hv}$ do not vary much with hail size at higher altitudes. Therefore, the membership functions of $Z_{DR}$ and $\rho_{hv}$ are the same for all three hail size categories (Fig. 1a). In the second altitude range, between the 0°C and -25°C wet-bulb temperature isotherms, $Z$ and $\rho_{hv}$ have major discriminative power, whereas $Z_{DR}$ weakly depends on maximal diameter of hailstones (Fig. 1b). All three radar variables possess strong classification capability below the freezing level with the discriminative power of $Z_{DR}$ increasing toward the ground (Figs. 1c–f).

The algorithm for hail detection and discrimination of its size is conceived as a natural extension of the existing NEXRAD HCA, which identifies the class “hail mixed with rain” (Park et al. 2009). The suggested algorithm splits this class designation into three categories of hail size using the aforementioned fuzzy-logic routine in locations where the HCA recognizes hail mixed with rain. For the time being, the values of all three membership functions are summed up with equal weights, although this can be changed in the future. Similar to the existing WSR-88D HCA, the HSDA algorithm provides classification results at all antenna elevations, and class designation at any particular elevation is performed independently; that is, the algorithm does not utilize full vertical profiles of radar variables. An experimental version of the S-band HSDA has been run on a large number of hail storms observed by newly upgraded dual-polarization WSR-88D systems, with ground truth collected as part of the NSSL SHAVE during 2012 and 2013 (Ortega et al. 2009, 2012). Following the SHAVE method, a team of meteorology students from the University of Oklahoma working under supervision of NSSL scientists use the Google Earth software tools to display experimental weather data and geographic information databases such as digital phonebooks. Using these data, the students then make verification calls to

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**Fig. 1.** Membership functions for small hail (black), large hail (blue), and giant hail (red) for the hail classification algorithm as a function of height $H$ of the radar sampling volume, showing values for (left) $Z_H$, (center) $Z_{DR}$, and (right) $\rho_{hv}$. Information is shown for the six height intervals that are discussed in the text: (a) $H > H(T_w = -25^\circ C)$, (b) $H(T_w = 0^\circ C) < H < H(T_w = -25^\circ C)$, (c) $H(T_w = 0^\circ C) - 1 km < H < H(T_w = 0^\circ C)$, (d) $H(T_w = 0^\circ C) - 2 km < H < H(T_w = 0^\circ C) - 1 km$, (e) $H(T_w = 0^\circ C) - 3 km < H < H(T_w = 0^\circ C) - 2 km$, and (f) $H < H(T_w = 0^\circ C) - 3 km$. Orange represents overlapped membership functions for small, large, and giant hail; purple represents overlapped membership functions for small and large hail.
residences and businesses throughout the United States that are thought to have been affected by severe weather. This way, a large amount of high-resolution hail data categorized into three size classes can be obtained. More than 8000 SHAVE hail reports were collected in 2012 alone (Ortega et al. 2012).

An example of the HSDA product with three categories of hail is presented in Fig. 2. A severe hailstorm producing giant hail with diameters exceeding 10 cm was observed with the KFWS WSR-88D west of Fort Worth, Texas, on 15–16 May 2013. The map in Fig. 2 shows a spatial distribution of the HSDA output integrated over a period of 2 h. The color of each pixel in the map indicates the maximal hail size estimated by HSDA within a 2-h period in this particular location. The overlaid circles of different size and color show locations of the SHAVE ground reports within the same time period. Small, large, and giant hail reports are designated as S in red circles, L in purple circles, and G in magenta circles, respectively. The reports with unknown hail size or “no hail reported” are marked with U and N, correspondingly. A cursory look at the map suggests an overall consistency of the radar classification product and surface observations, with some tendency for HSDA to over-predict giant hail. A detailed analysis of the validation results will be presented in Part III of this study.

