Freezing of Raindrops in Deep Convective Updrafts: A Microphysical and Polarimetric Model

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ABSTRACT

Polarimetric radar observations of convective storms routinely reveal positive differential reflectivity $Z_{DR}$ extending above the 0°C level, indicative of the presence of supercooled liquid particles lofted by the storm’s updraft. The summit of such “$Z_{DR}$ columns” is marked by a zone of enhanced linear depolarization ratio $L_{DR}$ or decreased copolar cross-correlation coefficient $r_{hv}$ and a sharp decrease in $Z_{DR}$ that together mark a particle freezing zone. To better understand the relation between changes in the storm updraft and the observed polarimetric variables, it is necessary to first understand the physics governing this freezing process and the impact of freezing on the polarimetric variables.

A simplified, one-dimensional explicit bin microphysics model of stochastic drop nucleation by an immersed foreign particle and subsequent deterministic freezing is developed and coupled with an electromagnetic scattering model to explore the impact of the freezing process on the polarimetric radar variables. As expected, the height of the $Z_{DR}$ column is closely related to the updraft strength and initial drop size distribution. Additionally, the treatment of the stochastic nucleation process can also affect the depth of the freezing zone, underscoring the need to accurately depict this process in parameterizations. Representation of stochastic nucleation and deterministic freezing for each drop size bin yields better agreement between observations and the modeled vertical profiles of the surface reflectivity factor $Z_H$ and $Z_{DR}$ than bulk microphysics schemes. Further improvements in the representation of the $L_{DR}$ cap, the observed $Z_{DR}$ gradient in the freezing zone, and the magnitude of the $r_{hv}$ minimum may require inclusion of accretion, which was not included in this model.

1. Introduction

Within warm-season, deep convective midlatitude storms, both warm and cold microphysical processes occur that can be important for determining the precipitation characteristics of such storms. Warm rain–generated liquid drops can be carried above the environmental 0°C level by the storm’s updraft, where they subsequently freeze into ice pellets. These frozen drops can grow further via accretion of supercooled liquid cloud water (riming), transforming them into graupel. The graupel and frozen drops can serve as an important source of hailstone embryos (e.g., Knight and Knight 1970, 1974, 2001; Federer and Waldvogel 1978; Knight 1981; Ziegler et al. 1983; Nelson 1983). Graupel particles are also known to play an important role in lightning production through charge transfer and separation [e.g., MacGorman and Rust (1998), and references therein]. Therefore, lofting of liquid
hydrometeors and subsequent freezing at supercooled temperatures is a critical process in producing precipitation ice necessary for electrification and lightning production in convective storms (e.g., French et al. 1996; Jameson et al. 1996; Ramachandran et al. 1996; Bringi et al. 1997; Blyth et al. 1997; Carey and Rutledge 2000).

Dual-polarization radar observations in convective storms routinely reveal columnar regions of enhanced differential reflectivity $Z_{DR}$ that extend above the environmental freezing level, in some cases by as much as 2.5 km or more. Such ‘‘$Z_{DR}$ columns’’ (Fig. 1) are indicative of liquid or mixed-phase oblate hydrometeors being lofted above the freezing level by the convective storm updraft. Since the first observations of such signatures (Hall et al. 1984; Illingworth et al. 1987; Caylor and Illingworth 1987; Tuttle et al. 1989), they have been widely documented (e.g., Meischner et al. 1991; Herzegh and Jameson 1992; Conway and Zrnić 1993; Höller et al. 1994; Brandes et al. 1995; Hubbert et al. 1998; Kennedy et al. 2001; Loney et al. 2002; Ryzhkov et al. 2005; Kumjian and Ryzhkov 2008). At the top of $Z_{DR}$ columns, polarimetric radar observations typically reveal a sharp decrease in $Z_{DR}$ owing to a substantial decrease in the complex dielectric factor and increased tumbling of hydrometeors as they freeze into solid ice pellets. Coincident with this freezing zone is a decrease in the copolar cross-correlation coefficient $\rho_{hv}$, which is sometimes called the $\rho_{hv}$ hole (e.g., Kumjian and Ryzhkov 2008). For radars operating in the mode of alternate transmission and reception of horizontally- and vertically-polarized waves, the hydrometeors and processes contributing to the $\rho_{hv}$ hole are observed as a pronounced increase in linear depolarization ratio $L_{DR}$, called the $L_{DR}$ ‘‘cap’’ (e.g., Jameson et al. 1996; Bringi et al. 1997; Hubbert et al. 1998; Kennedy et al. 2001). Though large changes in $\rho_{hv}$ and $L_{DR}$ at the summit of $Z_{DR}$ columns have been associated with the wet growth of graupel and hail (e.g., Herzegh and Jameson 1992; Jameson et al. 1996; Kennedy et al. 2001; Picca and Ryzhkov 2012), changes that are smaller in magnitude may be explained by the increased tumbling and diversity of particle species present during the freezing process (Bringi et al. 1997; Hubbert et al. 1998). These signatures have been implicated in the role of hail formation and development (e.g., Kennedy et al. 2001; Picca and Ryzhkov 2012), and thus studying them may reveal important practical information.

Severe continental storms are not the only convective storms to feature $Z_{DR}$ columns; tropical and subtropical convective storms are known to produce tall $Z_{DR}$ columns as well. For example, Ramachandran et al. (1996) used polarimetric radar data and in situ observations to demonstrate that Florida storms with stronger updrafts promoted drop freezing at colder temperatures ($\approx -5^\circ$ to $-10^\circ$C), which led to electrification through ice processes. Similarly, French et al. (1996) demonstrated that an increase in ice concentration was associated with an increase in electrification in Florida convective clouds. Drop freezing and subsequent growth of ice hydrometeors atop $Z_{DR}$ columns was also found in Australian convective storms by Carey and Rutledge (2000) and May et al. (2001, 2002).

The evolution of $Z_{DR}$ columns and their capping signatures can also provide insight into the storm’s behavior. For example, the evolution of $Z_{DR}$ columns has been recently investigated (Picca and Ryzhkov 2010; Picca et al. 2010) and has proven useful in the short-term forecast of severe convective storms and hail fall. These preliminary studies found strong positive lagged correlations between changes in the integrated volume of positive $Z_{DR}$ values above the environmental freezing level (‘‘$Z_{DR}$ column volume’’) and changes in the surface reflectivity factor $Z_H$ 60-40-dBZ ratio on time scales of
about 15–20 min. Picca and Ryzhkov (2012) also found that a deeper minimum in the $\rho_{hv}$ hole atop the $Z_{DR}$ column was followed by an increase in hail size at the surface. Analogously, Kennedy et al. (2001) found that $L_{\text{DR}}$ maxima aloft preceded the largest hail at the surface.

Aircraft penetrations through $Z_{DR}$ columns confirm the inferences based on polarimetric radar data: that $Z_{DR}$ columns comprise liquid raindrops and small, wet graupel and hailstones (e.g., Brandes et al. 1995; Smith et al. 1999; Loney et al. 2002; Schlatter 2003; Clabo et al. 2009). These observations demonstrate the existence of liquid or mixed-phase hydrometeors at subfreezing temperatures. A notable exception is Théiault and Stewart (2009), whose winter microphysics scheme includes a “slush” category for partially frozen/melted hydrometeors. Instead, many schemes convert liquid raindrops directly into ice pellets once they are lofted above the freezing level to sufficiently cold temperatures (e.g., Wisner et al. 1972; Ziegler 1985); Milbrandt and Yau (2005), representing instantaneous freezing. Because of this, some bulk microphysics parameterization schemes struggle to reproduce realistic tall $Z_{DR}$ columns (e.g., Jung et al. 2008, 2010). For example, Jung et al. (2010) use a two-moment parameterization scheme to simulate supercell polarimetric signatures. Though they reproduce a $Z_{DR}$ column, its maximum height (~1.5 km above the environmental 0°C level) is significantly less than observed maximum column heights (~2.5–3.0 km above the environmental 0°C level). Such model inadequacies may limit the utility of numerical investigations of the possible prognostic capabilities of $Z_{DR}$ columns; namely, the link between their evolution and storm behavior that has been recently revealed (e.g., Picca et al. 2010; Picca and Ryzhkov 2010).

