Effects of Altitude on Maximum Raindrop Size and Fall Velocity as Limited by Collisional Breakup

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ABSTRACT

Coalescence and breakup of drops are recognized as the main mechanisms determining raindrop size distributions on the ground. Full knowledge of these processes is hindered by the challenging difficulties both in the laboratory and tunnel experiments and during observations in the open air.

In real rain breakup is mainly due to collision between drops of different sizes (collisional breakup) and occurs when the collisional kinetic energy (CKE) is not absorbed by the colliding drops. In this work, the authors observe and measure the dependence on altitude of the occurrence of collisional breakup in real rainfall events, and then estimate the corresponding limit terminal velocities of drops and their size when breakup significantly takes place.

Data from Pludix, an X-band microwave disdrometer, were collected at three locations at different elevations: collisional breakup position in the power spectrum of Pludix increases toward higher frequencies with increasing altitude. Terminal velocities and sizes of the drops at breakup were determined consequently, with drop sizes resulting in 4.55 ± 0.35, 4.02 ± 0.32, and 3.16 ± 0.3 mm for altitudes of 15, 950, and 3300 m MSL, respectively. The authors computed the CKE of the colliding drops at the breakup, finding an upper limiting value of about 1.22 × 10^{-2} J for all three altitudes. This shows that most dominant collisional breakup signature occurs at similar CKE values for all three locations, corresponding to different drop diameters at different altitudes because of the effect of air density on the drop terminal velocity.

1. Introduction

After the formulation of a warm rain formation theory (Langmuir 1948), many research activities aimed at gaining deeper knowledge of the main processes that determine the microphysical structures of liquid precipitation. In particular, drop collision/coalescence and breakup were recognized as the main mechanisms to take place in the falling rain and able to define the drop size distribution (DSD) measured at the ground level, if evaporation (below cloud base) and diffusional growth (inside cloud) are neglected.

Collisional breakup (i.e., drop disruption after collision with a smaller drop) has been recognized as the main mechanism limiting the drop size in natural rain (Magarvey and Geldart 1962) below the spontaneous breakup size. Systematic experimental studies of collisional breakup are difficult in both wind tunnels and free air for many reasons, related to the difficulties in controlling collision/breakup parameters such as drop size and velocity, air velocity, and in focusing on the single collisional breakup event. The first attempts were made by Mc Taggart-Cowan and List (1975) using an aerodynamic drop accelerator to study binary collisions between five pairs of drops at terminal speed. Using a similar device, Low and List (1982a,b) increased the spectrum of drop sizes in colliding pairs in order to cover a wider range of events. The collisional kinetic energy
(CKE) and the Weber number $W$ are found to be relevant parameters in determining the breakup type, the number of fragments, and their size distribution. McFarquhar (2004) revisited these results and proposed a new parameterization for collisional breakup ensuring mass conservation and estimating the uncertainties. An equilibrium DSD is derived by a box model, characterized by two peaks around 0.26 and 2.3 mm. Along the same line, Barros et al. (2008) using Low and List (1982a,b) and McFarquhar (2004) parameterizations in a microphysical model, compared observed and simulated breakup fragment distributions and found good agreement in peak location, with higher discrepancies in the number of very small droplets (diameter below 0.2 mm).

More recently, List et al. (2009a) modified the McTaggart-Cowan and List (1975) device to study the collision–coalescence and collision–breakup characteristics at 50 kPa in comparison with those obtained at 100 kPa. For high and intermediate CKE values the reduced pressure is effective in increasing the number of breakup fragments. The time evolution of the DSD is sensitive to pressure: the equilibrium peaks are rapidly reached at 100 kPa, while at 50 kPa the equilibrium is reached more gradually—by a factor of 2 over time (List et al. 2009b).

In this work we provide for the first time evidence of the dependence on altitude of the rain drop velocity at the breakup in real rainfall. We exploit the capability of the Pludix sensor to measure the vertical speed of falling objects by the accurate measure of Doppler shift frequency. The drop speed is converted to drop diameter by an altitude-dependent drop size–drop velocity relationship and an estimate of the collision energy balance is derived.

