Hypercooling in the Atmospheric Boundary Layer: Beyond Broadband Emissivity Schemes

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ABSTRACT

In an accompanying paper by Ponnulakshmi et al., the prevailing flux emissivity scheme for nonblack surfaces was shown to include an erroneous reflected flux term. The error leads to a spurious cooling within the opaque bands; an expression for the correct broadband-reflected flux was given that eliminated this spurious cooling contribution. Herein, it is shown that the error is generic in nature, and is relevant to any frequency-parameterized radiation scheme applied to nonblack surfaces; such schemes are typically used in longwave radiation budget calculations. The correct reflected flux, previously developed within the framework of a broadband emissivity scheme in Ponnulakshmi et al., is generalized here so as to be applicable to any frequency-parameterized radiation model. The error is illustrated by comparing the bandwise fluxes, obtained using the prevailing and correct narrowband formulations, for a model (tropical) atmosphere. The flux discrepancy is the smallest for opaque bands within which the participating medium emits like a blackbody, and it is largest in frequency intervals where the medium is nearly transparent.

1. Introduction

Line-by-line calculations resolve the finest structure of the spectrum (Ellingson et al. 1991) and serve as a benchmark for other coarse-grained models. In the interest of computational efficiency, one resorts to frequency parameterization, leading to narrowband formulations that are typically used in calculations involving the longwave radiation budget. The fluxes in such a formulation are obtained by integrating the monochromatic fluxes over a frequency interval much larger than the average interline spacing but smaller than the length scale of variation of the Planck function, and are given by (Liou 2002)

$$F_j^{\uparrow}(u) = \pi B_j(T_g)\tau_j^f(u) - \int_0^u \pi B_j[T(u')]\dot{\tau}_j^f(u-u')\,du', \qquad (1)$$

$$F_j^{\downarrow}(u) = -\int_u^{u_t} \pi B_j[T(u')]\dot{\tau}_j^f(u'-u)\,du', \qquad (2)$$

for an atmosphere of height u_t above black ground ($\epsilon_{gj} = 1$). Here, B_j is the (approximately constant) Planck function in band j and the overdot denotes differentiation with respect to the argument. The net flux is $\mathbf{F}(u) = \sum_{j=1}^{M}[F_j^{\uparrow}(u) - F_j^{\downarrow}(u)]$, with M being the number of bands. The diffuse transmission function $\tau_j^f(u)$ in (1) and (2) is a band-averaged transmittance that, with the aid of statistical band models, is expressible in terms of averaged line characteristics (Goody 1964).

Earlier studies have employed narrowband formulations to study the evolution of both diurnal (Savijärvi 2006) and nocturnal boundary layers (NBL; Duynkerke 1999; Savijärvi 2009); the formulation is sometimes used as a benchmark to test the validity of simpler models (Rodgers and Walshaw 1966; Ramanathan and Downey 1986). The purpose of this paper is to point out the error that arises in extending (1)–(2) to a nonblack surface (ground). The error is due to an incorrect reflected flux, and it arises from not discriminating between the spectral content of ground emission and the atmospheric column emission reflected from the ground. It manifests as an intense cooling close to ground, the intensity being a function of the vertical resolution used in a given calculation. We present in section 2 the correct reflected flux that removes this spurious cooling in a narrowband formulation. Thereafter, the discrepancy in the bandwise

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fluxes, calculated using the erroneous and correct formulations, is determined for a standard tropical atmosphere. Section 3 summarizes the main points of the analysis and highlights the inherent superiority of the correlated-k methods in this regard.

2. The narrowband formulation for reflective ground

The existing generalization of (1) for ground with an emissivity ϵ_{gj} in the jth band is given by (Savijärvi 2006)

$$F_j^{\uparrow}(u) = \{\epsilon_{gj}[\pi B_j(T_g)] + (1-\epsilon_{gj})F_j^{\downarrow}(0)\}\tau_j^f(u) - \int_0^u \pi B_j[T(u')]\dot{\tau}_j^f(u-u')\,du'. \quad (3)$$