To fully explore and understand these apparent links between $Z_{DR}$ column evolution and storm behavior, we must first understand the physics governing the appearance of the columns. Whereas earlier studies have shown computations of the polarimetric variables for freezing drops without treatment of the microphysics (Bringi et al. 1997; Hubbert et al. 1998), or have treated the microphysics in a simplified manner without showing computations of the radar variables (e.g., Smith et al. 1999), the present study couples microphysics and electromagnetic-scattering calculations. To do this, a one-dimensional explicit bin microphysics and electromagnetic model is developed and used to quantify the polarimetric radar variables. Though lacking the generality of full three-dimensional full-physics models, this type of simplified modeling approach allows for a better understanding of the impact of the freezing process on the polarimetric radar variables in isolation from other processes. As mentioned above, bulk microphysics parameterization schemes often used in more general numerical weather prediction models inadequately describe the complete nucleation and freezing process, limiting their utility in investigations of $Z_{DR}$ columns. The layout of the rest of the paper is as follows. Section 2 provides an overview of the physics of raindrop freezing. The microphysics and electromagnetic model components are described in section 3. Section 4 presents results of the model, including sensitivity tests and comparisons with observations. The paper closes with a discussion and brief summary (section 5) of the main results.

2. Overview of the physics of freezing drops

In the atmosphere, supercooled liquid drops typically undergo heterogeneous nucleation because of the presence of impurities in the liquid water or aerosols in the air. Pruppacher and Klett (1997, hereafter PK97) describe four modes of heterogeneous nucleation: deposition, condensation, immersion, and contact. When the air is supersaturated with respect to ice, water vapor can be deposited directly as ice on the surface of a particle at temperatures below 0°C in the deposition mode. For situations where saturation with respect to water is attained, subfreezing ice nuclei may act as cloud condensation nuclei, where freezing occurs at some later stage during condensation. In the immersion mode, the foreign particle is submersed in the liquid drop at temperatures above 0°C, whereupon freezing occurs when the drop becomes sufficiently cold. In the contact mode, ice nuclei initiate nucleation instantaneously upon contact with a supercooled drop. The latter two modes are of interest for drop freezing in convective storms.

Heverly (1949), Bigg (1953a,b), Langham and Mason (1958), Barklie and Gokhale (1959), Pitter and Pruppacher (1973), and Vali (1971, 1994) among others have showed through observations that the median nucleating temperature $T_m$ of a population of drops is a function of the drop volume $V_d$. This can be expressed in terms of the supercooled median nucleating temperature $T_{sm} = T_0 - T_m$ (where $T_{sm} > 0$ and $T_0 = 0$°C):

$$T_{sm} = A - B \ln(V_d),$$  

(1)
where \(A\) and \(B\) are constants determined by the sample of water and its immersed foreign particles or impurities. There are two schools of thought on the process of how this freezing is initiated (PK97). In the “stochastic hypothesis,” it is assumed that, at a given temperature, all equal-sized ice embryos formed in a population of equal-sized supercooled drops will have an equal probability of reaching the critical size for freezing, as a result of random fluctuations among the water molecules. Immersed nuclei will enhance the freezing potential but otherwise will not affect the stochastic nature of freezing. In the other view, called the “singular hypothesis,” heterogeneous drop freezing is attributed entirely to the immersed nucleus with the warmest characteristic freezing temperature. In addition, the number of ice germs formed in a particular drop depends on the number of immersed nuclei in the drop that are activated. Though both hypotheses have some elements of experimental evidence supporting them, neither is complete by itself (PK97). Because the singular hypothesis requires knowledge of the number concentration of immersed nuclei in each drop (which is generally not known), the stochastic hypothesis is used in this study.

The freezing process occurs in two stages. The first stage generally is assumed to happen instantaneously upon nucleation, wherein dendritic ice crystal growth occurs in the core of the liquid drop, converting only a small portion of the liquid water into ice. The amount of ice formed depends on the environmental temperature as described quantitatively in the next section. The subsequent freezing occurs from the outside in, as an ice shell forms around the droplet and grows inward (PK97). The latter stage can take up to several minutes for the largest drop sizes (Fig. 2). The physical appearance of the drop (ice shell, with a core of liquid and ice) will be important for the electromagnetic scattering model presented in the next section.

Smith et al. (1999) investigated the freezing of supercooled raindrops atop the ZDR column in a convective storm. The authors hypothesized that the drop freezing contributed to the \(L_{\text{DR}}\) cap signature but did not show computations of the polarimetric radar variables. Though drop nucleation was treated in an overly simplistic manner, the study included the effects of accretion of supercooled water onto the freezing drops, conditions more realistic in convective storm updrafts. The heating of a particle due to accretion tends to prolong the freezing process. The authors also point out that remaining liquid water on the outside of freezing or frozen drops would also contribute to the increase in \(L_{\text{DR}}\) frequently observed in polarimetric radar data. The upgrade of National Weather Service Weather Surveillance Radar-1988 Doppler (WSR-88D) network will equip the radars with polarization diversity, but the radars will operate in the mode of simultaneous transmission and reception of horizontally and vertically polarized waves (e.g., Doviak and Zrnić 1993; Doviak et al. 2000). Therefore, \(L_{\text{DR}}\) measurements will not be available. Instead, \(\rho_{hv}\) is measured; this variable will be reduced (analogously to an enhancement of \(L_{\text{DR}}\)) for the same reasons as described by Smith et al. (1999). A diversity of hydrometeor types (ice pellets, partially frozen drops, and pure liquid drops) will also contribute to reducing \(\rho_{hv}\) atop \(Z_{\text{DR}}\) columns. In severe convective storms, Picca and Ryzhkov (2012) argue that significant reductions in \(\rho_{hv}\) atop \(Z_{\text{DR}}\) columns in the temperature region between \(-10^\circ\) and \(-20^\circ\)C indicate wet growth of giant (>5-cm diameter) hailstones, owing to strong non-Rayleigh-scattering effects. For the sake of simplicity, however, the current study will only consider the freezing of raindrops.