2. Experimental data analysis

We collected data from the Pludix sensor at three different altitudes above sea level during three field campaigns. Pludix (Prodi et al. 2000a,b; Caracciolo et al. 2006) is an X-band (9.5 GHz), continuous-wave, low-power (10 mW), bistatic, vertically pointing radar that measures the Doppler shift between transmitted and received radiation as backscattered by falling hydrometeors. The reflecting volume is the volume, just above the instrument, in which the falling hydrometeors reflect received radiation as backscattered by falling hydrometeors. The reflecting volume is the volume, just above the instrument, in which the falling hydrometeors reflect a detectable signal, and varies with drop size and velocity from few cubic meters to cubic decimeters for larger and smaller drops, respectively. The basic output of Pludix is the Doppler spectrum, or the spectral intensity of backscattered radiation, as function of the Doppler frequency shift, collected every minute and sampled from 0 to 1024 Hz in 1024 bins that are 1 Hz wide. The frequency of the signal depends on the velocity with which the target (drop) crosses the equiphasic surfaces (i.e., the set of points at equal distance from the radiative centers of the Pludix antennas). An inversion procedure, based on scattering simulations, converts the power according to the frequency in number of hydrometeors per unit of volume as a function of the terminal velocity and successively, via a diameter–velocity relationship, as a function of the drop diameter, sampled in twenty-one 0.3-mm-wide size classes from 0.8 to 7.1 mm (Caracciolo et al. 2006).

The data considered here stem from three experimental campaigns carried out in Ferrara (44.85°N, 11.63°E), northern Italy, at 15 m MSL (Caracciolo et al. 2006); Wasserkuppe (50.50°N, 9.94°E), central Germany, at 950 m MSL (Prodi et al. 2011); and LinZhi (29.77°N, 94.74°E), Tibetan Plateau (China) at 3300 m MSL (Caracciolo et al. 2011). All three campaigns lasted for several months in order to get a complete observation of the seasonal cycle of precipitation characteristics. To investigate the breakup, only the power spectra as a function of the Doppler frequencies are considered here. The breakup signature in the power spectrum is an abrupt decrease of the power in a very short frequency interval (Prodi et al. 2011) (e.g., an order of magnitude of power decrease in about 50 Hz).

The three datasets were inspected to select breakup cases: for each 1-min spectrum with rain rate larger than 10 mm h$^{-1}$ the minimum of the derivative of the received power with respect to the Doppler frequency is searched for and the minutes with the lower values of the derivative are selected as breakup minutes. For each of the minutes, the frequency corresponding to the power peak is considered as the frequency for which the power starts decreasing, assumed as a robust indication of the drop size where breakup becomes effective in modifying the precipitation structure.

The underlying assumption for the selection is that, as the drop size and rain rate increase, both power and frequency of the peak increase, but the increase in frequency is limited by the breakup as the maximum drop size is reached. We define this stage as “equilibrium breakup”: the received power sharply decreases with frequency above the power peak in the immediately higher frequencies, thus indicating the abrupt decrease of the drop concentration with frequency, when a given value is exceeded.

3. Results and discussion

In Fig. 1 two power spectra are shown for a breakup and a nonbreakup case (Fig. 1a), with the corresponding DSD (Fig. 1b). The two minutes have similar rain rates: 18.3 and 19.0 mm h$^{-1}$ for the breakup and nonbreakup
cases, respectively. The difference between the two spectra is in the shape of the maxima and in the slope of the power after them. For the nonbreakup case the drop size is limited by the coalescence effectiveness and results in a DSD with a nearly exponential decrease for drops larger than 1.5 mm (see Fig. 1b). The DSD for the minute that we classified as the breakup case is markedly different, showing an excess of drops above 2 mm and shaped as two maxima equilibrium DSD as seen in laboratory (Low and List 1982a) and modeling (Prat and Barros 2007) studies. Furthermore, a detailed examination of the selected minutes shows that all the considered spectra were recorded during daytime convective episodes occurred in warm months.

