Quantitative Precipitation Estimation of the Epic 2013 Colorado Flood Event: Polarization Radar-Based Variational Scheme

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ABSTRACT

The accuracy of rain-rate estimation using polarimetric radar measurements has been improved as a result of better characterization of radar measurement quality and rain microphysics. In the literature, a variety of power-law relations between polarimetric radar measurements and rain rate are described because of the dynamic or varying nature of rain microphysics. A variational technique that concurrently takes into account radar observational error and dynamically varying rain microphysics is proposed in this study. Rain-rate estimation using the variational algorithm that uses event-based observational error and background rain climatological values is evaluated using observing system simulation experiments (OSSE), and its performance is demonstrated in the case of an epic Colorado flood event. The rain event occurred between 11 and 12 September 2013. The results from OSSE show that the variational algorithm with event-based observational error consistently estimates more accurate rain rate than does the \( R(Z_{HH}, Z_{DR}) \) power-law algorithm. On the contrary, the usage of ad hoc or improper observational error degrades the performance of the variational method. Furthermore, the variational algorithm is less sensitive to the observational error of differential reflectivity \( Z_{DR} \) than is the \( R(Z_{HH}, Z_{DR}) \) algorithm. The variational quantitative precipitation estimation (QPE) retrieved more accurate rainfall estimation than did the power-law dual-polarization QPE in this particular event, despite the fact that both algorithms used the same dual-polarization radar measurements from the Next Generation Weather Radar (NEXRAD).

1. Introduction

Improving radar-based quantitative precipitation estimation (QPE) has been one of the major goals of the radar-meteorology community for more than 50 years. Retrieval of rain rate \( R \) requires accurate radar measurements and a relation that relates radar observations to rain rate. For example, Marshall and Palmer (1948) utilized the measured raindrop size distribution (RSD), the simulated reflectivity \( Z \), and a power-law relation \( Z = aR^b \) (\( Z-R \)) to estimate rainfall rate. However, \( Z \) is approximated by the sixth moment of the RSD, and \( R \) is approximated by the 3.67th moment. The \( Z-R \) relation varies vastly in convective and stratiform precipitation and in different climatological regions because of the natural variability in RSD (Battan 1973).

Seliga and Bringi (1976) proposed utilizing the dual-polarization radar measurement known as differential reflectivity \( Z_{DR} \) (dB) to reduce variability in the radar-based QPE. The \( Z_{DR} \) estimates the mean raindrop size by measuring the ratio between the horizontal reflectivity \( Z_{HH} \) and the vertical reflectivity \( Z_{VV} \). The axis ratios of raindrops decrease as the raindrop sizes become larger. The radar-based QPE is thus improved by...
including the RSD information from the \( Z_{\text{DR}} \) (Seliga et al. 1981; Gorgucci et al. 1995; Ryzhkov and Zrnić 1995). Furthermore, the dual-polarization radar measurement known as specific differential phase \( K_{\text{DP}} \left( \text{km}^{-1} \right) \) that is proportional to the fourth moment of the RSD is found to improve the accuracy of rainfall estimation (Ryzhkov and Zrnić 1996; Bringi and Chandrasekar 2001). The \( K_{\text{DP}} \) is unbiased by the transmitted power of the radar, and it is ideal for estimating rain rate when rain is mixed with hail or randomly tumbling ice particles (Bringi and Chandrasekar 2001). With the inclusion of \( \rho_{hv} \) (copolar correlation coefficient), the radar-data quality control is significantly improved through distinguishing rain echoes from ground clutter and other nonmeteorological signals. Thus, over-estimations of the radar-based QPE due to these artifacts are significantly reduced when the \( \rho_{hv} \) are used.

A number of dual-polarization QPE algorithms are proposed in the literature. These algorithms in various “power law” forms attempt to utilize the advantage of dual-polarization moments (DPMs) by using one parameter \([ R(K_{\text{DP}})]: \) Gorgucci et al. 1999; Bringi and Chandrasekar 2001; Ruzanski and Chandrasekar 2012, two parameters \([ R(Z_{\text{HH}}, Z_{\text{DR}})]: \) Ryzhkov et al. 2005a; Illingworth and Thompson 2005; \( R(K_{\text{DP}}, Z_{\text{DR}}) \): Ryzhkov and Zrnić 1995; Ryzhkov et al. 2005a; \( R(Z_{\text{HH}}, K_{\text{DP}}): \) Lee 2006, or three parameters \([ R(Z_{\text{HH}}, K_{\text{DP}}, Z_{\text{DR}}): \) Gorgucci et al. 2001; Bringi et al. 2002]. These various forms of dual-polarization QPE algorithms have shown pronounced improvements relative to \( Z-R \) relations.

To construct the aforementioned algorithms, specific RSD models (e.g., gamma distribution) or observed RSDs are combined with a rigorous electromagnetic-scattering model (Vivekanandan et al. 1991; Bringi and Chandrasekar 2001). The coefficients (e.g., \( a \) and \( b \) of \( R = a Z^b \)) of these power-law QPE algorithms are thus obtained by linear regression. Because of various RSDs and assumptions in the microphysical model (e.g., axis ratio and canting angle), a variety of power-law QPE algorithms with different combinations of DPMs are reported in the literature (Ryzhkov et al. 2005a; Lee 2006). Lee (2006) has shown that RSD variability is one of the major sources of error in radar-based QPE. Therefore, for reducing error in QPE, these power-law algorithms require additional “empirical tuning” to take into account spatiotemporal RSD variations as a function of climatological regions.

Errors in DPM measurements lead to errors in QPE. The \( Z_{\text{DR}} \) can be noisy because of the low sampling rate of a fast-scanning radar (Bringi and Chandrasekar 2001; Melnikov 2004). The \( K_{\text{DP}} \) is estimated from the measurements of differential propagation phase \( \Phi_{\text{DP}} \) as a range-derivative variable. The artifacts due to the fluctuations in \( \Phi_{\text{DP}} \) from backscattering Mie phase shift \( \delta \) introduced into \( K_{\text{DP}} \) are reduced by the filtering technique (Hubbert et al. 1993; Hubbert and Bringi 1995; Bringi and Chandrasekar 2001; Ryzhkov et al. 2005a; Wang and Chandrasekar 2009; Giangrande et al. 2013), but the filtering techniques introduce additional uncertainty in \( K_{\text{DP}} \)-related QPEs. Also, in the case of light rain, \( K_{\text{DP}} \) is too small to be estimated with sufficient accuracy. It is not uncommon that a particular dual-polarization QPE algorithm performs well in one rain event but fails in another event because of unsatisfactory DPM data quality.

