Predictability of Precipitation from Continental Radar Images. Part V: Growth and Decay

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ABSTRACT

In a Lagrangian frame of reference, the accuracy of rainfall systems predicted by nowcasting algorithms can be improved by incorporating the growth and decay of the rainfall. The scale dependence of predictability of growth and decay of continental-scale precipitating systems is studied with the help of the U.S. national radar composites. The growth and decay of precipitating systems is estimated in a time interval \( t + \tau \) by correcting the precipitation image for advection and rotation at time \( t + \tau \) with respect to the precipitation image at time \( t \) and then subtracting the former from the latter. Results show that the two-dimensional correlation of growth and decay has an elliptical structure, indicating that growth and decay is nonisotropic. The probability density function of precipitation intensities and of growth and decay follows a Gaussian distribution. The scale-dependence analysis of growth and decay patterns indicates that the growth and decay of rainfall may be predictable up to about 2 h for scales larger than 250 km.

1. Introduction

Weather radars play a vital role in detecting the echoes from various types of precipitating systems allowing quantitative nowcasts of rainfall. The most successful nowcasts of surface precipitation patterns are generated by algorithms based on the advection of rain fields observed by weather radar. The generation of automated short-term nowcasting is now widely implemented by many radar groups, and most of them are equipped with specialized nowcasting software (Browning et al. 1982; Walton et al. 1985; Dixon and Wiener 1993; Handwerker 2002; Seed 2003; Rasmussen et al. 2003; Mueller et al. 2003; Turner et al. 2004; Fox and Wikle 2005; Ruzanski et al. 2011). A comprehensive review of nowcasting techniques is found in Wilson et al. (1998). Nowcasting of rainfall fields can be obtained through three different types of approaches, namely, persistence in Lagrangian coordinates based on advection using radar data; conceptual models of growth and decay of rain cells using radar, satellite imagery of clouds, and/or lightning data; and quantitative nowcasts combined with numerical weather predictions (Bellon and Austin 1978; Mueller et al. 2003; Wolfson et al. 2008; Dupree et al. 2009; Bowler et al. 2004; Li and Lai 2004).

Browning et al. (1982) pointed out that the errors in the rainfall fields generated by Lagrangian extrapolation are mainly due to the development and decay of rainfall fields. Thus, the growth and decay of rainfall systems counteracts the basic assumption of a linear extrapolation approach.

More recently at McGill University, an algorithm for nowcasting of precipitating systems called McGill Algorithm for Precipitation Nowcasting by Lagrangian Extrapolation (MAPLE) was developed for operational use. In Part I of this series of papers illustrating the development and workings of MAPLE, Germann and Zawadzki (2002) studied the scale dependence of predictability for continental-scale Lagrangian nowcasts using Fourier (discrete cosine transform) low-pass filtering, but without invoking any assumptions of scaling. In Part II (Germann and Zawadzki 2004), the scale dependence for probabilistic Lagrangian nowcasting using a high-resolution field of echo motion is examined. In Part III (Turner et al. 2004), the operational nowcasting algorithms are improved by using the results of Part I and by rejecting the unpredictable scales. In Part IV, Germann et al. (2006) investigated the sources of forecast uncertainty and the dependence of predictability on location and scale.
In connection with these works, an attempt has been made to study the predictability of growth and decay of the rainfall. Since small scales (convective cells embedded in the system) change very rapidly, their lifetime is very short (a few tens of minutes; Wilson 1966; Henry 1993; Wilson and Megenhardt 1997). Bellon and Austin (1978) have used a statistical analysis to deduce the extent of rainfall growth and decay caused by the orography over the Montreal, Quebec, Canada, area and suggests that its incorporation would improve the accuracy of forecasts. Thus, the prediction of growth and decay of rainfall systems could play a major role in various fields, such as aviation, since at least half of the national airspace delays are caused by thunderstorms (Forman et al. 1999). But most attempts to incorporate growth and decay have failed (Tsonis and Austin 1981; Wolfsen et al. 1999, and the references therein). In fact, MAPLE incorporates some of the growth and decay, since the field of precipitation motion is affected by systematic and persistent changes in the precipitation patterns. Furthermore, utilizing boundary layer convergence lines and fuzzy logic technique, Mueller et al. (2003) incorporated the growth and decay term in the Auto Nowcaster System up to a lead time of 1 h. Yet, the incorporation of the growth and decay term in nowcasting systems to obtain the highest accuracy possible in the prediction represents the next frontier. Thus, this study attempts to understand the statistical properties of growth and decay, and its predictability.

The growth and decay of precipitating systems is estimated utilizing the existing radar observations. In Lagrangian persistence, we define growth and decay as simply the mismatch between the two rainfall fields. This definition of growth and decay includes all of the vertical changes of precipitation. Adding growth and decay to the extrapolated rainfall fields incorporates not only small-scale changes but also large-scale changes. Thus, the present study focuses on the predictability of growth and decay at different scales from a continental-scale radar dataset.

The present paper is organized as follows. Section 2 describes the events and data used for this study. The definition and estimation of growth and decay in the Lagrangian frame of reference are given in section 3. Statistical properties of growth and decay are discussed in section 4. Scale dependence of predictability of growth and decay is explained in section 5. The results are summarized in the last section.

