Nonlinear Wind Analysis of Single-Doppler Radar Observations within a DVAD Framework

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ABSTRACT

The application of the distance velocity azimuth display (DVAD) method to the retrieval of vertical wind profiles from single-Doppler radar observations is presented in this study. It was shown that Doppler velocity observations at a constant altitude can be expressed as a single polynomial function for both linear and nonlinear wind fields in DVAD. Only a one-step least squares fitting of a polynomial function is required to obtain the vertical wind profile of a real wind field. The mathematic formulation of DVAD results in two advantages over the traditional nonlinear VAD method used for the nonlinear analysis of single-Doppler observations. First, the requirement of only one-step least squares fitting leads to robust performance when Doppler velocity observations are contaminated by unevenly distributed data noise and voids. Second, the degree of nonlinearity to properly represent a real wind field can be directly estimated in DVAD instead of being empirically determined in the traditional method. A proper nonlinear wind model for approximating the real wind field can be objectively derived using the DVAD method. The merits of DVAD as a quantitative single-Doppler analysis method were compared with the traditional method using both idealized and real datasets. Results show that the simplicity and robust performance of DVAD make it a good candidate for single-Doppler retrieval in operational use.

1. Introduction

The ground-based weather Doppler radar has been an important observational tool for deducing kinematic structures of precipitating (Browning and Wexler 1968; Waldteufel and Cobin 1979; Lee et al. 1994), and more recently nonprecipitating (Kollias et al. 2001; Bluestein et al. 2004), cloud systems. The ability to observe a large region with higher spatial and temporal resolutions than those obtained from the rawinsonde and surface station measurements has greatly advanced our understanding of various mesoscale and convective weather systems (Houze 2004). Despite its wide applications in both research and operational communities, the direct interpretation of the three components \((u, v, w)\) of a wind field from the observed Doppler velocity is not straightforward because a Doppler radar only measures the component of a wind field projected along the radar beam direction. The retrieval of basic physical properties (horizontal velocity, vertical velocity, divergence, deformation, and vorticity) from single-Doppler radar observations can only be accomplished with certain assumptions on the structure of the actual wind field. Previous studies have used a linear assumption (e.g.,

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coordinate system.

plane, or simply the nonuniformity in the
tudes, here nonlinear means higher-order variations of winds in a
is focusing on the retrieval of horizontal winds at different alti-
viation VWP is saved in later parts of this paper to refer to its
study are not computed by the original VAD method, the abbre-
wind profile is just one particular vertical wind profile calculated by
VWP in operational radar product catalogs. The same abbreviation
Lhermitte and Atlas 1961; Browning and Wexler 1968;
both research and operations (e.g., Davis and Lee 2012;
VAD vertical wind profile1 (e.g., Fig. 1 in Chrisman and
observed by a single-Doppler radar. Except for these
larity is common in real weather situations (Rabin and
Doppler velocity can be expressed in analytic functions
limited physical information of these real simplified wind fields can be obtained by conducting a least squares fitting
(LSF) of the observations to the analytic functions. The
VAD method (Browning and Wexler 1968) expressed
Doppler velocities along “a ring of constant radius cen-
tered at the radar site” (hereinafter referred to as a VAD
ring) as a Fourier series, a function of azimuth angle. If
the wind field is linear, the resulting Fourier coefficients
of the LSF can be physically interpreted as the mean dis-
vergence, mean horizontal winds, and mean horizontal
deforation averaged over an area encircled by the
VAD ring. By processing the data of multiple VAD rings
on different radii and/or different elevation angles, the
VAD vertical wind profile1 (e.g., Fig. 1 in Chrisman and
Smith 2009) can be deduced and has been widely used in
both research and operations (e.g., Davis and Lee 2012;

The spatial linearity is the fundamental assumption for
deducing physical parameters (i.e., the Fourier co-
efficients) of a wind field from the VAD analysis. However, the departure of actual wind fields from lin-
earity is common in real weather situations (Rabin and
Zawadzki 1984). When nonlinear2 wind components

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1 The VAD vertical wind profile is commonly abbreviated as VWP in operational radar product catalogs. The same abbreviation is also used for the vertical wind profile, and logically the VAD wind profile is just one particular vertical wind profile calculated by the VAD method. Since the vertical wind profiles referred to in this study are not computed by the original VAD method, the abbrevi-ation VWP is saved in later parts of this paper to refer to its original meaning, “vertical wind profile.”

2 The word “nonlinear” is a common nomenclature meaning any variation in time and/or space that is not constant. Since this paper is focusing on the retrieval of horizontal winds at different alti-tudes, here nonlinear means higher-order variations of winds in a plane, or simply the nonuniformity in the X–Y plane of a Cartesian coordinate system.

exist, the Fourier coefficients deduced in the VAD
analysis are not representative of the mean kinematic properties in a real wind field (Waldteufel and Cobin
Unresolved nonlinear wind components could contam-
inate the retrieval of linear wind components
(Waldteufel and Cobin 1979; Koscielny et al. 1982) and
lead to significant errors in retrieved vertical wind pro-
files (VWPs). In their formulation of the extended VAD
(EVAD) method, Matejka and Srivastava (1991) also
noticed the necessity to include certain nonlinear wind
components to retrieve the mean divergence within a
VAD ring. Although real wind fields may not deviate
significantly from a linear distribution under certain
weather conditions, the intrinsic linear wind hypothesis
in the VAD analysis is unduly restrictive in obtaining
accurate kinematic properties for both research and
operational purposes (Caya and Zawadzki 1992, here-
inafter CZ92). Despite the documented detrimental ef-
effect of unresolved nonlinear wind components on VAD
analysis, there is no simple way to examine whether the
wind field is linear. Hence, the existence of nonlinear
wind components was mainly neglected in the VAD
analysis in practice.