The combining of various dual-polarization QPE algorithms using decision-tree logic is proposed (Cifelli et al. 2002, 2011; Ryzhkov et al. 2005a,b; Giangrande and Ryzhkov 2008). This composite method has been shown to be more accurate than an individual dual-polarization algorithm (Ryzhkov et al. 2005a). This method helps one to avoid using the \( K_{\text{DP}} \) for light-rain estimation. For example, \( R(Z) < 6 \text{ mm h}^{-1} \) is considered to be light rain in Ryzhkov et al. (2005b). Thus, \( R(K_{\text{DP}}) \) is only applied when \( R(Z) > 6 \text{ mm h}^{-1} \). Cifelli et al. (2011) suggest a more sophisticated method for detecting light rain using multiple thresholds: \( K_{\text{DP}} < 0.3\, \text{km}^{-1} \), \( Z_{\text{HH}} < 38 \text{ dBZ}, \) and \( Z_{\text{DR}} < 0.5 \text{ dB} \). Nevertheless, the threshold for differentiating light rainfall from moderate rainfall is dependent on the data quality of a particular radar and the RSD characteristics of a specific climatological value or rain event. Therefore, these thresholds may not be applicable across the board for all radars or for various rain climatological datasets for which RSDs change as a function of time and space.

Another major drawback of the decision-tree-logic QPE algorithm is that the transitions between individual algorithms introduce discontinuity in the rainfall estimate. Pepler et al. (2011) and Pepler and May (2012) introduced a blended technique that combined various QPE algorithms objectively according to their respective theoretical error characteristics. In this technique, observational error characteristics of each DPM are estimated theoretically as a function of radar samples and Doppler spectrum width (Bringi and Chandrasekar 2001). Thus, the theoretical errors of various QPE algorithms are derived accordingly. The optimal rain estimation is subsequently obtained by combining various rainfall estimations weighted by their respective errors. They found that the weighted combination of dual-polarization algorithms offers modest improvements over the decision-tree-logic method.

The quality of various DPMs is not only determined by the number of radar samples and Doppler spectrum width, however; it is also influenced by a range of factors under various measurement conditions. Possible
uncertainties in \(Z_{\text{DR}}\) of up to ±0.7 dB and in \(\Phi_{\text{DP}}\) of up to ±1° in the presence of nonuniform beamfilling (Ryzhkov 2007) introduce significant error in QPE (Bringi and Chandrasekar 2001; Lee 2006). In addition, the measurement volume may contain mixed-phase hydrometers (e.g., coexistence of rain and hail) or various artifacts (e.g., birds, insects, and elevated ground clutter via sidelobes). Hubbert et al. (2010a,b) demonstrated in the case of a simultaneously transmitted mode that the orientation of scatters causes cross coupling between horizontally and vertically polarized signals. Furthermore, \(Z_{\text{DR}}\) can be contaminated by a wet radome (Gorgucci et al. 2013). Consequently, even the advanced power-law-based QPE algorithms that make use of decision-tree and blended techniques become vulnerable to the aforementioned observation errors.

In summary, these power-law-based dual-polarization QPE algorithms are affected by two major factors: 1) natural RSD variability and 2) observational errors in DPMs. The variational algorithm that minimizes the above-mentioned two factors has the potential to improve the accuracy of dual-polarization QPE. A 1D variational scheme for retrieving rainfall rate using S-band polarimetric radar is introduced by Furness (2005) and Hogan (2007; hereinafter H07). This variational scheme dynamically adapts to natural variability of RSD, observational error, and rain climatological information. H07 shows that the variational algorithm can successfully utilize the DPMs and the background information concurrently for rainfall estimation. Also, it is relatively less sensitive to observation error than are power-law-based dual-polarization QPE algorithms.

Figueras i Ventura et al. (2010) applied the variational algorithm to French C-band radar network measurements, but the QPE validation against the rain gauges was unsatisfactory. They concluded that the higher attenuation in C band (especially in heavy-precipitation cases) and improper treatment of measurement error variance might have caused unsatisfactory QPE. Performance of the variational algorithm critically depends on the accuracy of the error variances of DPMs and background information.

To objectively estimate spatiotemporal error variances of DPMs, a statistical diagnostic method (Desroziers et al. 2005) was proposed by Chang et al. (2014; hereinafter C14). The usage of proper error variances of DPMs greatly improved the accuracy of the attenuation correction of X-band polarimetric radar data. Yet, the improvement to the QPE due to the usage of proper error variances was not investigated in C14. The main goal of this study is to apply the variational scheme from C14 (hereinafter referred to as variational QPE) and to investigate its applicability to QPE. The performance of the variational QPE is evaluated by comparing it with one of the most widely used “conventional” power-law dual-polarization QPE algorithms [hereinafter referred to as “dual-pol QPE,” or \(R(Z_{\text{HH}}, Z_{\text{DR}})\)]. An epic Colorado flood event that produced nearly 300 mm of rainfall in 48 h between 11 and 12 September 2013 (Hamill 2014) and that has data from operational S-band dual-polarization radar and rain gauges will be carefully examined in this study.

A brief description of the variational QPE algorithm is presented in section 2. The variational QPE is first evaluated with simulated DPMs by using observing system simulation experiments (OSSE) in section 3. Practical applicability of this method is demonstrated in the case of an epic Colorado flood event. The data used in this study are described in section 4. A comparison of the variational QPE and the polarimetric Denver (KFTG) Next Generation Weather Radar (NEXRAD) level-3 QPE products with gauge measurements is presented in section 5. Section 6 summarizes the results and describes future work.

2. Description of variational QPE

a. Variational method

In the power-law relation \(Z = aR^b\), parameters \(a\) and \(b\) represent variations in RSD (Steiner and Smith 2000; Steiner et al. 2004). The multiplicative parameter \(a\) is much more sensitive to RSD variations than is the exponent parameter \(b\) (Bringi and Chandrasekar 2001; H07). Previous studies (Furness 2005; H07; C14) have shown that it is adequate to fix \(b\) and that a value of 1.5 is satisfactory. Hence, the parameter \(a\) of \(Z = aR^{1.5}\) represents the RSD variations (H07; C14). The variational method retrieves the “target” parameter \(a\). In practice, the natural logarithm of coefficient \(a\) of \(Z–R\) [i.e., \(\ln(a)\)] is retrieved to avoid an unrealistic negative value of \(a\). Hereinafter, the symbol \(\tilde{a}\) represents \(\ln(a)\). The values of \(\tilde{a}\) are presented in linear scale.

The variational QPE consists of two major components. The first component is the forward model \(H\). The forward models use \(Z/R\) or \(a\) as a proxy for variation of the RSD and predict \(Z_{\text{DR}}, K_{\text{DP}}, A_{\text{H}}\) (one-way specific attenuation at horizontal polarization; dB km\(^{-1}\)), and \(A_{\text{DP}}\) (one-way specific differential attenuation; dB km\(^{-1}\)) along a beam. The forward models are based on a rigorous electromagnetic-scattering model (Vivekanandan et al. 1991; Bringi and Chandrasekar 2001) and a gamma RSD (mm\(^{-1}\) m\(^{-3}\)) model (Ulbrich 1983). The gamma RSD is shown as

\[N(D) = N_0 D^\alpha \exp\left[-(3.67 + \mu)(D/D_0)\right].\]  (1)
For a specified RSD, the rigorous electromagnetic-scattering and wave-propagation model computes $Z_{HH}$, $Z_{DR}$, $K_{DP}$, $A_{HH}$, and $A_{DP}$ at S-band frequency and the “referenced” rainfall rate is derived. To construct the forward models, the values of gamma parameters are varied for emulating natural variability of the RSD. The median volume diameter $D_0$ (mm) varies from 0.1 to 6 mm, and the value of the intercept parameter $\log_{10}(N_0)\text{ (mm}^{-1}\text{ m}^{-3})$ varies from 1.0 to 16.0. The shape parameter $\mu$ (dimensionless) is fixed at 5 (H07; C14). The forward models are implemented as lookup tables for numerical computations (Fig. 1). Cao et al. (2013) used a constrained-gamma RSD for implementing the forward model as a table lookup. The variational scheme considers rain below freezing level in this study. Mixed-phase precipitation, such as rain–hail or rain–snow, is not considered.