The methods for discrimination of hail size at C and X bands can be devised on similar principles. The challenges are that 1) attenuation/differential attenuation is stronger at shorter wavelengths, 2) $Z$ does not increase that much with maximal hail size as it does at S band and all radar variables are significantly less sensitive to hail with diameters larger than 25 mm (see Figs. 8a and 12 in Part I), and 3) $Z_{DR}$ may be equally high for smaller and larger hail. Some additional discrimination parameters can be utilized, however. For example, Kaltenboeck and Ryzhkov (2013) have found that the spatial variability of $Z_{DR}$ and $\rho_{hv}$ within high-reflectivity cores at C band may serve as additional useful parameters for discrimination between different hail sizes. The variability of $Z_{DR}$ and $\rho_{hv}$ increases with increasing hail size because of larger differential attenuation and lower $\rho_{hv}$.

3. Polarimetric attenuation correction in melting hail

Melting hail produces strong attenuation/differential attenuation at S, C, and X bands. The relation between specific attenuation $A_h$ and radar reflectivity in hail is different from the one in rain. We use the results of modeling in Part I to obtain best-fit power-law $A_h(Z)$.
relations in melting hail at S, C, and X bands and to compare them with the corresponding relations for pure rain retrieved from simulations using 47 144 drop size distributions measured in central Oklahoma (Schuur et al. 2005). The parameters of the power-law \( A_h(Z) \) relations for rain and hail at three different radar wavelengths are listed in Table 1, and the corresponding dependences of \( A_h \) on \( Z \) are displayed in Fig. 3. In all three plots in Fig. 3, the lines for hail for \( Z > 53 \) dBZ appear as a smooth continuation of the lines representing \( A_h(Z) \) dependences for pure rain encompassed between the lines for \( T = 0^\circ \text{C} \) and 30\(^\circ\text{C} \). Such smooth transition from the \( A_h(Z) \) relations for rain [based on measured drop size distributions (DSDs)] and model-based \( A(Z) \) dependences for hail may add credibility to the polarimetric model of melting hail in Part I.

The polarimetric methods for attenuation correction are based on the use of the relations (Bringi et al. 1990)

\[
\Delta Z = \alpha \Phi_{DP} \quad \text{and} \quad \Delta Z_{DR} = \beta \Phi_{DP},
\]

where \( \alpha = A_h/K_{DP} \) and \( \beta = A_{DP}/K_{DP} \). The factors \( \alpha \) and \( \beta \) in rain and their variability are well known. They are functions of \( Z_{DR} \) and raindrop temperature. These dependences are illustrated in Fig. 4, where the scatterplots of \( \alpha \) and \( \beta \) versus \( Z_{DR} \) for temperatures 0\(^\circ\text{C} \) and 30\(^\circ\text{C} \) simulated from the Oklahoma disdrometer dataset are shown. The dependence of \( \alpha \) on \( Z_{DR} \) is nonmonotonic, whereas \( \beta \) generally increases with \( Z_{DR} \). Simulations that are based on measured DSDs show that the magnitude of specific attenuation \( A_h \) in rain can be as high as 0.05, 0.4, and 2.2 dB km\(^{-1} \) at S, C, and X bands, respectively, for a rain rate of 100 mm h\(^{-1} \).

The values of \( A_h, A_{DP}, \alpha, \) and \( \beta \) are generally higher in melting hail than in rain. Attenuation/differential attenuation in melting hail can be significant even at S band. An example of anomalously large attenuation/differential attenuation experienced by the S-band KICT WSR-88D in a supercell storm east of Wichita on 30 May 2012 is illustrated in Figs. 5 and 6. The storm produced hail of large size, with reported maximal diameters between 2.5 and 4.5 cm. A composite plot of \( Z \),

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Rain</th>
<th>Hail</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( a )</td>
<td>( b )</td>
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<tr>
<td>S band (( \lambda = 11.0 ) cm)</td>
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<tr>
<td>0</td>
<td>( 1.54 \times 10^{-5} )</td>
<td>0.62</td>
</tr>
<tr>
<td>30</td>
<td>( 7.26 \times 10^{-6} )</td>
<td>0.61</td>
</tr>
<tr>
<td>C band (( \lambda = 5.45 ) cm)</td>
<td></td>
<td></td>
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<tr>
<td>0</td>
<td>( 4.27 \times 10^{-5} )</td>
<td>0.73</td>
</tr>
<tr>
<td>30</td>
<td>( 1.29 \times 10^{-5} )</td>
<td>0.77</td>
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<tr>
<td>X band (( \lambda = 3.2 ) cm)</td>
<td></td>
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<tr>
<td>0</td>
<td>( 1.62 \times 10^{-4} )</td>
<td>0.74</td>
</tr>
<tr>
<td>30</td>
<td>( 5.50 \times 10^{-5} )</td>
<td>0.86</td>
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</table>