2. Data and events description

Weather Surveillance Radar-1988 Doppler (WSR-88D) U.S. national composites cover small scales of few kilometers to continental-scale rainfall systems. The radar network was primarily intended to provide precipitation monitoring and to allow for short-term (0–30 min) warning of weather hazards. Radial wind and hydrometeor reflectivity data from these radars represent rich sources of data that can potentially be used to improve weather forecasts. The data obtained from these radar composites benefit from three levels of quality control. The WSR-88D network volumetric data are remapped by the National Severe Storms Laboratory (NSSL) using nearest-neighbor interpolation on range and azimuth plane, resulting in unified Cartesian 3D reflectivity fields with high temporal (5 min) and spatial (1 km) resolution. To avoid the presence of nonmeteorological targets (echoes from birds and insects) and to include the maximum vertical changes in the precipitating systems, the reflectivity maps of constant altitude plan position indicator (CAPPI) at 2.5 km are utilized rather than CAPPIs at 3.5 km or above and on the 2D composite maps possible at the possible lowest elevation. The spatial coverage of the radar network extends from 20° to 52° North latitude and from 130° to 60° West longitude. The time resolution is 5 min, providing an opportunity to generate rainfall accumulation maps over various time intervals. We have derived 1-h accumulation maps by accumulating the 5-min CAPPI rainfall rates and re-mapped them with a spatial resolution of 4 km over a domain of 3500 km × 7000 km. Maps of radar reflectivity factor $Z$ were converted to rain rate $R$ using a standard $Z-R$ relationship, $Z = 300R^{1.5}$ (Carbone et al. 2002), and instantaneous rain rate maps were accumulated to obtain hourly rainfall maps. The 1-h accumulations are then converted back into radar reflectivity factor utilizing the same $Z-R$ relation. In this study, we have used reflectivity (dBZ) rather than rainfall rate (mm h⁻¹) fields because reflectivity fields are smoother and follow a Gaussian distribution.

The present study is based on 312 h of rainfall data observed on 13 days during spring and summer seasons. All of the 13 events persist for more than 24 h in time and are forced by large-scales like fronts, cyclones, long-lived propagating organized convection or monsoon rains. To make sure that the estimated correlation function, lifetime and verification scores are based on a sufficiently large sample, the rainfall fields are selected with a minimum extent of about $2 \times 10^5$ km². The statistics of average spatial extent and rain fraction at two threshold levels for the 13 events are listed in Table 1. The rain fraction is defined as the ratio of area of rainfall greater than that threshold value to the total area of rainfall. The rain fraction at 40 dBZ threshold indicates the areal coverage of convection in rainfall field. The growth and decay of mesoscale rainfall systems varies.
Table 1. Statistics of the 13 rainfall events utilized in this study. Values are averages over 24 h. Extent is defined as the area with a rainfall larger than 15 dBZ (0.2 mm h\textsuperscript{-1}). Last two columns indicate the fraction of area that is covered with Z larger than 25 dBZ (1 mm h\textsuperscript{-1}) and 40 dBZ (10 mm h\textsuperscript{-1}), respectively.

<table>
<thead>
<tr>
<th>Date</th>
<th>Extent (10\textsuperscript{3} km\textsuperscript{2})</th>
<th>&gt;1 mm h\textsuperscript{-1} (%)</th>
<th>&gt;10 mm h\textsuperscript{-1} (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 Apr 2008</td>
<td>4.07</td>
<td>51.4</td>
<td>2.24</td>
</tr>
<tr>
<td>6 May 2008</td>
<td>2.37</td>
<td>44.7</td>
<td>4.51</td>
</tr>
<tr>
<td>24 May 2008</td>
<td>3.34</td>
<td>46.5</td>
<td>4.48</td>
</tr>
<tr>
<td>6 Jun 2008</td>
<td>4.51</td>
<td>49.9</td>
<td>2.97</td>
</tr>
<tr>
<td>9 Jul 2008</td>
<td>4.12</td>
<td>46.8</td>
<td>1.9</td>
</tr>
<tr>
<td>13 Jul 2008</td>
<td>3.42</td>
<td>49.3</td>
<td>2.3</td>
</tr>
<tr>
<td>18 Jul 2008</td>
<td>2.49</td>
<td>50.9</td>
<td>3.67</td>
</tr>
<tr>
<td>22 Jul 2008</td>
<td>2.8</td>
<td>51.4</td>
<td>5.39</td>
</tr>
<tr>
<td>24 Jul 2008</td>
<td>5.25</td>
<td>48.7</td>
<td>2.33</td>
</tr>
<tr>
<td>30 Jul 2008</td>
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<td>46.7</td>
<td>2.49</td>
</tr>
<tr>
<td>10 Aug 2008</td>
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<tr>
<td>13 Aug 2008</td>
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<td>40.3</td>
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</tr>
<tr>
<td>21 Aug 2008</td>
<td>2.96</td>
<td>34.5</td>
<td>0.68</td>
</tr>
</tbody>
</table>

with meteorological conditions prevailing in different seasons. To determine the variability of the predicted growth and decay of mesoscale rainfall systems in different seasons, the events are selected in two different seasons. Through visualizing the animations of all of the 13 events, we noticed that the motion field has a clear translation and rotation components with varying degree of freedom. The events on 9 July and 18 July 2008 have less rotation component compared to other events. The events on 5 May and 22 July 2008 are strongly influenced by relative vorticity and deformation. The events on 24 July and 21 August 2008 possess cyclonic type of rotation.