The second component of the variational QPE is the minimization of a cost function $J$. The $J$ is as shown in the following equation:

$$
J = 0.5[H(x) - y]^T\mathbf{O}^{-1}[H(x) - y] \\
+ 0.5(x - x_b)^T\mathbf{B}^{-1}(x - x_b).
$$

(2)

The function $J$ is composed of two components. The $J_{\text{obs}}$, the first rhs term in Eq. (2), is the component of the observational term. The $J_{\text{background}}$, the second rhs term in Eq. (2), is the component of the background term. The vectors $x$ and $y$ represent the target variable $\tilde{a}$—that is, RSD variation and the polarimetric measurements (i.e., $Z_{DR}$ and $\Phi_{DP}$)—and $x_b$ represents the background term. The $\mathbf{O}$ and $\mathbf{B}$ are the error covariance matrices of the observation and background terms.

For given first guess of $x$, the measured $Z_{HH}$ is first corrected for attenuation along the radar beam by a predicted $A_{HH}$ from the forward model. Using the corrected $Z_{HH}$, the forward model predicts an intrinsic $Z_{DR}$, $A_{DP}$, and $K_{DP}$. Subsequently, the intrinsic $Z_{DR}$ is reduced by the amount of $A_{DP}$. The predicted $\Phi_{DP}$ is integrated from the predicted $K_{DP}$ along the range. These predicted $Z_{DR}$ and $\Phi_{DP}$ values are compared with corresponding measured values (i.e., $y$). The optimal estimations of $x$ (i.e., $\tilde{a}$) as a function of range that retrieve the best rainfall estimation are obtained by minimizing the cost function. The minimization uses the Gauss–Newton iteration method (Rodgers 2000; H07; C14).
The background term \( \mathbf{x}_b \) is included as an a priori so that an appropriate value of \( \mathbf{a} \) is obtained in the regions of light rain where \( Z_{DR} \) and \( \Phi_{DP} \) are too small or useful polarimetric radar measurements are unavailable. A climatological value of \( \mathbf{a} \), for example, is usually obtained from locally tuned \( Z-R \) relations. The value of \( \mathbf{a} \) also can be obtained as an average from the previous scan or from an adjacent beam (H07). The contribution to the cost function from each component in Eq. (2) is weighted by the error covariance matrices (i.e., \( \mathbf{O} \) and \( \mathbf{B} \)). The variational QPE processes one radar beam at a time. Detailed information on the variational scheme can be found in H07 and C14.

b. Estimation of the observation and background error variance

The performance of the variational QPE critically depends on the accuracy of the error covariance matrices (i.e., \( \mathbf{O} \) and \( \mathbf{B} \)). Earlier studies (H07; Figueras i Ventura et al. 2010) simply used predetermined and fixed error covariance matrices. The \( \mathbf{O} \) was estimated by analyzing the radar measurements (Bringi and Chandrasekar 2001; Melnikov 2004; H07). The \( \mathbf{B} \) was determined from the variance of \( \mathbf{a} \), estimated from the disdrometer climatological data (H07). The values of predetermined \( \mathbf{O} \) and \( \mathbf{B} \) are mostly incorrect, however, because spatiotemporal uncertainties in the radar measurements and the natural variability of RSDs for a particular rain event are unknown without the detailed analysis of disdrometer measurements. Moreover, the climatologically derived \( \mathbf{a} \) of various precipitation types is not always available in practice, and \( \mathbf{B} \) is rarely studied in the literature. Also, higher weighting on the background term is required when ground clutter, partial beamfilling, and gradients in precipitation contaminate \( Z_{DR} \) and \( \Phi_{DP} \).

To estimate the \( \mathbf{B} \) that is in tune with RSD variation and data quality, predicted and measured radar variables are compared for a range of \( \mathbf{B} \) using a discrete-approximation method. The estimated \( \mathbf{B} \) corresponds to the value of \( \mathbf{B} \) that produces the least difference between predicted and measured radar observables (i.e., \( J_{\text{obs}} \)). A lower value of \( \mathbf{B} \) weights the background term more strongly than observations and vice versa. From the practical point of view, this approach guarantees a reasonable outcome in either poor observational data or when the forward model fails to represent the particular RSD.

Measurement error \( \mathbf{O} \) is obtained by a diagnostic method as described in C14. The diagnostic method (Desroziers et al. 2005; C14) combines observation, analysis, and background information for estimating observation errors for a particular rain event. Residuals \( \mathbf{d}_{\text{obs}}^\text{bg} \) between measurements and background and residuals \( \mathbf{d}_{\text{ana}}^\text{obs} \) between measurement and analysis are used for estimating observational error. This method assumes that the above-described residuals are the same as the residuals of the corresponding errors. They are defined as

\[
\begin{align*}
\mathbf{d}_{\text{obs}}^\text{bg} &= \mathbf{Y}_{\text{obs}} - H(\mathbf{X}^\text{bg}) = \mathbf{e}_{\text{obs}} - H(\mathbf{e}_{\text{bg}}) \quad \text{and} \\
\mathbf{d}_{\text{obs}}^\text{ana} &= \mathbf{Y}_{\text{obs}} - H(\mathbf{X}^\text{ana}) = \mathbf{e}_{\text{obs}} - H(\mathbf{e}_{\text{ana}}).
\end{align*}
\]

In the above equations, the superscripts \( \text{ana}, \text{bg}, \) and \( \text{obs} \) represent the analyses (i.e., optimal \( \mathbf{a} \) from the variational algorithm), the background terms (i.e., \( \mathbf{a} \)), and the observations from radar (i.e., \( Z_{DR} \) and \( \Phi_{DP} \)). \( \mathbf{e}_{\text{obs}}, \mathbf{e}_{\text{bg}}, \) and \( \mathbf{e}_{\text{ana}} \) are observation, background, and analysis errors. Since nonoverlapping or uncorrelated components in residuals are an observation error, the cross product as shown below is used for estimating the error:

\[
(\mathbf{d}_{\text{ana}}^\text{obs})^T(\mathbf{d}_{\text{bg}}^\text{obs}) = (\mathbf{e}_{\text{obs}}^\text{ana})^T(\mathbf{e}_{\text{obs}}^\text{bg}) - (\mathbf{e}_{\text{obs}}^\text{ana})^T H(\mathbf{e}_{\text{bg}}^\text{obs}) + [H(\mathbf{e}_{\text{ana}}^\text{obs})]^T H(\mathbf{e}_{\text{bg}}^\text{obs}).
\]