CZ92 extensively examined the relationship between the physical parameters of a nonlinear wind field and the
Fourier coefficients from the VAD analysis. It was shown that the Fourier coefficients (e.g., Browning and Wexler 1968) no longer represent those aforementioned kinematic properties for nonlinear wind fields, except for the divergence term. However, the Fourier co-
efficients become functions of radius at a constant alti-
tude in nonlinear wind fields. By utilizing this range
dependence, a second polynomial fit was used to re-
trieve the horizontal wind vector at the radar site for
nonlinear wind fields. The work presented in CZ92 not
only clarified the relations between the physical pa-
rameters of a wind field and the results of VAD analysis
under nonlinear conditions, but also proposed a new
method (hereinafter referred to as NVAD) to account for
the nonlinearity and obtain physically meaningful
results. However, CZ92 only tested the NVAD method
in limited cases in which its practical limitations were not
fully explored.

Mesoscale weather systems typically contain non-
linear winds. During the Southwest Monsoon Experiment
(SoWMEX) and the Terrain-Influenced Monsoon Rainfall Experiment (TiMREX) field campaign (Jou
et al. 2011), the NVAD method was implemented to deduce more frequent and accurate VWPs from
Doppler velocity observations collected by the National
Center for Atmospheric Research (NCAR) S-band
dual-polarization Doppler radar (S-Pol) to supplement
the rawinsonde VWPs, which were collected 4–8 times per day. After processing a large set of SoWMEX/TiMREX data, it was found that the NVAD method generated inconsistent VWPs in many circumstances. The NVAD method is sensitive to the distribution of data noise and voids in the Doppler velocity observations. The sensitivity is again related to the mathematical formulation and corresponding computation process used in NVAD.

The purpose of this study is to propose an alternative method, the distance velocity azimuth display (DVAD; Lee et al. 2014), to deduce VWPs in nonlinear wind fields. Instead of using Doppler velocity $V_d$ alone to analyze nonlinear wind fields observed by a single-Doppler radar, the DVAD method uses the quantity $r V_d$ ($V_d$ scaled by the distance from the radar to the gate, $r$) as a variable. A description of the general use of $r V_d$ to analyze single-Doppler radar observations can be found in Lee et al. (2014). In the DVAD method, the linear wind field is represented exclusively by a bivariate quadratic equation representing three basic types of conic sections in a Cartesian coordinate system. Nonlinear wind fields can be expressed in the same mathematical form with higher-order equations. In addition to its graphical intuition and mathematical conciseness, the potential of using DVAD to quantitatively retrieve physical parameters from both linear and nonlinear wind fields is exploited in this study, and the results are compared with the VWPs deduced from the NVAD method. It is shown that DVAD is consistent with NVAD under idealized conditions but is more robust when processing real observations. The concise mathematical expression of DVAD allows the estimation of the degree of nonlinearity of real wind fields in an objective way.

The rest of this paper is organized as follows. In section 2, a brief review of the mathematical formulation of NVAD and DVAD is provided. In section 3, an evaluation of VWPs retrieved from NVAD and DVAD using an idealized dataset is conducted, and the potential limitations of NVAD are discussed. In section 4, field campaign and operational Next Generation Weather Radar (NEXRAD) datasets are used to demonstrate the robust performance of DVAD. In section 5, an objective way of assessing the nonlinearity of actual wind fields is discussed. Discussion and conclusions are given in section 6.

2. Review of NVAD and DVAD

a. NVAD

Common volume scans of ground-based Doppler radars can be viewed as data collected on a series of conical surfaces with different elevation angles. The Doppler velocity $V_d$ at a point $P(x, y, z)$ centered at the radar can be expressed in terms of Cartesian velocities $(u, v, w)$, terminal fall speed $v_t$, azimuth angle $\alpha$, elevation angle $\beta$, and the slant range $r$, as illustrated in Armijo (1969):

$$V_d = u \frac{x}{r} + v \frac{y}{r} + (w + v_t) \frac{z}{r} = u \sin \alpha \cos \beta + v \cos \alpha \cos \beta + (w + v_t) \sin \beta. \quad (1)$$

The horizontal components $u$ and $v$ of a linear wind field can be expressed using their Taylor expansions at a plane of constant altitude by neglecting the vertical velocity $w$ and terminal fall speed of precipitation particles $v_t$ as

$$u = u_0 + u_x x + u_y y \quad \text{and} \quad v = v_0 + u_x x + u_y y. \quad (2)$$

Here, $u_0$ and $v_0$ are the constant horizontal wind at the radar site, and $u_x$, $u_y$, $v_x$, and $v_y$ are the first derivatives of the horizontal winds. By substituting (2) into (1), $V_d$ can be written as (assuming $\beta = 0$)

$$V_d(\alpha) = \frac{r}{2} (u_x + v_y) + u_0 \sin \alpha + v_0 \cos \alpha + \frac{r}{2} (u_x + v_y) \sin 2\alpha + \frac{r}{2} (v_y - u_x) \cos 2\alpha. \quad (3)$$

Fitting $V_d$ as a harmonic function of azimuth $\alpha$ at a fixed range $r$, the coefficients of (3) are related to the mean values of kinematic parameters of the real wind field within a VAD ring as

$$a_0 = \frac{r}{2} (u_x + v_y), \quad a_1 = u_0, \quad b_1 = v_0,$$

$$a_2 = \frac{r}{2} (u_x + v_y), \quad \text{and} \quad b_2 = \frac{r}{2} (v_y - u_x), \quad (4)$$

where $a_1$ and $b_1$ equal the mean horizontal wind and $a_0$, $a_2$, and $b_2$ equal the mean divergence, stretching, and shearing deformation scaled by a factor of $r/2$, respectively (Browning and Wexler 1968). The equations in (4) are the basis for interpreting the results of the
traditional VAD analysis and are only strictly valid under a linear wind assumption. C292 extends the traditional VAD analysis by adding nonlinear terms (to keep the expression simple, only nonlinear terms up to third order are included) into (2). Then, \( V_d \) is expressed in the following form:

\[
V_d = \frac{r}{2} \left( u_x + v_y + \frac{1}{8} u_{xxx} r^2 + \frac{1}{8} u_{yyy} r^2 + \frac{1}{8} u_{xyy} r^2 + \frac{1}{8} v_{xxx} r^2 + \frac{1}{8} v_{yyy} r^2 \right) \\
+ \left( u_0 + \frac{1}{4} u_{xy} r^2 + \frac{1}{4} v_{xy} r^2 \right) \sin \alpha \\
+ \left( v_0 + \frac{1}{4} u_{yy} r^2 - \frac{3}{8} v_{yy} r^2 \right) \cos \alpha \\
+ \frac{r}{2} \left( u_y + v_x + \frac{1}{4} u_{xyy} r^2 + \frac{1}{12} u_{xxx} r^2 + \frac{1}{4} v_{xxx} r^2 \right) \sin 2\alpha \\
+ \frac{r}{2} \left( v_y - u_x - \frac{1}{6} u_{xxx} r^2 + \frac{1}{6} v_{xxx} r^2 \right) \cos 2\alpha \\
+ \left( -\frac{1}{8} u_{xx} r^2 - \frac{1}{8} u_{yy} r^2 + \frac{1}{4} v_{xy} r^2 \right) \sin 3\alpha \\
+ \left( -\frac{1}{8} v_{xx} r^2 + \frac{1}{8} v_{xy} r^2 - \frac{1}{4} u_{xy} r^2 \right) \cos 3\alpha \\
+ \left( -\frac{1}{16} u_{xyy} r^2 + \frac{1}{48} u_{xxx} r^2 + \frac{1}{16} u_{xxx} r^2 - \frac{1}{48} v_{xxx} r^2 + \frac{1}{16} v_{xxx} r^2 \right) \sin 4\alpha \\
+ \left( \frac{1}{48} u_{xxx} r^2 - \frac{1}{16} u_{xxx} r^2 - \frac{1}{16} v_{xxx} r^2 + \frac{1}{48} v_{xxx} r^2 \right) \cos 4\alpha + \cdots ,
\]

(5)

where the subscripts \( x, y \) of variables \( u, v \) stand for the partial derivatives of the horizontal winds (e.g., \( u_{xyy} \) is equal to \( \partial^3 u/\partial x \partial y^2 \)). After adding nonlinear terms, the relations between Fourier coefficients and kinematic parameters of a wind field become

\[
a_0 = \frac{r}{2} \left( u_x + v_y + \frac{1}{8} u_{xxx} r^2 + \frac{1}{8} u_{yyy} r^2 + \frac{1}{8} u_{xyy} r^2 + \frac{1}{8} v_{xxx} r^2 + \frac{1}{8} v_{yyy} r^2 \right), \\
a_1 = u_0 + \frac{3}{8} u_{xx} r^2 + \frac{1}{8} u_{yy} r^2 + \frac{1}{4} v_{xy} r^2, \quad b_1 = v_0 + \frac{1}{4} u_{xy} r^2 + \frac{1}{4} v_{xx} r^2 + \frac{3}{8} v_{yy} r^2, \\
a_2 = \frac{r}{2} \left( u_y + v_x + \frac{1}{4} u_{xyy} r^2 + \frac{1}{12} u_{xxx} r^2 + \frac{1}{4} v_{xxx} r^2 \right), \quad \text{and} \\
b_2 = \frac{r}{2} \left( v_y - u_x - \frac{1}{6} u_{xxx} r^2 + \frac{1}{6} v_{xxx} r^2 \right).
\]

(6)

Meanwhile, the mean kinematic quantities for a third-order nonlinear wind field can be derived by expanding (2) and integrating with respect to azimuth \( \alpha \) from 0 to \( 2\pi \) as follows:

\[
(u_x + v_y)_{\text{mean}} = u_x + v_y + \frac{1}{8} u_{xxx} r^2 + \frac{1}{8} u_{yyy} r^2 + \frac{1}{8} u_{xyy} r^2 + \frac{1}{8} v_{xxx} r^2 + \frac{1}{8} v_{yyy} r^2, \\
u_{\text{mean}} = u_0 + \frac{1}{8} u_{xx} r^2 + \frac{1}{8} u_{yy} r^2, \\
v_{\text{mean}} = v_0 + \frac{1}{8} v_{xx} r^2 + \frac{1}{8} v_{yy} r^2, \quad \text{and} \\
(u_y + v_x)_{\text{mean}} = u_y + v_x + \frac{1}{8} u_{xyy} r^2 + \frac{1}{8} u_{xxx} r^2 + \frac{1}{8} u_{yyy} r^2 + \frac{1}{8} v_{xxx} r^2 + \frac{1}{8} v_{yyy} r^2.
\]

(7)
A comparison of terms on the right-hand sides of (6) and (7) indicates that, except for the mean divergence term, the physical wind parameters in (7) no longer possess the same relations to the Fourier coefficients in (6) as in the linear case shown in (4). The physical wind parameters cannot be solved by the same VAD technique by fitting $V_r$ to a higher-order Fourier series along a VAD ring. Although the mean physical parameters within a VAD ring cannot be obtained when the wind field is nonlinear, CZ92 noticed that each of the Fourier coefficients in (6) is a polynomial function of $r$. Therefore, these physical quantities at the radar site (i.e., $r = 0$) can be evaluated by a second LSF via the Fourier coefficients obtained at different ranges. This two-step LSF procedure that is based on (6) formed the basis for the NVAD analysis proposed by CZ92. Plausible VWPs in clear-air boundary layers were retrieved by NVAD (CZ92).

