Vertical Wavenumber Spectra of Gravity Waves in the Martian Atmosphere Obtained from Mars Global Surveyor Radio Occultation Data

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ABSTRACT
Vertical wavenumber spectra of Martian gravity waves were obtained for the altitude range 3–32 km from temperature profiles acquired by the Mars Global Surveyor (MGS) radio occultation experiments. The spectra, which cover vertical wavelengths from 2.5 to 15 km, generally show a decline of the spectral density with wavenumber similar to those obtained in the terrestrial stratosphere and mesosphere. The power-law spectral index is typically around $\frac{2}{3}$ except near the low-wavenumber end, and the spectra frequently lie along the theoretical spectrum for saturated gravity waves developed for Earth’s atmosphere. The results suggest that gravity wave saturation occurs in the atmosphere of Mars as well as that of Earth.

1. Introduction
Gravity waves generated in Earth’s lower atmosphere grow in amplitude with height, become convectively and/or dynamically unstable, and break in the stratosphere and mesosphere. Under such conditions the amplitude growth stops and the waves are considered “saturated” (Lindzen 1981; Fritts and Alexander 2003). Theory predicts that the superposition of saturated gravity waves over a broad spectrum results in the vertical wavenumber spectrum of gravity wave energy following a power law with an exponent $-3$ (e.g., Smith et al. 1987; Tsuda et al. 1991). Mesosphere–stratosphere–troposphere (MST) radars, radiosonde and Global Positioning System (GPS) radio occultation measurements have confirmed that the spectra of the stratospheric and mesospheric gravity waves are roughly consistent with theoretical predictions (e.g., Fritts et al. 1988; Tsuda et al. 1989, 1991; Tsuda and Hocke 2002), although there are reports that the spectra in the troposphere show smaller (negative) logarithmic spectral slopes and amplitudes greater than the saturation model (Nastrom et al. 1997).

The theoretical vertical wavenumber spectrum for saturated waves is based on several semiempirical considerations (Smith et al. 1987). The logarithmic spectral slope of $-3$ relies on the assumption that the spectrum is composed of wave packets with widths inversely proportional to the central wavenumber and that these packets are saturated due to convective instability. The apparent universality of the spectrum in Earth’s atmosphere does not guarantee the applicability of the theory to the atmospheres of other planets where the wave generation process and the propagation condition can be much different from those of Earth. Studies of the vertical wavenumber spectra of planetary atmospheres would shed light on the universality of wave saturation. Yamanaka (1995) suggested, using formulas based on observations of Earth’s gravity waves, that saturation of gravity waves determines the turbulence level commonly in planetary atmospheres.

Recently Fritts et al. (2006) obtained horizontal wave-number spectra of small-scale density fluctuations with horizontal wavelengths of 20–200 km in the Martian atmosphere around 100-km altitude using the aerobraking
data obtained in the *Mars Global Surveyor* (*MGS*) and *Mars Odyssey* missions, and suggested that gravity waves are saturated in this region. Heavens et al. (2010) suggested, from temperature profiles covering altitudes from ~5 to 85 km taken by the Mars Climate Sounder (MCS) onboard the *Mars Reconnaissance Orbiter*, that convective instability occurs around an altitude of 70 km due to the intrusion of gravity waves into a weakly stratified region created by thermal tides. Gravity waves with vertical wavelengths shorter than 10 km cannot be observed by MCS due to the limit of vertical resolution, although the saturation of such waves might contribute to the observed neutrally stratified layer.

The present study investigates Martian gravity waves with vertical wavelengths of 2.5–15 km using temperature profiles obtained by the *MGS* radio occultation experiment, which has a high vertical resolution of ~1.25 km (Hinson et al. 1999). Creasey et al. (2006) obtained the global distribution of the potential energy of gravity waves using *MGS* radio occultation data and found that the wave energy is maximized in the tropics and that the activity there is enhanced in northern summer. In this study, vertical wavenumber spectra are obtained for the first time to examine whether Martian gravity waves are saturated via convective instability. Section 2 describes the analysis procedure, section 3 presents results, section 4 examines the wave dissipation processes and the possible error in the measurements, and a summary is given in section 5.

### 2. Dataset and analysis procedure

*Mars Global Surveyor* was inserted into a Martian sun-synchronous polar orbit passing over approximately 0200 and 1400 local time (LT) in March 1999 and finished operation in January 2007. *MGS* radio occultation experiments were conducted using a 3.6-cm wavelength radio signal transmitted by the spacecraft and received on Earth. *MGS* sampled essentially all latitudes and longitudes during the mission. Each temperature profile extends from the surface to the 10-Pa pressure level, which is approximately 35-km altitude, with a sample spacing of about 1 km. Details of the measurement are described in Tyler et al. (1992) and Hinson et al. (1999).