3. The model

a. Freezing model physics and equations

The model simulates the freezing of raindrops in a simplified, steady-state one-dimensional column “updraft.” Here, we couple an explicit microphysics model with electromagnetic scattering calculations to compute vertical profiles of the polarimetric radar variables, in contrast to previous efforts that treat either the microphysics or electromagnetics, but not both (e.g., Bringi et al. 1997; Hubbert et al. 1998; Smith et al. 1999). The impact of the stochastic nucleation and deterministic freezing processes on the polarimetric radar variables
are of interest in this study. Thus, we neglect the growth of frozen particles by accretion and its associated contributions to the heat budget. Growth by accretion tends to prolong the freezing process and may contribute to the $L_{DR}$ “cap” observed atop $Z_{DR}$ columns (e.g., Jameson et al. 1996; Smith et al. 1999) but is not needed to explain it according to Brungu et al. (1997) and Hubbert et al. (1998). Additionally, collisions between particles of different sizes, a process which expedites freezing, are neglected (e.g., Khain et al. 2000; Lynn et al. 2005). Specifically, particle collisions would lead to more rapid nucleation and freezing of smaller drops, sharpening the vertical gradient of $Z_{DR}$ at the top of the $Z_{DR}$ column. Future studies including all physical processes may elucidate the relative importance of particle collisions and contact nucleation versus lofting of drops and freezing in the immersion mode in the appearance of $Z_{DR}$ columns. The temperature profile is prescribed as moist adiabatic to emulate in-cloud conditions, following a pseudoadiabat from a typical warm-season sounding in Norman, Oklahoma. At the first grid level, which corresponds to a height level of about 5.1 km MSL and a pressure level of about 552 hPa on the sounding, the temperature is 0°C. The relative humidity profile is fixed at 99%. The updraft speed increases quadratically with height as

$$w(h) = \frac{w_0 - w_{\text{max}}(h_{\text{max}} - h)^2 + w_{\text{max}}}{h_{\text{max}}}$$

(2)

where all symbols are explained in Table 1. Integration occurs every 10 m, though data are only stored (and thus polarimetric radar variables are computed) every 100 m. Any distribution of raindrops can be placed at the bottom of the domain and is advected upward by the updraft. Though any DSD can be selected, the “default” drop size distribution (DSD) is in the form of the three-parameter gamma model (e.g., Ulbrich 1983):

$$N(D) = N_0 D^\mu \exp(-\Lambda D),$$

(3)

with $N_0 = 0.1$ m$^{-3}$ mm$^{-2}$, $\mu = 1$, and $\Lambda = 0.62$ mm$^{-1}$. These values were selected to match a DSD predicted at the bottom of a simulated updraft in the spectral bin microphysics Hebrew University Cloud Model [HUCM; e.g., see descriptions in Khain et al. (2004), (2011); the storm is from the simulation by Ryzhkov et al. (2011)]. Sensitivity to the initial DSD is explored in the next section. The drop sizes are partitioned into 80 bins (0.05–7.95 mm in 0.1-mm increments). Liquid raindrop velocities are given by the Brandes et al. (2002) polynomial function of diameter $D$ (mm) with a density correction factor (Foote and duToit 1969; Beard 1985)

$$v_{\text{rain}} = \left(\frac{\rho_0}{\rho_{\text{air}}}\right)^B (-0.1021 + 4.932D - 0.9551D^2 + 0.07934D^3 - 0.002362D^4),$$

(4a)

whereas the velocity of ice pellets is determined by an expression fitting the velocity of ice pellets in the HUCM (e.g., Khain et al. 2000, 2001, 2004):

$$v_{\text{ice}} = \left(\frac{\rho_0}{\rho_{\text{air}}}\right)^B (0.2259 + 1.5954D - 0.0405D^2).$$

(4b)

Owing to a lack of observational data, we simply assume that the fall speeds of partially frozen drops change linearly from those of pure liquid to those of pure ice particles, based on mass water fraction. Because the model is steady state, the flux of particles is conserved at each level.

The initial nucleation is assumed to be in the immersion mode. Various foreign particles can be selected for the immersed nuclei by choosing different values of $B$ in Eq. (1) (e.g., Barklie and Gokhale 1959; Diehl and Wurzler 2004). For the present study, we use the value for nuclei typical for rainwater ($B = 2.0 \times 10^{-4}$ cm$^{-3}$ s$^{-1}$) from Barklie and Gokhale (1959). One can express Eq. (1) in terms of the fractional change (per unit time) in the number of liquid drops yet to be nucleated $N_u$ following Bigg (1953b):

$$-\frac{1}{N_u} \frac{dN_u}{dt} = V_d B [\exp(\alpha T_s) - 1].$$

(5)

Note that Barklie and Gokhale (1959) found the mean value of $\alpha = 0.65$ °C$^{-1}$, independent of the water sample. Equation (5) shows that the magnitude of the rate of change of the number of unnucleated drops increases with drop volume $V_d$ and supercooling temperature $T_s(>0)$. In other words, larger drops nucleate faster than smaller drops, and colder temperatures produce more rapid nucleation of drops of any given size. Suppose one now introduces a cooling rate,

$$\gamma_c = -\frac{dT}{dt} = \frac{dT_s}{dt}.$$

(6)

Under such conditions, Eq. (5) can be written as (PK97)

$$-\frac{1}{N_u} \frac{dN_u}{dt} \frac{dt}{dT_s} = V_d B [\exp(\alpha T_s) - 1].$$

(7)