**b. DVAD**

The DVAD method uses the quantity $rV_d$ to analyze nonlinear wind fields observed by a ground-based Doppler radar. The detailed mathematical expression and graphical interpretation of DVAD can be found in Lee et al. (2014). Here, the mathematical derivation related to quantitative analysis of DVAD is briefly reviewed. By multiplying both sides of (1) by $r$, the quantity $rV_d$ at a point $P(x, y, z)$ can be written as

$$rV_d = ux + vy + (w + u_j)z,$$

where the Cartesian coordinates ($x$, $y$, and $z$) are centered at the radar. The altitude $z$ of a data point is determined using the $\sqrt{2}$ Earth model with a constant gradient of refractivity (Doviak and Zrnic 2006, 18–23). The velocity components $u$, $v$, and $w$ at a constant altitude can be represented by their Taylor series with respect to the radar site as

$$u = u_0 + u_xx + u_yy + \frac{1}{2}u_{xx}x^2 + u_{xy}xy + \frac{1}{2}u_{yy}y^2 + \cdots,$$

$$v = v_0 + v_xx + v_yy + \frac{1}{2}v_{xx}x^2 + v_{xy}xy + \frac{1}{2}v_{yy}y^2 + \cdots,$$

and

$$w = w_0 + w_xx + w_yy + \frac{1}{2}w_{xx}x^2 + w_{xy}xy + \frac{1}{2}w_{yy}y^2 + \cdots.$$  

(9)

On the spatial scale (\(\sim 100 \text{ km}\)) covered by a Doppler radar, the mean magnitudes of $w$ and $v_t$ are usually one order smaller than those of $u$ and $v$. Moreover, the limited value of the maximum elevation angle (\(<20^\circ\)) scanned by an operational ground-based Doppler radar results in a small contribution of $w$ and $v_t$ into $V_d$. Therefore, both $w$ and $v_t$ can be neglected in (8) for the analysis of operational radar data. Although both $w$ and $v_t$ are neglected in the formulation and computation of DVAD, this simplification could have potential impact on the result of DVAD when $w$ and/or $v_t$ are large. The potential impact could be further amplified by the fact that some research radars have maximum elevation angles well beyond 20°. As a design criterion of DVAD, it is advised to discard Doppler observations beyond 20° elevation angle during computation.

Assuming the horizontal components $u$ and $v$ can be sufficiently approximated by finite terms of their Taylor series, then $rV_d$ can be rewritten by substituting (9) into (8) as follows:

$$rV_d = u_0x + v_0y + u_xx x^2 + (u_y + v_x)xy + v_y y^2 + \frac{1}{2}u_{xx}x^2 + \left( u_{xy} + \frac{1}{2}v_{xx} \right) x^2 y + \left( \frac{1}{2} u_{yy} + v_{xy} \right) y^2 x + \frac{1}{2} v_{yy} y^3 + \cdots.$$  

(10)

It is noted that $rV_d$ is a polynomial function of the Cartesian coordinate $x$ and $y$ without the complexity of trigonometrical basis functions. Equation (10) can be written in a more concise form using the summation notation:

$$rV_d = \sum_{i=1}^{n} \sum_{j=0}^{k} c_{ij}x^i y^j,$$

(11)

where $c_{ij}$ is the coefficient of the two-dimensional (2D) polynomial function and $n$ is the highest degree of the 2D polynomial function. Assuming there are $l$ Doppler velocity observations at the same height, the coefficient $c_{ij}$ can be solved for in a least squares sense. For convenience of discussion, the least-squares-solving process is represented in vector form:

$$b = Ac + \epsilon,$$

(12)

where $b$ is a vector with $l$ items each containing observed $rV_d$, $A$ is an $l \times m$ matrix ($l$ is the number of observations and $m$ is the total number of coefficients of the polynomial function) depending on the distribution of radar observations and the order of the nonlinear wind model, $c$ is a vector containing the coefficients to solve, and $\epsilon$ is a vector representing random error (e.g., noise in the radar hardware and sampling error) in the observations. The coefficients contained in $c$ can be solved for by minimizing the following cost function:

$$J = \|Ac - b\|_2.$$  

(13)

Since the basis functions of the polynomial series in (11) are not intrinsically orthogonal, an algorithm utilizing
singular vector decomposition (SVD) is used to solve the corresponding normal equation of (13) by setting \( \mathbf{VJ} = 0 \) (Boccippio 1995). In comparing (10) with (11), the coefficients of the polynomial series are related to the horizontal components of the wind field at the radar site as

\[
c_{10} = u_0 \quad \text{and} \quad c_{11} = v_0.
\]

A comparison of (11) and (5) shows that the quantitative analysis of Doppler radar observations has a concise mathematical form in DVAD. Linear and nonlinear wind fields are represented by the same polynomial basis functions with different orders. As a result, only a one-step 2D LSF is required to obtain the horizontal winds \((u_0 \text{ and } v_0)\) at a specific altitude above the radar site while two separated LSFs are required in NVAD to obtain the same information. Even though (11) is formulated for \( rV_d \) at a constant altitude, Doppler velocities within a certain vertical extent (e.g., 500 m) centered at a nominal altitude are used to solve the normal equation corresponding to (13) in practice. Therefore, the retrieved horizontal winds \( u_0 \text{ and } v_0 \) represent the wind above the radar within the corresponding vertical extent. The use of Doppler velocities in a finite vertical extent results in better computational stability.

