New Hailstone Physics. Part II: Interaction of the Variables

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ABSTRACT

The reduction of parameter dimensions in Part I is complemented by the compaction of parameter space in Part II. The range of diameters is $0.5 \leq D \leq 8$ cm, and the assumed liquid water content varies within $1 \leq W_f \leq 6$ gm$^{-3}$ for dry growth and $W_f \leq 6$ gm$^{-3}$ for shedding. Entirely new data throw new light onto HMT and growth.

Results are as follows: (i) dry growth is unimportant, since most hailstones grow spongy; (ii) radial growth is slow for dry and fast for spongy growth because less latent heat of freezing needs to be discarded if a smaller portion of the accreted water is frozen: this growth with shedding is particularly effective if the product $Y$ of the net collection efficiency and ice mass fraction of the deposit is $0.2 \leq Y \leq 0.6$; (iii) the lowest possible surface temperature $t_S$ for dry growth is $-32.3^\circ C$. For water-skin-covered, spongy particles $t_S > -5^\circ C$, and $t_S > -0.55^\circ C$ for shedding from wet surfaces without water skins; and (iv) the interplay between water-skin thickness and surface temperature allows interconnection of all variables. However, new icing experiments are necessary to prove the proposed sphere growth by special gyration, to quantify the components of $Y$, and to address water-skin properties and growth.

Radically redesigned dynamic cloud models need to incorporate hail packaging and rain spectra evolution in clouds. The latter will connect hailstone shedding with a warm rain process that is parallel to and interacts with hail formation.

1. Introduction

List (2014, hereinafter Part I) served to develop equations governing all aspects of heat and mass transfer (HMT) and growth of spherical hailstones. Part II presents the results of the calculations involving all variables, as classified in the three CASES [(i) dry; (ii) shedding from wet surfaces, not covered by permanent water skins; and (iii) surfaces with water skins] (Part I). They are sequenced in different variable combinations and represent the main body of the new results. In addition, the parameter space is minimized by restricting the range of variables to the one of importance for growth.

The reduction from six “old” to four newly defined variables for HMT allows a much more compact representation of the results—without any loss of accuracy and without any additional assumptions. The six old variables are air temperature $t_A$ ($^\circ C$) and hailstone surface temperature $t_S$ ($^\circ C$), cloud liquid water content $W_f$ (kg m$^{-3}$), net collection efficiency $E_{NC}$ ($-$), ice fraction of the spongy deposit $I_f$ ($-$), and Reynolds number $Re$. The four new variables are $X$ (g m$^{-3}$), $Y$ ($-$), $\Phi$ (kg m$^{-3}$), and $\Psi$ ($-$), with $X = W_f D^{3/4}/Re^{3/4}$ replaced by a proportional factor $X' = W_f D^{3/4}$, where $D$ (m) is the diameter of the spherical hailstone and $X'$ is preferable over $X$ because $Re$ for free-fall needs to be split into its many components to solve the main equation. The quantity $Y$ is the product of $E_{NC}$ and $I_f$, and $\Phi$ and $\Psi$ are functions only of $t_A$ and $t_S$.

Previous experiments with gyrating, water-skin-covered spheroids limited $t_S$ to $\approx -5^\circ C$ (Lesins and List 1986; García-García and List 1992; Greenan and List 1995; Zheng and List 1995). Shedding from wet surfaces that are not covered by water skins narrows the resulting temperature band, $0^\circ \leq t_S \leq -0.55^\circ C$. These temperature range reductions alone are equivalent to a reduction of parameter space by 87.5% and 98.5%, respectively. The growth speed of ice dendrites into water skins is a key to the growth of spongy ice. The hailstone’s radial growth

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speed \( V_R \) (m s\(^{-1}\)) is linked to the temperature gradient across the skin \([t_s - t_d/d]\), with \( d \) (m) the water-skin thickness. With the temperature of the sponge–water skin interface, at \( t_s \approx 0 ^\circ C \), the gradient can be reduced to \( t_d/d \). Supercooled water skins limit the HMT (CASE 3).

For CASE 1 the governing equations are simplified by setting the product of \( E_{NC} \) and the fractional ice content \( I_f \) of the deposit equal to unity (\( Y = 1 \)). In CASE 2 with heavy shedding, the group of four new variables is also reduced to three by setting \( t_s \approx 0 ^\circ C \). The variable reduction in HMT for CASE 3 is achieved by equating the speed of the ice sponge front, penetrating the water skin, to the growth speed of the skin–air interface (see Fig. 5 in Part I). This compensates for the addition of \( d \).

One of the major inputs into growth calculations is the free-fall velocity \( V \) (m s\(^{-1}\)) of the rough, spherical hailstones, calculated with measured drag coefficients \( C_D \) (\(-\)) (List et al. 1969). The spheres had various degrees of roughness, characterized by a ratio \( k_s \), between roughness element height and a hailstone diameter of 0.07 [typical for collected hailstones (List 1958b)]. This value is chosen for the present calculations. Note that the flow regime even around the smallest spherical hailstones \( (D = 0.5 \text{ cm}) \) is supercritical due to surface roughness. Thus, \( C_D \) is assumed to be constant at 0.5 for \( D \geq 2 \text{ cm} \), up to \( \text{Re} \approx 400,000 \), the upper limit of the drag experiments (List et al. 1969). At \( t_a = -20 ^\circ C \) the Re of hailstones with \( D = 8 \text{ cm} \) is \( \approx 100,000 \). However, for \( D = 0.5 \text{ cm} \) \( C_D \) is unity (List and Schemenauer 1971).

The most common small hail particles found as embryos in hailstones are water soaked but originally low-density conical graupel (List 1958a,b, 1961b, Part I). Small hail has a density between that of water and ice, with the mix determining the ice mass fraction \( I_f \) of the particle. Other embryos, such as frozen droplets, are also possible, but rare. There is no substantiation that embryos start from frozen droplets. The density of deposits directly grown on hailstones is in the same range \((915 < \rho_H < 1000 \text{ kg m}^{-3})\), with an average of 958 kg m\(^{-3}\), where \( \rho_H \) is the hailstone density). The density of (frozen) hailstones is very close to ice density (Vittori and Di Caporiacco 1959; Macklin et al. 1960; List 1961b; List et al. 1970). Low-density shells and cores have been observed after the draining of water from grown spongy ice, thus causing densities as low as \( 0.1 < \rho_H < 0.3 \text{ g cm}^{-3} \) (List 1978–79). Draining can occur either during contact with the ground or during free-fall by the Bernoulli effect. Such overlapping of sequential growth phases had first been explained by List (1961a). Thus, references in the literature of large low-density hailstones (Knight et al. 2008; Rasmussen and Heymsfield 1987a,b, c) need to be viewed with caution. They could have been caused by superposition of growth phases or plain size inflation by naming a \( D = 2 \text{ cm} \) hailstone as large.