Thus, at a given $T_s$, more rapid cooling (larger $\gamma_c$) leads to fewer nucleations per unit time. Consider the implications for an updraft: in the Lagrangian framework of
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Constant determined by water sample impurities</td>
</tr>
<tr>
<td>a</td>
<td>Outer radius of the entire particle</td>
</tr>
<tr>
<td>B</td>
<td>Constant determined by water sample impurities</td>
</tr>
<tr>
<td>$c_w$</td>
<td>Specific heat capacity of water</td>
</tr>
<tr>
<td>D</td>
<td>Particle diameter</td>
</tr>
<tr>
<td>$D_a$</td>
<td>Minor axis of the spheroid</td>
</tr>
<tr>
<td>$D_b$</td>
<td>Major axis of the spheroid</td>
</tr>
<tr>
<td>$D_v$</td>
<td>Diffusivity of water vapor in air</td>
</tr>
<tr>
<td>$f = \sqrt{D_b^2/D_a^2 - 1}$</td>
<td>Function of aspect ratio that determines shape factor</td>
</tr>
<tr>
<td>$f_{a,b}$</td>
<td>Scattering amplitude along minor/major axis</td>
</tr>
<tr>
<td>$f_b$</td>
<td>Ventilation coefficient of heat</td>
</tr>
<tr>
<td>$f_r$</td>
<td>Ventilation coefficient of water vapor</td>
</tr>
<tr>
<td>$f_v$</td>
<td>Unfrozen fractional liquid volume following nucleation</td>
</tr>
<tr>
<td>$h$</td>
<td>Height above the melting layer</td>
</tr>
<tr>
<td>$h_{max}$</td>
<td>Height of the maximum updraft speed, $w_{max}$</td>
</tr>
<tr>
<td>$k_a$</td>
<td>Heat conductivity of air</td>
</tr>
<tr>
<td>$k_i$</td>
<td>Heat conductivity of ice</td>
</tr>
<tr>
<td>$L_{a,b}$</td>
<td>Shape factors for oblate spheroids</td>
</tr>
<tr>
<td>$L_{DR}$</td>
<td>Linear depolarization ratio (dB)</td>
</tr>
<tr>
<td>$L_m$</td>
<td>Latent enthalpy of freezing/melting</td>
</tr>
<tr>
<td>$L_r$</td>
<td>Latent enthalpy of sublimation</td>
</tr>
<tr>
<td>$N(D)$</td>
<td>Number concentration of particles of diameter $D$ to $D + dD$</td>
</tr>
<tr>
<td>$N_0$</td>
<td>DSD intercept parameter</td>
</tr>
<tr>
<td>$N_f$</td>
<td>Number concentration of drops of diameter $D$ that are nucleated</td>
</tr>
<tr>
<td>$N_T$</td>
<td>Total number concentration of drops of diameter $D$</td>
</tr>
<tr>
<td>$N_u$</td>
<td>Number concentration of drops of diameter $D$ yet to be nucleated</td>
</tr>
<tr>
<td>$r$</td>
<td>Inner radius of the growing ice shell</td>
</tr>
<tr>
<td>$T$</td>
<td>Air temperature</td>
</tr>
<tr>
<td>$T_0$</td>
<td>273.15 K</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Temperature at the surface of the drop</td>
</tr>
<tr>
<td>$T_m$</td>
<td>Median nucleating temperature</td>
</tr>
<tr>
<td>$T_s^{\prime}$</td>
<td>Supercooled temperature (&gt;0)</td>
</tr>
<tr>
<td>$T_{\text{sun}}$</td>
<td>Median supercooled temperature (&gt;0)</td>
</tr>
<tr>
<td>$T_w$</td>
<td>Ambient environmental temperature far from the particle</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$V_d$</td>
<td>Volume of raindrops</td>
</tr>
<tr>
<td>$v_{\text{ice}}$</td>
<td>Velocity of ice pellets</td>
</tr>
<tr>
<td>$v_{\text{rain}}$</td>
<td>Velocity of raindrops</td>
</tr>
<tr>
<td>$w$</td>
<td>Updraft speed</td>
</tr>
<tr>
<td>$w^{\prime}$</td>
<td>Updraft-modified vertical velocity of drop</td>
</tr>
<tr>
<td>$w_0$</td>
<td>Updraft speed at the $0^\circ$C level</td>
</tr>
<tr>
<td>$W_{\text{max}}$</td>
<td>Maximum updraft speed</td>
</tr>
<tr>
<td>$Z_{\text{DR}}$</td>
<td>Differential reflectivity factor (dB)</td>
</tr>
<tr>
<td>$Z_{\text{H}}$</td>
<td>Radar reflectivity factor at horizontal polarization (dBZ)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Constant = 0.65 $^\circ$C$^{-1}$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Density correction factor for velocity (0.4)</td>
</tr>
<tr>
<td>$\Gamma_w$</td>
<td>Lapse rate within updraft (moist adiabatic)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_c$</td>
<td>Cooling rate</td>
</tr>
<tr>
<td>$\epsilon_i$</td>
<td>Complex dielectric factor of ice</td>
</tr>
<tr>
<td>$\epsilon_{\text{w}}$</td>
<td>Complex dielectric factor of liquid water</td>
</tr>
<tr>
<td>$\epsilon_c$</td>
<td>Complex dielectric factor of the core</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Complex dielectric factor of the freezing particle</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>DSD slope parameter</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Radar wavelength</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Volume fraction of the inner spheroid (liquid core)</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>Reference density $= 1.2$ kg m$^{-3}$</td>
</tr>
<tr>
<td>$\rho_{\text{air}}$</td>
<td>Air density at a given height level</td>
</tr>
<tr>
<td>$\rho_{\text{hv}}$</td>
<td>Co-polar cross-correlation coefficient</td>
</tr>
<tr>
<td>$\rho_{\text{sat}}(T_w)$</td>
<td>Water vapor density at the particle surface</td>
</tr>
<tr>
<td>$\rho_{\text{sat}}(T_w)$</td>
<td>Saturated vapor density at the ambient environmental temperature</td>
</tr>
<tr>
<td>$\rho_{\text{w}}$</td>
<td>Ambient water vapor density</td>
</tr>
<tr>
<td>$\phi_w$</td>
<td>Relative humidity</td>
</tr>
<tr>
<td>$\left(\frac{d\rho_{\text{w}}}{dT}\right)_{\text{sat},i}$</td>
<td>Average slope of the saturated vapor density (over ice) as a function of temperature in the neighborhood of $T_w$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Resonance parameter</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Fractional increase in updraft speed</td>
</tr>
<tr>
<td>$R$</td>
<td>Thermodynamic factor in Eq. (17)</td>
</tr>
</tbody>
</table>

In the framework of our model, at a given height level (or model grid point), the updraft velocity $w$ is constant. Thus, we can integrate the differential equation (7) at each level:

$$\int_0^{T_s} \frac{1}{N_u} \frac{dN_u}{dT_s} dT_s = \int_0^{T_s} \frac{V B}{w \Gamma_w} \left[ \exp(\alpha T_s) - 1 \right] dT_s,$$

which after performing a change of variables on the left-hand side of the equation...
\[
\int_{N_T}^N d\ln(N_u') = \frac{V_d B}{w' T_w} \int_0^T \left[ \exp(\alpha T') - 1 \right] dT',
\]
results in
\[
-\ln\left( \frac{N_u'}{N_T} \right) = \frac{V_d B}{w' T_w} \left[ \frac{\exp(\alpha T_s)}{\alpha} - T_s \right],
\]
(9)
where \(N_T\) is the total number of drops. Exponentiating both sides, we obtain an expression for the number of drops (at a given height) not yet nucleated as a function of \(T_s\):
\[
N_u(T_s) = N_T \exp\left\{ -\frac{V_d B}{w' T_w} \left[ \frac{\exp(\alpha T_s)}{\alpha} - T_s \right] \right\},
\]
(10)
So, the fraction of drops that are nucleated at a given height is given by:
\[
\frac{N_f(T_s)}{N_T} = 1 - \exp\left\{ -\frac{V_d B}{w' T_w} \left[ \frac{\exp(\alpha T_s)}{\alpha} - T_s \right] \right\},
\]
(11)
which is shown schematically in Fig. 3. Thus, for a given \(V_d\), the fraction of drops that nucleate at a given height is governed by \(T_s\) at that height and the upward velocity of the drops. All else being equal, a fractional increase in the updraft strength \(z\) will increase the fraction of un-nucleated drops by a power of \(1/(1 + \zeta)\).1

Following PK97, we can use an approximate form of Eq. (9), assuming \(\exp(\alpha T_s) - 1 \approx \exp(\alpha T_s)\), to find \(T_{sm}\), or the temperature at which 50% of the drops have been nucleated. Substituting \(N_T/2\) for \(N_u\) and \(T_{sm}\) for \(T_s\) in this form of Eq. (9) and solving for \(T_{sm}\), we obtain:
\[
T_{sm} = T_0 - T_m = \frac{1}{\alpha} \ln\left( \frac{\alpha T_{sc} T_w}{V_d B} \right)
\]
(12)
Note that \(T_{sm}\) is dependent on the updraft velocity; for an increase in updraft strength by a factor of \(\zeta\), \(T_{sm}\) increases by \(\ln(\zeta)/\alpha\).

Only a portion of drops within a given size bin will nucleate at a particular temperature [cf. Eq. (11)], capturing the stochastic nature of drop nucleation. These packets of drops will follow their own unique “growth trajectories” for the remainder of their ascent. Thus, at any given height level, partially frozen drops in the same size bin may have progressed to different stages of freezing, depending on their unique growth trajectories. It is expected that this diversity of particle liquid water fractions at a given level in the mixed-phase region of the updraft will contribute to decreased \(\rho_{hv}\), as is commonly observed, without invoking significant contributions from non-Rayleigh scatterers. However, without entrainment or “recycling” of ice particles into the updraft (which is possible in higher-dimensional models and the real atmosphere), this effect may be underrepresented in our model.