3. Growth and decay

It is vital to properly define the term growth and decay within rainfall systems because it is coupled with many complicated processes, starting from the cloud formation stage to the precipitation observed at the surface. Changes in microphysical processes or dynamics associated with vertical and horizontal winds lead to significant changes in the rainfall system at different levels. Cloud drops formed by vapor deposition can grow by aggregation, riming, etc.; they decay by evaporation/sublimation at different levels. After melting, the raindrop size can grow by collision–coalescence and decay by breakup or evaporation. Vertical and horizontal winds play a major role in the collision process. Thus, growth and decay is associated with the entire depth of a precipitating system and is very difficult to define. Hydrologists and meteorologists are interested in the final outcome of the precipitation systems, that is, the surface rain; consequently, we are defining the growth and decay of a rainfall system based on the rainfall accumulations near the surface. Hence, the growth and decay includes the microphysical changes, which are occurring aloft, and the dynamical changes during the descent of precipitating particles to the surface, and it is thus equivalent to the change in the rainfall field between two time periods under Lagrangian persistence.

Nowcasting of precipitation, considered here, is based on extrapolation of the present state by assuming persistence of the storm attributes over time (Lagrangian persistence). Errors in the nowcast are attributed to uncertainty and changes in the motion field and precipitation growth and decay. Let us first clearly define what is meant by growth and decay by reference to the conservation equation of precipitation content $M$ (Kessler 1969) as follows:

$$\frac{\partial M}{\partial t} + V_h \nabla_h M + (w + V_t) \frac{\partial M}{\partial z} + M \left( \frac{\partial V_t}{\partial z} + w \frac{\partial \ln p}{\partial z} \right) = S_M,$$

where $V_h$ and $w$ are the horizontal and vertical components of air velocity, respectively; $V_t$ is the terminal fall speed of precipitation; $\rho$ is the air density; and $S_M$ is the source/sink term that represents the microphysical processes leading to changes in precipitation content. Actual growth and decay of precipitation is described by the source term $S_M$. However, in nowcasting we refer to a quite different concept of growth and decay $S$ defined as

$$S = \frac{\Delta Z}{\Delta T},$$

where $\Delta Z/\Delta T$ is the time change (typically over 5 min or more) in precipitation intensity $Z$ in coordinates moving with the precipitation pattern; $Z$ is obtained from radar observations at some height above the surface. The motion of the pattern is derived before the nowcast is produced. Fields of motion vectors are obtained at a suitable spatial resolution for optimizing a nowcast by methods generally based on cross correlation of radar-observed precipitation at two time intervals and over an area that in general is much smaller than the area of the precipitation pattern. It is not possible to separate transport of precipitation from propagation of precipitation regions. The computed storm motion captures some of the systematic propagation that renders the concept of growth and decay purely empirical and dependent on the methods for determining storm motion. Evolution of cloud drops (or cloud ice) into precipitable water changes the volumetric...
growth and decay, and conversion of atmospheric water vapor into clouds and then into precipitable water is responsible for areal growth. The first process changes Z in the precipitating system itself, and the second process generates new cells and alters the coverage. These two processes change the observed reflectivity at near surface, which is accounted for through the change in Z; but with this definition of S, it is not possible to separate these two terms. Thus, S is the result of microphysical processes in the entire depth of the storm as well as the transport of precipitation by wind relative to the storm motion. It is quite a different concept from the one used in cloud physics and defined in Eq. (1).

The precipitating systems will advect and rotate with the background wind field and also evolve and diminish as time progresses. From Eq. (2), the evolution and decrease of surface rainfall at a particular grid point in a Lagrangian frame with time provides information about the growth and decay of the rainfall system at that grid point during that time interval. This can be mathematically represented as follows: Let \( P(x, y, t) \) indicate the precipitation field at time \( t \) with positional coordinates \((x, y)\) and \( P(x_1, y_1, t + \tau) \) be the new precipitation field after a time interval \( \tau \). Then,

\[
P(x_1, y_1, t + \tau) = \text{Advected and rotated } P(x, y, t) + \text{growth and decay } S. \tag{3}
\]

Storm transport by large-scale systems has a clear component of translation and rotation. Consequently, we will determine here storm motion by considering only a solid translation and a solid rotation of precipitation patterns in determining growth and decay, whereas residuals of rainfall \( S \) with respect to more complex motion fields used in nowcasting algorithms, such as MAPLE, will be considered elsewhere. The advection of a precipitation field can be estimated in two ways: 1) constant vector method (Austin and Bellon 1974), in which the total field is shifted by a constant vector; and 2) multiple-vector method (Barclay and Wilk 1970; Duda and Blackmer 1972; Germann and Zawadzki 2002), in which the field is displaced using a set of location-dependent vectors, sometimes determined by correlation in a defined set of subregions and sometimes using a variational echo-tracking method (semi-Lagrangian scheme).