In general, background and analysis errors of observations \([H(\mathbf{e}_{\text{bg}}^\text{obs})] \) and \([H(\mathbf{e}_{\text{ana}}^\text{obs})] \) are assumed to be unbiased and mutually uncorrelated with each other statistically. Therefore, the second–fourth terms on the rhs of Eq. (5) are assumed to be negligible. Thus, the mean value of the diagonal terms of cross product of the \( \mathbf{d}_{\text{obs}}^\text{bg} \) and \( \mathbf{d}_{\text{ana}}^\text{obs} \) is the observation error variance \( \sigma^2 \) as shown in the following:

\[
\sigma^2 = E[(\mathbf{d}_{\text{ana}}^\text{obs})^T(\mathbf{d}_{\text{bg}}^\text{obs})] \approx E[(\mathbf{e}_{\text{obs}}^\text{obs})^2].
\]

Technically, the observation error covariance matrix \( \mathbf{O} \) contains the radar measurement error and the forward-model error (C14). Therefore, the diagonal terms of the observation error covariance matrix \( \mathbf{O} \) for each beam are obtained iteratively by calculating \( E[(\mathbf{e}_{\text{obs}}^\text{obs})^2] \) of \( Z_{DR} \) and \( \Phi_{DP} \) as described in C14 (see their Fig. 5).

The \( \mathbf{O} \) and \( \mathbf{B} \) are diagonal matrices, and their non-diagonal error covariances are neglected in this study. The advantages of using diagnostic and discrete-approximation methods in the variational QPE are 1) predetermined or ad hoc \( \mathbf{O} \) and \( \mathbf{B} \) are no longer used and 2) a reasonable QPE is guaranteed even in low-quality observational data or the forward model fails to represent the particular RSD. The procedures for estimating \( \mathbf{O} \) and \( \mathbf{B} \) are summarized in the appendix. More detailed descriptions of implementing diagnostic and discrete-approximation methods can be found in C14.
3. Observing system simulation experiments

As discussed in H07, the performance of the variational scheme depends on the relative weights given to the observations and the a priori (i.e., background term). The improvements in the variational QPE due to the usage of objectively derived O and B have not been investigated. Also, the sensitivity of the variational QPE to the a priori is not well known. To understand the sensitivity and the performance of the variational QPE, OSSE is conducted (Gorgucci and Chandrasekar 2005; Gorgucci et al. 2006; C14). The main goals of the OSSE studies are to investigate 1) the sensitivity of the variational QPE to the \( \Delta \) background, 2) the performance of the variational QPE in the case of predetermined and diagnosed observation errors, and 3) the accuracy of the variational QPE as a function of \( Z_{DR} \) observation error.

The 96 NCAR S-Pol plan position indicator (PPI) data from the Dynamics of the Madden–Julian Oscillation (DYNAMO) field campaign (Yoneyama et al. 2013) were used to generate the “pseudo” DPMs—mainly, \( Z_{HH}^{PSE} \), \( Z_{DR}^{PSE} \), and \( \Phi_{DP}^{PSE} \). These data were collected on 16 October 2011. The constrained-gamma RSD retrieval algorithm (Zhang et al. 2001) was applied to these PPIs to obtain the gamma RSD parameters \( N_0, \mu, \) and \( D_0 \) at each range gate. The natural variability of the RSD was represented by these retrievals. The referenced rainfall rates \( R_{sim} \) were calculated from these retrieved RSDs. Thus, the retrieved gamma RSD parameters were applied to calculate a suite of radar observables \( (Z_{HH}, Z_{DR}, K_{DP}^{sim}, \text{and } \sigma_{sim}) \) and specific attenuation \((A_{HH}^{sim} \text{ and } A_{DR}^{sim})\) in S-band frequency (Vivekanandan et al. 1991; Bringi and Chandrasekar 2001).

The pseudo DPMs were consequently derived as follows:

\[
Z_{HH,k}^{PSE} = Z_{HH,k}^{sim} - \sum_{l} (2A_{HH,l,k}^{sim} \Delta r) + Z_{HH,k}^{ERR},
\]

\[
Z_{DR,k}^{PSE} = Z_{DR,k}^{sim} - \sum_{l} (2A_{DR,l,k}^{sim} \Delta r) + Z_{DR,k}^{ERR}, \quad \text{and}
\]

\[
\Phi_{DP,k}^{PSE} = \sum_{l} (2A_{DP,l,k}^{sim} \Delta r) + \delta_{sim} + \Phi_{DP,k}^{ERR}.
\]

These pseudo DPMs of each scan were constructed with the range resolution \( \Delta r \) of 0.15 km for a 70-km range. The \( Z_{HH}^{PSE} \) and \( Z_{DR}^{PSE} \) were obtained by including the attenuation effect \((A_{HH}^{sim} \text{ and } A_{DR}^{sim})\) and the random observational error \((Z_{HH,k}^{ERR} \text{ and } Z_{DR,k}^{ERR})\) to the simulated \( Z_{HH,k}^{sim} \) and \( Z_{DR,k}^{sim} \) at each range gate \( k \). The \( \Phi_{DP,k}^{PSE} \) was obtained by integrating the \( K_{DP}^{sim} \) along the range. The backscattering effect \( \sigma_{sim} \) and the random observational error \( \Phi_{DP,k}^{ERR} \) were added to the integrated \( \Phi_{DP} \). The values of random observational errors with standard deviation 1 dB, 0.2 dB, and 1° for \( Z_{HH,k}^{ERR} \), \( Z_{DR,k}^{ERR} \), and \( \Phi_{DP,k}^{ERR} \), respectively, were assumed. These standard deviations of random error satisfy minimum requirements of DPM for QPE, as suggested by Melnikov (2004). A total of 96 PPI scans of pseudo DPMs were generated. An example of the simulated PPI pseudo DPMs of reflectivity \( (Z_{HH}^{PSE}), \text{ differential reflectivity } (Z_{DR}^{PSE}), \text{ and differential phase } (\Phi_{DP}^{PSE}) \) are shown in Figs. 2a–c. The referenced rainfall rate \( R_{sim} \) obtained from the retrieved RSD is shown in Fig. 2d.

a. Sensitivity of variational-based QPE to the background term

In the light–medium–rain region \( (Z_{HH} < 40 \text{ dBZ}) \), S-band radar measurements exhibit weak \( \Phi_{DP} \) increment. Despite the fact that the attenuation effect is often considered to be undesirable in the application of DPMs, the estimation of \( \tilde{a} \) via the variational algorithm can additionally benefit from adjusting the attenuation-corrected \( Z_{DR} \) to 0 dB in the stratiform region (Smyth and Illingworth 1998). Therefore, \( \tilde{a}_{background} \) plays an important role when applying the variational QPE in the regions of light–medium rain where dual-polarization radar measurements are weak.

The sensitivity of the variational QPE to a set of \( \tilde{a}_{background} \) was investigated by using pseudo DPMs. Martner (1977) investigated the RSD of a thunderstorm and found that the \( \tilde{a} \) could vary from 120 during the convective stage to 436 at the stratiform stage of precipitation. To replicate the situation in which no adequate information on \( \tilde{a}_{background} \) exists, the parameter \( \tilde{a}_{background} \) was thus varied from 100 to 500 in steps of 20. A total of 21 retrievals were obtained for sensitivity analysis. The uncertainty in the variational QPE was examined by calculating the standard deviation of the retrieved \( \tilde{a} \) and rainfall rate from these 21 sets.