We used 21 174 vertical temperature profiles obtained from January 1998 to September 2006, which are released via the National Aeronautics and Space Administration Planetary Data System. Each temperature profile is interpolated (oversampled) at evenly spaced bins with 0.017-km intervals and a cubic function is fitted to the profile (Fig. 1). This fitted function is regarded as the background temperature $T_0$ and subtracted from the original temperature profile similarly to Creasey et al. (2006). The residual is regarded as the temperature perturbation $T'$ associated with gravity waves (Fig. 2). The minimum vertical wavelength resolved in this study is 2.5 km because the vertical resolution is ~1.25 km.

To study the dependence of the spectrum on the altitude, temperature distributions in the two altitude regions, 3–20 and 15–32 km, are analyzed separately with a 1024-point fast Fourier transform including a Welch window:

$$w_j = 1 - \left( \frac{j - N_d - 1}{N_d + 1} \right)^2 ,$$

where $j = 1, \ldots, N_d - 1$ denotes the bin number and $N_d (=1024)$ is the number of bins. Obtained spectra are classified into four seasons in terms of the solar longitude $L_s$ (315°–45°, 45°–135°, 135°–225°, and 225°–315°) and five latitude regions (75°–45°S, 45°–15°S, 15°S–15°N, 15°–45°N, and 45°–75°N). The spectra are then averaged in each season and latitude bin. The number of profiles used in the analysis and the background Brunt–Väisälä frequency are summarized in Table 1.
The vertical wavenumber spectrum of the normalized temperature perturbation for saturated gravity waves is predicted by theory as (Smith et al. 1987; Tsuda et al. 1991; Tsuda and Hocke 2002)

$$F_{T'/T_0} = \frac{N^4}{10g^2k_z^5},$$

where $g$ is the gravitational acceleration (m s$^{-2}$) and $N$ is the Brunt–Väisälä frequency (s$^{-1}$). The unit of $k_z$ requires attention: one $k_z$ in the cube of $k_z$ has a unit of cycles per meter (cpm), and other two $k_z$ have a unit radians per meter. Thus, a scaling factor $\frac{1}{4\pi^2}$ is required to convert to corresponding power spectral densities in terms of $k_z$ all in cpm. When we plot observed spectra for different background $N$ conditions at the same time and compare them with the theoretical spectrum of saturated gravity waves, it is convenient to divide the spectra by $N^4$ so that the spectrum of saturated waves does not depend on $N$. In the next section we compare observed wavenumber spectra divided by $N^4$ with the theoretical one above divided by $N^4$.

3. Results

Figures 3 and 4 show the vertical wavenumber spectra obtained for the altitude ranges of 3–20 and 15–32 km. Since the number of data averaged for each region $n$ is sufficiently large in most of the cases, the central limit theorem holds. Therefore, the half-width of the 95% confidence interval is given by 2 times the standard deviation divided by $\sqrt{n}$ and is typically 0.6%–3% for the number of data of 1000 and 2%–8% for the number 100.

We see a general tendency that the spectral density decreases with wavenumber similarly to those in the terrestrial stratosphere and mesosphere. The logarithmic spectral slope is close to $-\frac{3}{4}$ at $k_z = 0.12–0.4$ km$^{-1}$ (wavelengths 3–8 km) and becomes flatter at larger scales. The equatorial region tends to have the highest power, and the high latitudes tend to have the lowest power in all seasons; a notable exception is $L_\varphi = 135^\circ–225^\circ$ in the 15–32-km range, where the springtime southern middle and high latitudes show relatively high powers. The difference between the two altitude regions is small. The equatorial maximum is consistent with Creasey et al. (2006) even when the latitudinal variation of $N$ is considered.

Also plotted in the figures is the theoretical spectrum of saturated gravity waves given by (2). In the equatorial region ($15^\circ$S–15$^\circ$N) the observed spectra are close to the saturation curve at $k_z = 0.12–0.4$ km$^{-1}$ (wavelengths 3–8 km) in all seasons. Exceptions are $L_\varphi = 45^\circ–135^\circ$ and $L_\varphi = 135^\circ–225^\circ$ in the altitude range of 3–20 km, where the equatorial region shows spectral densities 3 or

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<th>Table 1. Number of data used in the analysis and the mean Brunt–Väisälä frequency (s$^{-1}$).</th>
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4 times larger than the saturation value. This apparent “excess” energy might be attributed to the influence of thermal tides. Hinson and Wilson (2004) suggested that thermal tides have large amplitudes in the equatorial region and are responsible for inversion layers that appear at pressures between 30 and 200 Pa (altitudes of about 10–30 km). Although the vertical wavelengths of the tides are considered to be ~35 km (Hinson and Wilson 2004) and longer than the maximum wavelength studied in our analysis, the tides include shorter wavelength components and might contribute to the excess energy.