FIG. 3. Average dependences of \( \log(A_h) \) on \( Z \) for rain (thin lines) and melting hail (thicker lines) at S, C, and X bands. The dependences for rain are obtained for \( T = 0^\circ \text{C} \) and \( T = 30^\circ \text{C} \) using 47 144 measured DSDs in Oklahoma. The dependences for melting hail are simulated from the model 1 described in Part I.
\( Z_{\text{DR}}, \Phi_{\text{DP}}, \) and \( \rho_{\text{hv}} \) at elevation 0.9° is presented in Fig. 5. Measured \( Z \) and \( Z_{\text{DR}} \) (before attenuation correction) are displayed in Figs. 5a and 5b. Large negative biases of \( Z \) and \( Z_{\text{DR}} \) are clearly visible in a narrow eastern sector where \( \Phi_{\text{DP}} \) is maximal and \( Z_{\text{DR}} \) drops below \(-5 \text{ dB}\). The reduction of \( \rho_{\text{hv}} \) is attributed to nonuniform beamfilling (Ryzhkov 2007), because \( \rho_{\text{hv}} \) is not affected by attenuation. We estimated median values of \( Z, Z_{\text{DR}}, \Phi_{\text{DP}}, \) and \( K_{\text{DP}} \) at each range gate within the narrow sector of 2.5° (between azimuths 89.2° and 91.7°) of anomalously high attenuation and then smoothed these data radially using a 7-gate running window (Fig. 6). Using median filtering in azimuth helps to reduce statistical errors in the estimates of polarimetric variables because \( \rho_{\text{hv}} \) is low within the hail cell. Notable is a very high value of \( K_{\text{DP}} \) approaching 7° km\(^{-1}\) in the middle of the hail core.

Utilizing Eq. (1) and the values of \( \alpha \) and \( \beta \) accepted for convective rain at S band is not sufficient to completely eliminate the biases in \( Z \) and \( Z_{\text{DR}} \) induced by attenuation because the factors \( \alpha \) and \( \beta \) are much higher in hail. Herein, we suggest a new method for attenuation correction in hail that is based on the ideas first presented by Carey et al. (2000), Ryzhkov et al. (2007, 2014), and Gu et al. (2011). According to this method, the hail-bearing convective core should be treated as a hot spot, and attenuation correction has to be performed separately using different factors \( \alpha \) and \( \beta \) in hail and surrounding rain. It is assumed that \( \alpha \) and \( \beta \) in rain are equal to their average climatological values, whereas \( \alpha \) and \( \beta \) in hail may vary from ray to ray and should be determined from the radar data behind the hail cell.
As a starting point, a propagation path through precipitation \((r_0, r_m)\) containing hail in the range interval \((r_1, r_2)\) has to be segmented into three parts: \((r_0, r_1)\), \((r_1, r_2)\), and \((r_2, r_m)\). We use the \(Z\) threshold of 50 dBZ to identify the segment \((r_1, r_2)\). Within a hot spot or hail segment \((r_1, r_2)\), specific attenuation \(A_h\) is determined using the so-called ZPHI method (Testud et al. 2000; Bringi and Chandrasekar 2001):
Ah(r) = \frac{[Z_a(r)]^b C(b, PIA)}{I(r_1, r_2) + C(b, PIA)/I(r, r_2)}
\tag{2}

I(r_1, r_2) = 0.46b \int_{r_1}^{r_2} [Z_a(s)]^b ds,
\tag{3}

I(r, r_2) = 0.46b \int_{r}^{r_2} [Z_a(s)]^b ds,
\tag{4}

C(b, PIA) = \exp\left(0.23b \text{PIA}(r_1, r_2)\right) - 1,
\tag{5}

the parameter $b$ is the exponent in the $A_h(Z)$ relation

$A_h = a Z^b,
\tag{6}$

valid in hail, and PIA is the two-way path-integrated attenuation within hail

$\text{PIA}(r_1, r_2) = 2 \int_{r_1}^{r_2} A_h(s) ds.
\tag{7}$

In Eqs. (2)–(4), $Z_a$ is the attenuated (biased) radar reflectivity factor.