All calculations in this paper are carried out for clouds represented by the Beckwith (1960) soundings of Denver, Colorado, hailstorms, with supercooling to the limit of \(-40 ^\circ C \), which is neither being postulated nor is it generally observed. Depending on the properties of the available ice nuclei, any appropriate freezing level can be chosen to further restrict the growth region. For figures involving HMT and growth, that flexibility will be maintained in this paper. Every attempt was made to achieve optimal accuracy. Hence, drag coefficients were tailored to hailstone size (see above).

The calculations are restricted to one set of conditions describing hail clouds. They are limited to spherical hailstones. The equations represent only the template for further studies. There was no intent to solve all the problems.

The approximations for the different material constants and their dependence on temperature are listed in the appendix. No attempt has been made to make the equation dimensionally correct.

**Nomenclature**

In the text the quantities for \( D \), \( W_f \), the “net accreted ice fraction” \( Y \), or any other quantity will often be specified by a suffix “\( i \)”, indicating the magnitude in centimeters, grams per cubic meter, or (\(-\)), respectively. A list of all symbols can be found in Part I.

The dimensions underlying all equations are in SI units, with the exception of the ones in the appendix. The dimensions used in the figures may be different from the SI units.

The sign “\( > \)” for temperature means “warmer than.” For temperature level “\( > \)” means “higher” or “colder.”

**2. The equations**

There are six equations to be evaluated for the understanding of the growth and heat and mass transfer of spherical hailstones.

The growth equation, linking radial growth to net accreted liquid water content \([22] \text{ of Part I}\), is

\[
V_R = \frac{1}{2} \frac{dD}{dt} = 0.25 \rho_H^{-1} E E_{NC} W_f V, \tag{1}
\]

where \( V_R \) and \( V \) are the radial growth and free-fall velocities (m s\(^{-1}\)), respectively, and \( EE_{NC} \) (\(-\)) is the product of collision and net collection efficiencies. The term \( E \) is set to unity in all other equations (however, it was carried in Figs. 1 and 10 as a warning for extrapolations for \( D < 0.5 \text{ cm} \)).

The HMT is given by its most inclusive form \([18] \text{ of Part I}\):
\[
K \times \left[ 0.535k(t_s - t_A) + 0.511D_{wa}L_F \frac{1}{RT_A}(e_S - e_A) \right]
\]
\[
0.25\nu L_F E_{NC}I_f \frac{e_S}{L_f}(t_s - t_A)
\]
\[
= W_f \text{Re}^{1/2} = X = C_{Re} W_f D^{3/4} = C_{Re} X'.
\] (2)

The lhs of (2) was derived from the ratio of the sum of the heat fluxes by conduction and convection, and evaporation over the sum of the accretion-related terms. The variable product \(E_{NC}I_f\) is the new variable \(Y\). In this equation \(\text{Re}^{1/2}\) for free-fall is substituted with
\[
\text{Re}^{1/2} = \frac{4g\rho_H}{3P_A C_{D} \nu^{3/4}} D^{3/4} = C_{Re} D^{3/4},
\] (3)
with \(C_{Re}\) linking \(\text{Re}^{1/2}\) and \(D^{3/4}\) by a fourth-root factor. At this time, information is required about the spherical hailstone’s density \(\rho_H\) and its \(C_D\). Note that the fourth-root factor will be of importance in error considerations. Term \(C_{Re}\) depends on \(t_A\) and air pressure \(P_A\). Using Denver hailstorm soundings by Beckwith (1960) removes \(P_A\) as a factor. Assuming further that the hailstones are spherical, both \(\rho_H\) and \(C_D\) need to be known to solve (3).

The meanings of the newly added variables in (2) and (3) are: \(K\) is the experimentally determined non-dimensional correction factor for HMT; \(k\) is the thermal conductivity of air (W m\(^{-1}\) K\(^{-1}\)); \(D_{wa}\) is the diffusivity of water vapor in air (m\(^2\) s\(^{-1}\)); \(L_F\) is the latent heat of evaporation from solids or liquids (J kg\(^{-1}\)); \(T_A\) is the absolute air temperature (K); \(e_S\) is the saturation vapor pressure at the surface (Pa); \(e_A\) is the water vapor pressure in air (Pa); \(\nu\) is the kinematic viscosity of air (m\(^2\) s\(^{-1}\)); \(c_w\) is the specific heat of water (J kg\(^{-1}\) K\(^{-1}\)); \(L_F\) is latent heat of fusion (J kg\(^{-1}\)); and \(g\) is gravity acceleration (m s\(^{-2}\)).

Here (2), which can be solved for \(X'\) or \(Y = E_{NC}I_f\), is abbreviated by using two relationships for the auxiliary variables \(\Phi\) and \(\Psi\), quantities that are only dependent on air and surface temperatures:
\[
K \times \left[ 0.535k(t_s - t_A) + 0.511D_{wa}L_F \frac{1}{RT_A}(e_S - e_A) \right]
\]
\[
0.25L_F \nu
\]
\[
= \Phi
\] (4)

and
\[
\frac{c_w}{L_F}(t_s - t_A) = \Psi.
\] (5)

Term \(\Phi\) is a measure of the heat contributed by convection and conduction to freeze the net accreted water, while \(\Psi\) is the fraction of heat contributing to freezing by the supercooled accreted water.

With the definitions of \(\Phi\) and \(\Psi\), (2) can be abbreviated
\[
\Phi = X(Y - \Psi).
\] (6)

Since both \(\Phi\) and \(\Psi\) are functions of \(t_A\) and \(t_S\) only, the four variables can also be given by the set \((X, Y, t_A, t_S)\). There is no algebraic solution for (4); thus, iteration procedures are required. The next equation links \(V_R\) (m s\(^{-1}\)) with the driving temperature gradient across the water skin of thickness \(d\):
\[
\frac{\Delta t}{d} = \frac{\rho_H L_F \chi^{-1} I_f}{k_w} V_R = C_3 \chi^{-1} I_f V_R,
\] (7)

with \(C_3 = 6.034 \times 10^8\) K s m\(^{-2}\) (\(\rho_H\) is set for the skin with 1000 kg m\(^{-3}\), the latent heat of fusion \(L_F = 3.337 \times 10^8\) J kg\(^{-1}\) for \(t_A = 0\) K, and \(k_w = 0.553\) W m\(^{-1}\) K\(^{-1}\)). The other new symbol, \(\chi\) (\(\cdot\)), represents the multiplier caused by turbulence in the water skin.

