Vortex–Vortex Interactions for the Maintenance of Blocking. Part II: Numerical Experiments

AKIRA YAMAZAKI* AND HISANORI ITOH
Earth and Planetary Sciences, Kyushu University, Fukuoka, Japan

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ABSTRACT

The selective absorption mechanism (SAM), newly proposed in Part I of this study on the maintenance mechanism of blocking, is verified through numerical experiments. The experiments were based on the non-linear equivalent-barotropic potential vorticity equation, with varying conditions with respect to the shape and amplitude of blocking, and characteristics of storm tracks (displacement and strength) and background zonal flow.

The experiments indicate that the SAM effectively maintains blocking, irrespective of the above conditions. At first, by applying a channel model on a β plane, numerical experiments were conducted using a uniform background westerly with and without a jet. The results show that the presence of a jet promotes the effectiveness of the SAM. Then, two types of spherical model experiments were also performed. In idealized experiments, the SAM was as effective as the β-plane model in explaining the maintenance of blocking. Moreover, experiments performed under realistic meteorological conditions showed that the SAM maintained a real block, demonstrating that the SAM is effective.

These results, and the case study in Part I, verify that the SAM is the effective general maintenance mechanism for blocking.

1. Introduction

In Yamazaki and Itoh (2013, hereafter Part I), a new block maintenance mechanism, the selective absorption mechanism (SAM), was proposed. The SAM is based on two principles: (i) supply of low (high) potential vorticity (PV) to a blocking anticyclone (cyclone), which is characterized by low (high) PV, and (ii) a vortex–vortex interaction between a blocking anticyclone (cyclone) and synoptic anticyclones (cyclones). The vortex–vortex interaction includes the attraction of synoptic eddies with the same polarities as well as vortex mergers between blocking anticyclones (cyclones) and synoptic anticyclones (cyclones). Thus, in the SAM, a blocking anticyclone selectively absorbs synoptic anticyclones transported by the westerly jet and excludes synoptic cyclones; the low PV supplied by the synoptic anticyclones extends the longevity of the blocking anticyclone.

In the same way, when a blocking cyclone coexists, it selectively absorbs synoptic cyclones, by which the high PV is supplied and prolongs its longevity.

Synoptic eddies are attracted by blocking vortices with the same polarity, in turn, strengthening them in the SAM. In other words, synoptic eddies and blocking interact with one another. Hence, the SAM is one of the eddy-feedback mechanisms (see Part I for more details). Furthermore, when the amplitude and/or size of a blocking vortex is larger, it exerts a stronger attraction on eddies of the same polarity; that is, the PV supply increases. Thus, the SAM works effectively with the higher strength (amplitude and size) of the blocking.

Through Part I, the SAM for blocking maintenance was investigated in the context of the eddy straining mechanism (ESM) introduced by Shutts (1983, hereafter S83) and via trajectory analyses of real blocking events. The former is performed by comparing the formulation of the ESM with that of the SAM because the effectiveness of the ESM is strongly constrained in realistic situations (Arai and Mukougawa 2002, hereafter AM02;
see Part I for more details). It was found that the ESM excludes vortex–vortex interactions in its formulation, whereas they are included in the SAM. This difference highlights the importance of vortex–vortex interactions in block maintenance. In Part I, the origin of air parcels within maintained blocking anticyclones was also investigated via the forward and backward trajectory analyses on an isentropic surface. These analyses verified the selective absorption in real block maintenance situations. Thus, the results in Part I supported theoretical, qualitative, and quantitative bases of the proposed SAM.

However, further quantitative analyses are required to generalize the SAM. To do so, it is most relevant to systematically conduct numerical experiments, including variations in many parameters that may influence the SAM, thus extracting characteristics of the SAM from various perspectives. Thus, numerical experiments are conducted and the results are presented here.

This paper is organized as follows. Section 2 describes various numerical models that are necessary and sufficient for the verification of the SAM. In section 3, we conduct numerical experiments in a β-plane channel model with two idealized settings: a uniform background westerly and that with a jet. Spherical model experiments are performed in section 4, examining both idealized and realistic conditions. Section 5 presents some discussion and implications for the SAM. Finally, in section 6, a general summary of Parts I and II is presented, as well as remarks for future work.

2. Design of a series of numerical models

All numerical models used in this study are based on an equivalent-barotropic, quasigeostrophic PV equation. The basis of the equivalent-barotropic model is that (i) blocks have equivalent-barotropic structures in the troposphere (see Fig. 3 of Part I) and that (ii) synoptic eddies interacting with blocking are fully mature, becoming barotropic downstream of storm tracks (Simmons and Hoskins 1978; Nakamura and Wallace 1993).

As a consequence of the use of barotropic models, wavemakers must be placed to generate synoptic eddies upstream of blocking. Thus, the models can reproduce the interaction between blocking and synoptic eddies.

The objective of numerical experiments is to extract characteristics of the SAM and to verify the SAM. Therefore, we must consider model settings to properly achieve the objective. First, two types of models are necessary—β-plane channel models and spherical models. The former is useful to quantify the SAM as it is the simplest model for describing the dynamics of blocking (S83; Haines and Marshall 1987; AM02). On the other hand, the latter model is appropriate to describe real blocking situations. Second, because the block maintenance and its characteristics in the SAM are mainly determined by the three factors [i.e., initial blocking patterns, synoptic eddies (wavemaker settings), and basic fields] these three factors should be systematically varied in the two models. That is, all settings that can influence the block maintenance must be taken into account; nevertheless, some experiments can be omitted in cases where the results can easily be predicted from other results.

From the above considerations, the settings and purposes of the main numerical models used in this study are summarized in Table 1. Other minor settings are also used and detailed explanations for these settings will be presented where necessary. This section provides a general perspective on the numerical experiments.

Initial blocking patterns of modon (like) and rider solutions, both of which are (quasi) stationary solutions, are adopted in all numerical experiments except those involving real blocking. The former mimics dipole-type blocking, whereas the latter models Ω-type blocking. The adoption of these solutions is reasonable because blocking has a quasi-stationary nature, as was shown in the appendix of Part I. Also, as many previous numerical experiments have used the same setting (e.g., S83; Haines and Marshall 1987; AM02), the use of quasi-stationary solutions thereby facilitates comparison between our results and those of other researchers.

Although synoptic eddy (i.e., wavemaker) settings are not explicitly shown in Table 1, we adopt several settings, including standard and no-wavemaker scenarios. The role of synoptic eddies can be directly evaluated from the difference in the block maintenance between the last two settings. One of several settings is the meridional shift from standard settings of wavemakers. Because the ESM is sensitive to this shift (see Part I), experiments in this setting are important to discriminate between the SAM and the ESM.

In the basic field, it is especially important to clarify the role of a jet. Part I suggested that the jet acts as a waveguide along which synoptic eddies propagate (e.g., Schwierz et al. 2004) to locations west of the blocking location. Therefore, we can expect that the jet would increase the effectiveness of the SAM. This feature should be ascertained through numerical experiments.

It is also important to examine wave-breaking patterns in association with blocking and a jet. Some recent studies have suggested that the jet is related to low-frequency variabilities (e.g., Martius et al. 2007), including those of blocking (Woollings et al. 2008). From a dynamic perspective, because a jet affects the behaviors of synoptic eddies by changing their positions or strengths (e.g., Thornicroft et al. 1993), the effectiveness
of the eddy feedback may vary as a consequence of jet dynamics (e.g., Rivière and Orlanski 2007; Rivière 2009). Thorncroft et al. (1993) suggested that the jet, or its horizontal shear, causes wave-breaking patterns that result in the bifurcation of synoptic waves into either anticyclonic or cyclonic types. In addition, Altenhoff et al. (2008) investigated the types of wave breaking in the vicinity of blocking during the formation and maintenance periods of blocks using a composite analysis of real blocking events; they concluded that cyclonic-type breaking frequently occurs to the southwest of blocking anticyclones during the maintenance period. Thus, there may be a relationship between the SAM and the cyclonic-type breaking. This relationship is also verified in numerical experiments here, which is the third purpose of the jet setting (Table 1).