To find PIA in hail, we have to estimate the difference between a “true” (not biased by attenuation) value of $Z(r_2)$ and its measured value $Z_a(r_2)$. This can be done using the ZPHI estimate [Eq. (2)] of $A_h$ within rain interval $(r_2, r_m)$, where

$\text{PIA}(r_2, r_m) = a [\Phi_{\text{DP}}(r_m) - \Phi_{\text{DP}}(r_2)].
\tag{8}$

As shown by Ryzhkov et al. (2014), the estimate of $A_h(r_2)$ is not affected by $Z$ bias caused by radar mis-calibration, attenuation, partial beam blockage, or wet radome. Hence, the unbiased value of $Z(r_2)$ can be estimated from $A_h(r_2)$ using Eq. (6). The intercept $a$ in the $A_h(Z)$ relation is notoriously prone to DSD variability and temperature. It is, however, reasonable to assume that the $A_h(Z)$ relation does not vary much in rain surrounding a hail core; hence, the same $A_h(Z)$ relation is valid for radials affected and unaffected by the presence of hail. In other words, intrinsic values of $Z$ for a given $A_h$ should be the same in the radials free of hail and the ones in the shadow of hail core. A similar assumption is utilized by Zhang et al. (2013) to correct reflectivity...
biases caused by partial beam blockage. After specific attenuation $A_b$ is obtained in all three segments of the propagation path through precipitation $(r_0, r_m)$, the corrected radar reflectivity factor can be computed as

$$Z(r) = Z_a(r) + 2 \int_{0}^{r_m} A_b(s) \, ds.$$  \hspace{1cm} (9)

After correction for attenuation, the area of $Z > 50 \text{ dBZ}$ appears extended farther away from the radar in the rear side of the hail cell and a hail-containing range interval expands so that its end range increases from $r_2$ to $r_2^{(c)}$. Then the previously described procedure should be repeated after $r_2$ is replaced by $r_2^{(c)}$. In the radials that are free of hail, attenuation correction is performed using Eq. (1).

To correct differential reflectivity along the radials containing hail, the estimate of $Z_{DR}$ bias in the interval $(r_2, r_m)$ should be made first. This is done by comparing the measured and expected (true) $Z_{DR}$ in the range gates where corrected $Z$ is between 20 and 30 dBZ. The expected or reference $Z_{DR}$ in this range of reflectivities is obtained from the corrected $Z$ as

$$Z_{DR} = 0.585 - 0.0507Z + 0.00165Z^2.$$  \hspace{1cm} (10)

The reason for using low reflectivities for such a comparison is that the $Z_{DR}(Z)$ relation is less affected by

$$\Delta Z_{DR}(r) = \begin{cases} \beta \Phi_{DP}(r) & \text{if } r < r_1 \\ \beta \Phi_{DP}(r) - (\beta_h - \beta) \Phi_{DP}(r_1) & \text{if } r_1 < r < r_2 \\ \beta \Phi_{DP}(r) + (\beta_h - \beta) [\Phi_{DP}(r_2) - \Phi_{DP}(r_1)] & \text{if } r > r_2 \end{cases}$$  \hspace{1cm} (11)

Again, Eq. (1) can be used to correct $Z_{DR}$ along the radials that are free of hail.