4. Discussion

a. Dissipation of gravity waves

The result suggests that the upper limit of the spectral density is determined by the theoretical spectrum of saturated gravity waves. This implies that small-vertical-scale waves are frequently saturated also in the Martian atmosphere and that the saturation spectrum, which has been constructed quasi empirically and tested in Earth’s atmosphere, is applicable also to Martian gravity waves. Saturated gravity waves will contribute to the acceleration of the mean flow and the generation of turbulence to induce diffusive transport of energy, momentum, dust, and other atmospheric constituents.

Power-law spectral indices near $-3$ are seen also in the spectra that have up to one order of magnitude less power than the saturation value. One possible explanation is that the mean wind variation with altitude, depending on its direction, reduces spectral densities or leaves the spectrum saturated. Eckermann (1995) argued that departures of wavenumber spectra in Earth’s atmosphere from the theoretical one arise when a spectrum of gravity waves propagates upward and encounters mean wind changes that cause intrinsic horizontal phase speeds to change. On average this effect tends to reduce spectral densities.

Another possibility is that radiative damping influences the amplitude. Zhu (1994) argued that a $k_z^{-3}$ spectrum can be produced by scale-dependent radiative damping for long-wavelength waves and a $k_z^{-1}$ spectrum can be produced by scale-independent transparent-limit damping for short-wavelength waves. Imamura and Ogawa (1995) and Eckermann et al. (2011) showed that the radiative damping rate depends on the vertical wavenumber in the Martian lower atmosphere, implying that radiative damping there is not at the transparent limit.

Eckermann et al. (2011) examined theoretically the effects of radiative damping, saturation, and molecular viscosity on the vertical propagation of Martian gravity waves. They concluded that radiative damping is the
dominant process in dissipating the waves and that saturation occurs only at >50 km depending on the vertical wavelength. Their result, however, depends on the parameters adopted in the model. For example, if the vertical displacement amplitude given near the surface (100 m was adopted) is higher or the horizontal wavelength (100 km was adopted) is shorter, the wave would start to saturate at lower altitudes. The importance of radiative damping will depend on the source characteristics of the gravity waves.

It should be noted that Doppler spread theory (Hines 1991) might also explain the observed spectra. Hines suggested that nonlinear interaction between the waves of the full spectrum causes the Doppler shifting of the local intrinsic frequency of any given wave in the wind field imposed by all waves, and spread the waves in vertical wavenumber, particularly into the large \( k_z \) tail, to create the \( k_z^{-3} \) spectral form. In this sense, the observed \( k_z^{-3} \) spectra do not necessarily mean saturation through convective instability.

b. Influence of horizontal averaging

The temperature profiles used in the analysis rarely show neutral stability layers, which are indicative of gravity wave saturation. We should note, however, that physical quantities measured by radio occultation technique are horizontally averaged and that signatures of small-scale waves will be smoothed out to some extent. Here we examine the effect of this averaging on the wavenumber spectrum based on the method of Tsuda and Hocke (2002).

The horizontal resolution in radio occultation is given by (Kursinski et al. 1997)

\[
h = 2\sqrt{2d(R_M + z)},
\]

where \( d = 2\sqrt{\lambda L} \) is the first Fresnel radius with \( \lambda \) being the wavelength of the radio wave and \( L \) the distance between the position of the spacecraft and the limb of the planet, \( R_M \) is the radius of Mars, and \( z \) is the observed altitude. In the MGS radio occultation experiments, by substituting \( \lambda = 3.56 \) cm, \( L = 18000 \) km, \( R_M = 3396 \) km, and \( z = 40 \) km into (3), we obtain \( h \sim 210 \) km. Here we neglect the anisotropy of gravity waves in the azimuthal direction. Following Fritts and VanZandt (1993), it is assumed that the spectrum can be written as a separable function of the vertical wavenumber and the intrinsic frequency \( \omega \) as

\[
F(k_z, \omega) = E_0 A(k_z) B(\omega).
\]

We further assume that \( B(\omega) \propto \omega^{-p} \) with \( p = \frac{5}{3} \) between the inertial frequency \( f \) and \( N \). By using the dispersion
relation $|\omega| = |k_h N/k_z|^p$. $F(k_z, \omega)$ is converted into the spectrum with respect to the horizontal wavenumber $k_h$ and the vertical wavenumber $k_z$:

$$G(k_h, k_z) = \int F(k_z, \omega) d\omega dk_h = E_0^{\frac{p}{4}} A(k_z) k_z^{p-1} N_0^{1-p} k_h^{-p}.$$  

(5)

Integrating $G(k_h, k_z)$ with respect to $k_h$ after multiplying an observation filter function of $k_h$, we obtain the vertical wavenumber spectrum to be observed. The limb averaging does not influence the logarithmic spectral slope of the vertical wavenumber spectrum provided that $F$ is written as a separable function of $k_z$ and $\omega$ and that $B$ is given by a power of $\omega$. In more general cases the spectral slope would be affected by the averaging.