Through these experiments, it will be shown that synoptic eddies are absorbed into blocking vortices with the same polarity, and hence, blocking is maintained as long as there is interaction with synoptic eddies, irrespective of the settings. Demonstrating this characteristic is the most important for verification of the SAM. Furthermore, we will be able to confirm that phenomena predicted by the SAM actually occur in numerical experiments, and that phenomena in numerical experiments correspond to real-world events. We suggest that these series of numerical experiments necessarily and sufficiently verify the SAM.

Finally, one comment should be noted about the setting of (quasi) stationary solutions as initial conditions. This setting may be considered reasonable, as stated above; however, it generally leads to overestimation of the longevity of blocks compared to more realistic settings with (quasi) stationary patterns plus disturbances, in which blocking patterns decay much more quickly. This should be kept in mind when examining the longevity of model blocking.

3. Numerical experiments for channel models

Numerical experiments using a β-plane channel model are conducted in this section. As mentioned in the previous section, two basic fields are used: uniform westerly and nonuniform westerly with a jet. The former will be discussed in section 3a, and the latter will be in section 3b. Also, the SAM is compared with the ESM, because the ESM is formulated in the β-plane channel model.

a. Uniform background westerly

The SAM was evaluated using an equivalent-barotropic, quasigeostrophic PV equation on a β plane, which was also the approach used by S83 and AM02. To include the asymmetry between synoptic anticyclones and cyclones, we used the nonlinear equation

\[
\left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) q + J(\psi, q) + \beta_a \frac{\partial \psi}{\partial x} = F - \varepsilon \nabla^2 \psi - \nu \nabla^6 \psi,
\]

(1)

where \( U \) is the uniform basic westerly, \( \beta_a = \beta + UL_D^{-2} \), \( \beta \) is the Rossby parameter, \( L_D \) is the Rossby deformation radius of the background (zonal) field, \( F \) is the wavemaker forcing, \( \varepsilon \) is the Ekman friction coefficient, and \( \nu \) is the hyperdiffusion coefficient. Here, \( q \) and \( \psi \) represent the perturbation from a constant basic westerly, having the relationship

\[
q = \nabla^2 \psi - \gamma^2 \psi,
\]

(2)

where \( \gamma^2 = 1/L_d^2 \) and \( L_d \) denotes the Rossby deformation radius of the disturbance.

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1 They present wave breaking as a “streamer,” which has a PV-filament structure.
The parameter values, except for $F$ and $\mathcal{E}$, are the same as those used by AM02:

\[
\begin{align*}
U &= 13.8 \text{ m s}^{-1}, \\
\beta &= 1.6 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1}, \\
\beta_b &= 1.2\beta \text{ m}^{-1} \text{ s}^{-1}, \\
L_d &= 845 \text{ km}, \\
\nu &= 1.0 \times 10^{10} \text{ m}^4 \text{ s}^{-1}, \\
2\pi L_\lambda &= 42000 \text{ km}, \\
\pi L_y &= 21000 \text{ km}.
\end{align*}
\]

The value of $\beta_b$ here corresponds to $L_D = 2077$ km. The numerical integration was performed using a fourth-order Runge–Kutta scheme with a time step of 30 min. The integration period was 20 days. The initial day was denoted as day 0, then continuing day 1, . . . , to day 20. Other details can be found in Yamazaki and Itoh (2009).

The experimental design was as follows. A blocking flow was placed in the center of the channel as the initial condition; high-frequency eddies generated by a wavemaker upstream of a block drift to the blocking region. For the initial block, we used a modon solution or a rider solution that is a modified form of the modon solution (both solutions together will be sometimes referred to simply as modons). Moreover, we assigned three wavemaker settings with different meridional positions for each blocking anomaly. In all, six experimental designs were used and compared.

The wavemaker was usually placed in a region centered 7875 km upstream of the block, according to

\[
F = \begin{cases} 
\hat{F} \sin \left[ \frac{\pi (x-x_0)}{\Delta x} \right] \cos \left[ \frac{4\pi (x-x_0)}{\Delta x} - \omega t \right] \sin \left[ \frac{\pi (y-y_0)}{\Delta y} \right] & \text{for } x_0 < x < x_1 \text{ and } y_0 < y < y_1, \\
0 & \text{otherwise},
\end{cases}
\]

where $\Delta x = x_1 - x_0 = 10500$ km, $\Delta y = y_1 - y_0 = 2625$ km, $\hat{F} = 6.48 \times 10^{-10}$ s$^{-2}$, and $(x_0, y_0) = (5250, 9187.5)$ km; these parameters correspond to an eddy diameter of 2625 km with an amplitude equal to or less than the amplitude of the initial blocking and a period $\omega/2\pi = 4.5$ days. Our experiments examined how eddies generated by the wavemaker interact with the blocking. In these experiments, the blocking duration was compared between the following three wavemaker settings: 1) no wavemaker placed (No-eddy Exp), 2) wavemaker placed (No-shift Exp), and 3) wavemaker shifted 1000 km southward from that of the No-shift Exp (Shift Exp). Note that this shift is much larger than that specified by AM02 (the shift in AM02 was about 400 km).

The application of modon solutions to blocking was first used by McWilliams (1980). He further investigated the necessary conditions for the basic field under which these solutions can exist, as well other details. The reason that modon solutions are applied is that they are exact nonlinear solutions of the equivalent-barotropic, quasigeostrophic equation, and shapes of modon solutions resemble those of blocks (McWilliams 1980).

However, there is one problem in the application of modon solutions. Modons are unstable for $\mathcal{E} = 0$ (see, e.g., AM02). If blocking is amplified by the instability, we cannot judge whether it is the unstable amplification or the synoptic eddies that maintain the blocking. This problem, however, can be overcome by understanding that the unstableness of a modon acts as a basis for its decay; Haines and Marshall (1987) showed that a modon loses its energy by Rossby wave radiation into downstream regions. Also, as will be shown later in our experiments, with $\mathcal{E} = 0$ and unforced conditions, modons dissipate (by Rossby wave radiation) and then drift downstream. The same is true for rider solutions and spherical modon solutions (described in the next section), as both have almost the same characteristics as modon solutions (e.g., Swenson 1987; Neven 2001). Thus, based on these reasons, it is plausible to adopt modons as blocking flows in our models.

A modon solution and a rider solution are given for the initial values of dipole-type and $\Omega$-type blocks, respectively (Fig. 1). It may appear that the rider solution is not a pure $\Omega$-type block because of its cyclonic anomaly. This solution is nevertheless applied because when we add a constant zonal westerly to the streamfunction field, the field appears in the form of an $\Omega$-shape (Fig. 1d), and rider solutions are also exact solutions of the equivalent-barotropic, quasigeostrophic equation (Flierl et al. 1980). The derivation of modon and rider solutions is described in the appendix. We used the same parameters for the solutions as those used by AM02 and Haines and Marshall (1987); $\zeta = -U$ is the intrinsic phase speed, $a = 2430$ km is the radius of the modon, and $d_{\text{in}} = 3.9 \times 10^{-12}$ m$^{-2}$. One more parameter for the rider solution, the constant PV value $Q_r = 2.064 \times 10^{-5}$ s$^{-1}$ (called the “rider”) was given in the region $r \leq a$ ($r$ is the radius from the modon center).