As shown in Fig. 3a, the standard deviation of the retrieved \( \tilde{a} \) varies from less than 5 to 150. The higher values (above 100) are associated with reflectivity < 15 dBZ. The lack of sufficient polarimetric information (e.g., \( Z_{DR} \)) causes higher standard deviation in the retrieved \( \tilde{a} \). The accuracy of a retrieved \( \tilde{a} \) depends on the \( \tilde{a}_{background} \) in the presence of weak of polarimetric radar measurements (H07). The standard deviation of retrieved rainfall rate \( \sigma_{R} \) varies between 0.25 and 5 mm h\(^{-1}\) (Fig. 3b).

The uncertainty in retrieved rainfall rate was further examined using the normalized mean standard deviation (NMSD) as function of reflectivity:
Only the values of $\sigma_R$ (Fig. 3a) and $R_{\text{sim}}^\text{dr}$ (Fig. 2d) associated with a $Z_{\text{HH}}$ (Fig. 2a) of greater than the reflectivity threshold were included for the calculation of NMSD. As shown in Fig. 3c, the NMSD is about 7% for the reflectivity threshold between 5 and 25 dBZ. The NMSD reduces to 4.5% for reflectivity of greater than 38 dBZ. The decrease in NMSD is due to the supplementary RSD information from $Z_{\text{DR}}$ and $\Phi_{\text{DP}}$ DPMS at higher reflectivity. Rather than choosing $\bar{a}_{\text{background}}$ arbitrarily in this OSSE study, however, physically derived and event-dependent $\bar{a}_{\text{background}}$ (e.g., H07; C14) is usually used in practice to reduce the uncertainty of retrieved rainfall rate.

\[
\text{NMSD} = \left[ \frac{E(\sigma_R)}{E(R_{\text{sim}}^\text{dr})} \right] \times 100\%
\quad \text{when } Z_{\text{HH}} > \text{threshold.}
\]

(10)

**b. The performance of variational QPE with regard to radar measurement errors**

The performance of the variational QPE was investigated by comparing it with one of the most widely used dual-pol QPE algorithms:

\[
R = a_1 Z_{\text{HH}}^b \cdot Z_{\text{dr}}^c.
\]

(11)

These coefficients $a_1$, $b_1$, and $c_1$ vary depending on the climatological RSD (e.g., Ryzhkov et al. 2005a; Bringi and Chandrasekar 2001). Therefore, these coefficients are obtained by analyzing the RSD measurements from a local disdrometer in practice. In this OSSE study, these coefficients were obtained by a linear regression of simulated $Z_{\text{HH}}^\text{sim}$, $Z_{\text{DR}}^\text{sim}$, and $R_{\text{sim}}^\text{dr}$. As shown in Fig. 4, the corresponding coefficients obtained from 96 PPIs have...
large variability. This spread indicates that the OSSE data realistically emulate natural RSD variability.

For retrieving rain using Eq. (11), the mean values of $a_1$, $b_1$, and $c_1$ (Fig. 4) from 96 PPIs were used. The two-way total attenuations ($A_h$ and $A_{hv}$; dB) in $Z_{HH}$ and $Z_{DR}$ were corrected with a $\Phi_{DP}$-based method (Bringi et al. 1990; Anagnostou et al. 2006):

\[
A_h = \alpha \Delta \Phi_{DP} \quad \text{and} \quad A_{hv} = \beta \Delta \Phi_{DP},
\]

The coefficients $\alpha$ and $\beta$ were also obtained from the simulated data, and they are ideally suited for this particular simulated set of DPMs. For the variational QPE, the attenuation correction is included automatically because it is a standard product in the current version of the variational algorithm. The value of $\hat{a}_{\text{background}}$ for simulated data was 166.

The dual-pol and variational QPE algorithms were applied to the 96 PPIs of pseudo DPMs. The computed rainfall rates $R_{sim}$ from the retrieved RSDs at each gate were considered to be the referenced rainfall rate. Plots of the number density function (NDF) between the estimated and the referenced rainfall rate are shown in Fig. 5. The rainfall estimation from the variational algorithm (Fig. 5a) shows less scatter than does that from the dual-pol algorithm (Fig. 5b). Because the variational algorithm utilizes the $Z_{DR}$, $\Phi_{DP}$, and $\hat{a}_{\text{background}}$ and dynamically adapts to spatiotemporal error variance concurrently, it is less vulnerable to the random observational errors of $Z_{DR}$. On the contrary, dual-pol QPE is more sensitive to the observational error of $Z_{DR}$ (Fig. 5b).

To examine the accuracies of QPEs, the relative bias (RBIAS) and relative root-mean-square error (RRMSE) as a function of rainfall rate threshold were derived with

\[
\text{RBIAS} = \left[ E(T_R - T_G)/E(T_G) \right] \times 100\% \quad \text{when} \quad T_G > \text{threshold} \quad \text{and}
\]
\[
\text{RRMSE} = \left[ E[(T_R - T_G)^2]/E(T_G)^2 \right]^{0.5} \times 100\% \quad \text{when} \quad T_G > \text{threshold}.
\]

Here, $T_R$ is radar estimate and $T_G$ is referenced rainfall rate $R_{sim}$. Only the QPEs associated with a $T_G$ of higher than a specified threshold were included for calculation of RBIAS and RRMSE. The lowest threshold was set to 0.2 mm h$^{-1}$ for eliminating weak polarimetric radar data.

Three different rainfall estimates were retrieved from the pseudo DPMs for comparison: 1) the variational

\[
\text{Fig. 3. The standard deviations of (a) retrieved } \hat{a} \text{ and (b) retrieved rainfall rate (mm h}^{-1}\text{) due to variation in background } \hat{a}. \text{ (c) The normalized mean standard deviation of retrieved rainfall rate (})\%	ext{) as a function of reflectivity.}
\]
QPE with an improper $O$ (referred to as VAR$_{i}$O), 2) the variational QPE with diagnosed $O$ (referred to as VAR$_{d}$O), and 3) the dual-pol QPE [referred to as $R(Z_{HH}, Z_{DR})$]. For the VAR$_{d}$O, the values of the $O$ of $Z_{DR}$ and $F_{DP}$ were obtained as described in section 2b. On the other hand, the values of the $O$ of $Z_{DR}$ and $F_{DP}$ were fixed as 0.1 dB and 2.0°, respectively, in the VAR$_{i}$O, even though the actual random observational errors were 0.2 dB and 1.0° for $Z_{DR}$ and $F_{DP}$ as mentioned earlier.

Both of the variational QPEs (VAR$_{d}$O and VAR$_{i}$O) used the same $a$background. The values of $B$ in VAR$_{d}$O were obtained by the discrete-approximation approach as described in section 2b. The estimated values of $B$ were between 0.9 and 1.0. To examine the QPE difference between VAR$_{d}$O and VAR$_{i}$O solely due to improper and dynamically varying observational errors, the $B$ for VAR$_{i}$O was fixed at 1.0.