3. Evaluation of NVAD and DVAD using an idealized dataset

The mathematical expressions of NVAD and DVAD are formulated to retrieve the same VWP at the radar, so the same result is expected when applying the two methods to the same dataset. Despite their theoretical equivalence, the different solving processes associated with the different mathematical formulations of NVAD and DVAD could lead to different results when processing real Doppler velocity observations. Since the true VWP above the radar site is generally unknown for real observations, a synthetic dataset with a known true VWP is used to evaluate the accuracy of VWPs produced by NVAD and DVAD, respectively. Common characteristics of Doppler velocity in real observations, in terms of data quality, are simulated on the synthetic dataset by sequentially adding measurement noise and voids. The VWPs deduced from NVAD and DVAD could be dramatically different in these situations and possible causes are investigated.

a. Comparison of VWP

The synthetic dataset was obtained from an observation simulation experiment (OSE). The original 3D wind field in the OSE was extracted from a numerical simulation of a squall line (Sun and Zhang 2008), and the wind vectors at three different altitudes with three \( V_d \) plan position indicators (PPIs) are shown in Fig. 1. The choice of model output is to provide a reference wind field with realistic variations in the horizontal and vertical directions. The results presented in this section are not limited to any particular choice of reference wind field. The Doppler velocities are resampled from the model wind field by placing a radar at the center of the domain (black plus signs in Fig. 1) using volume coverage pattern 21, consisting of nine elevation angles from 0.5° to 19.5°. Each PPI scan extends 150 km along the beam direction with an azimuthal resolution of 0.5° and a radial resolution of 0.5 km. The VWPs at the radar deduced from both NVAD and DVAD are compared with the simulated VWP to evaluate the performance on the basis of three different experiments. Various degrees of artifacts are sequentially added to the idealized resampled Doppler velocities that may be encountered in real radar observations as follows:

- \( E_1 \): no artifacts;
- \( E_N \): random noise drawn from a normalized distribution is added to the ideal data in \( E_1 \);
- \( E_{N+V} \): data voids are added to the noisy data in \( E_N \) to simulate a realistic radar observation.

The results are demonstrated in Fig. 2. Figure 2a shows that the retrieved VWPs from both methods are essentially the same with negligible deviations (<1 m s\(^{-1}\)) from the true values when the data are free of artifacts (\( E_1 \)). The minor deviations are caused by the fact that the original wind field has an infinite number of non-linear terms, while the results of both methods shown in Fig. 2a (the same for Figs. 2b and 2c) are based on a fourth-order nonlinear wind model. The deviation from the true value is mainly caused by the unresolved higher-order terms.

After adding random noise with a standard deviation of 2 m s\(^{-1}\), the retrieved VWPs from both methods in Fig. 2b show little changes. It is worth pointing out that the random noise is added directly to the resampled Doppler velocities. As shown in (12), the normally distributed noise has been taken into account in the least squares formulation. As a result, adding random noise in evenly distributed data (without data void) will not affect the result.

Nevertheless, the results can be dramatically different when the same noisy data are combined with data voids. The data voids added in \( E_{N+V} \) are missing data points taken from a real volume scan collected during the SoMWEX/TiMREX field campaign. Each PPI scan of the resampled \( V_d \) field was matched with that of the real volume scan; then, the resampled \( V_d \) points were
removed when the corresponding data points in the real volume scan have missing values. Four $V_d$ PPIs of the resultant volume data used in $E_{N+V}$ are shown in Fig. 3.

Figure 2c shows that the VWP derived from NVAD deviates significantly from the true value, especially at upper levels, while the VWP derived from DVAD is generally not influenced by the same data noise and voids. Possible reasons for the distinct results are discussed in the next subsection.

b. Limitations of NVAD method

The mathematical formulation of NVAD requires two sequential LSFs to solve for the horizontal wind at the radar site. The first step is similar to the traditional VAD analysis with higher-order Fourier series used to fit $V_d$ on a VAD ring. The second step uses polynomial-based functions to fit the Fourier coefficients obtained in the first step at different ranges. After the nonlinearity is taken into account explicitly, the two LSFs in NVAD could both introduce uncertainties when Doppler velocities contain noise and voids (e.g., $E_{N+V}$).

The uncertainty in the first-step VAD analysis of NVAD is closely related to data voids and noise in observed Doppler velocities. As illustrated in Daley (1993, 45–49), fitting observations of uneven density (due to the existence of data voids) and noise is prone to the overfitting problem, especially when higher-order mathematical models are used. Data noise and voids are inevitable in Doppler radar observations as a result of weak signal, ground clutter, nonmeteorological objects, abnormal propagation, beam blockage, and velocity aliasing. Figure 4 and Table 1 present an example of an incorrect VAD analysis caused by the overfitting problem in real observations. Figure 4a shows a PPI plot of Doppler velocities observed at 0.5° elevation angle during the SoWMEX/TiMREX field campaign. The large black circle in Fig. 4a indicates the VAD ring used to perform the VAD analysis. The fitted curves of different orders from this VAD ring are shown in Fig. 4b, and the corresponding values of the first five Fourier coefficients and their standard deviations (SDs) of fitting are listed in Table 1. It is noted that the SD values of fitting decrease monotonically with higher-order VAD analyses, which indicates higher-order wind models fit better to the observed data. The fitted curves in Fig. 4b also demonstrate the same tendency where the curves of
higher-order models are closer to the observed data. Nevertheless, a comparison of the two coefficients $a_1$ and $b_1$ between the third- and fourth-order fitting reveals an abrupt change. Since the estimated wind field from the single-Doppler velocity pattern in Fig. 4a is approximately southwesterly, it is clear that the results of the second- and third-order fitting are reasonable, while the fourth-order fitting is clearly incorrect (the wind direction is reversed) due to the overfitting problem.

The uncertainty in the second-step polynomial analysis of NVAD can be understood by examining the SD values listed in Table 1. Despite the incorrect results of the fourth-order VAD analysis, its SD value of fitting is nevertheless the smallest. NVAD uses the SD values of the first step fitting as weights for the Fourier coefficients used in the second step polynomial fitting. These physically incorrect coefficients will yield higher weights and further contaminate the final results.