We conducted a linear stability analysis to determine the value of the Ekman friction coefficient $\mathcal{E}$. The analysis checks whether a blocking flow is stable or not. Results are shown in Fig. 2. The blocking flows for both
the modon and rider solutions are unstable for $\mathcal{E} < 0.24$ day$^{-1}$. On this basis, it may appear that experiments should be conducted under the stable conditions of $\mathcal{E} \geq 0.24$ day$^{-1}$. However, in these conditions, the blocking duration and the eddy feedback are not evident because of the strength of the damping. Therefore, the value of $\mathcal{E} = 0.09$ day$^{-1}$ was chosen for our experiments; using this value, the interaction between blocking and eddies is not contaminated by damping; the instabilities affect the decay of the blocking flows but not their maintenance.

The reason that the instability of the modon and rider solutions is the basis for their decay may be as follows. Patterns of unstable modes similar to those of blocking occur only at the fourteenth and fifteenth modes for the modon solution (not shown). As these modes show the two smallest growth rates (Fig. 3a), they are probably not a dominant contribution to the blocking flow. On the other hand, the patterns of the fastest growing modes for both the modon and rider solutions are largely different from those of the initial blocking pattern, thereby causing their decay (Figs. 3b and c).

The time evolution of the dipole blocking in the No-eddy Exp, No-shift Exp, and Shift Exp are shown in Fig. 4. In the No-eddy Exp (Fig. 4a), the block cannot maintain its amplitude and it is therefore drifted downstream by the background flow (note that the blocking center on day 0 is $x = 21000$ km) because its energy is transported downstream by Rossby wave radiation, which is manifested as wave activity fluxes (not shown). On the other hand, in the No-shift Exp, eddies are absorbed into the blocking; that is, anticyclonic (cyclonic) eddies are attracted and absorbed by the

![Fig. 1. Initial streamfunction of (a) dipole-type and (b) $\Omega$-type blocks. The contour interval is $9.225 \times 10^6$ m$^2$ s$^{-1}$; dashed lines show negative values. (c),(d) As in (a) and (b), respectively, but for the streamfunction fields with a background zonal wind $U$. The zero contour is omitted in (a) and (b). The interior of the outermost thick lines in (a) and (b) is the area $S$ for the calculation of the TBPV defined in (5).](http://journals.ametsoc.org/jas/article-pdf/70/3/743/3806633/jas-d-12-0132_1.pdf)

![Fig. 2. Fastest growth rates (day$^{-1}$) of the modon (circles) and rider (triangles) solutions as functions of the Ekman friction coefficient $\mathcal{E}$ (day$^{-1}$).](http://journals.ametsoc.org/jas/article-pdf/70/3/743/3806633/jas-d-12-0132_1.pdf)
blocking anticyclone (cyclone). Although this absorption can be easily recognized in the streamfunction field (e.g., during day 10–day 12.5), it is more evident in the PV field (Figs. 4b and c) because PV is a Lagrangian conserved quantity; the passive tracer (circle) with negative PV anomaly placed just upstream of the block on day 10 is absorbed into the blocking anticyclone with conserving the PV anomaly. As the same feature is also observed on other days and in the Shift Exp, the selective absorption of eddies by blocking vortices with the same polarities is therefore confirmed. In both the No-shift Exp and the Shift Exp, the block maintains large amplitudes and the downstream drift is minimal, even on day 15. Furthermore, compared with the No-shift Exp, the meridional position of the block in the Shift Exp is remarkably locked on day 15. The above results are consistent with the SAM, which predicts that block maintenance can be accomplished irrespective of meridional shifts of a wavemaker (section 3 of Part I).

To objectively quantify the blocking duration, we defined the total blocking PV (TBPV) as the retention rate of the total amount of PV at the initial blocking position. The definition of the TBPV $q_{\text{block}}$ is

$$q_{\text{block}} = \frac{1}{S} \int (q_S - q_N) dS,$$

where $S$ is the initial area of the blocking interior, $q_S$ represents positive values of $q$ in the region of the initial streamfunction $\psi_{\text{init}} < -9.225 \times 10^6$ m$^2$ s$^{-1}$, and $q_N$ represents negative values of $q$ in the initial streamfunction $\psi_{\text{init}} > 9.225 \times 10^6$ m$^2$ s$^{-1}$. The sum of these two regions corresponds to $S$—that is, the inner regions of the outermost solid and dashed contours represented by the thick lines in Fig. 1. The TBPV decreases as the amplitude of blocking decreases and/or blocking is shifted from its initial position. Thus, the retention of this value is a reasonable measure of the maintenance.

In addition, to consider the phase dependence of synoptic eddies, integrations with different initial phases of the wavemaker were carried out, and the means of these ensemble members were taken (Arai 2002). This improves the statistical confidence of our results. Thus, phase-independent time evolutions of the TBPV were obtained. Here, the forced term in (4) is replaced by

$$F = \tilde{F} \sin \left[ \frac{\pi(x-x_0)}{\Delta x} \right] \cos \left[ \frac{4\pi(x-x_0)}{\Delta x} - \omega t - \phi \right] \cos \left[ \frac{\pi(y-y_0)}{\Delta y} \right],$$

where $\phi$ is a phase factor. By changing the phase factor $\phi$ in an increment of $\pi/5$ over the range $0 \leq \phi < 2\pi$, 10 integrations were performed.

Time changes of the TBPV for the ensemble members and means in the No-shift Exp and Shift Exp are illustrated in Fig. 5. The TBPV in the No-eddy Exp continuously decreases, as expected from Fig. 4; blocking in this model cannot be maintained without synoptic eddies. On the other hand, the ensemble means in both the No-shift Exp and Shift Exp show that the TBPV maintains almost the same value from day 5, when the
first eddies arrive at the block, to about day 10. Even after that, the TBPV of some members in the Shift Exp has almost the same values as those of the ensemble mean in the No-shift Exp. These results indicate that the SAM can effectively work even when there is a large shift of the wavemaker. As to phase dependency, because the spread of the ensemble members in each experiment is small, the SAM does not depend on the phase of the wavemaker. The above results are the objective evidence of blocking maintenance by the SAM.

The behavior of upstream eddies is noteworthy. Anticyclones (cyclones) shift southward (northward) from the mean storm track (Figs. 4b and c). This behavior occurs on account of the \( \beta \)-gyre effect, by which the anticyclones (cyclones) meridionally drift toward the equator (polar) side, because “a huge cyclonic vortex” of the earth repels (attracts) smaller anticyclonic (cyclonic) vortices (see section 2 of Part I). This shift has a tendency to reduce the TBPV value. Nevertheless, the effect of selective absorption overcomes the \( \beta \)-gyre effect, thus maintaining the large TBPV value. We can also confirm the robustness of the blocking maintenance without depending on the meridional shift of the wavemaker by comparing an ensemble-mean snapshot of the streamfunction field on day 15 in the No-shift Exp with that in the Shift Exp (Fig. 6). Even on this day, blocks (especially blocking cyclones) in the No-shift Exp and the Shift Exp were maintained at nearly the same amplitudes and initial phases (at 21 000 km).

The evolution of \( \Omega \)-type blocks was analyzed next (Fig. 7). As in the maintenance of dipole blocks, selective absorption is also evident in \( \Omega \)-type block maintenance; however, the amplitude of the \( \Omega \)-type block is stronger than that of the dipole-type block. This can be explained as follows: since the blocking anticyclone of the rider solution is stronger than that of the modon, the former causes stronger vortex–vortex interaction than
the latter (see the introduction or section 2 of Part I). Thus, the SAM is more effective for $\Omega$-type than for dipole-type blocking.

Figure 8 shows the time changes of the TBPV for an $\Omega$-type block. The decay in terms of the TBPV is somewhat more rapid than that of the dipole-type block (note that the initial amplitude of the TBPV in a $\Omega$-type block is different from that in the dipole type), but the blocking is maintained by eddy absorption, which counteracts dissipation and Rossby wave radiation. In addition, the spread of the TBPV in the No-shift Exp and the Shift Exp is small, and the ensemble-mean TBPV values in both experiments are nearly the same. These indicate that eddy absorption is insensitive to initial phases and meridional shifts of synoptic eddies, which probably reflects the characteristic that the large amplitude of the $\Omega$-type blocking anticyclone can lead to large supply of low PV from synoptic anticyclones because of the strong vortex–vortex interaction irrespective of the relative position between synoptic and blocking anticyclones. Because the large PV supply delays the decrease of the amplitude of blocking against dissipation, blocking can maintain its large amplitude and hence the quasi stationarity against the downstream advection by the background westerly.