In Figs. 2a, 2b, and 2c the 10 spectra with lower derivatives are plotted for the Ferrara, Wasserkuppe, and LinZhi sites, respectively. The maximum decrease of power spectrum occurs in rather narrow frequency intervals for the three sites (see Fig. 2), showing that these frequency limits might be regarded as a characteristic of
the experimental site and, thus, the altitude. It can also be observed that the equilibrium breakup can be reached at different power values, indicating that the equilibrium breakup can occur at different rain rates. This is clearly evident for LinZhi data (Fig. 2c) where the power intensity for the two minutes is much higher than the others (the rainfall rate is around 500 mm h⁻¹ for these minutes and around 20 mm h⁻¹ for the others), but the equilibrium breakup frequencies are very similar.

For each site the breakup Doppler frequency is computed as the average of the frequencies of the power peaks for the selected spectra, and also the standard deviation is computed to describe the natural variability of the process in the further calculations; the values are reported in Table 1.

The relationship between drop fall speed and Doppler frequency is clearly independent of any assumption on the drop shape and hydrodynamic properties, as follows:

\[ V = \frac{\lambda f}{2}, \]

where \( V \) is the drop terminal velocity (m s⁻¹), \( \lambda \) is the Pludix wavelength (\( \lambda = 3.15 \) cm), and \( f \) is the Doppler frequency shift (Hz). The drop velocity is shown in Fig. 2 as the top horizontal axis. The reduction of air density from Ferrara to Wasserkuppe and LinZhi causes a marked increase in the terminal speed of the largest drops, as reported in Table 1.

The power spectra reported in Fig. 2 show slightly different shapes for the LinZhi site, except for the two more-intense minutes (with rain rate about 500 mm h⁻¹; see Fig. 2c); here the drops are comparatively smaller and the backscattering cross sections are closer each other. Therefore, the spectra show a slow decrease for frequencies smaller than the frequency of the power peak. At lower altitudes, drops are larger and the differences among backscattering cross sections are more marked, resulting in a sharper power peak, since the backscattering cross sections in the Mie theory depend on fifth and lower power of the drop radius.

To estimate the breakup diameter, a relationship between drop diameter and terminal velocity is necessary. At sea level the Gunn and Kinzer (1949) \( V-D \) relationship can be used, but for the other two sites there is a need to take into account the reduction in air density with height and viscosity with temperature, and we used the \( V-D \) relationship suggested by Beard (1977). In Fig. 2 the vertical lines show the correspondence among frequency, drop velocity, and drop diameter. In Fig. 3 the drop terminal velocity is plotted as a function of the diameter of the equivolumetric sphere, computed for reference values for the three experimental sites of indicated air temperature and pressure, remarking that the pressure dependence on the drop shape is still unknown.

For drops of the same size the reduced air density is effective in increasing the terminal velocity, especially for larger drops. Combining these estimates with the velocities derived from the Pludix measurements, an estimate of the equilibrium breakup diameter at different heights can be obtained, as reported in Table 1.

The effect of altitude is clearly to reduce the size of the breakup diameter, which decreases as altitude increases. Because the air viscosity and the water surface tension depend slightly on temperature, which is very similar in the three sites, the main factor affecting the breakup mechanism is air density: a reduction in air density results in an increase of the drop terminal velocity and thus in an increase in the kinetic energy of the collision between larger and smaller drops.

![Fig. 3. Relationship between drop terminal velocity and drop diameter for the three sites considered in this work. The Gunn and Kinzer (1949) \( V-D \) relationship is used for the sea level pressure and the Beard (1977) \( V-D \) relationship for higher altitudes.](http://journals.ametsoc.org/doi/pdf/10.1175/JAS-D-12-0100.1?cookieSet=1)
In Table 1 the uncertainties estimated for each parameter are also reported. The standard deviation of the distribution of the power peak frequencies is considered as the uncertainty of the equilibrium breakup frequency; this error is then propagated to velocity and diameter and reported in Table 1. These uncertainties are originated by natural fluctuations of cloud microphysics and by the deviations of the environmental conditions from the values reported in Fig. 3, used for the diameter retrieval.