The instantaneous rainfall estimates from 96 PPIs were obtained for evaluating the performances of the dual-pol and the variational QPE algorithms. As shown in Fig. 6a, the VAR$_{d}$O has the lowest RRMSE for all rain intensities. The VAR$_{i}$O has an RRMSE that is comparable to that of $R(Z_{HH}, Z_{DR})$ and is much higher than the RRMSE of VAR$_{d}$O. This suggests that the diagnosed $O$ does play a crucial role in improving accuracy of rainfall rate in the variational algorithm. The improperly assumed $O$ leads to assigning higher weight on $Z_{DR}$ and lower weight on $F_{DP}$, and it is the main reason why RRMSE of VAR$_{i}$O is higher than the corresponding value of VAR$_{d}$O. The values of RRMSE of all three algorithms decrease with the increase of rain intensity. The RRMSE of VAR$_{d}$O is 43% for rain intensity > 60 mm h$^{-1}$. On the other hand, the RRMSEs of VAR$_{i}$O and $R(Z_{HH}, Z_{DR})$ are 61% for rain intensity > 60 mm h$^{-1}$.

In Fig. 6b, the RBIAS of the VAR$_{d}$O is the lowest for all of the rain intensities > 0.2 mm h$^{-1}$. It varies between 0% and ~6% as a function of rain intensity. The RBIAS of VAR$_{i}$O are between 17% and 21%. The $R(Z_{HH}, Z_{DR})$ overestimates by 17% for rain intensities > 0.2 mm h$^{-1}$. The RBIAS of $R(Z_{HH}, Z_{DR})$ decreases with increasing rain intensity and underestimates by approximately 1% for rain intensity > 60 mm h$^{-1}$.
To quantify the effect of observation error, the rainfall estimation was obtained using ideal DPMs ($Z^\text{sim}_{\text{HH}}$ and $Z^\text{sim}_{\text{DR}}$) without any observation error. As shown in Fig. 6b, the $R(Z_{\text{HH}}, Z_{\text{DR}})$ from perfect DPMs has nearly no bias if all of the rainfall intensities are included. This result is mainly due to the fact that coefficients of $R(Z_{\text{HH}}, Z_{\text{DR}})$ were derived from simulated measurements and are ideally suited for this particular case. RBIAS decreases with increasing rain intensity and reaches 21% for rainfall intensity $\leq 60 \text{ mm h}^{-1}$. These results show that the $R(Z_{\text{HH}}, Z_{\text{DR}})$ algorithm is sensitive not only to observation error but also to the fixed power-law formulation.

The consistently smaller values of RRMSE and RBIAS indicate that the variational algorithm is less vulnerable to $Z_{\text{DR}}$ observation error than is the $R(Z_{\text{HH}}, Z_{\text{DR}})$ power-law algorithm. Previous studies (Blackman and Illingworth 1997; Lee 2006) have suggested that spatiotemporal smoothing reduces uncertainty in $Z_{\text{DR}}$ and hence errors in hourly and daily averaged QPE. In Fig. 6c, the hourly VAR_d_O has the lowest RRMSE (21%–50%). On the other hand, the RRMSE of hourly VAR_i_O and $R(Z_{\text{HH}}, Z_{\text{DR}})$ are 27%–75% and 24%–57%, respectively. The $R(Z_{\text{HH}}, Z_{\text{DR}})$ has smaller RRMSE than does the VAR_i_O, except for rain intensity $>55 \text{ mm h}^{-1}$. In Fig. 6d, RBIAS of the VAR_d_O shows no significant change for instantaneous and hourly rain estimations since the bias could not be removed by the smoothing.

The OSSE results for the daily rainfall estimates are shown in Figs. 6e and 6f. The RRMSE for VAR_d_O is the lowest (12%–14%). The RRMSE for VAR_i_O (27%–32%) is the highest for all rain intensities. The RBIAS values for VAR_i_O are similar to the instantaneous and hourly rainfall estimates. The values of RBIAS are 0.0%–2.5% for VAR_d_O, 14%–16% for $R(Z_{\text{HH}}, Z_{\text{DR}})$, and 22%–23% for VAR_i_O. The lowest RBIAS of VAR_d_O is noteworthy since all three estimates used the same radar measurements.

The OSSEs show that the observational errors of simulated DPMs have a distinct influence on various QPE algorithms. The variational algorithm consistently performs better than $R(Z_{\text{HH}}, Z_{\text{DR}})$ in terms of the RRMSE and RBIAS when dynamically varying spatiotemporal errors were used. With the increase of time averaging in the hourly and the daily rainfall, the $R(Z_{\text{HH}}, Z_{\text{DR}})$ outperformed the variational algorithm when an improper $O$ was used. The result indicates that a dynamically estimated $O$ is essential for the accurate variational QPE. The OSSE study demonstrates the superiority of the variational technique over a power-law-based dual-pol algorithm [e.g., $R(Z_{\text{HH}}, Z_{\text{DR}})$].

c. Accuracy of variational QPE as a function of $Z_{\text{DR}}$ observation error

Sensitivity of the variational QPE to the $Z_{\text{DR}}$ observation error was examined by varying the standard deviation of random observation error of $Z_{\text{DR}}$ from 0.2 to 0.5 dB. The observation error of $Z_{\text{HH}}$ and $\Phi_{\text{DP}}$ remained the same at 1 dB and 2°, respectively. As shown in Fig. 7a, the RRMSE of instantaneous QPE increases with an increase in the observation error of $Z_{\text{DR}}$ for both the variational and $R(Z_{\text{HH}}, Z_{\text{DR}})$ algorithms. The RRMSE of $R(Z_{\text{HH}}, Z_{\text{DR}})$ is 150%–750% for $R^\text{sim}_{\text{HH}} > 0.2 \text{ mm h}^{-1}$ and 100%–400% for $R^\text{sim}_{\text{HH}} > 10 \text{ mm h}^{-1}$. The RRMSE of the variational algorithm with the diagnosed $O$ is smaller (i.e., 100%–200%) for $R^\text{sim}_{\text{HH}} > 0.2 \text{ mm h}^{-1}$ and is 80%–100% for $R^\text{sim}_{\text{HH}} > 10 \text{ mm h}^{-1}$. RBIAS is
smaller for the QPE retrieved using the variational method, as shown in Fig. 7b. The RBIAS for the variational algorithm is 0%–15% for $R_{\sim \text{m}} > 0.2 \text{ mm h}^{-1}$ and 8%–10% for $R_{\sim \text{m}} > 10 \text{ mm h}^{-1}$. The RBIAS of $R(Z_{\text{HH}}, Z_{\text{DR}})$ increases with an increase in the observation errors of $Z_{\text{DR}}$. It is 18%–100% for $R_{\sim \text{m}} > 0.2 \text{ mm h}^{-1}$ and 10%–90% for $R_{\sim \text{m}} > 10 \text{ mm h}^{-1}$.