To confirm our results, we performed the same experiments using $\mathcal{E} = 0.25$ day$^{-1}$, which satisfies the conditions for linear stability (see Fig. 2). We obtained

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**Fig. 5.** Time changes (days) of TBPV ($s^{-1}$) in (a) the No-shift Exp and (b) the Shift Exp for the dipole block. The gray lines show the No-eddy Exp results, the thick black lines the ensemble means, and the thin black lines the ensemble members for each experiment. In (a), there only appear to be five thin lines; however, this is because two of the lines are exactly the same.

**Fig. 6.** Snapshots of the ensemble-mean streamfunction ($m^2 s^{-1}$) fields in (a) the No-shift Exp and (b) the Shift Exp. The contour interval is $4.612 \times 10^6 m^2 s^{-1}$; dashed lines show negative values. The snapshots displayed are from day 15.
similar results, but with a very fast damping of the blocks (not shown). We can say that block maintenance by the SAM works for any input value of $E$.

Finally, we confirmed the PV-supply rate that is predicted by the SAM. In the SAM, the total amount of supplied PV is important; it is expected that, because the amplitude and size of the wavemaker affect the total amount of PV supplied to the block, an increase of either will improve the maintenance of blocking. We performed sensitivity analyses for the amplitudes and sizes of the wavemaker resulting in dipole-type blocking in the No-shift Exp.

We first performed experiments to change the amplitudes of the wavemaker; the standard value of $F = 6.48 \times 10^{-10} \text{s}^{-2}$ (F1) was replaced with values equal to $1/2$ (F1/2), $1/4$ (F1/4), or $1/8$ (F1/8) the amplitude of the original $F$. Figure 9a shows that stronger amplitudes correlate with larger TBPV. Next, experiments were performed using different sizes of the wavemaker; in addition to the standard experiment with the eddy diameter of 2625 km (R2625), we also used eddy diameters of 2100 km (R2100) or 1750 km (R1750). Figure 9b shows that the magnitude of the TBPV is generally correlated with the size of the eddy. These results are consistent with the PV-supply mechanism of the SAM. Concerning the spread of the TBPV, the spreads in the two experiments are still small as compared with absolute values of the TBPV. Hence, we confirmed once again that phase dependence does not contaminate the SAM.

Since it was shown in the above experiments that the eddy feedback in the SAM is viable and robust, the hypothesis that blocking flows themselves determine their own longevities, as suggested by S83 and Pierrehumbert and Malguzzi (1984), is consistent with the SAM. Moreover, in the SAM, the configuration of blocking flows can be chosen flexibly, because the only condition required for maintenance is that blocking is a large-amplitude stationary anticyclone that can interact with synoptic eddies. In fact, even when similar experiments were performed using modon and rider solutions with different parameters and a Gaussian vortex (i.e., an anticyclonic PV anomaly given by the Gaussian distribution), consistent results were obtained. In addition, experiments

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**Fig. 7.** As in Fig. 4, but for $\Omega$-type blocking.
using a channel twice as wide as that used above were performed, and nearly identical results were achieved.

b. Background westerly with jet

Experiments in which a westerly jet was added to the model used in the previous subsection are described here. The addition of a westerly jet makes the \( \beta \)-plane model more realistic, since blocks most frequently occur downstream of strong jet regions (e.g., Barriopedro et al. 2006). By comparing results of these experiments with those in the previous subsection, the effect of the jet was evaluated. Because the jet acts as a waveguide along which the synoptic eddies propagate to blocking regions, it is expected that a jet makes the SAM more effective.

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**FIG. 8.** As in Fig. 5, but for \( \Omega \)-type blocking.

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**FIG. 9.** Time changes of the ensemble-mean TBPV for a dipole-type block in the No-shift Exp with (a) different amplitudes and (b) sizes of the wavemaker. The gray line is the TBPV in the No-eddy Exp and the black lines represent, from thin black to thick black, those corresponding to (a) amplitudes that are \( 1/8, 1/4, 1/2, \) and equal to the original value, and (b) waveform sizes of 1750, 2100, and 2625 km. (Note that the line for F1/8 is almost over the gray line.)
The wavemaker is placed at the latitude of the center of the jet \((y = y_c)\), which is the same as its placement in the No-shift Exp above. The amplitude, period, and size of the wavemaker are the same as in the earlier analysis. Note that amplitudes of eddies are smaller than those in the experiments assuming a uniform westerly, as the wind speed of the jet core is twice as large as that of the uniform westerly; thus, the lapse time for an air parcel to pass over the wavemaker is shorter. We conducted experiments without and with the wavemaker (No-eddy Exp and No-shift Exp, respectively). In addition, to ensure that the jet acted as a waveguide, the wavemaker was shifted 1000 km (West-shift Exp) upstream relative to its location in the No-shift Exp. Thus, three conditions for the two types of blocks were compared. Other details for the numerical integration are identical to those in the previous analysis.

Figure 11 shows time sequences of PV in the No-eddy Exp, No-shift Exp, and West-shift Exp for the dipole-type block. The time evolution of the No-eddy Exp indicates that the blocking decays and drifts downstream more rapidly than in the experiment with uniform background flow (Fig. 11a), perhaps because the initial blocks in the experiments with the jet are no longer exactly stationary but quasi stationary. In the time sequence of the No-shift Exp (Fig. 11b), on the other hand, the blocking maintains its amplitude and position even on day 15, by selectively absorbing eddies carried from upstream along the jet (e.g., anomalies A and B). We found that eddies from upstream approach the block with fewer meridional shifts than was observed in the experiments without the jet. Thus, the SAM is more effective in the presence of a jet.

We here consider the shape of the eddies prior to absorption. In one case (anomaly B), a cyclonic eddy became oriented (tilted) in a northwest–southeast direction. This corresponds to the cyclonic wave breaking of a trough upstream of the block, which is consistent with the wave-breaking pattern on the southwest side of a blocking anticyclone, as observed by Altenhoff et al. (2008). Therefore, such a wave-breaking pattern can occur when synoptic cyclones are selectively absorbed by a blocking cyclone while drifting downstream. On the
other hand, an anticyclonic eddy (anomaly A) became oriented (tilted) in a southwest–northeast direction; in this situation, the eddy was absorbed with anticyclonic wave breaking. Although there are many examples with orientations that vary from the above examples, Altenhoff et al. (2008) present statistical results that indicate the two types of tilting can coexist in real situations.

The reason for tilting can be explained as follows. Although eddies generally drift downstream trapped in the jet, anticyclonic (cyclonic) eddies shift to the southern (northern) side of the jet because of the β-gyre effect; the former are transported into an anticyclonic shear field while the latter are transported into a cyclonic shear field. Thus, the tilting occurs before the eddies are absorbed by the blocking. Such a viewpoint can be inferred from the patterns in Fig. 11.

The positions and amplitudes of the block and the patterns of selective absorption of eddies in the West-shift Exp are nearly identical to those in the No-shift Exp. Because eddies are carried by the jet (waveguide) to a position west of the block, the selective absorption is effective in the vicinity of the block (anomalies C and D); the block maintains the blocking phase and amplitude, despite the upstream shift of the wavemaker.

The results show that the existence of the jet is favorable for block maintenance. Synoptic eddies traveling along the jet can reach the block with only small meridional shifts, because the jet acts as a waveguide. On the other hand, large meridional shifts of synoptic eddies by the β-gyre effect are unfavorable for the interaction of the eddies with the block, and are characterized by an anticyclone (cyclone) on the north (south) side. Thus, the suppression of meridional shifts by the jet is favorable for operation of the SAM.