The constant vector method uses the maximum correlation between the fields for estimating the mean motion vector. This method assumes that the advection is uniform in space and time, which is not valid in real atmosphere. In the multiple-vector method, the advection is divided into \( N \) steps of time length \( \tau \), and for each time step advection is calculated iteratively (Germann and Zawadzki 2002). The final displacement vector is the vector sum of the \( N \) vectors of the individual time steps. The domain size is a strong constraint in the multiple-vector method. The tracking of individual cells is a highly difficult task when growth and decay is the dominant process because this process has a large impact in determining accurate motion vectors. Large-scale storms are made up of clusters of single cells, and net storm motion is a result of propagation of discrete cells that is often very different similar to the individual cell motion. Wilson (1966) found that cells tended to move with the mean wind between 10 000 and 20 000 ft (between about 3000 and 6100 m), but the large-scale features move more slowly and to the right of the small-scale features (Browning et al. 1982). Thus, determining the envelope motion separate from the storm cell motion is a long-standing problem in weather radar research (Chornoboy et al. 1994). For very short-term predictions, this cell motion is accurate; but, for longer-term predictions, the envelope must be tracked.

The present study is focused on determining the predictability of growth and decay, and to make the results more generalizable, we decided to stay away from using highly sophisticated algorithms, such as multiple-vector methods (which are not available for all the scientific community), to track echoes and to compensate for the movement of echoes at the continental scale, at least to a first order. We hence introduce a new scheme called the rotation and translation (referred to as the R-T) method to estimate the large-scale motion and, in turn, to preserve Lagrangian persistence.

The basic assumption in the R-T method is that the advection of continental-scale precipitating systems is associated with large-scale waves. This method is a refinement of the constant-vector method. Since the advection is not constant in space and time, we estimate a solid mean advection of the rainfall system at each 1-h time step. We also included a solid rotation of the rainfall system at each time step. The rotation term takes into account the velocity variations in space, such as those associated with the evolution of a large-scale low pressure weather system. Note that these two terms may or may not be constant at each time interval and that it depends on the background wind field.

If \( \theta \) is the angle of rotation (clockwise direction) with respect to the center of mass \((c_x, c_y)\) of the system within time interval \( \tau \) (in our case \( \tau = 1 \) h), \( P(x_1, y_1, t + \tau) \) can be represented by a function \( P(x_0, y_0, t + \tau) \), where

\[
x_\theta = c_x + (x_1 - c_x) \cos \theta + (y_1 - c_y) \sin \theta, \tag{4}
\]

\[
y_\theta = c_y + (y_1 - c_y) \cos \theta - (x_1 - c_x) \sin \theta. \tag{5}
\]

Here \( x_\theta \) and \( y_\theta \) represent the predictive coordinates of \( x_1 \) and \( y_1 \) after rotating the field for an angle \( \theta \) and are
equal to \( x + \alpha \) and \( y + \beta \), respectively. Letting \( \mathbf{U} \) and \( \mathbf{V} \) represent the zonal and meridional precipitation field translation vectors, respectively, and then the zonal and meridional displacement vectors are given by

\[
\alpha = \mathbf{U} \tau,
\]

\[
\beta = \mathbf{V} \tau;
\]

\( \mathbf{U} \) and \( \mathbf{V} \) are estimated from the advection term of the R-T method, and the solid advection vector is obtained using the maximum cross correlation between the fields. The cross correlation between the fields is estimated using the following formula given in Zawadzki (1973):

\[
C(\tau)_{(\alpha\beta)} = \frac{\sum \sum P(x, y, t) \times P'(x, y, t + \tau)}{\sum \sum P(x, y, t)^2 \times \sum \sum P'(x, y, t + \tau)^2}.
\]

The lag displacement \((\alpha, \beta)\) corresponding to the maximum value of \( C(\tau) \) provides the advection term. Here \( P'(x, y, t + \tau) \) is the predicted field of \( P(x_1, y_1, t + \tau) \) in Lagrangian persistence, which is equal to \( P(x_0 - \alpha, y_0 - \beta, t + \tau) \). Since the pixel size is 4 km \( \times \) 4 km, the estimated advection vector may be in error of \( \pm 4 \) km. Using these \( \alpha \) and \( \beta \) values of maximum \( C(\tau) \), the zonal \( (\mathbf{U} = \alpha \times 4 \text{ km h}^{-1}) \) and meridional \( (\mathbf{V} = \beta \times 4 \text{ km h}^{-1}) \) echo motion vectors are estimated. The growth and decay \( S(x, y, t + \tau) \) of the rainfall system within the time interval \( \tau \) is estimated using the following expression:

\[
S(x, y, t + \tau) = P(x, y, t + \tau) - P(x, y, t).
\]