Illustrations of background information are presented in Figs. 1 and 2. Figure 1 shows \(V_R\) as a function of the accreted liquid water content according to (1), with the terminal velocity substituted by (15) of Part I. The cutoff is for radial growth speed, its maximum, is set at 4 mm min\(^{-1}\). The auxiliary functions \(\Phi\) [(4)] and \(\Psi\) [(5)] for dry growth and growth with shedding are displayed in Fig. 2. The function \(\Psi\) is so close to linear that it was drawn as linear. Minor deviations are caused by the different temperature dependencies of the \(c_w\) and \(L_F\).

3. Range restrictions of variables

In Part I, the HMT of a single hailstone has been reduced to the motion of a point in four-parameter space. The purpose of range restriction, stressed in this part, is to minimize the parameter volume, that is, to concentrate on the most probable growth conditions. In the following all the limiting conditions will be included starting with Fig. 3, even if some are substantiated only later. The reason for this step is simple: there will be no need for additional, upgraded diagrams. The reader is also faced with the real (reduced) range of conditions right from the beginning.

The following bounds and ranges are introduced: 0.5 ≤ \(D\) ≤ 8.0 cm, 0° ≤ \(t_S\) ≤ \(t_A\) ≤ −40°C. Further for CASE 1, 1 ≤ \(W_f\) ≤ 3 g m\(^{-3}\); \(\rho_H = 915\) kg m\(^{-3}\); the upper limit of \(W_f\) for ice growth is set lower than the adiabatic value because of the depletion of cloud water by the hailstones that grow while competing for the available cloud droplets.
For CASES 2 and 3, \(1 \leq W_f \leq 6 \text{ g m}^{-3}\), and \(915 \leq \rho_f \leq 1000 \text{ kg m}^{-3}\) or average \((958 \text{ kg m}^{-3})\); the \(W_f\) limit is doubled because of shedding—and shed drops can be recollected by growing hailstones. For CASE 3, \(t_S \geq -5^\circ\text{C}\), based on experiments with gyrating spheroids (Lesins and List 1986; García-Garcia and List 1992; Greenan and List 1995; Zheng and List 1995) and supported later in section 5 (Fig. 10). Doubling could involve hail accumulation zones as envisaged by Kessler (1969) and as seen in the packaged hail, identified by frequency analysis of return signals from a vertically pointing Doppler radar. Luckily, the antenna was hit by hail [Thomson 1997; Thomson and List 1999; and, reprocessed in color by Thompson (List 2004)].

The upper limit for both \(\varepsilon\) and \(I_{t}\) is unity, while the lower limit has to accommodate “mushy” hailstones with \(I_{t}\) as low as 0.1–0.3 (such hailstones maintain their physical integrity at free-fall speeds, as had been shown in icing experiments in wind tunnels). The limits \(L_i\) of \(X^\prime\), the product of \(W_f\) and \(D^{3/4}\), are \(L_1 = 0.59\) for \(W_1\) and \(D_{0.5}\), \(L_3 = 1.78\) for \(W_3\) and \(D_{0.5}\), \(L_6 = 3.56\) for \(W_6\) and \(D_{0.5}\), and \(L_{\text{max}} = 28.5\) for \(W_6\) and \(D_6\). Dimensions of \(L\) are \(\text{g m}^{-3}\) cm\(^{3/4}\).

There is a combined requirement, namely, that both lower limits of \(W_f\) (1 g m\(^{-3}\)) and \(D_0(0.5\text{ cm})\) are adhered to at all times. In the figures with \(X^\prime(=W_fD^{3/4})\) as an abscissa, a gray band appears that contains those restrictions. Last but not least, a limit to \(V_R\) of 4 mm min\(^{-1}\) is suggested that is roughly in agreement with radar observations.

These restrictions may be changed by any user of the equation set.

4. The \(X^\prime, Y, t_A, t_S\) domains

a. The \(t_A-X^\prime\) domain

1) CASE 1 (Fig. 3)

The main properties of and differences between the three CASES are shown in Fig. 3. For CASE 1 the governing (2) is reduced by setting the product \(Y(=\varepsilon\rho_fI_{t})\) equal to unity. The \(t_S = 0^\circ\text{C}\) line represents the limit of dry growth at ice density. It is not the Schumann–Ludlam limit (SLL) because the SLL requires a wet surface due to the shedding of nonfreezing accreted water. Lower ice densities will require even lower surface temperatures. Higher air temperatures are less conducive for low-density ice growth. The lowest surface temperature for dry growth is \(-32.3^\circ\text{C}\). It is observed for \(D_{0.5}\) hailstones with \(W_1\). Dry growth (riming) of large hailstones at \(-40^\circ\text{C}\) is possible up to \(X^\prime = 7.26\). This is the low limit for cloud water supercooled to \(t_A = -40^\circ\text{C}\). Less supercooling moves \(t_S\) to a higher temperature. With \(D_8^{3/4} = 4.76\) the corresponding \(W_1\) is \(1.67 \text{ g m}^{-3}\).