The higher RRMSE caused by the $Z_{\text{DR}}$ observation error can be minimized by applying various filtering techniques (Blackman and Illingworth 1997; Lee 2006). For example, the NEXRAD operational QPE algorithm smooths out nonmeteorological variability in the measurements using the most frequent value in the nine adjacent radar sample bins along each radial. This
technique is referred as a “Mode-9 filter” (Istok et al. 2009). These filtering techniques are customized to a particular radar and scan strategy, however. There is no consensus in the research community as to the usage of one filtering technique versus another. Since evaluation of the performance of the filtering technique is not the main goal of this study, it will not be discussed in this paper. Note also that, without any filtering, the variational QPE outperforms the $R(Z_{\text{HH}}, Z_{\text{DR}})$ algorithm consistently in the presence of large $Z_{\text{DR}}$ errors.

4. Data

In the previous section, accuracy of the variational and the power-law dual-pol QPE algorithms has been compared using the OSSE studies. To demonstrate performance of the variational QPE, the measurements collected during an epic Colorado flood event that occurred between 11 and 12 September 2013 (Hamill 2014) were analyzed. This event produced nearly 300 mm of accumulated rainfall in 48 h. The results from the variational QPE were compared with NEXRAD operational products and rain gauge data.

a. Rain gauge data

The rain gauge data from an “automated local evaluation in real time” (ALERT; http://alert5.udfcd.org) network, maintained by the Urban Drainage and Flood Control District (UDFCD) of Denver, Colorado, was used for comparison. Raw ALERT gauge data were converted from tip counts to physical precipitation amounts. Then time-history plots for individual ALERT gauge sites were carefully reviewed to flag sites with data spikes. Hourly rainfall maps that are based on these data were also examined to detect anomalous data values. In situ measurements from 100 rain gauges were used for validation.

b. Parsivel disdrometer data

Data were analyzed from two Particle Size Velocity (Parsivel) disdrometers (Löfler-Mang and Joss 2000) installed in Fourmile Canyon, Colorado, and owned by NCAR (their locations are marked in Fig. 9). The RSD measurements by the Parsivel instruments during the flood event were compared with the nearby rain gauges, operated by UDFCD, located less than 2 km from the Parsivel. In general, the peak rainfall intensity and the total accumulation during the flood event were consistent with the operational gauge measurements.

c. KFTG level-2 data

The level-2 data at 0.9° elevation scan from KFTG were processed by the variational algorithm. The ground-clutter contamination at 0.9° was less than at 0.5° elevation scan. Radar measurements associated with $\rho_{hv} < 0.8$ were flagged as nonmeteorological data, and they were deleted. Some poor-quality and noisy suspicious $Z_{\text{DR}}$ measurements remained even after the aforementioned $\rho_{hv}$ threshold.

d. KFTG level-3 products

The dual-pol QPE algorithm in the NEXRAD Radar Product Generator (RPG) processes the data from the Radar Data Acquisition computer at “super resolution” of 0.5° by 0.25 km. The RPG then low-pass filters the data and decreases the spatial resolution to 1° by 0.25 km before further processing. The NEXRAD dual-pol QPE level-3 rain product—namely, the Digital Precipitation Rate (DPR)—is the final instantaneous precipitation rate as based on the dual-pol QPE algorithm. The DPR was downloaded from the National Climatic Data Center data archive. The hourly QPE product from the $Z–R$ legacy Precipitation Processing Subsystem was also considered for comparison.

The NEXRAD dual-pol QPE algorithms have been investigated by several researchers (e.g., Ryzhkov et al. 2005a,b; Giangrande and Ryzhkov 2008). The operational dual-pol QPE synthesizes a number of algorithms using the hydrometer classification (Vivekanandan et al. 1999) and results for optimal estimates. These algorithms are
A number of studies (Goodall et al. 2012; Cocks et al. 2012; Berkowitz et al. 2013) have shown that NEXRAD dual-pol QPE performs profoundly better than the conventional Z–R algorithm. During the 2013 Colorado flood event, the hydrometeor classifications obtained from the best/lowest available radar scans at each location indicated that the hydrometeors were primarily raindrops and detected no mixed-phase precipitation. Therefore, the dual-pol QPE was mainly derived from Eq. (18) rather than from Eq. (17).

5. Comparison between radar-based QPEs and rain gauge

As discussed in section 3a, the \( a^{\text{background}} \) plays an important role in regions in which no useful polarimetric radar measurements are available. The spatio-temporal characteristics of \( a \) also vary widely as the natural variability of RSD. Statistics of climatologically derived \( a^{\text{background}} \) are not reported in the literature. For the first radar scan, the \( a^{\text{background}} \) was initially assumed to be 300. Then the averaged retrieved \( a^{\text{background}} \) from high-data-quality regions (\( \rho_{\text{HV}} \geq 0.99 \)) and \( Z_{\text{HH}} \geq 40 \) are used as \( a^{\text{background}} \) for subsequent scans. The values of \( O \) and \( B \) were obtained as described in section 2b. The \( a^{\text{background}} \) for each individual scan is shown in Fig. 8a. The values varied between 50 and 450. The mean value of \( a^{\text{background}} \) was 132.8, and it was close to the \( a = 145 \) derived from Parsivel disdrometers located 80 km northwest from radar. It is important to note that the retrieved \( a \) from the variational algorithm is in agreement with the value obtained from the Parsivel disdrometer.

The NEXRAD was operated in “volume coverage pattern” 12, with 15 sample pairs of simultaneously transmitted polarimetric radar returns in 4.2 min per cycle during this event. The \( Z_{\text{DR}} \) had a higher observation error because of the lower number of samples collected during the fast scanning strategy. Figure 8b shows the time series of NDF (in logarithm scale) of the theoretical standard deviation of \( Z_{\text{DR}} \) observation error for each PPI (color shades) for the same time period, with the mean values of the standard deviation of \( Z_{\text{DR}} \) observation error for each PPI as estimated from the diagnostic and theoretical methods being shown as the blue and black lines, respectively.

\[
R(Z) = 0.0171Z^{0.714},
\]

\[
R(K_{\text{DP}}) = 44/K_{\text{DP}}^{0.822} \text{sign}(K_{\text{DP}}), \quad \text{and}
\]

\[
R(Z, Z_{\text{DR}}) = 0.0067Z^{0.927}Z_{\text{DR}}^{3.42}.
\]

Equation (16) is applied to light rain in the absence of dual-polarization radar measurements and when hail is above the melting layer, dry snow, and graupel. Equation (17) is applied when hail is below the melting layer. For the rain category only, Eq. (18) is applied (Giangrande and Ryzhkov 2008; Berkowitz et al. 2013).
### a. Total accumulated rainfall validation

The 48-h total accumulated rainfall from Z–R, dual-pol, and the variational QPE is shown in Fig. 9. The Z–R QPE ($Z = 250R^{1.2}$ during the 2013 Colorado flood event) in Fig. 9a shows the lowest accumulated rainfall estimation. In regions 1 and 2, the total accumulated rainfall is 250–300 mm (as measured by rain gauges); the Z–R QPE estimation is ~200 mm in region 1, however. The dual-pol QPE estimation of ~250 mm is higher than the Z–R QPE, as shown in Fig. 9b. Underestimation of dual-pol QPE in regions 1 and 2 is noticeable. On the other hand, the variational QPE of 300 mm is the closest to the rain gauge observation, as shown in Fig. 9c.