Thus, the growth and decay is just the mismatch of the precipitation patterns in a Lagrangian frame of reference with positive (negative) values indicating growth (decay). The \( S \) obtained using Eq. (9) is in Lagrangian persistence with respect to the previous-hour accumulated rainfall map. The schematic diagram illustrating the R-T method is shown in Fig. 1. For example, the growth and decay estimated at 0 UTC, where \( n \) being the time from 1 to 23, is \( S(x_{n-1}, y_{n-1}, t + n \tau) \), which is the mismatch between \( P'(x_{n-1}, y_{n-1}, t + n \tau) \) and \( P(x_{n-1}, y_{n-1}, t + [n - 1]\tau) \). \( P'(x_{n-1}, y_{n-1}, t + n \tau) \) is nothing but \( P(x_{n}, y_{n}, t + n \tau) \) in Lagrangian persistence with respect to \( P(x_{n-1}, y_{n-1}, t + [n - 1]\tau) \). To obtain \( S(x, y, t + n \tau) \)—that is, \( S(x_{n-1}, y_{n-1}, t + n \tau) \) in Lagrangian persistence with respect to \( S(x, y, t) \)—\( S(x_{n-1}, y_{n-1}, t + n \tau) \) is further adjusted for the rotation and advection vectors in a stepwise fashion. These rotation \((\theta_{n-1} \rightarrow \theta)\) and advection vectors \((\alpha_{n-1} \rightarrow \alpha \) and \( \beta_{n-1} \rightarrow \beta)\) are estimated using the R-T method by considering the respective rainfall maps at particular time intervals. Since the center of mass of the system \((c_x, c_y)\) varies at each time step, instead of adjusting for cumulative rotation and advection, we adjusted for stepwise rotation followed by advection, as illustrated in Fig. 1.

The performance of the R-T method is illustrated with the help of Fig. 2. Radar composites shown in Figs. 2a and 2b are hourly accumulated rainfall maps observed at 0000 and 0100 UTC 18 April 2008, respectively. From these two radar images, it is clear that the cold front of the system is moving toward the east, while the warm front is almost stationary. To maintain Lagrangian persistence—that is, to eliminate the rotation and advection from the precipitation field—the R-T method is applied to the radar data at 0100 UTC. The outcome, that is, the field at 0100 UTC in 0000 UTC rainfall map coordinates (in Lagrangian persistence) is shown in Fig. 2c. The correlation and root-mean-square (RMS) values between the two rainfall fields at 0000 and 0100 UTC before and after applying the R-T method are 0.69, 18.8 dB(Z) and 0.9, 16.2 dB(Z), respectively. The higher correlation and lower RMS values suggest that the Lagrangian persistence is maintained by means of the R-T method. The growth and decay of the rainfall during the 1-h time interval is estimated using Eq. (9) and is shown in Fig. 2d. During this time interval, the positive reflectivity values are observed in the front end and negative reflectivity values are observed in the rear end of the cold front of...
the system, suggesting that the growth process is predominant toward the direction of motion of the system, in agreement with the results of Wolfson et al. (1999). The event observed on 18 April 2008 is a frontal system. The positive and negative reflectivity values are observed in the front end, and the rear end of the cold front of the rainfall system suggests that along the direction of motion, the cold front is associated with growth process. When cold air moves toward the warm air, the warm air is lifted up and condenses and forms cloud drops and, in turn, into rain drops by increasing the area of precipitation, which is observed in the 18 April 2008 event. In contrast, at the rear end of the system due to the subsidence of air motion, one can expect decay of rainfall.

4. Characteristics of rainfall fields and associated growth and decay

The extreme variable nature of rainfall in space and time makes it difficult to estimate the area or time averages and higher moments of the rainfall amounts directly from the observations (Martin 1989). Thus, to characterize precipitation in space and time, the statistical properties and higher moments of the rainfall are useful.

a. Probability density function

One way of characterizing the rainfall is in terms of the mean and variance of the probability density function (PDF) fitted to the rainfall data with any one of the functional forms. To estimate the best fit to the PDFs of radar-observed mesoscale systems, we have fitted the gamma, lognormal, and Gaussian distribution functions to the measured values. PDFs of rainfall field are estimated for $Z \geq 15$ dBZ and with an interval of 1 dBZ. The PDFs of $S$ is obtained through Eq. (9) with a time delay of 1 h because the growth and decay presented in our analysis is the mismatch of hourly accumulated rainfall patterns in the Lagrangian persistence. The outcome of the R-T method provides mismatches at the boundaries of the precipitation field with intensities greater than or equal to 15 dBZ, thus yielding a high occurrence of $S > 15$ dBZ. To avoid this artificial effect caused by thresholding, pixels having intensities less than 15 dBZ—that is, in our case, no echo pixels in the
common area—are assigned a value of 15 dBZ. Note that in the present study, pixels having no rain in both fields are not considered in the computations of PDFs. The RMS deviation between the observed and fitted PDFs for the 13 events analyzed is shown in Table 2.