The \(t_A-X^\prime\) domain is the key diagram for dry growth. It is further partitioned by giving the \(X^\prime\) ranges of \(D_i\) hailstones. The horizontal \(D_i\) bars cover the range that starts at the left with \(W_i\) and extends to \(W_6\). The bars are aligned to touch the \(t_A = 0^\circ\text{C}\) line. The \(W_i\) ends define the corresponding \(X_i^\prime\). This limits the dry conditions for the \(D_i\) bar chosen as long as \(t_S \leq 0^\circ\text{C}\) (as octagons). The possible growth regions are between \(X_1^\prime\) and \(X_6^\prime\). A \(D_{0.5}\) hailstone can grow dry for \(W_1\) at \(t_A = -2.45^\circ\text{C}\) and for \(W_3\) at \(t_A = -8.7^\circ\text{C}\); a \(D_2\) hailstone can grow dry for \(W_1\) at \(t_A = -8.2^\circ\text{C}\) and for \(W_2\) at \(t_A = -28.2^\circ\text{C}\); a \(D_4\) hailstone can grow dry for \(W_1\) at \(t_A = -14.9^\circ\text{C}\) and \(W_3\) at \(-41.4^\circ\text{C}\) (for graphic purposes only). For \(D_4\) the biggest \(W_f\) for dry growth (at \(t_A = -40^\circ\text{C}\)) is 2.6 g m\(^{-3}\). A \(D_8\) hailstone reaches the limit of dry growth with \(W_1\) at \(-26.5^\circ\text{C}\); at \(-40^\circ\text{C}\), the upper limit moved to \(W_f = 1.67 \text{ g m}^{-3}\). At \(-40^\circ\text{C}\) and \(X^\prime = 7.26 \text{ g m}^{-3}\) cm\(^{3/4}\), all hailstones reach the uppermost limit for dry growth. However, \(X^\prime\) is reduced for \(t_A > -40^\circ\text{C}\), as Fig. 3 (CASE 1) clearly shows. Hence, the domain in which hailstones...
grow dry is within the range designated by octagons and the gray area. There is another point: The low limit for a $D_4$ hailstone gives the upper limit for all $D_i$ hailstones with $D_i < D_4$ at that $t_A$ level; that is, all hailstones with sizes $D < 4$ cm grow dry. At every level a different maximum size is allowed and each size is coupled to a different $W_f$.

By definition $X'$ provides the link between the two pairs $\{t_A; t_S\}$ and $\{W_f; D\}$. The pairs are equal if they have the same $X'$. Each pair produces a multitude of combinations for one $X_i'$, for every $X_i'$. But choosing $W_f$ or $D$ starts to unravel a situation if $t_A$ or $t_S$ is chosen.

In CASE 1 the upper limit of dry growth at $-40^\circ$C is at $X' = 7.26$ g m$^{-3}$ cm$^{3/4}$, but it does not indicate the beginning of spongy growth. This is dealt with by CASES 2 and 3, where the hailstones are wet and involve the relevant latent heat of evaporation from the liquid phase and not from ice. Thus, the limit for wet growth at $-40^\circ$C is at $X' = 6.88$ g m$^{-3}$ cm$^{3/4}$.

One general observation about the occurrence of large hailstones: They must be products of the most effective icing conditions. Thus, it is unlikely that they grow at low $W_f$ and close to $-40^\circ$C because that would require extended residence time in hail growth regions. Maintaining continuous updrafts of up to $\sim 70$ m s$^{-1}$ for sufficient time is not likely—which would kill the notion of low-density large hailstones.

The reason why large hailstones grow dry at high levels (low $t_A$) is because all components of the HMT increase substantially with increasing size. According to (1), radial growth speed is proportional to both $V$ and $W_f$. Higher HMT translates into faster freezing of accreted water (and less sponge) or higher $I_f$ or both. Falling to the 0°C level will favor spongy growth above the $t_S = 0^\circ$C height level (Fig. 3, CASE 1)—unless the fall occurs outside the updraft in a cloud part with low $W_f$. Further fall from the 0°C level to the ground will lead to melting, with the meltwater being drawn into

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**FIG. 2.** Air temperature dependence of $\Phi$ and $\Psi$, based on (4) and (5), respectively. CASE 1 for dry surfaces; CASE 2 for (heavy) shedding from wet surfaces at $t_S \sim 0^\circ$C (surfaces not covered by permanent water skins); CASE 3 for surfaces covered by supercooled water skins, with surface temperatures between 0° and $-5^\circ$C. No restricted growth regions given.
any remaining loose ice deposit—another reason for questioning references to finding low-density hailstones at the ground.

Proof by radar is quite difficult because of the ambiguity of the reflectivity of spongy, spherical hailstones (Joss and List 1963; Joss 1964). Considering that $X_{\text{max}} = 28.5 \text{ g m}^{-3} \text{ cm}^{3/4}$ (at $W_6$ and $D_6$), and that the limit to dry growth is at $X_{\text{max}} = 7.26 \text{ g m}^{-3} \text{ cm}^{3/4}$, dry growth covers only a small region of the whole growth domain. This region becomes even smaller if the nucleation temperature is moved from $24.8 \degree C$ lower height levels with higher $t_A$.

This situation points to the importance of numerically exploring the size sorting mechanism and the related evolution of size spectra in pulsating updrafts.

2) CASE 2 (FIG. 3)

Figure 3, CASE 2, shows the slightly supercooled bands ($0^\circ \leq t_S \leq 0.55^\circ$), together with the sponginess/ice fraction of the deposits—as given by $Y$. The lower temperature ($-0.55^\circ$) represents the limit of the radial growth speed, set at $4 \text{ mm min}^{-1}$. This is in agreement with the postulated fastest radial growth speed of dendrites ([19] of Part I). The hailstone surfaces are neither dry nor covered by a continuous water skin. They result from shedding of small partial surface skins, blown away and disintegrating into drop spectra immediately after shedding. All those narrow bands in Fig. 3, CASE 2, are indicative of the magnitude of $Y$, the product of $E_{\text{NC}}$ and $I_f$. As expected, smaller $Y$ values are found at higher $t_A$, and dry growth is moved into the top-left corner of the diagram with smaller $X'$ and low $t_A$.

As an example of a more detailed analysis, CASE 2 is further explored by breaking up $X'$ into its components, as is displayed in the $t_A$–$W_f$ domain for different $D_i$ and $Y_i$ (Fig. 4). At given $W_f$ and $Y$, the air temperature is markedly lower for larger hailstones (this figure could be extended to CASE 3 by expanding the bands from...
on $Y$. Air temperatures for spongy growth are generally at much warmer $t_A$. Temperatures for low “accreted spongy fraction” ($Y < 0.2$) are mostly at $t_A > -10^\circ C$.

### b. The $t_S-X'$ domain (Fig. 5)

1) **CASE 1**

The lower limit $X_1'$ sets $t_S$ at $-32.3^\circ C$ (Fig. 5, top left, CASE 1). For easier interpretation $X'$ has been decomposed into $W_f$ and $D$ (bottom left). The limits for $X_1'$ are indicated in all panels of Fig. 5 and later, as long as $X'$ is a coordinate. The associated panel (bottom left) shows that $D_8$ hailstones start to grow dry for the interval $4.77 < X' < 7.26$ g m$^{-3}$ cm$^{-3/4}$. The lower limit is for $t_A = -29.2^\circ C$; the upper is $-40^\circ C$.