As argued in an accompanying paper (Ponnulakshmi et al. 2012), an error arises in using $\tau_j^f(u)$ in (3) to attenuate both ground emission $\epsilon_{gj}[\pi B_j(T_g)]$ and the reflected flux $(1-\epsilon_{gj})F_j^{\downarrow}(0)$ despite the sharp contrast in the respective spectral energy contents. The use of $\tau_j^f(u)$ in the latter case, despite the non-Planckian spectral energy distribution of $F_j^{\downarrow}(0)$, leads to a spurious deficiency of radiant energy in the opaque bands, and thereby an intense cooling close to ground. The error first arose in the context of a broadband emissivity scheme (Garratt and Brost 1981), the incorrect ($F_{\rm rgw}^{\uparrow}$) and correct ($F_{\rm rgc}^{\uparrow}$) reflected flux in this case being

$$F_{\rm rgw}^{\uparrow}(u) = (1-\epsilon_g)F^{\downarrow}(0)\tau^f(u);$$
$$F_{\rm rgc}^{\uparrow}(u) = (1-\epsilon_g)\int_0^{u_t}\sigma T^4(u')\dot{\tau}^f(u+u')\,du', \quad (4)$$

for gray ground with emissivity ϵ_g, and with $F^{\downarrow}(0) = -\int_0^{u_t}\sigma T^4(u')\dot{\tau}^f(u')\,du'$. Here, $\tau^f(u)$ is the diffuse broadband transmissivity used (incorrectly) to attenuate both ground emission and the reflected flux. However, the transmissivity appropriate for the reflected flux is

$$\tau_r^f(u) = -\frac{\int_0^{u_t}\sigma T^4(u')\dot{\tau}^f(u+u')\,du'}{F^{\downarrow}(0)}, \quad (5)$$

and not $\tau^f(u)$.

Before generalizing (4) to the narrowband formulation above, we obtain the condition under which the erroneous broadband reflected flux equals the correct one. The resulting constraint clearly points to the generic nature of the error, and thence its relevance to any frequency-parameterized radiation scheme. Equating the reflected fluxes in (4), one finds that the relation

$$\int_0^{u_t} du'\,T^4(u')[\tau^f(u)\dot{\tau}^f(u') - \dot{\tau}^f(u+u')] = 0 \quad (6)$$

must hold for the spurious cooling error to be absent. Since the temperature profile in (6) is arbitrary, one must have

$$\tau^f(u)\tau^f(u') - \tau^f(u+u') = G(u), \quad (7)$$

with $G(u)$ being an arbitrary function of u; using $u'=0$ in (7), one concludes that G is identically zero. Hence,

$$\tau^f(u+u') = \tau^f(u)\tau^f(u'), \quad \forall \quad u, u'. \quad (8)$$

The only nontrivial continuous solution of this functional equation is an exponential (Aczel 1996). Thus, for the spurious cooling error to be absent regardless of the particular (atmospheric) temperature profile, the participating medium must be gray, implying an exponentially decaying broadband transmissivity function; that is, $\tau^f(u) = e^{-\alpha u}$, where α^{-1} is the photon mean free path. It may be seen from (5) that $\tau_r^f(u) = \tau^f(u)$ in this case.

The water-vapor-laden atmosphere (water vapor is the principal participating component in a cloud-free atmosphere) is, however, pronouncedly nongray due to the enormous wavelength sensitivity of the water vapor absorption in the infrared and the resulting disparity in photon pathlengths even within small spectral intervals; thus, $\tau^f(u)$ for water vapor departs significantly from a decaying exponential [see Figs. 1 and 2 in Ponnulakshmi et al. (2012)]. One, therefore, expects the erroneous reflected flux to lead to a spurious cooling error in any atmospheric calculation with nonblack bounding surfaces and with radiation modeled using an emissivity scheme. An unlikely scenario may arise if (6) holds despite the broadband transmissivity not being an exponential; in which case, $T(u')$ would have to closely approximate the null eigenfunction of (6) (a Fredholm integral equation of the first kind). Rather than attempt to calculate this eigenfunction, it is easier to verify if such an exception occurs for typical atmospheric profiles. Figure 1 shows this not to be the case; the flux difference for a model tropical atmosphere remains comparable to the individual fluxes. In summary, a necessary and sufficient condition for the spurious cooling error to arise in a broadband emissivity scheme is for $\tau^f(u)$ to deviate from an exponential.