The partially frozen drops are assumed to have an ice germ in their core, with the remaining unfrozen fractional liquid volume \(f_v\) surrounding the ice germ given by
\[
\frac{f_v}{L_m} = 1 - \frac{T_{sc} c_{w}}{L_m}
\]
(13)
following PK97. From this level, the drops continue their ascent in the updraft and begin to freeze according to PK97, who assume ventilated drops undergoing isotropic heat loss via conduction and evaporation to the environment, neglecting riming. The governing heat balance equations are as follows
\[
4\pi r^2 \rho_w r m \frac{dr}{dt} \left( 1 - \frac{T_{sc} c_{w}}{L_m} \right) = -4\pi k_r [T_0 - T_a(r)]
\]
and
\[
4\pi r^2 \rho_w r m \frac{dr}{dt} \left( 1 - \frac{T_{sc} c_{w}}{L_m} \right) = -4\pi \frac{k_r [T_0 - T_a(r)]}{a - r},
\]
(14)
\[ \frac{4\pi k_a r [T_0 - T_h(r)]}{a - r} = 4\pi a k_a [T_a(r) - T_h] \frac{\bar{f}}{T_h} + 4\pi a L_s D_v (\rho_{w} - \rho_{\text{mix}}) \frac{\bar{f}}{T_h}, \]

(15)

where \( a \) is the outer radius of the entire particle, \( r \) is the inner radius of the growing ice shell, and \( T_0 \) is the surface temperature of the particle. See Table 1 for a description of all other symbols. Because \( T_0 \) is generally unknown, we solve for it in Eq. (15) and use the following relation from PK97 to eliminate it:

\[ \rho_{w} - \rho_{\text{mix}} = (1 - \phi) \rho_{w,\text{sat}}(T_0) + [T_a(r) - T_\text{sat}(r)] \left( \frac{d\rho}{dT} \right)_{\text{sat}}, \]

(16)

Solving the remaining equations for \( dr/dt \), one obtains an expression for the growth rate of the ice shell:

\[ \frac{dr}{dt} = \frac{a k_a [T_0 - T_\text{sat}]}{\rho_{L} - \rho_{w,\text{sat}}(T_\text{sat}) \left( \frac{dT}{dr} \right)_{\text{sat}}} + \frac{L_s D_v (\rho_{w} - \rho_{\text{mix}})}{\rho_{L} - \rho_{w,\text{sat}}(T_\text{sat}) \left( \frac{dT}{dr} \right)_{\text{sat}}}, \]

(17)

where we have defined

\[ \bar{r} = k_a \frac{\bar{f}}{L_s D_v} \left( \frac{dT}{dr} \right)_{\text{sat}}. \]

(18)

By dividing both sides of the expression by the particle vertical velocity \( w' \), one obtains an expression that is used to compute the ice shell thickness at each height level. Once \( r \to 0 \), the particle is entirely frozen and is considered an “ice pellet.” After this transition, the ice pellet does not grow or interact with any other particles for the remainder of its ascent. In reality, accretion of supercooled liquid water may contribute to particle growth but is neglected here to focus on the isolated impact of freezing.

**b. Electromagnetic model**

The next portion of the model involves computing the complex scattering amplitudes of the particles as they ascend and freeze. To preserve the physics of freezing as described in the previous section, the hydrometeors are treated as two-layer spheroids, with an outer ice shell (the thickness of which is determined explicitly) surrounding an inner core consisting of a mixture of liquid water and ice. Note that the choice of the distribution of ice and water can strongly affect the scattering characteristics of particles (see the appendix for a discussion of the various ways to distribute liquid water on or in a particle and how it impacts the resulting computations of the polarimetric radar variables). The complex dielectric factors of water and ice \( \varepsilon_w \) and \( \varepsilon_i \) are functions of temperature and radar wavelength and are computed following the formulas in Ray (1972). The complex dielectric factor of the particle inner core \( \varepsilon_c \) is given by the Maxwell-Garnett (1904) formula for a mixture of water (as the background or “matrix”) and ice (as the “inclusions”) in order to represent the unfrozen liquid water and embedded ice germ. The radar wavelength is assumed to be \( \lambda = 10.97 \) cm (S band). The Rayleigh approximation to the scattering amplitudes of a two-layer spheroid is used, following Bohren and Huffman (1983):

\[ f_{a,b} = \frac{\pi^2 D^3}{6\lambda^2} \frac{(\varepsilon_i - 1)[\varepsilon_i + L_{a,b}(1 - \xi)(\varepsilon_c - \varepsilon_i)] + \xi\varepsilon_i(\varepsilon_c - \varepsilon_i)}{[1 + L_{a,b}(\varepsilon_i - 1)] + \xi L_{a,b}(\varepsilon_i - 1)}. \]

(19)

In Eq. (19), \( \xi \) is the volume fraction of the inner spheroid (i.e., the liquid–ice mixture) and \( L_{a,b} \) are shape factors for the oblate spheroids with major axis \( D_b \) and minor axis \( D_a \):

\[ L_a = \frac{1 + f^2}{f^2} \left( 1 - \tan^{-1} f \right), \quad L_b = \frac{1 - L_a}{2}, \]

(20)

where

\[ f = \sqrt{D_b^2/D_a^2 - 1}. \]

(21)

The axis ratios of all particles are defined for the individual diameters size bin by the Brandes et al. (2002) relation for raindrops:

\[ \frac{D_a}{D_b} = 0.9951 + 0.02510D - 0.03644D^2 + 0.005303D^3 - 0.0002492D^4, \]

(22)

where \( D \) is in mm. The S-band polarimetric radar variables are calculated from the scattering amplitudes for each hydrometeor species (rain, partially frozen drop, and ice pellet), following Ryzhkov (2001) and Ryzhkov et al. (2011). The particles are assumed to have a mean canting angle of 0° (e.g., Saunders 1971; McCormick and Hendry 1974; Brussard 1976; Beard and Jameson 1983; Ryzhkov et al. 2002), with a canting angle distribution width \( \sigma \) dependent on the mass water fraction of the particle: \( \sigma = 10° \) for pure rain, increasing to \( \sigma = 40° \) for...
pure ice pellets linearly as a function of mass water fraction (e.g., Ryzhkov et al. 2002, 2009, 2011). Note that there is uncertainty in this parameterization of $\sigma$ for mixed-phase particles because of a lack of observations, and that $\sigma$ can affect the polarimetric variables (especially for particles with larger complex dielectric factor). Because Eq. (19) is valid only for the Rayleigh condition, one must determine the appropriate range of diameters for which the Rayleigh approximation is valid at C and X bands using the resonance parameter

$$\mathbf{R} = D|\epsilon|^{1/2}/\lambda.$$  

(23)

When the Rayleigh approximation is no longer valid, one must use more sophisticated scattering calculations, such as the T-matrix method (e.g., Mishchenko 2000). In addition to the Rayleigh approximation, we have computed the polarimetric radar variables for selected model runs using a T-matrix method similar to the one used by Aydin and Zhao (1990). Only selected model runs were converted to radar variables using the T-matrix method because it is computationally expensive as there are numerous ($>100,000$) possible particles owing to the various growth trajectories. The computational parameters are treated as before, with raindrops and ice pellets treated as oblate spheroids (composed of pure water and pure ice, respectively) and partially frozen drops treated as two-layer spheroids, with an outer shell made of ice and the inner core a mixture of water and ice. The Rayleigh approximation results (at S band) agree well with the T-matrix calculations. The Rayleigh-computed profiles of $Z_{DR}$, $Z_{H}$, $\rho_{hv}$, and $L_{DR}$ are within about $1\,\text{dBZ}$, $0.1\,\text{dB}$, $0.0005$, and $0.5\,\text{dB}$, respectively, of the T-matrix profiles.