2) **CASES 2 and 3**

The solutions for CASE 2 are restricted to the thin $t_S$ band, limited by $0^\circ C \geq t_S \geq -0.55^\circ C$, at the bottom of Fig. 5a, while CASE 3 is for conditions within a band $0^\circ C \geq t_S \geq -5^\circ C$ that relates $t_S$ to $X'$ for different values of $t_A$ and $Y$. The general condition is $t_S > t_A$. Figure 5 also shows that very low $t_A$ can be tolerated easier, $t_S \geq -5^\circ C$, if $Y$ is closer to unity. At high air temperatures and low $X'$, $t_S$ is decreasing fast, while the decline is slowing with decreasing $t_A$.

### c. The $Y-X'$ display, CASES 2 and 3 (Fig. 6)

Since $Y$ is equal to unity for CASE 1, the study of the $Y-X'$ domain is restricted to CASES 2 and 3. CASE 2 limits the surface temperature to the gray bands with $0^\circ C \geq t_S \geq -0.55^\circ C$ for every $t_A$, whereas CASE 3 produces wider bands with $0^\circ C \geq t_S \geq -5^\circ C$ (octagons + gray). The results are represented by Fig. 6. The region $X' < X'_1$ is cut off, so is the hatched area at high values of $Y$ and $X'$ because $t_S$ is limited by $-40^\circ C$.

The hyperbolic character of the type $X'Y$ is imposed by (2). Lowering $t_A$ decreases the values of $X'$ and/or $Y$. The magnitude of $Y$ can be reduced by shedding through decreasing $E_{SC}$ and/or a lower fractional ice content of the deposit. Actually, Lesins and List (1986), García-García and List (1992), Greenan and List (1995), and Zheng and List (1995) have shown that increasing ice fraction and shedding are strongly correlated for gytating spheroids. Their diagrams for $E_{SC}$ and $I_f$ versus $W_f$ could be used to form products $Y$ for spheroids. Such data would be suggestive for spherical hailstones.

d) **The $t_S-t_A$ domain (Fig. 7)**

The quasi-linear relationship (5) between $t_A$ and $t_S$, for constant $W_f$ and $D$ (or $X'$) is displayed in Fig. 7. There are three cutoffs: (i) at $t_S = -32.3^\circ C$ (CASE 1 at $D_{th}$ and $W_1$) because $W_f < W_1$ is not allowed; (ii) at $t_S \geq -0.55^\circ C$ (CASE 2); and (iii) at $t_S \geq -5^\circ C$ (CASE 3). Increasing
size and/or liquid water content (or \(X\)) increases \(t_S\). Because of the lower limit of \(W_f\), a \(D_{0.5}\) hailstone requires a \(t_A\) colder than \(-2.45^\circ C\) (CASE 1) or \(-2.68^\circ C\) (CASE 3) for growth with \(t_S = 0^\circ C\). For \(W_f = 0^\circ C\), \(t_{A\max} = -8.67^\circ C\). CASE 1 shows how \(t_A\) and \(t_S\) relate to different \(D_i\). Low \(t_A\) and \(t_S\) are favoring high \(D_i\).

The \(D_{0.5}\) hailstones can only grow dry at cold temperatures, coupled with high \(t_S\), and most importantly, only at \(W_f\) above but close to \(W_1\).

For CASES 2 and 3, only three combinations of diameter \(D_i\) and liquid water content \(W_i\) are given for three \(Y\) each. Depending on \(Y\), \(D_{0.5}\) hailstones can grow over a \(t_A\) range covering \(35^\circ C\) in CASE 3, while \(D_{0.5}\) ice particles only barely cover \(10^\circ C\). For CASE 2 the difference between the two ranges is \(25^\circ C\) versus \(5^\circ C\). The insert in Fig. 7 refers to this range assessment. This figure makes it quite clear how much more cooling is required to freeze larger proportions of the deposit, as expressed by lower \(t_A\). In general, increasing \(D\) or \(W_f\) at a given \(t_S\) will cool \(t_A\) to the limit. Note that the relationship between \(X\) and \(t_S\) is quite identical in CASES 2 and 3 because the former has no permanent water skin, while the latter does.

e. The \(V_R-t_A\) domains (Fig. 8)

1) CASE 1

In (1) relating \(V_R\) to \(W_f\) and \(V\) can also be connected to the HMT equations either through \(t_A\) or \(t_S\). This provides valuable new insights about the differences between CASE 1 and the contrasting CASES 2 and 3, as well as the different parameter ranges and restrictions. CASE 1 is relatively simple because it is for dry growth, whereas CASES 2 and 3 have \(Y\) as an additional dimension. Figure 8, CASE 1, is a plot of the \(V_R-t_A\) domain for \(Y = 1.0\). There are three wedge-type data arrays with origins \(O_i\), where “i” gives \(W_f\) in grams per cubic meter. To visualize trends the origin \(O_{4.5}\) and the corresponding \(W_{4.5}\) wedge is also shown. Note that \(t_A\) associated with the \(O_1\)
is −2.45°C, caused by the warming due to the freezing of the accreted water with \( W_f \). The lines forming the wedges at \( t_A \) are the intersections of \( t_S = 0°C \) and \( D_{0.5} \). The data are divided by lines for lower \( t_S \) and larger \( D_i \). The highest growth rate for \( W_2 \) is \(<2 \text{ mm min}^{-1} \) for \( D_{\text{max}} \), slightly larger than 2 cm. It occurs at the lowest supercooling for dry growth. The \( D_8 \) hailstones can only grow slightly above \( W_1 \), with \( t_S \) near 0°C and \( t_A \) in the −30°C range. It is the size that lowers the hailstones’ \( t_S \) out of range—even at \( t_A = −40°C \).

2) CASE 3 (AND 2)

The \( V_{R−t_A} \) domain for the two shedding CASES has a more complex structure because of the added variable \( Y \). This is demonstrated by Fig. 8, CASE 3, with the splitting of \( t_S \) into bands with \( t_A \) at different \( Y \). Two groups of constant \( W_i \) are displayed, in contrast to the groups of \( D_i \) of CASE 1. CASE 2 is not displayed, but its bands of \( 0° \leq t_S \leq −5°C \) would shrink to \( 0° \leq t_S \leq −0.55°C \); that is, they would have a width of approximately one-tenth of that of CASE 3. The wedges of CASE 1 are becoming more open as \( Y_{1.0} \) decreases to \( Y_{0.2} \). Noteworthy is the shift of the \( Y_i \) lines for different \( D_i \). With decreasing \( Y \) the bands for \( 0° \leq t_S \leq −5°C \) vary less with \( t_A \), and they move to higher air temperatures. There are two limiting features: the two data frames are for \( W_1 \) and \( W_6 \), each covering the range of \( D_{0.5}−D_8 \). Again, the cutoff was set at \( V_R = 4 \text{ mm min}^{-1} \). In contrast to \( W_6 \), the \( V_R \) for \( W_1 \) varies only within a restricted range up to \( −1 \text{ mm min}^{-1} \).