To quantitatively evaluate block maintenance in the jet experiment, the TBPV for cases in which the jet was present were examined (Fig. 12a). The time changes show that the TBPV in the No-shift Exp and the West-shift Exp were larger and more sustained than in cases...
where the jet was absent (Fig. 5). This result is consistent with the fact that the jet acts as a waveguide. On the other hand, the TBPV in the No-eddy Exp with a jet present decayed more rapidly than in cases where the jet was absent (Fig. 5). Therefore, the existence of the jet augments the effectiveness of the SAM. The TBPV in the West-shift Exp was somewhat smaller than that in the No-shift Exp because the first arrival of eddies was delayed on account of the 1000-km shift of the wavemaker. After the first arrival of eddies, TBPV values of both experiments varied almost in parallel.

Selective absorption was also found in the case of the \( V \)-type block in the presence of a jet (not shown); thus, the TBPV is larger and more maintained in situations involving a jet than in situations in which the jet is absent (Figs. 12b and 8).

4. Numerical experiments for spherical models

As indicated in Part I, blocking occurs in high and middle latitudes. Moreover, blocking anticyclones dominate over blocking cyclones. These indicate that the sphericity is an important factor for blocking, and the use of a spherical model is more appropriate for real blocking situations. Two experiments using a spherical model are conducted; one uses an idealized block with the wavemaker (section 4a), and the other uses a real blocking flow and synoptic eddies obtained from reanalysis data (section 4b). The former can be compared with the results of the \( \beta \)-plane channel model, and the latter leads the more realistic quantification of the SAM than those in the previous settings.

a. Idealized blocking

Modon solutions on a \( \beta \) plane were extended to those on a sphere—that is, spherical modon solutions. On a sphere, dipole vortices of the modon are asymmetric because of the constraint of angular velocity (Neven 1994). We here show results of numerical experiments with the same design as those of the channel model, but for cases in which the initial block is a spherical modon. A spherical modon solution was adopted from Verkley (1984) and Neven (2001). This solution, its derivation, and its parameters are found in the appendix. As for the parameters, the outer wavenumber of the modon is \( n_{\text{out}} = 1 \), the inner wavenumber of the modon is \( n_{\text{in}} = 10 \), and the modon radius is \( \theta_s = 21.15^\circ \). These values were adopted because they gave a similar amplitude and size of the initial block as was used in the \( \beta \)-plane modon solution. The center latitude and longitude of the modon was placed at 45\( ^\circ \)N and 180\( ^\circ \), respectively. Using these parameters, the spherical modon solution is shown in Figs. 13 and 14a. Note that this figure displays the stream-function; the north-side anticyclone would be further highlighted for geopotential height because the geopotential approximately corresponds to the stream-function multiplied by the Coriolis parameter.

For spherical model experiments, we used a spectral model with triangular truncation at wavenumber 42 (T42).
Fig. 13. As in Fig. 1a and b, but for the spherical modon in the spherical model. The contour interval is $9.225 \times 10^3 \text{ m}^2 \text{s}^{-1}$; the dashed lines show negative values. The zero contour is omitted.

$$
\left(\frac{\partial}{\partial t} + \frac{U_s}{R} \frac{\partial}{\partial \lambda}\right) q + J(\psi, q) + \beta \frac{1}{R} \frac{\partial \psi}{\partial \lambda} = F - \varepsilon \nabla^2 \psi - \nu_H \left( \nabla^2 - \frac{2^4}{R^8} \right) \nabla^2 \psi, \quad (9)
$$

where $U_s$ is the background zonal wind speed at the equator; $\lambda$ and $\phi$ are the longitude and latitude, respectively; $R = 6371 \text{ km}$ is the radius of the sphere; $\beta_0 = (2/R^2 + 1/L^2_0)U_s + 2\Omega$; $\Omega = 7.29 \times 10^{-5} \text{ s}^{-1}$ is the angular velocity of the sphere; and $\nu_H$ is the hyperdiffusion coefficient; this is the same equivalent-barotropic, quasigeostrophic PV equation that was used in the channel model equation. We set the parameter values $L_d = 822 \text{ km}$ and $\nu_H = 2.22 \times 10^3 \text{ m}^2 \text{s}^{-1}$, and the other parameters were given the same values as those for the channel model, except for $U_s = 19.52 \text{ m} \text{s}^{-1}$, which gives $U = 13.8 \text{ m} \text{s}^{-1}$ at a latitude of $45^\circ \text{N}$, which coincides with the value of $U$ in the channel model.

The wavemaker was placed in the same way as it was placed in the channel model, and was given the same amplitude and size as in the channel model: $\Delta x = x_1 - x_0 = 90^\circ$, where $x_1 = 135^\circ \text{E}$ and $x_0 = 45^\circ \text{E}$, and $\Delta y = y_1 - y_0 = 20^\circ$, where $y_1 = 55^\circ \text{N}$ and $y_0 = 35^\circ \text{N}$. Only the meridional shift was different: a southward $10^\circ$ shift (Shift Exp) and, in addition, a northward $10^\circ$ shift (N-shift Exp) caused by the north–south asymmetry of the spherical model.

We examined the time evolution of PV and the streamfunction in the No-eddy, No-shift, Shift, and N-shift Exps. The time evolution in the No-eddy Exp shows that by day 10, the blocking anticyclone had already lost its vortexlike structure; then, on day 15 this anticyclone became rather zonally uniform (Fig. 14b). This was caused by strong Rossby wave radiation from the blocking, associated with the spherical geometry; this was confirmed by the wave activity flux (not shown).

In the No-shift Exp, on the other hand, the initial blocking pattern remained until day 15 (Fig. 14c). Thus, the difference between the blocking duration in the No-eddy Exp and the No-shift Exp was more distinct than that on the $\beta$ plane. The time evolution in the Shift Exp and N-shift Exp (Figs. 14d and e) show that the initial blocking position and amplitude are still present on days 10 and 15, though the blocking cyclone considerably drifts downstream in the N-shift Exp. The absorption of anticyclonic eddies into the blocking anticyclone occurs in every experiment with wavemaker forcing. Our conclusion from the above results is that, because the difference between the amplitude of blocking with and without the wavemaker for the spherical model is larger than that for the $\beta$-plane model, the SAM on a spherical surface operates more effectively than it does on the $\beta$ plane, and that the spherical surface contributes to a more realistic blocking structure.

In the spherical model, we chose parameter values of the modon solution and the value of the Rossby deformation radius in an arbitrary manner, to some extent. Then, we performed the same experiments with values that were not greatly different from the initially chosen parameters. Results from these experiments were essentially the same. The phase dependence of the wavemaker was also checked; however, the results were largely independent of the phase of the wavemaker.

b. Realistic blocking

In this subsection, we examined a realistic blocking event as an initial condition on the maintenance of the block. The initial condition is taken from the streamfunction field at 250 hPa in the Japanese 25-yr Reanalysis (JRA-25) data (Onogi et al. 2007) when a block is maintained.

Numerical experiments were performed for the A-1996 event that occurred in the Atlantic region (see Table 1 of Part I). This blocking occurred on 6 March 1996 and lasted until 27 March 1996; it was chosen because of its prolonged duration, as a long-lasting block is desirable for numerical modeling. Another blocking event was simultaneously occurring in the Pacific region at this time; therefore, an initial condition was chosen at 0000 UTC 8 March, when the Pacific blocking was relatively weak. Figures 15a,b, and c show the streamfunction at 250 hPa at the initial time, 7.5 days later (1200 UTC 15 March), and 10 days later (0000 UTC 18 March), respectively. Although the blocking lasted for another 10 days, it had been advected quite far downstream (Fig. 15c).