In all 13 cases, the Gaussian fits have a lower RMS deviation than both the lognormal and gamma distribution functional fits. In all three functional fits, the RMS deviation is larger when 13 cases are combined than the individuals. As the width of the distribution changes from case to case and thus combining all of the cases (last entry of Table 2) produces more variance and, in turn, RMS deviation around the fit than for individual cases. The PDFs of rainfall field observed during the 13 cases and the fitted Gaussian distribution functional form are shown in Fig. 3. The average of 24 PDFs for each case is shown in Fig. 3. The average of 24 PDFs for each case is shown in Fig. 3. From Fig. 3, it is clear that the PDF of radar reflectivity of rainfall field follows a Gaussian distribution with slightly varying mean and standard deviations, that is, the PDFs of the reflectivity associated with rainfall of mesoscale convective systems are reasonably stable. The result is in good agreement with the PDFs obtained from rainfall observed from space over large areas (Meneghini and Jones 1993).

The PDFs of $S$ for the 13 cases are shown in Fig. 4. As with the PDFs of rainfall intensities, the PDFs of growth and decay intensities approximately follow Gaussian distributions. The Gaussian fit to the observed PDFs of $S$ is also shown in Fig. 4 with the continuous line. The RMS deviation between the observed and fitted PDFs and the skewness of the observed PDFs of $S$ are tabulated in Table 3. Positive skewness is observed in all 13 cases. A poor selection of events—that is, selecting cases just at the beginning of the development phase but then terminating them long before the complete dissipation—could also produce this type of results. However, we have selected events in such a way that they include complete growth and decay phases to avoid such an effect. As shown in Table 3, the mean value of the $S$ field is $-0.11$ dBZ day$^{-1}$, indicating that the observations are biased toward the decay phase by 2.7% with respect to the growth phase. Thus, the observed positive skewness indicates that the growth phase is associated with more rapid changes than the decay phase. As seen from Fig. 2d, the dynamical and microphysical processes associated with the growth phase result into a more rapid variability than is the case with the decay phase, thus generating a long tail toward the distribution of larger growth.

b. Correlation structure

To generate the plausible ensemble members of the forecasts the information of spatial and temporal correlation structures of that particular fields are essential. The correlation structure provides the information of the order in the fields. Thus to know the correlation structure of the rainfall fields and growth and decay a two-dimensional autocorrelation function of the precipitation field and of growth and decay [in terms of dB(Z) units] is

![Fig. 3. Normalized probability distribution values as a function of radar reflectivity factor of rainfall field (dBZ) averaged over 24 h for 13 days and the mean distribution of all. Dots indicate observed probability distribution values, and the solid line represents the best Gaussian fit.](http://journals.ametsoc.org/jas/article-pdf/69/11/3336/3630851/jas-d-12-029_1.pdf)

![Fig. 4. As in Fig. 3, but for intensities of growth and decay.](http://journals.ametsoc.org/jas/article-pdf/69/11/3336/3630851/jas-d-12-029_1.pdf)
estimated with the help of fast Fourier transforms using the following formula

\[ C(P)_{(x,y)} = \text{IFFT}[\text{FFT}^C(P) \times \text{FFT}(P)], \quad (10) \]

where \( C(P)_{(x,y)} \) stands for 2D autocorrelation function of \( P \) at lag \( (x, y) \) and \( \text{FFT}^C \) represents the conjugate of 2D FFT. A two-dimensional exponential decaying correlation function is fitted to the outcome of the autocorrelation function in order to estimate the correlation lengths. Let \( a \) and \( b \) be the correlation lengths (where the correlation drops to \( 1/e \)) along major and minor axes, respectively, and \( \theta \) be the angle of orientation with respect to the positive \( y \) axis (north), then the exponentially decaying two-dimensional correlation function is given by

\[
C(P)_{(x,y)} = \exp \left\{ -\left[ \frac{(x \cos \theta + y \sin \theta)^2}{a^2} + \frac{(-x \sin \theta + y \cos \theta)^2}{b^2} \right] \right\}. \quad (11)
\]

The two-dimensional correlation of the rainfall fields and of the growth and decay patterns estimated from Eq. (10) and fitted with Eq. (11) for the 18 April 2008 case is shown in Figs. 5 and 6, respectively. The two-dimensional correlation structure of the rainfall field and of growth and decay is elliptical, which indicates that both the rainfall field and growth and decay are nonisotropic in space. The event observed on 18 April 2008 is a frontal system with a warm front that is almost stationary and a cold front that is evolving and translating with time. During the initial 12 h, the surface area of rainfall associated with the cold front increases and the warm front decreases. In contrast, an increase in the areal precipitation is observed over the entire system after 1200 UTC, which indicates that this event is strongly forced by the diurnal convection. The areal variation of rainfall associated with the microphysical, dynamical changes due to diurnal forcing alters the order of the rainfall and, in turn, the correlation structure that can be