Figure 8 documents the strong dependences of \( V_R \) on both \( W_i \) and \( D_i \) for all CASES. CASE 3 shows why \( Y \rightarrow 1 \) is limiting sponge growth and also reduces the radial growth rate. Term \( V_R \) is halted by the lowest \( t_A \) possible. More heat needs to be removed when \( I_f \) increases.

There is an interesting conclusion, namely, that \textit{high growth rates} are only possible for low net accreted ice fractions \( Y \), that is, preferably within \( 0.2 \leq Y \leq 0.6 \).

The broken lines of Fig. 8 show that the dependencies on \( D \) have been linearized. In addition, some lines for constant \( D \) have been slightly shifted to cover up small differences in dependence of \( Y \).

f. The \( V_{R−t_S} \) domains (Fig. 9)

1) CASE 1

The radial growth velocity for dry growth of spherical hailstones in the \( V_{R−t_S} \) domain is displayed in Fig. 9, CASE 1, as subdivided by \( D, W_f \), and \( t_A \). Its abscissa is limited to \( t_S = −32.3°C \) for \( t_A = −40°C \). The lowest trapezoidal array represents \( D_{0.5} \), bounded at the bottom by \( W_1 \) and \( W_3 \) at the top. The dashed second array represents the conditions for \( D_2 \), while the third is for \( D_8 \). With the larger sizes such as \( D_8 \), the growth conditions with the highest \( V_R \) (≥3 mm min\(^{-1} \)) occur with warming \( t_S \) and lowering \( t_A \). Augmenting \( D_i \) increases the radial growth speed. The \( t_S \) cutoff is at the end of the displayed scale.

2) CASES 2 AND 3

Shedding is always associated with spongy growth. Thus, as with Fig. 8, CASE 3, a \( Y \) substructure is added to the diagram (Fig. 9, CASE 3). There are two extreme frames in the \( V_{R−t_S} \) domain: one is for \( W_1 \), the other \( W_6 \). The range limits at \( D_{0.5} \) and \( D_8 \) are different, with the exception of the basis at the \( D_{0.5} \) trapezes. The trapezoidal arrays for \( Y_{1.0} \), \( Y_{0.6} \), and \( Y_{0.2} \) carve out the conditions for \( W_6 \) (top array) and \( W_1 \) (bottom array). This makes it clear that, for high radial growth speeds, \( Y \), as in Fig. 9, has to be in the range of \( 0.6 \leq Y \leq 0.2 \). Both Figs. 8 and 9 suggest that the cutoff of \( V_R \), presently at 4 mm min\(^{-1} \), could be moved to 5 or even 6 mm min\(^{-1} \). This would lower the limiting surface temperature to \( t_S = −0.6° \) or −0.65°C, respectively, which is equivalent to a very slight shift of the \( V_R \) cutoff away from the \( t_S = 0°C \) line in CASE 2 of Figs. 3–9.

Figure 9, CASES 2 and 3, shows that most of the domain covering \( t_S \) to −40°C is irrelevant, except that it
better explains the data trends in the limited bands. It is a great example for showing the drastic restriction of the parameter space that is basically caused by shedding. For both CASES, $V_R$ increases with increasing $W_f$ and $D$. $V_R$ is greatest for $t_S = 50^\circ C$ and the warmest possible $t_A$.

A significant observation in Fig. 8 is that freezing a smaller fraction of the accreted water allows faster radial growth speeds because less latent heat of fusion needs to be removed. The consequence of further increasing $Y$ is to reduce $V_R$. Note that the scale of $V_R$ is different in Figs. 9a and 9b because dry growth is much slower for CASE 1. Most of this shift is a direct consequence of the doubling of $W_3$ to $W_6$.

5. Overview

Figure 10 represents all CASES, thus allowing comparisons. The original hailstone is given by size and height level by a point in Fig. 10a, together with the resulting free-fall speed. Selecting an accreted liquid water content produces a $V_R$ in Fig. 10b. CASE 1 displays $t_A$ for $W_1$ and $W_3$ and different $D_i$ in Fig. 10c (a more detailed version of this display is given in Fig. 8, CASE 1). Figure 10b also covers CASE 2 by the display of the ice front temperature $t_s$, opposite the corresponding radial growth speed. This is also to underline that $V_R$ in CASE 2 should not differ by much from CASE 3. The story line for CASE 3 continues directly from Fig. 10b to Fig. 10d, where $V_R$ is related to the temperature gradient across the water skin, after choosing $I_i$. Neglecting the interface temperature $t_i$, this gradient is composed of the ratio $-t_S$ over $d$ (Fig. 10e). At this point, no information is available about water-skin thickness. Nevertheless, the possible range of conditions is highlighted (octagons). Thus, guesses have to be made based on the icing of spheroids ($d$ may have a thickness of magnitude $0.1 \text{ mm}$). The results are surprising, considering the suggestion of very large temperature gradients across the water skin of up to $40^\circ C \text{ mm}^{-1}$. This value is, however, reduced for smaller hailstones, smaller $W_f$ and smaller $I_i$. CASE 3 is then developed beyond Fig. 10e, into Fig. 10f, which relates $t_S$ to $X$. Figure 10f is an expansion of the bottom-left corner of Fig. 5, CASE 3. The complex matching of $t_A$ with $t_S$ in Fig. 10a is not attempted. The key parts of Fig. 10 serve for comparison of the different CASES, the relationship between $t_S$ and $d$, and the connection to the new variable $X'$.

6. Summary and comments

a. General findings

The four new variables $X$, $Y$, $\Phi$, and $\Psi$, established in Part I, are fully representing the HMT of spherical hailstones over their whole size range $0.5 \leq D \leq 8 \text{ cm}$. The corresponding equations were developed without any restrictions or approximations. They are based on
the recognition that the six variables of the HMT can be simply reduced to four new ones. It was inherently built into the six-variable equation. The possible simplifications were just not recognized before. The auxiliary variables $F$ and $C$ are functions of $t_A$ and $t_S$ only. Thus, the HMT is equally determined by $X$, $Y$, $t_A$, and $t_S$. The variable $X_0$ contains the factors $CD$ and $rH$ in a fourth-root term. This considerably reduces specific errors. Most importantly, it allows a replacement of $X$ by $X_0$ according to $X_5 = Wf^{Re^{1/2}} = CRe D^{3/4} = CRe X'$, that is, $WfD^{3/4} = X'$. Thus, the free-fall Re is being replaced by $D$. This step was made possible by assuming a coupling of $t_A$ and $p_A$ to height in Denver hailstorms. Errors are introduced by approximations (hailstone density, drag coefficient) and assumptions about 16 water-skin properties, gyration effects, etc.