Because there is no basic state, the basic equation is

$$
\frac{\partial \psi}{\partial t} + J(\psi, q) = -\varepsilon \nabla^2 \psi - \nu_H \left( \nabla^2 - \frac{2^4}{R^8} \right) \nabla^2 \psi. \quad (10)
$$
FIG. 14. As in Fig. 4, but for the spherical model. (a) Initial streamfunction (on day 0) and time sequences of (b) the No-eddy Exp and (c) the No-shift Exp on days 5, 10, and 15, and of (d) the Shift Exp and (e) the N-shift Exp on days 10 and 15. For (c), (d), and (e), the PV field (color shades; 10^-5 s^-1) is also displayed. The longitudes (lon) displayed on (left) also apply to (right).
Parameter values were the same as those in the idealized experiment. Two experiments were performed, each with a different initial condition. In one, the initial condition was the pattern shown in Fig. 15a; (10) was time integrated from this condition. This was named the realistic experiment (Real Exp). The other initial condition was that the streamfunction at 90°E–30°W was replaced by its zonal-mean value; it however was linearly interpolated to approach the original values at both ends (90°E and 30°W) in the ranges of 90°–110°E and 30°–50°W. For this initial condition, upstream synoptic eddies were completely suppressed; therefore, this experiment was called the no-eddy experiment (No-eddy Exp). The initial pattern for the No-eddy Exp is illustrated in Fig. 15d.

Snapshots of the Real Exp and the No-eddy Exp are shown in Fig. 16. In the No-eddy Exp, the blocking had already disappeared by day 7.5 (Fig. 16a). The blocking experienced a rapid change over time, as the initial condition was not a quasi-stationary solution. On the other hand, in the Real Exp, the blocking was still maintained on days 7.5 and 10 (Figs. 16b and c), corresponding to Figs. 15b and c. It is clear that synoptic eddies contributed to the block maintenance. Furthermore, the PV originating from synoptic anticyclones was absorbed by the blocking anticyclone (not shown). We infer from this experiment that the SAM is effective in real blocking. However, after day 10, the blocking in the Real Exp rapidly decayed and disappeared, which is a departure from the real situation; however, the
dissipation in the numerical result is to be expected because eddies in the barotropic model are not self-excitely generated upstream of the blocking in the Real Exp. In fact, features upstream of the blocking were considerably different between Figs. 15 and 16.

5. Apparent straining and straining as one process of absorption

a. Apparent straining

The reason that the eddy straining mechanism (ESM) has received broad support is that, not only is the logic of S83 elegant, but the eddy straining has also been identified in actual data (e.g., Hoskins et al. 1983; Nakamura and Wallace 1993; Higgins and Schubert 1994). However, it will be shown below that the observed eddy straining is apparent, and caused by the use of time filtering to see the interaction between blocking and synoptic eddies.

Figure 17 shows the time sequence of the PV field in the No-shift Exp with a jet and a dipole-type block on the $\beta$ plane. Two fields are shown, the unfiltered PV (unfiltered component) and the high-pass-filtered PV (filtered component), with a cutoff period of 8 days. Figure 17 shows the following two aspects: (i) Just upstream of the block, unfiltered eddies with negative (positive) polarities drift (are attracted) toward the north (south) side, whereas filtered eddies appear to be strained almost symmetrically in the north (south) direction (stars in Fig. 17). On the other hand, although unfiltered eddies are also distorted, they are never stretched symmetrically. (ii) Inside the blocking anticyclone (cyclone), filtered cyclonic (anticyclonic) eddies can be seen, but unfiltered ones cannot.

Concerning aspects (i) and (ii), the differences in the behaviors of filtered and unfiltered eddies can be interpreted using Fig. 18. The essential point is that filtered eddies represent the “deviation” of the unfiltered

---

2 High-pass-filtered streamfunction with a cutoff period of 6 or 10 days also shows the same manner as that of 8 days.
component from the local time-mean (low-pass filtered) component, and thus generally have different values or sometimes different signs from those of unfiltered eddies. We define a true eddy as a filtered eddy with the same polarity as an unfiltered eddy and in the same position, and an apparent eddy as an eddy with the opposite polarity as an unfiltered eddy.

As for aspect (i), Fig. 18a is a schematic snapshot showing the interaction between synoptic eddies and a dipole-type block. In this figure, the unfiltered eddies show selective absorption, while the filtered ones show eddy straining. The eddy straining is caused by the true cyclonic (anticyclonic) eddies upstream of the blocking cyclone (anticyclone) connected to the apparent cyclonic (anticyclonic) eddies upstream of the blocking anticyclone (cyclone). These apparent eddies can be explained as follows. The unfiltered streamfunction along the gray line in Fig. 18a has positive values on unfiltered anticyclonic eddies but a zero value at other points (Fig. 18b). When time filtering is applied to this streamfunction field, the unfiltered distribution decreases to the dotted line in Fig. 18b. Thus, the filtered eddy field shows an anticyclone, cyclone, and anticyclone in a row, from west to east, as in the upper part of Fig. 18b. The filtered cyclonic eddy at the center of this row is the apparent cyclonic eddy upstream of the blocking. The apparent anticyclonic eddy upstream of the blocking cyclone can be explained in the same way when the gray line is drawn on the south side of the block.

As for aspect (ii), the apparent cyclonic (anticyclonic) eddies passing inside the blocking anticyclone (cyclone) (the filtered anticyclonic eddy south of the blocking cyclone in Fig. 18a) can be explained in the same manner. Thus, when we see the ensemble of filtered eddies—that is, a storm track—it appears to be spreading and splitting in a north–south direction around the blocking.

In addition, amplitudes of the high-pass-filtered eddies just upstream of and inside the blocking become smaller than those of unfiltered ones. The decreasing amplitudes are just the amplitude of low-pass-filtered component (same as the time mean in this case). In other words, the low-frequency component is overestimated. This low-frequency component is not related to upward cascade of energy from the high-frequency component because synoptic eddies in this situation are just passing through the same gray line in Fig. 18a.

It has been shown that time filtering with cutoff periods of about 6–10 days detects apparent eddies immediately upstream of and inside blocking, with behaviors that are different from those of PV parcels and real synoptic eddies. In some studies employing these high-pass filters, the eddy straining or the splitting of a storm track can therefore be seen in the synoptic field (e.g., Hoskins et al. 1983, their Fig. 16; Higgins and Schubert 1994, their Fig. 4b). When we apply a high-pass filter with a cutoff period of 8 days to PV fields of real blocking flows, then the eddy straining can be seen (not shown). It should be emphasized that superficial views of high-pass-filtered parameter fields, such as a streamfunction or geopotential field, would lead to the concept of eddy straining as a physical entity.
To many observational studies using high-pass filters to detect synoptic eddies, the SAM suggests that we must not discuss “shapes” of eddies in the analysis of the interaction between blocking and synoptic eddies. On the other hand, the drawbacks of using these filters are relatively minor when investigating amplitudes or activities of detected eddies around the blocking (e.g., Nakamura and Wallace 1993). Nonetheless, these amplitudes still do not represent real ones; true amplitudes are generally underestimated by high-pass filters (Fig. 18b).

Apparent eddies (shapes and amplitudes) lead to the estimation of apparent eddy forcing. For example, when estimating the influence of eddy forcing on blocking dynamics, the use of apparent eddies may lead to an incorrect quantitative estimation of eddy forcing. In addition, because the amplitude of the high (low) frequency component is underestimated (overestimated) in the vicinity of blocking, evaluations of the contributions of these frequencies to the dynamics of blocking or synoptic eddies may be biased.