The two limitations of NVAD are mutually related and essentially determined by the distribution of data noise and voids. To better illustrate the impact of data noise and voids on the accuracy of NVAD and DVAD retrievals, an azimuthally continuous gap with increasing width is added to $E_N$. The results in Table 2 show that, as the width of the gap and the order of fitting increase, the error of retrieval by NVAD increases accordingly while those of DVAD remain small. The higher-order results of NVAD become unusable when the gap width exceeds $150^\circ$. It is noted that when the maximum gap width is greater than $180^\circ$ (not shown), the errors of both methods become significant and unusable. Table 3 shows that the DVAD result is not sensitive to the azimuthal location of the gap. In real weather observations, data noise and voids are not only inevitable but also variable in both time and space. It is difficult to examine the data quality of each VAD ring of a volume scan used in NVAD and to rule out those that could potentially lead to incorrect results.

Within the DVAD framework, linear and nonlinear wind fields at a constant altitude are expressed as a single polynomial function only with different orders. Unlike CZ92, the VWP at the radar is directly resolved with only a one-step 2D LSF whether the wind field is linear or nonlinear. The greater redundancy of data used in the 2D fitting process effectively reduces the overfitting problem and is able to overcome the limitations of NVAD as demonstrated in $E_N+V$. Although the above experiments demonstrated the robust performance of DVAD when processing simulated Doppler velocity observations with noise and voids, they only provided a specific scenario of the distribution of noise and voids, while their distributions could vary greatly in real weather systems. A more comprehensive comparison between NVAD and DVAD as well as the limitations of DVAD, using both SoWMEX/TiMREX field campaign
and operational NEXRAD datasets, is presented in the next section.

4. Evaluation of NVAD and DVAD using real datasets

a. VWP during SoWMEX/TiMREX

The SoWMEX/TiMREX field campaign was designed to address the physical processes leading to the heavy precipitation events in southwestern Taiwan influenced by the interactions between the onset and northward advances of the Asian summer monsoon in late spring and Taiwan’s steep topography (Jou et al. 2011). The primary radar data used in this section were collected with the NCAR S-Pol radar from 1800 UTC 2 June to 0630 UTC 3 June 2008. Rawinsondes were released four times daily between 15 May and 30 June 2008, and the frequency increased to eight times daily during selected intensive operation periods (IOPs). These rawinsonde data provided an independent reference for evaluating the VWPs obtained from NVAD and DVAD. Figure 5 shows sample $V_d$ PPI plots (in color) at 0.5° elevation angle from four volume scans overlaid on the topography of Taiwan (in gray). The Doppler velocity observations suffered from a large area of missing data in the northeast direction because of the complex topography of Taiwan’s Central Mountain Range (CMR). Data voids over the regions other than the CMR also changed with time as a result of the translation and evolution of weather systems. A quantitative description of the data voids and their temporal evolution over the analysis period, characterized by the data coverage and max gap ratio in the 0.5° elevation angle PPI scan, is shown in Fig. 6. Figure 6 indicates that during the analysis period the dataset had small coverage and large contiguous gaps before 2200 UTC and after 0300 UTC 3 June 2008 while the data coverage was...
relatively improved between those times. Only basic data quality control procedures (ground-clutter removal and Doppler velocity unfolding) were applied to this dataset to create a more challenging set of conditions for the tested methods. As will be shown in the following analysis, the ground clutter and other artifacts have important impacts on the retrieved winds at lower altitudes.

Figure 7 shows retrieved VVPs using second-order nonlinear wind models to approximate the real wind field. For the purpose of verification, the sounding data from the Pingtung rawinsonde site in southern Taiwan (blue plus sign in Fig. 5) are included for comparison with results from NVAD and DVAD. It is noteworthy that the rawinsonde measures a wind profile along the trajectory of the balloon, while the VVPs obtained from Doppler radar (both NVAD and DVAD) represent compatible winds over the large area covered by the radar. Close agreement between the two types of VVPs is not expected (CZ92). Nevertheless, good agreement between the radar-derived VVP with that of the rawinsonde confirms the representativeness of the VVP obtained from NVAD and/or DVAD.

During the analysis time period, a cold front approached the S-Pol domain from the northwest and passed the S-Pol site at 0100 UTC 3 June 2008. The southwest-to-northwest veering of wind vectors over time associated with the passage of this cold front was clearly delineated in both VVPs from NVAD and DVAD. Both VVPs showed greater variations of wind direction and speed over time at lower altitudes than at higher altitudes, consistent with the passage of a trough at low to midlevels. There were four soundings released from the Pingtung rawinsonde site during the same time period. When compared with the sounding profile (red wind barbs), both VVPs from NVAD and DVAD compared favorably to those from the Pingtung rawinsonde site. It is noted that the VVPs deduced by both

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methods consistently have the greatest error at lower altitudes. This error is related to the increased non-linearity of the wind field and the decreased data quality at lower levels.

Figure 8 shows retrieved VWPs using fourth-order nonlinear wind models to approximate the real wind field. When the fourth-order nonlinear model was used, there were significant differences between the results of the two methods. As shown in Fig. 8a, the result of the NVAD method had abnormal and extreme values at different altitudes and times (e.g., 2000, 2230, and 0200 UTC). The missing wind barbs between 2000 and 2100 UTC were outliers where wind speed was greater than 40 m s\(^{-1}\). It is noted that the occurrences of inconsistent and unreasonable results were intermittent, which was probably caused by the temporal variation of data voids. The result of DVAD using a fourth-order nonlinear model was consistent with that of the second-order model and compared favorably to the sounding data. The southwest-to-northwest veering of wind direction over time was still clearly shown by the time series of VWPs in Fig. 8b. Even-higher-order nonlinear models were used to retrieve VWPs at the S-Pol site, and the

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**TABLE 3. Speed errors (m s\(^{-1}\)) of DVAD subjected to a 180° azimuthally continuous gap placed in different directions.**
results (not shown) were similar to those in Fig. 8, where NVAD showed more incorrect results while DVAD continued to show consistent results with the sounding data.