All combinations of variables are explored, but only after limitation of the ranges of variables, such as $1 \leq W_f \leq 6$ g m$^{-3}$ (3 g m$^{-3}$ for dry growth, 6 g m$^{-3}$ for shedding), $0.5 \leq D \leq 8$ cm, and $0^\circ \geq t_S \geq t_A \geq -40^\circ$C, and, most surprisingly, $t_S \geq -5^\circ$C for hailstones covered by water skins. While cooling cloud water to $t_A = -40^\circ$C is extreme, occasional scavenging of all nuclei in an updraft is possible. However, the lowest $W_1$ limits $t_S$ to $>-32.3^\circ$C for dry growth (CASE 1). All these $t_S$ limits allow a substantial compaction of the results to the most probable growth conditions, and further reduce the parameter volume covered by growing hailstones. This is in addition to the reduction of parameter space from six to four dimensions (CASE 2) or three in CASES 1 and 3 (see Part I). For CASE 2 a maximum (dendritic) growth speed of $V_R = 4$ mm min$^{-1}$, corresponding to $t_S = -0.55^\circ$C, was assumed. This limit can be raised, but may necessitate an increase in $W_f$ to $>6$ g m$^{-3}$. The reader may redesign such limits.

The $V_R$ in CASE 3 is highly dependent on diameter; it is fastest for $0.2 < Y (= E_{NC}I_f) < 0.6$. This expresses the fact that spongy growth is required for high radial growth speeds and the creation of large hailstones. The reason for this is that complete freezing ($I_f = 1$) requires a time-consuming transfer of a latent heat of freezing to the air, which slows radial growth. However, if less water is frozen in the deposit, $V_R$ is increased. The thickness of the hailstone-covering supercooled water skins has been
related to \( t_S \) and \( X' \). In laboratory experiments with icing of gyrating spheroids, photography and fast-scanning infrared cameras did not reveal details of the water skin (such as thickness or turbulence-induced effects by impinging cloud droplets). Expected effects are estimated to be \( \approx 20\% \). This figure is supported by the combined effect of (air) turbulence, roughness, and gyration, which produced an overall correction factor of \( K = 1.29 \).

Experiment-based data on spheres, including limited icing studies, and results and implications from experiments with gyrating spheroids have been applied and extrapolated, thus creating a robust background for the calculations and conclusions.

b. Specific results

1) The four-variable equations are solved and all results for all parameter combinations are given; this also includes the breaking down of \( X' \) into \( W_f \) and \( D \). The separation of \( Y \) into \( E_{NC} \) and \( I_f \) is possible but premature because of the lack of relevant experimental data.

2) Calculation of radial growth speed gave an amazing result: The surface temperature of a spherical hailstone that grows while shedding from a permanent water skin is warmer than \(-5^\circ C\) (CASE 3) (in confirmation of previous icing experiments).

3) The surface temperature of fast-shedding spherical hailstones without water skins is deduced to be warmer than \(-0.55^\circ C\) (CASE 2).

4) Most hail growth conditions in a thunderstorm are associated with spongy growth.

5) CASES 1 and 2 are relatively simple. In CASE 1 the surface is dry, but its (surface) temperature varies. In CASE 2 \( t_S = 0^\circ C \) is assumed, while the hailstone’s substrate is spongy with varying \( I_f \). CASE 3 has both a varying surface temperature, \( t_S \leq 0^\circ C \) and a spongy substrate. Further, CASES 2 and 3 are very similar because their surface temperatures are arranged in bands of different width, both bordering \( 0^\circ C \leq t_S \leq -5^\circ C \). The biggest \( \Delta t_S \) of CASE 3 is \( \approx 10 \times \) the bandwidth of CASE 2.

6) Dry growth at \( t_S = 0^\circ C \) produces deposits with ice density (minus air bubbles caused by originally dissolved air); lower densities require \( t_S < 0^\circ C \). Large hailstones can only grow dry (with ice density) at high altitudes with low \( W_f \). Such conditions, however, are not conducive to the growth of large hailstones.
Further, if formed at high levels, “low density” hailstones would have to descend through fewer cold-air layers with \( t_A \) still below \( 0^{\circ} \text{C} \) and where dry growth is replaced by spongy growth. Then low-density particles would be compacted by accreted water, even at \( t_A = 0^{\circ} \text{C} \).

7) The three types of experimentally determined effects by air turbulence, roughness, and gyration can be safely built into the HMT equations with one single multiplicative factor, \( K = 1.29 \).

8) CASE 2 is built up by bands with surface temperatures \( 0^{\circ} \geq t_s \geq -0.5^{\circ} \text{C} \) over a background of \( Y \) that decreases at constant \( t_A \) with increasing \( X' \). CASE 3, the most complex CASE, is similar but allows a larger supercooling of the water skins’ surface, with bands bracketed by \( 0^{\circ} \geq t_s \geq -5^{\circ} \text{C} \).

9) The gradients across the water skin are up to \( 40^{\circ} \text{C} \) mm\(^{-1} \), a value that is reduced for smaller hailstone sizes, smaller \( W_f \), and smaller mass fractions of ice in the deposit \( I_f \). Water skins may have thicknesses of \( 0.1 \) mm.

c. Errors

The errors in using an average \( C_D = 0.75 \) in the calculation of the growth velocity \( V_R \) and the heat and mass transfer components of growing hailstones are within \( \pm 5\% \). However, this error is not random; it has
a negative sign for smaller hailstones ($\leq -5\%$) because the fall velocities are smaller, whereas a positive sign applies for larger hailstones. For radial growth speed calculations in all figures of this Part II, the correct $C_D$ has been applied (1.0 for 0.5 cm stones and 0.5 for $D_1 \approx 2$ cm). The assumed factor $k_r = 0.07$ (roughness element height 7\% of diameter) is reflected in $C_D = 0.5$ for $D \approx 2$. (The drag increases to $C_D = 0.67$ for $k_r = 0.15$.) Hailstone density errors are <5\%. There is an error expected in the calculation of the effective dendritic growth speed because accreted droplets penetrate the water skins and make it more turbulent. No effect of impinging drops has ever been observed in the laboratory, but their influence needs to be quantified in future experiments—they are far from being easy! Considering the magnitude of $k_r$, however, it is not expected that this skin turbulence effect exceeds 20\%.