### Table 3. RMS deviation between observed and fitted PDFs and skewness of the observed PDF.

<table>
<thead>
<tr>
<th>Event</th>
<th>RMS deviation</th>
<th>Skewness</th>
<th>Field mean ( \delta [\text{dB(Z)} \text{ day}^{-1}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 Apr 2008</td>
<td>0.013</td>
<td>1.65</td>
<td>0.118</td>
</tr>
<tr>
<td>6 May 2008</td>
<td>0.032</td>
<td>1.93</td>
<td>0.618</td>
</tr>
<tr>
<td>24 May 2008</td>
<td>0.029</td>
<td>1.58</td>
<td>−0.336</td>
</tr>
<tr>
<td>6 Jun 2008</td>
<td>0.025</td>
<td>1.57</td>
<td>−0.675</td>
</tr>
<tr>
<td>9 Jul 2008</td>
<td>0.027</td>
<td>1.37</td>
<td>−0.179</td>
</tr>
<tr>
<td>13 Jul 2008</td>
<td>0.024</td>
<td>1.36</td>
<td>−0.069</td>
</tr>
<tr>
<td>18 Jul 2008</td>
<td>0.030</td>
<td>1.44</td>
<td>−0.331</td>
</tr>
<tr>
<td>22 Jul 2008</td>
<td>0.026</td>
<td>1.14</td>
<td>0.279</td>
</tr>
<tr>
<td>24 Jul 2008</td>
<td>0.019</td>
<td>1.28</td>
<td>−0.428</td>
</tr>
<tr>
<td>30 Jul 2008</td>
<td>0.031</td>
<td>1.53</td>
<td>−0.12</td>
</tr>
<tr>
<td>10 Aug 2008</td>
<td>0.025</td>
<td>1.56</td>
<td>0.008</td>
</tr>
<tr>
<td>13 Aug 2008</td>
<td>0.025</td>
<td>1.65</td>
<td>−0.152</td>
</tr>
<tr>
<td>21 Aug 2008</td>
<td>0.024</td>
<td>1.7</td>
<td>0.12</td>
</tr>
<tr>
<td>All cases</td>
<td>0.045</td>
<td>1.51</td>
<td>−0.11</td>
</tr>
</tbody>
</table>

![Fig. 5](http://journals.ametsoc.org/journals/jas/article-pdf/69/11/3336/3630851/jas-d-12-029_1.pdf) Fig. 5. 2D spatial correlation of 1-h accumulations of rainfall field (dBZ) during different hours on 18 Apr 2008. Overlaid ellipse indicates the decorrelation distance to \( 1/e \).
seen from Figs. 5 and 6. Following the extent of areal rainfall, the spatial correlation also drops to smaller correlation lengths during the initial hours and vice versa at latter hours of the day. Thus, under the assumption of Lagrangian persistence, the correlation lengths and orientation of the correlation of the rainfall field and growth and decay vary with time.

The 2D correlation structure parameters of the precipitation field and of growth and decay—that is, the correlation lengths along the major and minor axes, the axial ratios, and orientation angle for the 13 case studies—are presented in Figs. 7 and 8, respectively. The correlation parameters are obtained by fitting Eq. (11) to the correlation functions estimated from Eq. (10). The correlation functions are obtained utilizing 1-h accumulated radar reflectivity fields. The correlation parameters of the precipitation field, presented in Fig. 7, show a high degree of variability in space with time during each case.
The dynamical and microphysical processes associated with the entire depth of the precipitating system produce variability in the surface precipitation, and in turn in the growth and decay, and alters the coverage of the rainfall from time to time. The diurnal variation of the correlation length suggests that the diurnal forcing also changes the coverage of rainfall. All of these processes introduce randomness into the rainfall fields. Variations in the coverage and degree of randomness of the rainfall and of growth and decay produce large variability in the correlation length from time to time, which are clearly represented in Figs. 7 and 8. Though the precipitating systems are forced by various large-scale mechanisms, the correlation parameters are not varying much from event to event.

The correlation length of radar reflectivities of rainfall field along the major (minor) axis varies from 40 (24) to 500 (156) km with a mean correlation distance of 128 (68) km, whereas the axial ratio varies from 1 to 5 with a mean value of 1.9. In contrast, the correlation length of growth and decay varies from 16 to 44 km with a mean value of 24 km along the major axis and from 12 to 36 km with a mean value of 16 km along the minor axis. The axial ratio varies from 1 to 2.5 with a mean value of 1.5. This indicates that the autocorrelation function of mesoscale rainfall fields forced by various large-scale mechanisms is elongated along the major axis by 1.5 times than the minor axis but varies with time due to the diurnal forcing. The degree of randomness is about 6 and 4 times more in the growth and decay field when compared to the rainfall field along major and minor axes, respectively. At some times, as seen in Figs. 5–8, the orientation of the correlation function of growth and decay differs from that of the rainfall field. The prevalence of the growth and decay process changes the internal structure of the correlation function and, in turn, the orientation axis, as indicated in Figs. 5 and 6. Thus, although the rainfall field and growth and decay possess similar PDFs, the characteristics of the correlation functions vary from each other.

5. Scale dependence of predictability of rainfall and of growth and decay fields

The nowcasting algorithms’ prediction accuracy is improved through extrapolating only the predictable scales. To generate nowcasts of dBZ values (for simplicity denoted by $Z$), the predictable low-frequency scales $Z_L$ (low-pass dBZ) are separated from those unpredictable smaller scales $Z_H$ (high-pass dBZ). Thus, $Z = Z_L + Z_H$. Then, to improve the accuracy of nowcasts, the term $S$ should be introduced into the existing extrapolated $Z_L$ field. To incorporate $S$ into $Z_L$, the predictability of $S$ at different scales should be known. Thus, the scale dependence of the predictability of growth and decay is examined using 1-h composite radar accumulations. Germann and Zawadzki (2002) studied the scale dependence of the predictability of precipitation with Lagrangian persistence using a variational echo-tracking technique. A similar approach is attempted here but instead of the variational echo-tracking method, we used the R-T method to retain the Lagrangian persistence.