The fields of $Z$ and $Z_{DR}$ corrected for attenuation are shown in Figs. 5e and 5f. It is evident that the correction procedure efficiently eliminates the $Z$ and $Z_{DR}$ biases caused by attenuation/differential attenuation. If total attenuation along the range interval $(r_0, r_1)$ is negligible, then a path-integrated attenuation in hail $\Phi_{PA}[r_1, r_2^{(c)}]$ is equal to $\Delta Z$ where $\Delta Z = Z(r_2^{(c)}) - Z_0(r_2^{(c)})$. The magnitude of $\Delta Z$ as a function of azimuth is displayed in Fig. 7 (thicker solid line) together with $\Phi_{DP} = \Phi_{DP}[r_2^{(c)}] - \Phi_{DP}(r_1)$ (thin solid line) and the bias in differential reflectivity $\Delta Z_{DR}$ (dashed line). Figure 7 shows that the maximal absolute values of attenuation-induced biases of $Z$ and $Z_{DR}$ are about 17 and 6 dB, respectively. Such anomalously high values of $\Delta Z$ and $\Delta Z_{DR}$ at S band correspond to relatively modest differential phase shift $\Phi_{DP}$ barely exceeding $150^\circ$ (cf. Fig. 6c). The estimates of the factors $\alpha$ and $\beta$ in hail can be obtained as the ratios $\alpha_h = \Delta Z / \Delta \Phi_{DP}$ and $\beta_h = \Delta Z_{DR} / \Delta \Phi_{DP}$.

Median values of $\alpha_h$ and $\beta_h$ in the azimuthal sector from $87^\circ$ to $92^\circ$ are about 0.1 and 0.04 dB $^{-1}$, respectively, which are an order of magnitude higher than typical values in rain (see Fig. 4). Theoretical simulations using the model 1 of melting hail described in Part I yield the values of $\alpha_i$ and $\beta_i$ for large hail at S band within ranges of $(0.07-0.13 \text{ dB }^{-1})$ and $(0.013-0.027 \text{ dB }^{-1})$ at the height levels 1.5–2 km below the freezing level. The maximal value of $\alpha_h$ estimated from the radar measurements illustrated in Fig. 7 is just in the middle of the theoretical range for maximal hail size of 3.5 cm, which gives credibility to our theoretical model 1. The maximal observed $\beta_h$ is about 2 times that predicted by model 1 for this size of hail, however. This is not surprising because model 1 does not explicitly treat the effects of vigorous size sorting in rain that affect $Z_{DR}$.
much more than \( Z \). Additional size sorting, which is more adequately treated by the more sophisticated model 2, usually increases \( Z_{\text{DR}} \) and the intensity of differential attenuation (i.e., magnitude of \( \beta \); see Fig. 17 in Part I).

Radial profiles of \( Z \) and \( Z_{\text{DR}} \) before and after correction for attenuation along the radial with maximal attenuation at azimuth = 88.7° are shown in Fig. 8. Corrected values of \( Z_{\text{DR}} \) (bottom panel in Fig. 8) are lowest where \( Z \) is maximal within the hail core, which makes perfect sense (Bringi and Chandrasekar 2001). The minimal value of \( Z_{\text{DR}} \) is about 0 dB where the corrected value of \( Z \) is 75 dB\( Z \) at a range of 28.5 km from the radar. Specific attenuation \( A_h \) in hail can be estimated by comparison of uncorrected and corrected radial profiles of radar reflectivity. The \( Z \) bias of 17 dB is accumulated within a distance interval between \( r = 18 \) and 30 km that corresponds to a net value of \( A_h \) of \(-0.71 \text{ dB km}^{-1} \), which is more than an order of magnitude higher than the maximal value in rain at S band. Notable are very high values of \( Z_{\text{DR}} \) approaching 5–6 dB at the periphery of the hail cell, which are indicative of strong size sorting (Kumjian and Ryzhkov 2008; Ryzhkov et al. 2011; Kaltenboeck and Ryzhkov 2013).

Anomalously high attenuation is relatively rare at S band (even in hail). It is more common at C and X bands. Borowska et al. (2011) evaluated specific attenuation in melting hail at C band through direct comparisons with simultaneous measurements at S band and showed that \( A_h \) in wet hail is highly variable and can be an order of magnitude higher than in pure rain with an intensity of 100 mm h\(^{-1}\). This conclusion is very similar to our findings at S band. Substantial differential attenuation at C band (up to 2 dB km\(^{-1}\)) was also reported in that study.