The differences in the behaviors of unfiltered and filtered eddies around blocks, as discussed above, is one of the reasons why quantitative discussions on the eddy feedback are difficult. Because of these differences, for example, the detection of synoptic eddies in the trajectory analysis in Part I was conducted well upstream of the blocks, where apparent eddies do not appear. Furthermore, for the same reason, quantitative analyses to extract vortex–vortex interactions using high-pass filters are difficult, and thus special care is required for quantitative assessments, such as budget analyses. A detailed method for the assessments will be reported in the next paper.

b. Straining as one process of absorption

The eddy-feedback mechanism requires the straining of eddies for the inverse cascade of energy from eddies to blocking (S83). This straining process is, of course, included in the SAM, which is one process of absorption. This is because when a synoptic eddy is being absorbed by a blocking vortex, the eddy is strongly strained by the rotation of the blocking. Such straining of a vortex while being absorbed by another vortex with the same polarity is a well-known process (e.g., McWilliams 1984). Here, this process is named “eddy breaking,” or the breaking of an eddy shape by a blocking vortex, and is distinguished from eddy straining (as defined by S83).

Eddy breaking can be observed in the case studies and numerical experiments presented in this study (Fig. 19). We first review a case study from section 4 of Part I. Event A-2001, a typical eddy breaking event, is analyzed as a representative example of the eddy breaking (note that the other events also show eddy breaking). Figures 19a,b, and c show snapshots of the anticyclonic parcels (black circles) in event A-2001; the snapshot in Fig. 19a (Fig. 19b) precedes that in Fig. 19b (Fig. 19c) by 1.5 days. In Figs. 19a and b, the anticyclonic parcels have not come into contact with the blocking anticyclone (the low-PV region) west of Europe, but they gather near one another; that is, the synoptic anticyclones that they represent are not deformed. Figure 19c shows that the anticyclonic parcels are already touching the block, and that they are strained by it; this is how eddy breaking occurs. Eddy breaking in the numerical experiment is shown in Figs. 19d and e. These panels show snapshots of contours with negative PV values (displaying blocking and synoptic anticyclones) in a channel model experiment without the background flow, in which a single anticyclonic eddy is placed upstream of a dipole-type block. Figure 19d is a snapshot before the contact, while Fig. 19e is a snapshot 2 days later, when the eddy is just colliding with the blocking anticyclone. As in the case study, the colliding eddy is broken by the blocking anticyclone. It should be noted that the anticyclonic eddy is never strained to the direction of a blocking cyclone south of the blocking anticyclone, as opposed to the ESM. Furthermore, the time scales of these eddy breaking events are short as about 1.5 and 2 days in the case study and the numerical experiment, respectively, which coincides with the results of McWilliams (1984). Thus, we can conclude that the straining of eddies is an eddy breaking process in the SAM.

6. Conclusions and remarks

a. Conclusions

In Parts I and II of this study, the SAM is verified on the basis of both observations and numerical experiments. This paper (Part II) indicates the quantitative importance of the SAM in a series of barotropic model experiments.

We checked blocking duration using an equivalent-barotropic, quasigeostrophic PV equation model. Two channel models on a β plane without and with a westerly jet, and on a sphere in idealized and realistic conditions, were used. In these models, the effectiveness of the SAM was evaluated by measuring the retention rate of the total amount of PV at the initial blocking position and so on.

First, through numerical experiments using the β-plane model without a jet (section 3a), the absorption of synoptic anticyclones (cyclones) into a blocking anticyclone (cyclone) occurred irrespective of the meridional shift of storm tracks and of sizes and amplitudes of synoptic
eddies, which is completely different from the ESM. Second, results of the experiments with a jet (section 3b), compared with those without the jet, indicate that the jet enhances the SAM, because the jet acts as a waveguide to efficiently carry synoptic eddies near blocking.

Finally, experiments using a spherical model were conducted for an idealized block with a wavemaker (idealized experiment) and a real blocking flow obtained from the reanalysis data (realistic experiment) in section 4. Comparing the idealized experiments using the spherical model and the channel model, we found that the SAM works more effectively for the spherical model than the channel model. In the realistic experiment, the SAM maintained blocking at realistic time scales. We thus conclude that the SAM is a robust mechanism in a wide variety of situations.

b. Remarks

We suggest that the SAM will operate not only in the case of synoptic eddies but also in the case of "Rossby eddies" with planetary spatial scales and low-frequency time scales. When Rossby eddies are stationary, anticyclonic and cyclonic "eddies" are in turn generated eastward from a source region; if an anticyclonic Rossby eddy is generated near a blocking anticyclone, they can interact with each other. Because Rossby eddies generally have a large size, the SAM could be very effective. Nakamura et al. (1997) showed that the dynamics of Atlantic and Pacific blocking differs in terms of the relative contributions of high-frequency (synoptic eddies) and low-frequency (Rossby waves) components. Hence, the effectiveness of the SAM in the low-frequency situations should be investigated.
Vortex–vortex interactions may also be important for blocking formation. This possibility is based on previous studies indicating that a climatological stationary ridge generated by orography or “thermography” of an oceanic diabatic heat source can trigger the formation of blocking. For example, Luo (2005) demonstrated in a numerical model that blocking can be formed on a stationary ridge generated by forcing mimicking the large-scale topography of the Northern Hemisphere by interactions involving synoptic eddies drifting from a wavemaker upstream of the ridge. For this formation, the mechanism of vortex–vortex interaction could be adopted: a stationary ridge can strengthen, by mechanisms such as local resonance near the ridge and interaction with Rossby wave anticyclones from upstream, to form a PV minimum region at blocking scales. This region may then become an “anchor” for gathering synoptic anticyclones and for promoting vortex–vortex interactions of anticyclones.

The SAM may be applied more generally to other large-scale phenomena in middle- and high-latitude regions, using “PV thinking” as a powerful tool in analyses of persistent anomalies at spatial and temporal scales larger than those of blocking, such as stationary lows (e.g., the Aleutian and Icelandic lows) and low-frequency variabilities (e.g., the North Atlantic Oscillation and the Pacific–North American pattern). These stationary lows and low-frequency variability patterns show large-scale PV anomalies in geographically fixed positions and many sources of synoptic and/or Rossby eddies around them. In addition, it has been found that some large-scale phenomena correspond to Rossby wave–breaking patterns in the upper troposphere (e.g., Woollings et al. 2008). Thus, it is possible that in such wave-breaking patterns (i.e., by PV mixing that repeatedly occurs in a persistent anomaly) a PV extremum region might be formed at the scale of the anomaly. Thus, the interactions of these large-scale phenomena with PV extrema, and the absorption or repulsion mechanisms involving synoptic eddies in vortex–vortex interactions, may be important for the dynamics of persistent anomalies. The applications of the SAM to other phenomena should be investigated in future studies.

In this study, the importance of the SAM for blocking maintenance has been confirmed. However, quantitative estimations of this mechanism on real blocking maintenance, especially in the baroclinic atmosphere, are not discussed or considered here. For example, Haines (1989) shows that the duration of blocking modeled by a modon solution becomes shorter in a baroclinic model than it does in a barotropic one. Also, some previous case studies show the importance of the baroclinicity upstream of blocks, especially for the formation of blocks, as discussed in section 6 of Part I. Quantitative studies of the SAM in the baroclinic atmosphere should be addressed in future studies.

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APPENDIX

Derivations of the Modon, Rider, and Spherical Modon Solutions

The modon and rider solutions on a β plane and sphere are derived in nearly the same way. The anomaly equations in (1) and (9) from the background zonal flows under the unforced nondissipative condition are, respectively,

\[
\frac{\partial q}{\partial t} + J(\mathbf{v} \cdot \mathbf{\nabla} q) = 0 \quad \text{(β plane)} \quad \text{(A1)}
\]

\[
\frac{\partial q}{\partial t} + J(\mathbf{v} \cdot \mathbf{\nabla} q) = 0 \quad \text{(sphere)} \quad \text{(A2)}
\]

where

\[
q = (\nabla^2 - 1/L_D^2) \psi, \quad \beta_\phi = \beta + UL_D^2,
\]

and other parameters are defined in the main text. Note that the condition \(L_D > L_A\) (where \(L_D\) is the Rossby deformation radius of the background field) is required for modons to remain in westerties. The modon here indicates the localized modon, not the wavelike modon (Neven 2001).