In the calculation of Figs. 7 and 8, all Doppler velocity observations within the maximum range (150 km) of the S-Pol radar were used. It is possible to select only Doppler observations within a certain distance from the radar. Figure 9 shows VWPs retrieved in the same way as those in Fig. 8, but using Doppler velocity observations within 100 km from the radar. It is clear that the retrieved VWPs are consistent with those in Fig. 8, which indicates that neither method is sensitive to the range of data used for calculation. Figures 7–9 demonstrate the improvement of DVAD over NVAD when higher-order models are used; the performance of the different orders of DVAD will be addressed in section 5.

b. VWP using NEXRAD datasets

The results using the dataset from the SoWMEX/TiMREX field campaign demonstrate that DVAD is generally more robust than NVAD when dealing with real observations contaminated by noise and data voids.
In this section, two winter storm cases collected by the operational NEXRADs were used to examine the performance of NVAD and DVAD under different weather regimes and geographic locations.

The dataset of the first winter storm was collected during the fourth IOP of the Profiling of Winter Storms (PLOWS; Rauber et al. 2014) field campaign. It consists of Doppler velocity observations from 0100 to 2000 UTC 8 March 2009 collected by the NEXRAD located at Davenport, Iowa (KDVN), and high-resolution sounding data every 2 h during the same time period compiled by the NCAR Earth Observation Laboratory. Figure 10 shows four sample $V_P$ PPIs during the period. The data coverage was good before 1800 UTC. Figure 11 shows the VWPs deduced by NVAD and DVAD using a fourth-order wind model. The maximum altitude of VWPs is limited to 5 km because there were not enough data for computation above this height. The VWPs from both the sounding and radar show easterly winds at lower levels and westerly winds above 2-km altitude. The lower-level winds turned westerly after 1800 UTC, which corresponds to the passage of a cold front. The VWPs computed by DVAD show consistency over time and are generally close to those of the sounding observations. The VWPs computed by NVAD are also consistent with those of the sounding observations, but contain missing and abnormal values especially when the data coverage deteriorates at the beginning and end of the analysis period. Both methods show greatest disagreement with the sounding observations at 1.5-km altitude, which is located in a transition zone between the upper-level westerlies and lower-level easterlies.

The second winter storm was observed on 29 November 2006. The synoptic chart for this day is characterized by two highs in the west and east directions and a north–south-oriented shear line.
The NEXRAD dataset during 0800–2000 UTC 29 November 2006 from KDVN is used because there is a Global Telecommunication System (GTS) sounding site near the radar for verification. The sounding site records observations 2 times per day at 0000 and 1200 UTC. Thus, there is only one sounding VWP for verification during the period of radar observation. Figure 12 shows four sample Vd PPIs at different times. Before 1500 UTC (and after 1700 UTC, which is not shown), there are large data voids in the radar domain as a result of the precipitation structure of the storm. Figure 13 shows the computed VWPs from both NVAD and DVAD. The VWPs from DVAD show consistent southwesterly winds above 1-km altitude that correspond well to the upper-level trough in the synoptic chart. Horizontal winds veer over time and correspond well to the passage of a cold front over the radar site at around 1230 UTC. The winds at 1200 UTC are consistent with those observed by the nearby sounding site. The upper-level southwesterly and veering lower-level winds are also revealed by NVAD, but the results are contaminated by missing and abnormal values.

5. Objective estimation of nonlinearity

When extending the single-Doppler analysis from linear to nonlinear wind models, one of the critical questions is how many nonlinear terms of a wind model are necessary to sufficiently represent the real wind field. CZ92 did not specify how to determine the optimal order of NVAD to represent the wind field, and the degree of nonlinearity was empirically estimated before the computation. Previous studies (Waldteufel and Cobin 1979; Koscielny et al. 1982; Boccippio 1995) have shown that the results of single-Doppler analyses can be biased by the underfitting or overfitting problem due to the choice of an improper wind model. One must be able to dynamically and objectively adjust the nonlinear wind model for best results, especially for operational purposes.

The mathematical models for different orders of nonlinear wind fields used in NVAD and DVAD are nested, which means that the coefficients of a lower-order model are exactly a subset of those of a higher-order model. The standard way of determining the optimal model out of such nested models is to perform an F test (Wilks 2011). An F test basically compares the residual sum of squares (RSS) between higher- and lower-order models to determine if the higher-order model is still necessary. Since two LSFs are used in the NVAD method, and each LSF has its own RSS value, it is not straightforward to determine the goodness of fit of the two steps combined. Furthermore, incorrect Fourier fitting may still have small RSS values (RSS is proportional to SD) because of the overfitting problem shown in previous sections; the small RSS values in NVAD may not truly indicate the goodness of fit. In the DVAD method, the RSS of the entire domain at each analysis altitude can be directly obtained since only a one-step fitting is required, and the RSS is more representative of the goodness of fit of the mathematical model to the true wind field. Moreover, since the calculation of DVAD uses Doppler velocity observations within a finite vertical extent, the differences in nonlinearity at different altitudes can be estimated independently. The independent evaluation of nonlinearity at different altitudes is not possible in NVAD because the vertical variation of the wind field is forced to be the same as those of the horizontal winds a priori. It has been shown that real wind fields have different degrees of nonlinearity at different altitudes (CZ92); therefore, the DVAD method is likely to produce more accurate VWPs throughout the vertical extent of the analysis domain.