Vertical profiles of the factors \( \alpha \) and \( \beta \) at C band simulated from model 1 are displayed in Figs. 9a and 9b in the cases of no hail (NH), small hail (SH), moderate hail (MH), and large hail (LH) (see definitions of these categories in Part I). Both factors exhibit a strong dependence on height, with maximal simulated values located at about 1.5–2.0 km below the freezing level. The specific attenuation \( A_h \) increases dramatically with increasing hail size, whereas \( A_{\text{DP}} \) and \( K_{\text{DP}} \) are less affected by hail and are primarily determined by large raindrops and small water-coated hailstones. Hence, the ratio \( \alpha \) is more sensitive to the presence of hail than is \( \beta \).

A comparison of the variability of ranges of \( \alpha \) and \( \beta \) in pure rain and melting hail mixed with rain at C band is illustrated in Figs. 9c and 9d. It is assumed that the factor \( \alpha \) varies between 0.05 and 0.18 dB\( \circ^{-1}\) in pure rain, with a median value of 0.08 dB\( \circ^{-1}\), whereas the factor \( \beta \) changes within the interval 0.008–0.1 dB\( \circ^{-1}\), with a median value 0.02 dB\( \circ^{-1}\) (Ryzhkov et al. 2007; Tabary et al. 2009). The ranges of variability of simulated \( \alpha \) and \( \beta \) for NH, SH, MH, and LH are shown as shaded polygons using the data displayed in Figs. 9a and 9b. It is apparent that the bulk of variability of \( \alpha \) is due to the presence of hail while the factor \( \beta \) is already very variable in rain and the presence of hail does not add much to its overall variation. The simulated values of \( \alpha \) and \( \beta \) are in good agreement with the estimates from observations by Ryzhkov et al. (2007) using the hot-spot method for attenuation correction that is described in Gu et al. (2011). The estimation of \( \alpha \) and \( \beta \) from direct comparison of S- and C-band measurements with closely located radars by Borowska et al. (2011) shows generally higher values of the ratios \( \alpha \) and \( \beta \) than are predicted by model 1.

Attenuation/differential attenuation correction in hail at C and X bands can be performed using the approach that is applied for S band in this study. Testing such an approach at shorter wavelengths is a subject for future investigation. The proposed method for attenuation correction in hail implies the estimation of the average magnitude of specific attenuation \( A_h \) within the hail core, which is sensitive to maximal hail size and can be potentially utilized in HSDA. This hypothesis requires further exploration.

4. Polarimetric rainfall estimation in the presence of hail

The observational data provide ample evidence that the rainfall estimation algorithm that is based on the use of specific differential phase \( K_{\text{DP}} \) is the best choice in the situations of rain mixed with hail because \( K_{\text{DP}} \)
is relatively insensitive to the presence of hail (e.g., Ryzhkov et al. 2005). The results of our modeling study generally support this notion, although with certain reservations. Vertical profiles of true rain rates retrieved from the model of melting hail described in Part I and their $R(K_{DP})$ estimates for different hail sizes at the three radar wavelengths are shown in Fig. 10. The $R(K_{DP})$ relation has a power-law form:

\[ R(K_{DP}) = A_{DP}/K_{DP} \]