4. Model results

a. Sensitivity tests

Figure 4 is a four-panel display of vertical profiles of the polarimetric radar variables based on model calculations with varying updraft intensities. It is evident that the shape of the $Z_{H}$ profile (Fig. 4a) is most strongly affected by the updraft intensity, as this variable is dependent on the number concentration of particles. This is because the model is steady state, so the number flux of particles is conserved at each level. The shape of the $Z_{H}$ profile reflects this inverse proportionality to the updraft intensity: larger $w$ corresponds to smaller $Z_{H}$. Superposed on this shape is an approximately 7 dBZ decrease owing to the difference in refractive index between liquid water and solid ice (e.g., Smith 1984) as drops nucleate and freeze.

The impact of updraft intensity on $Z_{DR}$, $\rho_{hv}$, and $L_{DR}$ is much less apparent, owing to their independence from total number concentration. A striking feature in the $Z_{DR}$ profiles (Fig. 4b) is a zone marked by a sharp decrease in values from over $4\,\text{dB}$ at heights below about $1200\,\text{m}$ above the melting layer (AML) to $0.4\,\text{dB}$ at heights above $3000\,\text{m}$ AML. This zone with a sharp decrease in $Z_{DR}$ is coincident with the approximately $7\,\text{dBZ}$ decrease in $Z_{H}$ and is what we will be referring to as the “freezing zone.” Note that the location of this freezing zone is shifted upward slightly with stronger updrafts. To eliminate the effects of updraft shape, sensitivity tests were performed using a constant updraft speed $w$ throughout the domain and are discussed below.

The vertical profile of $\rho_{hv}$ (Fig. 4c) reveals a minimum located at the bottom of the freezing zone, followed by an increase within the freezing zone. Like $Z_{DR}$, the height of these features increases with increasing updraft strength. The reason for the minimum $\rho_{hv}$ is the coexistence of a mixture of particle species in the larger size bins: pure liquid raindrops, solid ice pellets, and partially frozen drops that have progressed through various stages in the freezing process are all collocated at the same level (Fig. 5). Because all particles have diameters less than $1\,\text{cm}$, reductions in $\rho_{hv}$ at $S$ band are small. At C and X bands, reductions may be larger in magnitude. Additionally, larger particles and/or wet growth of ice particles probably are necessary for substantial $\rho_{hv}$ reductions at all radar wavelengths. Similar to the profile of $\rho_{hv}$, the profile of $L_{DR}$ (Fig. 4d) reaches its maximum at the freezing zone, after which it decreases sharply as the particles become entirely ice. Note that the large maximum ($>5\,\text{dBZ}$ enhancement) in $L_{DR}$ ($L_{DR}$ “cap”) often observed in deep convective storms is not reproduced, which suggests that larger particles and/or wet growth is required to reproduce this type of feature (e.g., Jameson et al. 1996; Smith et al. 1999). In other words, freezing itself is not sufficient for producing the largest observed magnitudes of $L_{DR}$ ($>20\,\text{dB}$) in convective storm updrafts, though it can explain more modest enhancements.

An important assumption of previous studies relating changes in the $Z_{DR}$ column to changes in storm intensity (e.g., Picca et al. 2010; Picca and Ryzhkov 2010) is that the changes in the $Z_{DR}$ column height are because of changes in updraft intensity. Though intuitive, this relation has not been quantified. In the next set of idealized calculations, the updraft intensity is varied, but the updraft speed is held constant in height to remove the effects of the shape of the updraft profile. The vertical extent of the $Z_{DR}$ column is quantified using the height of the freezing zone above the melting layer. The height of the freezing zone is determined by the height of the
maximum absolute value of the vertical gradient of $Z_{DR}$. Results of these tests (Fig. 6) demonstrate an increasing freezing zone height (meaning a “taller” $Z_{DR}$ column) with increasing updraft speed. This indicates the use of $Z_{DR}$ column height as a potential metric for estimating updraft speed, at least for weaker updrafts (10–20 m s$^{-1}$) where the sensitivity is much higher. This is because these weaker updrafts yield very slow ascent speeds for the largest raindrops, allowing ample time for them to nucleate and freeze at lower heights. The leveling off at higher updraft speeds indicates a limitation of using this type of metric for estimating updraft intensity in stronger storms. It is not known how the effects of accretion (which become increasingly important for stronger updrafts) will alter the results for the larger updraft speeds.

To investigate the sensitivity of the results to the initial DSD, we varied the initial DSD in three ways. First, the value of the shape parameter $\mu$ is varied while holding the slope parameter $\Lambda$ constant at the default value (0.62 mm$^{-1}$). The height of the freezing zone, given by the maximum absolute value of the vertical $Z_{DR}$ gradient, does not change with shape parameter (Fig. 7a). Aside from the initial $Z_{DR}$ in rain increasing with increasing $\mu$, the impact on the vertical profile of $Z_{DR}$ is negligible. In contrast, varying $\Lambda$ (while holding the shape parameter constant at the default value of 1.0) produces noticeable changes in the vertical profile of $Z_{DR}$ (Fig. 7b). Namely, the height of the freezing zone increases with increasing $\Lambda$, and the absolute value of the vertical $Z_{DR}$ gradient becomes smaller with increasing $\Lambda$. An increase in $\Lambda$ corresponds to a decrease in the median raindrop size. Thus, DSDs characterized by smaller drop sizes will produce slightly taller $Z_{DR}$ columns owing to the delayed nucleation and freezing of smaller drops.

**Fig. 4.** Vertical profiles of (a) $Z_H$, (b) $Z_{DR}$, (c) $\rho_{hv}$, and (d) $L_{DR}$ computed from output of the freezing model, shown in meters above the melting level (AML). The model parameters used for the calculations include the default DSD with varying updraft maximum intensity: 19 (solid curves), 25 (dashed curves), and 30 m s$^{-1}$ (dotted curves). These calculations are for S band, employing the Rayleigh approximation. In each case, $w_0 = 15$ m s$^{-1}$, and $h_{max} = 3$ km. The black arrows represent what we call the “freezing zone.”
Also, the gradient of $Z_{\text{DR}}$ in the freezing zone is weaker for the smaller-drop DSDs. These results suggest that storms with substantial warm cloud depth (i.e., cloud beneath the environmental $0^\circ$C level) in which greater coalescence growth of drops can occur may have shorter $Z_{\text{DR}}$ columns for a given updraft strength than those storms with less warm cloud depth and thus limited coalescence growth.

Some have argued that $\Lambda$ and $\mu$ are not independent, suggesting that a relation exists between $\Lambda$ and $\mu$ (e.g., Zhang et al. 2001, 2003; Cao et al. 2008). If we repeat the sensitivity analysis by varying $\Lambda$ and adjusting $\mu$ to the value given by the $\Lambda$-$\mu$ relation of Cao et al. (2008), the results are nearly identical to varying $\Lambda$ and holding $\mu$ constant (not shown). In other words, changing $\Lambda$ of the initial DSD produces the most significant effect on the simulated $Z_{\text{DR}}$ profiles.

The model sensitivity to the nucleation scheme is explored next (Fig. 8). Here, the “default” settings correspond to those outlined in the model description above, containing the size-dependent $T_m$ as well as a probability distribution function (PDF) of nucleating temperatures to capture the stochastic nature of drop nucleation [cf. Eq. (11)]. First, the drop-size dependency of $T_m$ is removed, and all drops are assigned $T_m = -9.6^\circ C$. This corresponds to $T_m$ of the largest drop size bin in the default settings. Using this scheme (which still includes the PDF and thus stochastic effects) results in a sharper $Z_{\text{DR}}$ gradient in the freezing zone, leading to complete freezing of the drops about 400 m lower than the default settings. The difference between this scheme and the default nucleation setting is most evident at the top of the freezing zone, which is mostly affected by the freezing of smaller drop sizes; in this case, the smaller drops are nucleated (and thus begin the freezing process) at lower heights. The difference is smallest at the bottom of the freezing zone, where the largest drops are the most significant contributors to $Z_{\text{DR}}$.