An F test requires that the data samples used in the fitting model are independent. Since spatially adjacent radar observations are essentially dependent, the standard F test is not applicable here. Following the principle of the F test, the coefficient of determination $R^2$, defined as

$$R^2 = 1 - \frac{RSS}{TSS},$$

FIG. 11. Time series of VWPs computed by (a) NVAD and (b) DVAD using Doppler velocity observations collected by KDVN on 8 Mar 2009. VWPs from a nearby rawinsonde site (the blue plus sign in Fig. 10) are shown as red wind barbs. Wind barbs are plotted every 0.5 km from 0.5 km above radar level and every hour in time. The vertical dashed line indicates the approximate time when the cold front passed the radar site.
is used to determine a proper nonlinear model for real wind fields. Here, TSS is the total sum of squares of the observation data. The value of $R^2$ measures the ratio of the explained variance by the nonlinear wind model to the total variance of observations, which is consistent with the idea of the $F$ test. By comparing the values of $R^2$ for two consecutive orders of a fitting model, defined as

$$I_m = \frac{R^2_{\text{high}}}{R^2_{\text{low}}} - 1,$$  \hspace{1cm} (16)

the relative improvement of using a higher-order model can be estimated. The practical procedure of determining the optimal order is as follows: the $R^2$ value of a linear wind model is first calculated as a reference value, and then $R^2$ values of higher-order models are calculated sequentially. After calculating the $R^2$ value of a higher-order nonlinear wind model each time, the value of $I_m$ is calculated according to (16). The same procedure is repeated until $I_m$ reaches a predefined threshold, which indicates that no more variance in the data can be explained by the higher-order model.

Table 4 shows an example of $I_m$ calculated with respect to different orders (up to seven) and altitudes using the same OSE data ($E_{N+V}$) shown in the previous section. The large values of $I_m$ of the lower-order models, especially at the lower levels, indicate that higher-order models are necessary. In other words, higher-order models of DVAD generally give more accurate results. Note that the $I_m$ values stayed small after reaching a small value, which means that increasing the order of DVAD will not bring any significant improvement beyond a certain order. This characteristic can be used as a criterion for determining the optimal order for a nonlinear wind field. After experimenting with a large set of SoWMEX/TiMREX data, it was
found that the value of 0.5% was a reasonable threshold for determining the optimal order. Table 4 shows a higher degree of the nonlinear wind field at low levels, as well as a quasi-linear wind field above 5 km. This result was consistent with the characteristics of wind fields shown in Fig. 1.

The objective procedure was applied to the same Doppler velocity observations from the SoMWEX/TiMREX field campaign. The retrieved VWPs alongside the objectively determined orders of nonlinear winds are shown in Fig. 14. The retrieved VWPs in Fig. 14a were consistent with Fig. 8b using fourth-order uniformly and compared favorably to the sounding VWPs. The determined orders of the nonlinear wind model shown in Fig. 14b provide some insights into the structure and variation of the wind field being examined. Figure 14b first shows that a higher-order wind model is generally required at lower levels. This is consistent with the increased variabilities of the wind field near the ground. Before 0100 UTC 3 June 2008 when the cold front moved into the radar scope, the nonlinearity of the wind field at lower levels was generally lower. During the passage of the cold front, the wind field with the radar scope consisted of both northwesterly and southwesterly winds, and Fig. 14b shows an increased order of the nonlinear wind field accordingly.

6. Summary and conclusions

In this study, the DVAD method was used to quantitatively analyze nonlinear wind fields observed by a single-Doppler radar. DVAD uses $rV_d$ as the variable for analyzing Doppler velocity observations at a constant altitude. The $rV_d$ field of linear and nonlinear wind fields observed by a single-Doppler radar at each altitude was expressed in a single and concise 2D polynomial function. Only a one-step LSF of a 2D polynomial function to the observed Doppler velocities was required to obtain the physical parameters of a real wind field. The new method was compared with the traditional nonlinear wind analysis method, namely NVAD, which uses two separate LSFs to solve for the same information. It was demonstrated that the VWPs retrieved by both DVAD and NVAD were consistent in ideal conditions. Both methods were robust to the existence of random noise without data voids. Nevertheless, when noisy data were unevenly distributed because of data voids, the difference between the VWPs deduced by the two methods became significant. The NVAD-derived VWP showed large deviations from the rawinsonde VWP, while DVAD showed consistent and robust results. The different VWPs were attributed to the overfitting problem when higher-order VAD analyses were used with unevenly distributed noisy data.

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FIG. 13. As in Fig. 11, but using data from a winter storm collected by KDVN on 29 Nov 2006. The wind barbs are plotted every 40 min in time.

FIG. 14. Time series of (a) VWPs computed by DVAD in which the order of the nonlinear wind model is determined objectively, and (b) the magnitudes of those orders. VWPs from the Pingtung rawinsonde site at four different times are shown as red wind barbs.
robust performance of DVAD was further demonstrated with real Doppler velocity observations from the SoWMEX/TiMREX field campaign and the operational NEXRAD network.

The use of only a one-step LSF in DVAD allows for the direct estimation of the degree of nonlinearity in real wind fields. The same estimation is not feasible in NVAD because two separate LSFs are used. An optimal nonlinear wind model to approximate the real wind field can be objectively determined during the analysis by comparing the $R^2$ values of different wind models. The degree of nonlinearity at different altitudes can also be assessed because the analyses at different altitudes are independent in DVAD. This feature is instrumental in understanding the vertical structure of the wind field and is important in alleviating both overfitting and underfitting. The degree of nonlinearity at different altitudes determined by DVAD was shown to be consistent with the characteristics of the original wind field.

The study shows the potential of DVAD as a quantitative analysis tool for single-Doppler radar observations beyond the qualitative interpretation presented in Lee et al. (2014). Although the case studies presented in this study demonstrate that DVAD is generally more robust than NVAD, they by no means suggest that the performance of DVAD is universally robust. In addition to the operational limitations mentioned in previous sections, DVAD is subject to the same fundamental limitation as that of VAD and NVAD. All of these single-Doppler analysis methods are based on a prescribed wind model, which results from a truncated Taylor expansion. The methods are essentially limited by the extent to which real wind fields can be represented by such a truncated Taylor expansion. In addition, as discussed in Lee et al. (2014), although DVAD simplifies the interpretation of single-Doppler observations, the vorticity still cannot be retrieved regardless of whether the linear or nonlinear wind fields. Therefore, the method proposed in this study cannot be used in flow fields where the vorticity dominates (e.g., mature tropical cyclones). Despite its own limitations, the simplicity and robust performance of the DVAD method make it a good candidate for single-Doppler analysis in operational use.

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