For a disturbance moving at a constant (intrinsic) phase speed \(c\) or angular velocity \(\omega\),
\[
\frac{\partial q}{\partial t} = -(\dot{\epsilon} + U) \frac{\partial q}{\partial x} \quad (\beta \text{ plane}) \quad \text{and} \quad (A3)
\]
\[
\frac{\partial q}{\partial t} = -\left( \frac{\dot{\omega}}{R} \right) \frac{\partial q}{\partial \lambda} \quad (\text{sphere}). \quad (A4)
\]

Substitution of (A3) and (A4) into (A1) and (A2), respectively, gives
\[
J(\dot{\epsilon} y + \psi, \beta_0 y + q) = 0 \quad (\beta \text{ plane}) \quad \text{and} \quad (A5)
\]
\[
J(-\dot{\omega} R^2 \sin \phi + \psi, \Omega_0 \sin \phi + q) = 0 \quad (\text{sphere}). \quad (A6)
\]

These equations have the form \(J(\Psi, Q) = 0\), where \(\Psi\) and \(Q\) are the left and right terms in the Jacobian, respectively. Their solutions are \(Q = d\Psi\), where \(d\) is a constant. Furthermore, rider solutions can be obtained by adding the rider \(Q_r\) (constant) to the right term in the Jacobian of (A5); that is, \(J(\Psi, Q_r) = 0\).

Given that the disturbance has a closed streamline, and that the streamline is located at a “distance” \(a\) or \(\theta_a\) from the disturbance center on the \(\beta\) plane or sphere, respectively, the solutions can be written as
\[
Q = \begin{cases} 
  d_{out} \Psi & (r > a \quad (\beta \text{ plane}) \quad \text{or} \quad \theta > \theta_a \quad (\text{sphere})], \\
  d_{in} \Psi & (r \leq a \quad (\beta \text{ plane}) \quad \text{or} \quad \theta \leq \theta_a \quad (\text{sphere})], 
\end{cases} \quad (A7)
\]

where \(r\) and \(\theta\) are the distance (m) and angular distance (°) from their centers on the \(\beta\) plane and sphere, respectively.

We first solve the modon and rider solutions on a \(\beta\) plane. Given proper boundary conditions at \(r \to \infty\) and \(r = a\) leads to the modon solution
\[
\psi = \begin{cases} 
  -K_1(s_{out}^2)X_1 y - y_0 \frac{y - y_0}{r} 
  & (r > a), \\
  J_1(s_{in}^2)Y_1 \frac{y - y_0}{r} + J_0(s_{out}^2)Y_0 + Q_r \frac{y - y_0}{s_{in}^2} 
  & (r \leq a), 
\end{cases} \quad (A12)
\]

where \(K_1\) and \(J_1\) are the first-order Bessel function of the second kind and the modified first-order Bessel function of the first kind, respectively. However, because the background flow determines \(s_{out}\) (considering the boundary condition at \(r \to \infty\)), the given value of either \(a\) or \(s_{in}\) determines the modon solution. To solve the rider solution, some terms related to the rider (monopole) component are added to (A8), giving the rider solution
\[
X_r = \frac{Q_r}{s_{in}^2} \frac{s_{in} J_1(s_{in}^2)}{s_{in}^2 K_0(s_{out}^2) J_1(s_{in}^2) - s_{out}^2 K_1(s_{out}^2) J_0(s_{in}^2)} \quad (A13)
\]

and
\[
Y_r = \frac{Q_r}{s_{in}^2} \frac{s_{out}^2 K_1(s_{out}^2)}{s_{in}^2 K_0(s_{out}^2) J_1(s_{in}^2) - s_{out}^2 K_1(s_{out}^2) J_0(s_{in}^2)}, \quad (A14)
\]

where \(K_0\) and \(J_0\) are the zeroth-order Bessel function of the second kind and the modified second-order Bessel function of the first kind, respectively. Equations (A9), (A10), and (A11) are common to the rider and modon solutions.

We next solve the spherical modon solution. The coordinate moving with the modon (disturbance) is represented as \((\lambda', \phi')\) and the modon center is located at \(\phi' = \pi/2\), giving \(\theta = \pi/2 - \phi'\) and the modon radius \(\theta_0 = \pi/2 - \phi_0\), where \(\phi' = \phi_0\). The cosine and sine rules for a spherical triangle give the relationship between \((\lambda, \phi)\) and \((\lambda', \phi')\):
\[
sin \phi' = \sin \phi \sin \phi_0 - \cos \phi \cos \phi_0 \cos(\lambda - \lambda_0), \quad (A15)
\]
\[
\sin \phi = \sin \phi' \sin \phi_0 - \cos \phi' \cos \phi_0 \cos \lambda', \quad (A16)
\]

\[
\cos \phi' \sin \lambda' = \cos \phi \sin (\lambda - \lambda_0), \quad (A17)
\]

where \( \lambda_0 \) and \( \phi_0 \) are the longitude and latitude of the modon center in the nonmoving coordinate, respectively. As in the case on a \( \beta \) plane, for given boundary conditions, the solution is

\[
\psi = A P^1_\kappa (\sigma \sin \phi') \cos \lambda' + B P^0_\kappa (\sigma \sin \phi') - C \sin \phi + D, \quad (A18)
\]

Here, \( P_m^\kappa \) is the \( m \)th order Legendre function of the \( \kappa \)th degree and \( A, B, C, D, \kappa, \) and \( \sigma \) are coefficients that take different values outside and inside of the modon radius, as represented by the subscripts “out” and “in,” respectively. By introducing

\[
\sigma = \begin{cases} 
-1 & (\theta > \theta_a) \\
1 & (\theta \leq \theta_a)
\end{cases} \quad (A19)
\]

and

\[
\kappa = \begin{cases} 
\frac{1}{2} - i(n_{\text{out}}) & (\theta > \theta_a) \\
n_{\text{in}} & (\theta \leq \theta_a)
\end{cases} \quad (A20)
\]

where \( i \) is the imaginary unit, the solution can be simply expressed. The coefficients \( n_{\text{out}} \) and \( n_{\text{in}} \) correspond to the outer and inner wavenumbers of the modon, respectively. We now give \( p = \kappa (\kappa + 1) \) [noting the relationship \( p = (d + \gamma^2) R^2 \)] and the other coefficients are determined as

\[
A = R^2 \frac{[ (2 + R^2 \gamma^2) \omega + \Omega_s ] \cos \phi_0 \cos \phi_0 \cos \lambda'}{2 + p} P^1_\kappa (\sigma \sin \phi_a), \quad (A21)
\]

\[
B = R^2 \frac{[ (2 + R^2 \gamma^2) \omega + \Omega_s ] \cos \phi_0 \sin \phi_0 \sigma (2 + p) P^0_\kappa (\sigma \sin \phi_a)}{2 + p}, \quad (A22)
\]

\[
C = R^2 \omega \frac{[ (2 + R^2 \gamma^2) \omega + \Omega_s ]}{2 + p}, \quad \text{and} \quad (A23)
\]

\[
D = R^2 \frac{[ (2 + R^2 \gamma^2) \omega + \Omega_s ]}{2 + p} \times \left[ \sin \phi_a \cos \phi_0 + \sigma \cos \phi_0 \sin \phi_0 \frac{P^0_\kappa (\sigma \sin \phi_a)}{P^1_\kappa (\sigma \sin \phi_a)} \right] + \mathcal{R}. \quad (A24)
\]

Here, \( \mathcal{R} \) is a standard value of the streamfunction field and is determined so that \( D_{\text{out}} = 0; \)