Next, the stochastic nucleation process is turned off so that all drops in each size bin nucleate at the same $T_m$. This causes a slightly sharper $Z_{\text{DR}}$ gradient. Note that in this case, the $Z_{\text{DR}}$ does not begin decreasing until a higher level (1600 m above the melting layer). This is because the larger drops do not nucleate at temperatures warmer than $T_m$, in contrast to the stochastic nucleation simulations. However, after nucleation, drop freezing still takes place at the same rate. Thus, the entire freezing process occurs within a 1100-m-deep layer. If all drops were to freeze instantaneously at a given $T_m$, the resulting $Z_{\text{DR}}$ profile would be a step function (i.e., no freezing zone). Therefore, it is clear that the treatment of the stochastic nucleation process can also affect the depth of the freezing zone—that is, the gradient of $Z_{\text{DR}}$ atop $Z_{\text{DR}}$ columns.

In addition to the factors affecting the characteristics of the freezing zone discussed above, we see from Eq. (11) that the PDF governing drop nucleation is dependent on the parameter $B$. The parameter $B$, which represents the chemical properties of the immersed nucleus in the drops, can vary by several orders of magnitude depending on the type of immersed material (e.g., Diehl and Wurzler 2004). Numerical experiments

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Fig. 5. Vertical profiles of the number concentration ($m^{-3}$) of the sixty-first size bin corresponding to a liquid drop of size 6.05 mm. The concentration of raindrops is shown by the black solid line, partially-frozen drops (or “slush”) by the dashed dark gray line, and completely frozen ice pellets by the dotted gray line. The default DSD and updraft with $w_{\text{max}} = 25$ m s$^{-1}$ are used, with $w_0 = 15$ m s$^{-1}$ and $h_{\text{max}} = 3$ km.

Fig. 6. Height of the freezing zone above the melting level as a function of the updraft speed. The default DSD is assumed. For these idealized calculations, the updraft profile is constant throughout the domain, varying from 11 to 50 m s$^{-1}$.
were performed assuming the values for rainwater, leaf material, pollen, and various mineral dusts (not shown). Though the shape of the $Z_{\text{DR}}$ profile was not significantly altered by varying $B$, the height of the freezing zone was affected. Smaller values of $B$ lead to fewer drop nucleations at a given height level (or, equivalently, to colder $T_m$, cf. Equation 12), which causes the freezing zone to appear at higher altitudes. The freezing zone height increases by about 400 m for every order of magnitude decrease of $B$, demonstrating how immersed materials characterized by smaller $B$ are less efficient at inducing drop nucleation at relatively warmer temperatures. Note that stronger updrafts also can be thought of in terms of increasing $T_{sm}$, leading to higher freezing zones.

b. Comparison with observations

The updraft-freezing model predicts the vertical profiles of the polarimetric radar variables within and above $Z_{\text{DR}}$ columns, but how well do these model predictions match the observations? The cross section in Fig. 1 of the prominent $Z_{\text{DR}}$ column was reconstructed from several elevation-angle sweeps and thus involves some degree of interpolation. Instead, genuine vertical cross-section scans [or range–height indicators (RHIs)] are desirable for direct comparisons with the model. Such data were collected on the afternoon of 24 April 2011 by the C-band University of Oklahoma Polarimetric Radar for Innovations in Meteorology and Engineering (OU-PRIME; see Palmer et al. 2011). Data were collected using 0.1° elevation-angle spacing. The melting layer is evident in the data at about 2.7 km AGL (Fig. 9). At a range of about 29 km, a storm cell displays a pronounced $Z_{\text{DR}}$ column that extends above the environmental 0°C level. Vertical profiles extracted from this storm cell are compared to model output (Fig. 10). The initial DSD was selected to approximately match the $Z_H$ and $Z_{\text{DR}}$ at the freezing level using the $\Lambda-\mu$ relation of Cao et al. (2008), with values $N_0 = 1000 \text{ m}^{-3} \text{ mm}^{-0.462}$, $\Lambda = 1.35 \text{ mm}^{-1}$, and $\mu = -0.537$, with $w_{\text{max}} = 14 \text{ m s}^{-1}$ at $h_{\text{max}} = 5.0 \text{ km}$.
Data were collected on 24 June 2011, with 0.1° spacing in elevation, when isolated convective storms were affecting the area. The nearest rawinsonde observations from Essen (approximately 90 km to the northwest) at 1200 UTC show the environmental freezing level at about 1.8 km AGL (not shown). Vertical profiles extracted from these RHI scans compare favorably with the modeled $Z_{DR}$ profile, using $A = 1.4$ mm$^{-1}$, $\mu = -0.495$, $w_{\text{max}} = w_0 = 12$ m s$^{-1}$, and $B = 2.0 \times 10^{-2}$ (Fig. 12). Again, the gradient of $Z_{DR}$ (especially at the top of the observed freezing zone) is sharper than the modeled profile, possibly owing to particle interactions being neglected in the model.

Despite the simplicity of the model employed in this study, it is able to reproduce key features in the upper portions of $Z_{DR}$ columns—namely, an accurate height above the melting level, similar gradient of $Z_{DR}$ in the freezing zone, and correct $Z_{DR}$ values within and above the column. Additionally, the simulated $Z_H$ profile is in general agreement with the observed profiles, notwithstanding the uncertainty in the actual updraft profile. This implies that, despite neglecting certain processes, the key physics governing the appearance of the top of $Z_{DR}$ columns are captured in our implementation of the stochastic nucleation process and subsequent explicit treatment of the deterministic freezing of drops.

5. Discussion and summary

The major characteristics of $Z_{DR}$ columns include the values of $Z_{DR}$ within and above the columns, the vertical extent of the column above the melting level, and the properties of the freezing zone (i.e., the gradient of $Z_{DR}$, $\rho_{hv}$ minimum, $Z_{DR}$ maximum, etc.). The theoretical model presented in this study is capable of reproducing many of these observed features. The $Z_{DR}$ values within and above the column can be matched with an initial DSD. The impact of updraft intensity on the vertical extent of the $Z_{DR}$ column is quantified, demonstrating that stronger updrafts lead to taller $Z_{DR}$ columns. The $Z_{DR}$ gradient within the freezing zone is in reasonable agreement with observed $Z_{DR}$ columns, indicating that the important physical processes (stochastic nucleation followed by deterministic freezing) are captured in the modeled framework. Discrepancies between the modeled and observed profiles (observations show a sharper $Z_{DR}$ gradient) are instructive, as they may indicate the role of particle interactions in expediting the freezing process, especially for the smaller drops. Such a process expedites the freezing process by promoting nucleations via the contact mode at relatively warmer temperatures, particularly for smaller and medium drop sizes for which fall speed differences are largest. A $\rho_{hv}$ minimum is
produced in the freezing zone, even using the Rayleigh approximation for calculations. This $r_{hv}$ reduction is due to a mixture of different particle types (liquid drops, partially frozen drops, and ice pellets) collocated at the same level (cf. Fig. 5), as well as the gradual increase in the width of the distribution of hydrometeor canting angles as freezing progresses. The latter also leads to an increase in $L_{DR}$. In convective storms containing larger particles, growth by accretion serves to further decrease $r_{hv}$ and increase $L_{DR}$ (Jameson et al. 1996; Bringi et al. 1997; Hubbert et al. 1998; Smith et al. 1999; Picca and Ryzhkov 2012). More general models with larger particles and/or those that account for wet growth should produce enhanced $L_{DR}$ and $r_{hv}$ extrema that are in better agreement with observations.