Hourly accumulated rainfall maps observed with the U.S. radar network and the corresponding growth and
decay patterns in Lagrangian persistence are decomposed into scales by means of a low-pass filter in the spectral domain using discrete cosine transforms (DCT; Ahmed et al. 1974; Gonzalez and Woods 1992; Germann and Zawadzki 2002). The lifetime or predictable time (the time when the correlation drops to $1/e$) of the rainfall field and of the growth and decay patterns for different cases at various scales and the best linear fit to each case data are represented in Fig. 9. The average fit to all the cases is represented by the thick solid line.

As shown in Germann and Zawadzki (2002), the lifetime of precipitating systems increases when smaller scales are eliminated. Similar to the rainfall field, the lifetime of growth and decay also increases with the removal of the small scales. The predictability of small scales is considerably less for both rainfall and growth and decay patterns, and hence filtering these scales by spatial averaging improves the correlation and thus the predictability. The average fit for the lifetimes $T_{\text{Field}}$ and $T_S$ in hours, as a function of scale yields

$$T_{\text{Field}} = 0.05 \lambda + 4.54$$  \hspace{1cm} (12)$$

$$T_S = 0.0055 \lambda + 0.45,$$  \hspace{1cm} (13)$$

where $\lambda$ is the cutoff wavelength in kilometers. From Eqs. (12) and (13), the predictability of rainfall and of growth and decay increases linearly with increasing scale. Compared with the rainfall field, the lifetime as a function of scale is 10 times less for growth and decay. For four of the selected days, the lifetime of the rainfall fields exceed 20 h at scales greater than 200 km. The longevity of rainfall fields during these 4 days (18 April and 10, 13, and 21 August) is also seen from the forecast verification scores. Following Germann and Zawadzki (2002), probability of detection (POD), false-alarm rate (FAR), critical success index (CSI), and equitable threat score (ETS) are computed for 1-h accumulated radar reflectivity fields as a function of lead time in Lagrangian persistence and are shown in Fig. 10. In all cases, the verification scores are decreasing with increasing lead time; but, for four long-lived events the verification scores, after an initial decrease, maintain a nearly constant magnitude for a long lead time.

The scale-dependence analysis of growth and decay suggests that, by neglecting scales less than 250 km in Lagrangian persistence, growth and decay can be predictable up to a lead time of about 2 h. Thus, for continental-scale precipitating systems, large-scale information on growth and decay can be added to the extrapolated fields up to a lead time of 2 h. This result does not contradict past works, showing the limited skill of including growth and decay. Since past studies focused primarily on scales smaller than the range of single radar, on the order of 100–200 km, they correctly found that it was difficult to take growth and decay into account. However, when multiple radars are combined, including growth and decay has some limited skills for relatively short time periods. We expect this result to be applicable whichever nowcasting algorithm will be used.

A comparison of the predictability of $S$ obtained in Lagrangian persistence with the benchmark Eulerian is shown in Fig. 11; $S$ in Eulerian persistence is simply the mismatch between the radar observations separated by
time period \( t \). A similar analysis is carried out to illustrate the scale dependence of predictability of \( S \) in Eulerian persistence. The analysis shown in Fig. 11 illustrates that the predictability of \( S \) at various scales is lesser than in the Lagrangian by 7%. The average lifetime of \( S \) in Eulerian persistence as a function of scale is given by

\[
T_{S,\text{Euler}} = 0.0051 \lambda + 0.42. \tag{14}
\]

Thus, removing the motion field from the rainfall pattern does not improve much in terms of predictability. This shows that our results are not excessively dependent on the method of advection and consequently using MAPLE would lead to similar conclusions as with the R-T method. The overall conclusion is that growth and decay is pretty chaotic at the mesoscale.

6. Conclusions

In an attempt to gauge the possibility of including growth and decay successfully in nowcasting systems, the statistical properties and the predictability of growth and decay of the rainfall systems in the Lagrangian frame of reference over continental scales were evaluated. What we found were the following:

1) Similar to radar reflectivity of rainfall fields, the PDFs of growth and decay follow the Gaussian distribution functional form;

2) The PDFs of growth and decay are not symmetric and are skewed toward growth, indicating that the growth phase is associated with more rapid variations than the decay phase;

3) Growth and decay patterns tend to be slightly more isotropic than rainfall fields, with the major-to-minor-axis ratios of their autocorrelation function being on average 1.5 versus 1.9 for rainfall;

4) The spatial correlation length of growth and decay is about 5 times smaller than that of the rainfall field;

5) The predictability of growth and decay patterns increases linearly with scale. For scales larger than 250 km, growth and decay is predictable up to a lead time of about 2 h; and

6) The predictable time of the growth and decay is 10 times smaller than that of the rainfall field.

Based on the above-mentioned findings, we believe that, for networked radar data, slight improvements to the...
skill of nowcasts could be achieved by extrapolating growth and decay for up to a few hours.

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