![Fig. 9. The factors $\alpha = A_{DP}/K_{DP}$ and $\beta = A_{DP}/K_{DP}$ at C band for different hail size categories: (a),(b) vertical dependences of $\alpha$ and $\beta$ for the NH, SH, MH, and LH cases and (c),(d) ranges of variability of $\alpha$ and $\beta$ for hail of different sizes (shaded polygons). The corresponding ranges and median values of $\alpha$ and $\beta$ for pure rain are depicted by solid and dashed lines. Asterisks and diamonds represent results of observational estimates by Ryzhkov et al. (2007) and Borowska et al. (2011).](https://journals.ametsoc.org/doi/abs/10.1175/JAMC-D-13-074.1)
where the factors $a$ and $b$ depend on radar wavelength. Theoretical simulations of $R$ and $K_{\text{DP}}$ for a large set of DSD measurements in Oklahoma yield the values of $a = 44.0$, $25.3$, and $16.9$ and $b = 0.822$, $0.776$, and $0.801$ at S ($\lambda = 11.0$ cm), C ($\lambda = 5.33$ cm), and X ($\lambda = 3.2$ cm) bands, respectively. The S-band $R(K_{\text{DP}})$ relation is utilized for polarimetric WSR-88D systems in the United States. In the cases of no hail and small hail, the vertical profiles of $R(K_{\text{DP}})$ (dashed curves in Fig. 10) follow the actual profiles of rain rate (solid curves in Fig. 10) very well. For moderate and large hail sizes, the $R(K_{\text{DP}})$ relation tends to overestimate rain rate close to the surface at S and C bands. Such overestimation is more pronounced at S band and is very mild at C band. This is in full accordance with results of theoretical simulations in Part I. Indeed, Figs. 11 and 12 in Part I show that the relative contribution of melting hail (i.e., hydrometeors with sizes exceeding 8 mm) to $K_{\text{DP}}$ is more significant at S band than at C band. The utilization of the $R(Z)$ relation with $Z$ capped at the level 53 dBZ (as is often done in operational practice to mitigate hail contamination) results in significant overestimation of rain for smaller-size hail and underestimation for larger-size hail. The $R(K_{\text{DP}})$ relation tends to overestimate rain rate close to the surface at S and C bands. Such overestimation is more pronounced at S band and is very mild at C band. This is in full accordance with results of theoretical simulations in Part I. Indeed, Figs. 11 and 12 in Part I show that the relative contribution of melting hail (i.e., hydrometeors with sizes exceeding 8 mm) to $K_{\text{DP}}$ is more significant at S band than at C band. The utilization of the $R(Z)$ relation with $Z$ capped at the level 53 dBZ (as is often done in operational practice to mitigate hail contamination) results in significant overestimation of rain for smaller-size hail and underestimation for larger-size hail. The maps of 6-h rain totals retrieved by the two algorithms are displayed in Fig. 11. The conventional algorithm yields visibly higher overall rain totals than the polarimetric algorithm and tends to overestimate the actual rain accumulation reported by gauges, as Fig. 12 shows. This overestimation is particularly significant for higher rain totals. In contrast, the polarimetric algorithm utilizing $K_{\text{DP}}$ in hail cores produces almost unbiased estimates of rain accumulation over a whole range of rain totals (right panel in Fig. 12). The bias of the 6-h rain total estimate has been reduced from 27% down to 4%, whereas the fractional RMS error decreases from 32% to 19% if the combination of $R(K_{\text{DP}})$ and $R(Z)$ is utilized instead of the stand-alone $R(Z)$ relation.

We evaluated the performance of the $Z$- and $K_{\text{DP}}$-based algorithms at S band for rainfall estimation in the mixture of rain and hail for the hailstorm on 30 May 2012 (illustrated in Fig. 5) using comparison with 36 rain gauges. The conventional $R(Z)$ algorithm utilizes a standard relation

$$R(Z) = 1.70 \times 10^{-2}[10^{0.0714Z(dBZ)}],$$

where $Z$ is capped at the 53-dBZ threshold. The polarimetric algorithm combines the $R(K_{\text{DP}})$ relation in Eq. (12) for heavy rain and hail with $Z$ exceeding 45 dBZ and the $R(Z)$ relation in Eq. (13) for $Z$ less than 45 dBZ. The maps of 6-h rain totals retrieved by the two algorithms are displayed in Fig. 11. The conventional algorithm yields visibly higher overall rain totals than the polarimetric algorithm and tends to overestimate the actual rain accumulation reported by gauges, as Fig. 12 shows. This overestimation is particularly significant for higher rain totals. In contrast, the polarimetric algorithm utilizing $K_{\text{DP}}$ in hail cores produces almost unbiased estimates of rain accumulation over a whole range of rain totals (right panel in Fig. 12). The bias of the 6-h rain total estimate has been reduced from 27% down to 4%, whereas the fractional RMS error decreases from 32% to 19% if the combination of $R(K_{\text{DP}})$ and $R(Z)$ is utilized instead of the stand-alone $R(Z)$ relation.