Figure 9d compares the 48-h accumulated radar rainfall estimations with data from rain gauges. The RBAIS and RRMSE were derived using Eqs. (14) and (15). The radar estimations were averaged within a 2-km radius of rain gauges. The results indicate that the variational QPE performed better than dual-pol and Z–R KFTG level-3 products. The variational QPE underestimates by 11% while the dual-pol and Z–R QPEs underestimate by 24% and 51%, respectively. Moreover, the RRMSE of the variational QPE is 29%, which is less than that for the KFTG level-3 dual-pol and Z–R QPEs.

Sensitivity of the variational QPE to fixed $O$ of $Z_D$ was further examined. As shown in Table 1, the values of RRMSE and RBAIS increase dramatically when a lower value of $O$ of $Z_D$ (0.2 and 0.3 dB) was used in the variational QPE. This result indicates that a properly diagnosed $O$ of $Z_D$ is crucial to retrieve accurate QPE from the variational algorithm. This result is consistent with the OSSE studies in section 3b.
b. Hourly rainfall validation

Figure 10 compares hourly rainfall from the variational and dual-pol QPE. The variational QPE has smaller values of RRMSE (2.15%) and RBIAS (−12.83%) than the dual-pol QPE RRMSE (2.45%) and RBIAS (−24.91%) for all rain intensities > 0.2 mm h⁻¹. The RRMSE and RBIAS of hourly rainfall estimates were further examined by gradually increasing the rain-intensity threshold as shown in Fig. 11. In general, RRMSE decreases as rainfall intensity increases, as shown in Fig. 11a. The RRMSEs for rainfall rate > 20 mm h⁻¹ are about 26% and 34% for the variational and dual-pol QPE, respectively. The variational QPE consistently outperforms dual-pol QPE. Figure 11b shows that RBIAS of the variational QPE is always smaller (from −9% to −14%) than that of the dual-pol QPE (from −24% to −29%). Overall, the results indicate that the variational QPE estimates more accurate hourly rainfall.

c. The validation of instantaneous rainfall rate

Comparisons of 5-min instantaneous QPEs from dual-pol and the variational method with the rain gauge data (5 min) are shown in Fig. 12. The RRMSEs are similar for both QPEs. One of the major reasons for high RRMSEs of instantaneous QPE is the large difference in sampling volumes between the radar and the rain gauges (Ryzhkov et al. 2005a).

The RBIAS of the variational QPE is approximately −18%, whereas the RBIAS of dual-pol QPE is approximately −29%. The underestimation of dual-pol QPE is more pronounced with increasing rainfall rate. Significant overestimation of dual-pol QPE is associated with low rain gauge values, as shown in the red-outlined box of Fig. 12a. This mismatch is caused by noisy $Z_{DR}$ due to the low sampling rate of KFTG in the regions of weak dual-polarimetric radar measurements.

6. Summary

Detailed investigations of variational-based QPE algorithm using OSSE studies and NEXRAD S-band polarimetric radar in a Colorado flood event between 11 and 12 September 2013 are presented in this paper. The results indicate that the variational algorithm retrieves more accurate rain estimation than does a conventional dual-pol power-law algorithm. Because the variational method dynamically combines the forward model, measurements, and background information using appropriate error variance, its retrieval is more accurate. The OSSE studies show that, even though $a^{\text{background}}$ is varied between 100 and 500, uncertainty in the variational QPE is less than 7.0%. The OSSE studies indicate that the performance of the variational QPE is degraded if improper error variance is used. It is essential to obtain spatiotemporal measurement and background error variances objectively for the specific radar data so as to obtain accurate rainfall estimation using the variational QPE.

### Table 1. RRMSE and RBIAS of 48-h accumulated rainfall derived by the variational QPE algorithm using diagnosed and fixed $Z_{DR}$ errors.

<table>
<thead>
<tr>
<th>$Z_{DR}$ observation error</th>
<th>RRMSE (%)</th>
<th>RBIAS (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diagnosed</td>
<td>29.39</td>
<td>−11.37</td>
</tr>
<tr>
<td>0.3 dB</td>
<td>58.92</td>
<td>47.83</td>
</tr>
<tr>
<td>0.2 dB</td>
<td>108.61</td>
<td>93.87</td>
</tr>
</tbody>
</table>
Even in the presence of a large observation error as high as 0.5 dB in $Z_{DR}$, the RBIAS of the variational QPE remains < 15%. On the other hand, the RBIAS of conventional dual-pol QPE [i.e., $R(Z_{HH}, Z_{DR})$] is >100%, and it is highly vulnerable to the $Z_{DR}$ measurement noise. Smoothing the data helps to reduce measurement noise, but it degrades spatiotemporal resolutions of precipitation and underestimates the heavier rainfall.

The variational QPE was compared with NEXRAD level-3 $Z$–$R$ and dual-pol QPE operational products for the epic 2013 Colorado flood event. The results indicate that the dual-pol QPE has significantly improved the rainfall estimation relative to the $Z$–$R$ QPE primarily because of the utilization of DPMs. The dual-pol QPE operational radar products underestimated the rain rate, however. It is postulated that empirical power-law relations that are based on predetermined rain microphysics are not appropriate to this particular “tropical rain” event. The variational QPE retrieved more accurate rainfall estimation than did dual-pol QPE in this particular event despite the fact that both algorithms used the same dual-polarization radar measurements from the KFTG radar.

The advantage of the simple but robust operational dual-pol QPE power-law algorithm is its capability to retrieve rain in mixed-phase precipitation such as rain–hail or rain–graupel. In contrast, the variational algorithm presented in this paper is applicable only to rain that is below the melting layer. One of the limitations of the variational algorithm is that it cannot be implemented for real-time operational application in the current configuration, because it is computationally expensive relative to the conventional dual-pol QPE algorithm. In the future, possible ways to reduce the computation time will be investigated by thinning the radar data. Also, performance of the variational algorithm in a variety of rain events that include cold- and warm-season events at various geographical locations will be investigated.

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recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

APPENDIX

Implementation of the Discrete-Approximation and Statistical Diagnostic Methods

The flowchart of processing procedures of the variational QPE algorithm is shown in Fig. A1. The data quality-control processes, including system bias in $Z_{HH}$ and $Z_{DR}$, and nonmeteorological-signal removal, are performed before applying the variational QPE algorithm (step 1). Subsequently, the first guess of $B$ is determined by discrete approximation using a prescribed $O$ at step 2. The prescribed $O$ is determined approximately on the basis of the number of radar samples and radar hardware characteristics. By using the first guess of $B$, the procedures shown in steps 3–5 diagnose the optimal $O$ by replacing with $E[(ε^{obs})^2]$ iteratively. The optimal $O$ is obtained when the values of $E[(ε^{obs})^2]$ converge. Another discrete approximation for $B$ is performed again at step 6, since the first guess $B$ at step 2 is obtained, because the prescribed $O$ did not include the actual radar measurements. This procedure ensures that an estimated $B$ is derived with an “optimal” $O$ in step 7 that is consistent with natural variation in raindrop size distribution and actual radar measurements.

REFERENCES