The vertical extent of $Z_{DR}$ columns above the melting level is directly related to updraft strength in two ways. First, stronger updrafts lead to reduced median nucleating temperatures $T_m$, causing drops to nucleate at relatively colder temperatures. Second, the faster ascent velocity of lofted drops allows the drops to ascend farther before total freezing occurs, in effect reducing the freezing rate. Thus, strong updrafts may loft particles to heights of about 3 km above the updraft-perturbed 0°C level before complete freezing, in agreement with numerous observed cases. In addition, strong updrafts produce positive temperature perturbations owing to the release of latent enthalpy during condensation; for example, simple parcel theory considerations reveal parcels reaching 0°C at heights 1 km or more above the environmental 0°C level in cases of large (>3000 J kg$^{-1}$) CAPE. Therefore, in extreme cases, one may expect $Z_{DR}$ columns to extend as much as 4 km or more above the environmental melting level (cf. Fig. 1). Additionally, the DSD sensitivity analysis suggested that distributions characterized by smaller drop sizes may have taller $Z_{DR}$ columns for a given updraft strength than those characterized by larger drop sizes.

The height of the freezing zone also increases with decreasing parameter $B$, which describes the impact of the chemical properties of the immersed material on the median nucleating temperature. According to values found in Barklie and Gokhale (1959) and Diehl and Wurzler (2004), the parameter $B$ can vary over several orders of magnitude, depending on the immersed foreign particle. For every order of magnitude decrease of $B$, the height of the freezing zone increases by about 400 m. Within a convective storm updraft, such an
increase corresponds to a decrease in temperature on the order of 2–3°C, in agreement with Wisner et al. (1972). Because the formation and growth of hailstones is sensitive to environmental temperature and the amount of supercooled liquid water and ice present (e.g., PK97; Nelson 1983; Ziegler et al. 1983), such changes in height (and temperature) of the freezing zone may have an impact on these hail growth

![Graphs showing vertical profiles of Z_{DR} and Z_H](image)

**FIG. 10.** (a) Fifteen observed vertical profiles of $Z_{DR}$ extracted from the RHI in Fig. 9, from the $Z_{DR}$ column centered at a range of about 29 km (gray lines), compared to the model $Z_{DR}$ profile (thick black curve). (b) As in (a), but $Z_H$ profiles are shown.

![Graphs showing vertical cross sections of Z_H and Z_{DR}](image)

**FIG. 11.** Vertical cross sections of (left) $Z_H$ and (right) $Z_{DR}$ taken along the 309.5° azimuth at 1034 UTC 24 Jun 2011 by the X-band BOXPOL radar. Data are courtesy of the Meteorological Institute of the University of Bonn. The $Z_{DR}$ column of interest is centered at about 23-km range.
processes. There have been recent indications that the concentration of aerosols may affect hail growth (e.g., Khain et al. 2011); here, we speculate that in addition, the type of aerosols that become immersed in liquid raindrops that are lofted above the environmental freezing level may also play a role in the hail growth process. For example, immersed particles leading to freezing of drops at relatively warmer temperatures would allow more efficient growth into graupel particles by riming, whereas delayed freezing until colder temperatures may result in a larger relative proportion of frozen drops being lofted into the prime hail-growth region of \(2 \times 10^8\) to \(2 \times 10^9\) C. Incorporating a probabilistic nucleation and deterministic freezing scheme into a more general three-dimensional storm-scale numerical model will help verify or refute this speculation.

The ability of the simplified modeling approach to reproduce key observed features in \(Z_{DR}\) columns improves confidence in our understanding of the dominant processes involved in raindrop freezing and provides a simple yet powerful tool for exploring these processes. We confirm earlier inferences from observational studies that the depth of the layer above 0°C in which particles freeze, effectively the \(Z_{DR}\) column height, is linked to updraft strength (e.g., Hallett et al. 1978; Illingworth et al. 1987; Tuttle et al. 1989; Brugi et al. 1997). We found a strong relation between the modeled \(Z_{DR}\) column height and updraft speed for updrafts weaker than 20 m s\(^{-1}\), which may be used as a metric for storm intensity pending empirical confirmation. Future research should investigate the role of accretion and wet growth on the \(Z_{DR}\) column height, particularly for stronger updrafts. Appropriate treatment of raindrop nucleation and freezing is a necessary precursor to employing more general three-dimensional numerical weather prediction models in investigations of \(Z_{DR}\) columns, especially those aiming to elucidate the suggested link between storm behavior and the evolution of \(Z_{DR}\) columns.

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APPENDIX

Scattering Calculations of Mixed-Phase Particles

For a particle of a given shape and size, the distribution of liquid water on or in the particle can significantly affect its scattering characteristics, which then affects the dual-polarization radar variables. Thus, the choice of how this is represented in the scattering component of a microphysical model should accurately reflect the physics. For example, in the present study, the choice was made to reflect the physical appearance of drops undergoing freezing. However, other particle models can be used in other situations. For S, C, and X bands, we calculate the \(Z_{DR}\) of an oblate, 6-mm hydrometeor (with the axis ratio of a raindrop of that size), varying the liquid water volume fraction from 0 to 1.0 for a variety of liquid water distributions: ice coated with a water core, spongy with ice as the matrix and water as the inclusions, water coated with an ice core, and spongy with water as the matrix and ice as the inclusions. These different distributions of water and ice can be used to model different particle types. For example, ice-coated water is
similar to freezing drops; the spongy mixtures can be used for melting low-density ice particles (hail, graupel, snow); the water-coated ice is used for melting of high-density hailstones. Note that the spongy particles are treated as mixtures, with the complex dielectric factors calculated following Maxwell-Garnett (1904). Calculations are performed using a T-matrix code.

Figure A1 displays the results of these calculations. Aside from agreeing at liquid water volume fractions of 0.0 and 1.0, the differences between the different models are quite substantial. At S band (Fig. A1a), there is a nearly 2-dB spread of $Z_{DR}$ values for liquid volume fractions of about 15%–35%, with the water-coated particle providing the largest $Z_{DR}$ values, and the ice-coated particles the smallest. The behavior is more complex at shorter wavelengths; at C band, the spread of $Z_{DR}$ values for about 40% liquid volume fraction is nearly 3 dB, and the curves begin to intersect at 60% liquid volume fraction. The behavior at X band is even more complicated, with the curves intersecting just above 20% liquid volume fraction. Also note the peak (>7 dB) caused by resonance effects in spongy particles with ice as the matrix, which exceeds all other models (including water coated) by nearly 3 dB.

Thus, it is clear that the choice of how liquid water is distributed in mixed-phase particles can strongly affect the polarimetric radar variables. The choice should accurately reflect the relevant microphysics. In the present study, freezing drops are represented as two-layer spheroids, with an outer shell of ice and an inner core mixture of water and ice, as this closely resembles the physical appearance of drops undergoing freezing (Fig. A2). For shorter radar wavelengths, the Rayleigh approximation is no longer valid, requiring use of more sophisticated (e.g., T matrix) electromagnetic-scattering
calculations to appropriately handle resonance scattering effects.

REFERENCES