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Note that in the middle of cells containing large hail exceeding 25 mm in size, the $R(K_{\text{DP}})$ relation may overestimate rain as predicted by the simulations shown in Fig. 10. Indeed, measured $K_{\text{DP}}$ values that are over 7° km$^{-1}$ (as in Fig. 6) would result in rain rates of higher

![Fig. 11. The fields of 6-h rain accumulations obtained by the KICT WSR-88D using (left) the conventional $R(Z)$ algorithm and (right) a combination of $R(Z)$ (for $Z < 45$ dBZ) and $R(K_{\text{DP}})$ (for $Z > 45$ dBZ) for the period from 30 May 2012 (2300 UTC) through 31 May 2012 (0500 UTC).](http://journals.ametsoc.org/doi/abs/10.1175/JAMC-D-13-074.1?journalCode=jamc)
than 218 mm h$^{-1}$, which is apparently too high, even for the case of severe hail. Such local overestimation does not have much of an impact on the overall performance of the $K_{DP}$-based algorithm for estimation of hourly or multihour rainfall accumulations (Fig. 12), however. Anomalously high local values of $K_{DP}$ exceeding 6–7 km$^{-1}$ at S band may be used as indication of large or giant hail to complement the algorithm for hail detection and determination of its size described in Part I.

The advantage of utilizing $K_{DP}$ for quantification of rain in mixture with hail was also demonstrated at X band in the recent study by Matrosov et al. (2013), where the relation $R(K_{DP}) = 14K_{DP}^{0.77}$ was used. It has to be kept in mind that the $K_{DP}$ data in hail can be very noisy if the cross-correlation coefficient $r_{hv}$ drops too low, which is more likely at shorter wavelengths. Backscatter differential phase $\delta$ in melting hail can be significant, ranging from 5$^\circ$ to 15$^\circ$ according to our simulations (not shown). Hence, large positive and negative excursions of the $K_{DP}$ estimate may occur if the effects of forward propagation and backscattering are not separated in the measurements of differential phase. Such problems with accurate estimation of $K_{DP}$ are particularly pronounced at C band, where large raindrops resulting from melted hailstones cause appreciable resonance scattering effects.

### 5. Conclusions

Any polarimetric algorithm for detection of hail and determination of its size has to take into account the height of the center of the radar resolution volume with respect to the freezing level. The algorithm should be also radar wavelength-dependent because the radar variables at C and X bands are much less sensitive to the presence of large hail with sizes exceeding 2.5 cm than at S band, and the effects of resonance scattering differently impact the radar variables at the three frequency bands. A new hail size discrimination algorithm is suggested and implemented at S band. The algorithm utilizes $Z$, $Z_{DR}$, and $\rho_{hv}$ and is designed to distinguish between small hail (with diameter $D$ less than 2.5 cm), large hail ($2.5 < D < 5.0$ cm), and giant hail with size exceeding 5.0 cm. The algorithm is based on the principles of fuzzy logic. Different membership functions for each of the three radar variables are utilized in six height intervals, two of which are above the freezing level and four of them are below the freezing level. The results of validation tests for the newly proposed algorithm will be presented in Part III of this study.

Melting hail may significantly enhance attenuation/differential attenuation of the radar signal, even at S band. A new method for attenuation correction is introduced and tested for the case of severe hail observed by the dual-polarization S-band WSR-88D. The examined hailstorm produced biases of $Z$ and $Z_{DR}$ up to 17 and 6 dB, respectively. The method allows one to quantify the attenuation-correction factors $\alpha = A_{\rho}/K_{DP}$ and $\beta = A_{DP}/K_{DP}$ in hail, which turned out to be an order of magnitude higher than typical values in rain at S band. The values of $A_{\rho}$ and $\alpha$ are in good agreement...
with results of theoretical simulations using the model 1 described in Part I. The observed maximal values of $\beta$ and $Z_{DR}$ are higher than predicted from model 1 and are in better agreement with predictions from the more sophisticated model 2, which takes into account the effects of size sorting and particle collisions. Theoretical values of $a$ and $\beta$ in hail at C band simulated by model 1 are generally consistent with but somewhat lower than their estimates in the experimental studies of Ryzhkov et al. (2007) and Borowska et al. (2011).

It is shown that the $R(K_{DP})$ relation is a good choice for estimating rainfall in the presence of hail at all three microwave frequency bands, but it may produce positive bias at S band for larger-sized hail. Such local overestimation of rain mixed with large hail may not compromise the accuracy of hourly or multihour rain totals obtained for hail-bearing storms, however.

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