A key parameter in characterizing the collisional breakup process is the CKE (Low and List 1982a), defined as the energy involved in the collision between a large drop (of diameter $D_L$ and terminal velocity $V_L$) and a smaller drop (of diameter $D_S$ and terminal velocity $V_S$) in the drop's center of mass frame:

$$\frac{\pi \rho}{12} \frac{D_S^3 D_L^3}{D_S^3 + D_L^3} (V_L - V_S)^2,$$

where $\rho$ is the density of the water ($\rho = 1000 \text{ kg m}^{-3}$).

Following Low and List (1982a), the collisional breakup takes place when the total amount of energy supplied to the system during the collision is not dissipated by the system itself by viscous deformation and oscillations of the merged drop, whose diameter of the equivolumetric sphere is $(D_S^3 + D_L^3)^{1/3}$. A detailed balance of the collision energy budget is hindered by the impossibility to properly define, both experimentally and theoretically, relevant features determining the budget. As an example, the difference between surface energy of the two drops before the collision and the drop after the collision may vary by one order of magnitude, depending on the drop shape, known to be strongly irregular and rapidly changing across and immediately after the collision process. Furthermore, the drag on the drop surface depends on the drop shape and the section perpendicular to the motion direction. Finally, oscillations and internal circulation dynamics are also unknown for large, unstable drops close to their breakup size. In this uncertainty frame, we provide a strong constraint, determining a reliable value for the maximum drop velocity, from which the breakup diameter can be straightforwardly derived, relying on well-established $V-D$ relationships. These results can be exploited by estimating CKE (not dependent on the drop shape) for the three sites considered in this study. In Fig. 4 we plotted CKE values for the three sites as function of $D_S$ (mm) for the respective breakup diameters reported in Table 1: two curves are reported for each site, one for $D_L + \sigma(D_L)$ and one for $D_L - \sigma(D_L)$ to indicate the range of possible breakup drop sizes, within our uncertainties. The maximum CKE is reached when $D_S$ is between 1.5 and 2.0 mm and its maximum value is similar for the three sites (within 5%), if $D_L + \sigma(D_L)$ curves are considered, indicating that a CKE of about $1.22 \times 10^{-5}$ J is able to disrupt the drops regardless of the altitude and can be assumed as a limiting value of the kinetic energy that can be absorbed by a drop collision. Of course, the disruption may also occur for lower diameters, owing to the particular alterations of the breakup mechanisms that are likely to take place in real rain.

4. Conclusions

In this work the maximum terminal velocity of natural raindrops close to the breakup size is inferred from disdrometric data collected at different altitudes above sea level. One-minute power spectra of Pludix disdrometer Doppler frequency are analyzed to select minutes when breakup takes place: breakup Doppler frequency and terminal velocity for three sites at 15, 950, and 3300 m MSL are derived, and, finally, the maximum diameter reached by the raindrops is evaluated through an altitude-dependent diameter–terminal velocity relationship.

The increase in altitude greatly impacts on the drop terminal velocity, owing to the effect of reduced air density on the drop hydrodynamics, especially for larger drops. Assuming that the breakup is a result of the collision of a large and a smaller drop, the air density reduction increases the difference between large and smaller drop terminal velocity: the CKE gap for a given pair of drop diameters increases with altitude, reducing the breakup diameter accordingly. A limiting value (altitude independent) on the maximum CKE gap that can be absorbed by a drop after a collision is estimated to be about $1.22 \times 10^{-5}$ J.
The measure of the limiting terminal velocity in natural rain can be used as a basis for further investigations of breakup, and, more generally, of precipitation microphysics. A prompt consequence of our findings is that, in the interpretation of field data, a modification in the drop size distribution with height below the melting layer should be expected with altitude above sea level, owing to the variation of breakup size.

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