\section*{d. Main implication}

The importance of shedding, quantized by $E_{NC}$, and $I_f$ of spongy deposits are clearly established as main factors controlling hailstone growth. These two variables are key to rapid growth of deposits of low ice fraction, because fast partial freezing requires less latent heat to be discarded. They are as important as $W_f$ and $D$ (or Re).

The surface temperatures $t_S$ and $t_A$ control the HMT of hailstones with $X^\prime$ and $Y$ through the two new variables $\Phi$ and $\Psi$. In CASE 1 any two out of $X^\prime$, $t_A$, and $t_S$ will determine the value of the third.

For free-fall $D$ can be isolated with a known $C_D$, thus circumventing the problems associated with the application of Re.

Shedding, enhanced by spongy ice growth, can initiate and maintain a warm rain process parallel to hail formation [proposed by Joe et al. (1980) and Joe (1982)]. Efficient rain in midlatitudes is normally formed in convective clouds that contain the ice phase, that is, the embryos of hailstones, small hail or hailstones per se ($\geq 5$ mm). According to the field experience of the author in Malaysia and Indonesia, the convective clouds are the major “vehicles” for hail-coupled warm rain.

\section*{e. Outlook}

1) The equations and data provided can serve as input into entirely new dynamic cloud models. Thereby, it is imperative that turbulence is not characterized as a mathematical continuum of singularities, evenly distributed over the whole updraft. Updrafts are rather consisting of vortices, two to four stacked on top of each other (a scenario supported by the radar-observed packaging of hailstones). They need to be treated by Navier–Stokes equations. Inspiration and guidance may be found in the rich literature on wakes. Extensive future vertical Doppler radar frequency soundings, as taken by Thomson (1997) and Thomson and List (1999) and refined by Thomson (List 2004), are necessary to obtain more size-spectrum information on hail evolution. The pulsing observed is similar to the one described for tropical rain by McFarquhar and List (1991). New dynamic cloud models, with packaging of hail and spectra evolution, need to be designed that also include shedding. A parallel evolution of warm rain [based on 100- and 50-kPa collision–coalescence–breakup experiments by List et al. (2009a,b)] may provide the link between hail and rain.

The quasi-steady state, conveyor belt–rigged giant hailstorms of the high plains of Colorado are different from the above-mentioned scenario.

2) The new HMT for spherical ice particles is perfectly suited to serve as the basis for an expansion to spheroidal and ellipsoidal hailstones.

3) There is a need for consolidation of the HMT and growth of ice or ice/water particles with sizes of 0.1–1.0 cm. This needs to address the biggest questions and errors that occur in that size range. It would also shine more light onto the transition from ice crystals to graupel to small hail to hailstones, or drops to hailstones. Free-fall behavior, growth, and HMT of such embryo particles have been treated over the whole size range by List and Schemenauer (1971), Schemenauer and List (1978), Cober and List (1993), and Youk et al. (2006). There is also a vast literature on such particles. One key issue will involve the replacement of the unsatisfactory definitions of the ice particles in the Glossary of Meteorology (Glickman 2000).

4) Another major task is the improvement of the knowledge base of spinning tops, as described by Klein and Sommerfeld (1965). Many questions, such as the onset of gyration and dependence of spin and nutation/precession frequencies on size and shape, are still open. Does gyration stop, as indicated in the latest growth stage of the giant hailstones (Knight and Knight 2005)? Most important, however, are experiments in icing wind tunnels, confirming that the special gyration proposed in Part I can produce spherical hailstones. Further, net collection efficiencies, coupled with ice contents of deposits, need to be measured, together with spectra of shed drops and water-skin properties.

5) The pioneering field study by Rasmussen and Heymsfield (1987a,b,c) stressed the role of water drops shed from hailstones acting as hail embryos when frozen. That work should be continued because the step to rain needs to be further substantiated in field experiments and refined by models.
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APPENDIX

Approximations

The following approximations have been used:

Thermal conductivity of air (W m\(^{-1}\) C\(^{-1}\)):

\[ k = 0.02382 + (7.118 \times 10^{-5}) t_A; \]

Dynamic viscosity of air (kg m\(^{-1}\) s\(^{-1}\)):

\[ \mu = (1.718 + 0.0049t_A - 1.2 \times 10^{-5} t_A^2) \times 10^{-5}; \]

Kinematic viscosity (m\(^2\) s\(^{-1}\)):

\[ \nu = \rho A \mu; \]

as with \( \rho A \) (kg m\(^{-3}\)) as air density;

Diffusivity of water vapor in air (m\(^2\) s\(^{-1}\)):

\[ D_{wa} = 4.01 \times 10^{-5} T_A^{1.94} p_A^{-1}; \]

Saturation pressure over water (Pa):

\[ e_v(t) = 10^b, \]

where \( b = 25.547 - 4.9283 \log_{10}(273.15 + t) - 2937.4 \times (273.15 + t); \]

Saturation pressure over ice (Pa):

\[ e_i(t) = 10^b, \]

where \( b = 12.55 - 2667 \times [2667 \times (273.15 + t)]; \]

Latent heat of vaporization of water (J kg\(^{-1}\)):

\[ L_v(t) = 2.5008 \times 10^6 \times (273.15 + t) \times (273.15 + t); \]

Latent heat of sublimation of ice (J kg\(^{-1}\)):

\[ L_s(t) = 2.8345 \times 10^6 - 190t; \]

Specific heat of water (J kg\(^{-1}\)°C\(^{-1}\)):

\[ c_w(t) = 4217.8 + 0.3471 t; \]

The parameterizations of \( k, \mu, D_{wa}, L_v, L_s, \) and \( e_v \) are found in Pruppacher and Klett (1978), whereas \( e_i \) and \( e_i \) are from Iribarne and Godson (1981). The value for \( \eta \) has been approximated by the author.

It is recommended that two master files are set up, one for hailstones with wet surfaces and the other for dry (ice) surfaces. As a check, it is desirable to have two quantities monitored: (i) the sum of all four heat transfer components (= 0) and (ii) the equality of the auxiliary functions \( \Phi \) and \( \Psi \), as calculated directly with \( t_A \) and \( t_S \) or through (6).

REFERENCES