The maximum wavelength $\lambda_{max}$ can be written as

$$\lambda_{max} = \frac{N}{k_{z_sat}^p},$$

(6)

where $k_{z_sat}$ is the characteristic wavenumber (cycles per kilometer) below which the spectrum becomes flat, and $f$ is the Coriolis parameter at the latitude under consideration. Letting $N = 10^{-2}$ s$^{-1}$ and $k_{z_sat} = 0.12$ km$^{-1}$, we obtain $\lambda_{max} = 680$ km and 1175 km for the latitude 60$^\circ$ and 30$^\circ$, respectively. Assuming that the filter function is a rectangular box with zero contribution at wavelengths below $h$, the ratio of the observed spectral density to the true spectral density is given by

$$R = \frac{\int_{2\pi h}^{2\pi \lambda_{max}} k_h^{-p} dk}{\int_{2\pi h}^{2\pi \lambda_{max}} k_h^{-p} dk},$$

which is calculated to be 0.54 and 0.68 at 60$^\circ$ and 30$^\circ$, respectively. This reduction will partly explain the tendency that the observed spectrum shows slightly less power than the saturation spectrum. We expect a similar situation holds in the equatorial region, where the assumption (6) fails because of the vanishing of $f$, since the observed spectral feature is similar among the latitudes.

5. Summary

The seasonal, latitudinal, and altitude dependences of the vertical wavenumber spectrum of Martian gravity waves were studied for the first time using the temperature profiles obtained by the MGS radio occultation experiment. The spectra, which cover vertical wavelengths from 2.5 to 15 km, generally show remarkable similarities to those in the terrestrial stratosphere and mesosphere. The spectral density decreases with wavenumber with a characteristic slope of around $-3$ at wavelengths shorter than $\sim 8$ km, and the spectrum slightly flattens near the long wavelength end. The spectrum tends to be close to the theoretical spectrum for saturated gravity waves at wavelengths shorter than $\sim 8$ km, especially in the equatorial region. The spectra at high latitudes tend to have less power than the saturation curve by up to one order of magnitude. No notable altitude dependence is observed. The suggested wave saturation will lead to diffusive transport of energy, momentum, and various atmospheric constituents.

The fact that the amplitudes of Martian gravity waves seem to be limited by saturation similarly to the terrestrial gravity waves has important implications. As described in section 1, the theoretical saturation spectrum was developed semiempirically based on the observations in Earth’s atmosphere, and thus the applicability of the theory to the Mars atmosphere would assure the true “universality” of the spectrum. Note, however, that the spectra obtained in this study differ from severe observational limitations: the vertical resolution is coarser than the usual meteorological observations in the terrestrial atmosphere, and the temperature field is horizontally smoothed. Moreover, the strong radiative damping in the Martian atmosphere must influence the wave characteristics. Further observations are needed to clarify both the similarity and the difference between the Martian and terrestrial gravity waves. Another promising statistical approach might be a comparison of the horizontal wavenumber spectrum; Imamura et al. (2007) analyzed small-scale horizontal structures of the temperature around 20-km altitude by using the infrared radiance data obtained by the Thermal Emission Spectrometer (TES) onboard MGS. They argued that the overall features of the horizontal wavenumber spectra are similar to those of the terrestrial stratosphere and that gravity waves have similar statistical properties between these planets.

Medvedev et al. (2011) suggested, by using a Martian GCM that includes a spectral parameterization of gravity waves, that convectively generated gravity waves would play an important role in driving the atmospheric circulation in the high-altitude region, especially the upper mesosphere (100–130 km). It should be noted, however, that they assumed a model spectrum of gravity waves based on observational results in Earth’s atmosphere. The results of the present spectral analysis will enable more realistic assessment of the effect of gravity waves in the Martian atmosphere.

Gravity wave breaking is likely the major source of upper-level atmospheric turbulence on Mars. The turbulence is a major hazard for potential instrument damage or loss during aerobraking phases of spacecraft orbital capture and during atmospheric entry of probes. Studies
of gravity wave saturation on Mars will also contribute to the safety of future Mars missions.

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