We now generalize (4) to a smaller frequency interval: that corresponding to the jth band in a narrowband formulation. Accounting for the non-Planckian energy distribution of the downwelling surface flux, in the same manner as in (4), one obtains the following expression

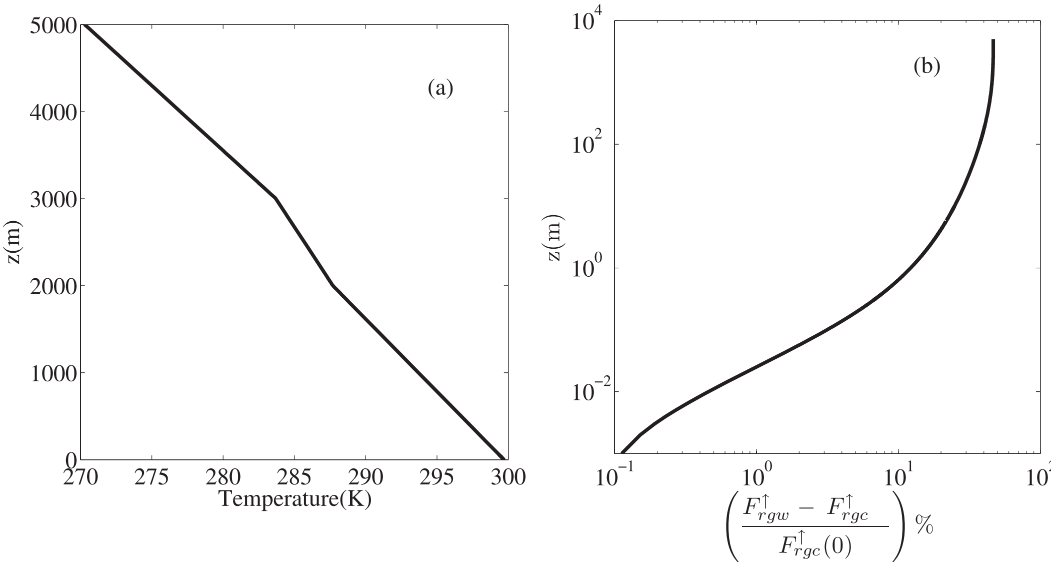

FIG. 1. (a) The temperature profile for a standard tropical atmosphere. (b) The corresponding difference between the erroneous ($F_{\rm rgw}^{\uparrow}$) and correct ($F_{\rm rgc}^{\uparrow}$) broadband-reflected fluxes [normalized by $F_{\rm rgc}^{\uparrow}(0)$] as a function of z.

for the upward bandwise flux to be used in a narrowband formulation for an atmosphere with reflective ground:

$$F_j^{\uparrow}(u) = \epsilon_{gj}\pi B_j(T_g)\tau_j^f(u) - \int_0^u \pi B_j[T(u')]\dot{\tau}_j^f(u-u')\,du' - (1-\epsilon_{gj})\int_0^{u_t} \pi B_j[T(u')]\dot{\tau}_j^f(u+u')\,du'. \tag{9}$$

Equating (9) and (3), one obtains that

$$\int_0^{u_t} du' B_j[T(u')][\tau_j^f(u)\dot{\tau}_j^f(u') - \dot{\tau}_j^f(u+u')] = 0, \tag{10}$$

for the spurious cooling error to be absent in a narrowband calculation. For an arbitrary temperature profile, (10) implies a condition analogous to (8) but one involving $\tau_j^f(u)$ instead. Thus, similar to the emissivity scheme above, an error will arise in a narrowband formulation if and only if the band-averaged transmittance departs from an exponential. As mentioned in section 1, $\tau_j^f(u)$ typically involves an average over a frequency interval much larger than the interline spacing, and it may be obtained in terms of effective line characteristics using statistical band models. For instance, a band model with identical Lorentzian profiles and random line positions (Goody 1964; Rodgers and Walshaw 1966) yields

$$\tau_j^f(u) = \exp\left[\frac{-Su}{\delta}\left(1 + \frac{Su}{\pi\alpha_L}\right)^{-1/2}\right], \tag{11}$$

where δ is the average line spacing, S is the mean line intensity, and α_L denotes the Lorentzian half-width. As is well known (Goody 1964), (11) exhibits three asymptotic regimes—the weak-line approximation when $1-\tau_j^f \propto u$ (for $Su \ll \pi\alpha_L$), the strong-line approximation with $1-\tau_j^f \propto u^{1/2}$ (for $\pi\alpha_L \ll Su \ll \delta^2/\pi\alpha_L$) when the line centers are strongly absorbed with additional absorption occurring in the wings, and subsequent saturation ($\tau_j^f \to 0$) once the pathlength exceeds the mean interline spacing ($Su \gg \delta$). An exponentially decaying transmittance is only realized in the heavily overlapping limit ($\delta \ll \pi\alpha_L$) when there is a direct transition from a linear decrease for small u to an exponential one for large u. Thus, the existence of a sensible strong-line regime is direct evidence of fine structure in the absorption spectrum and the related significance of ‘‘wing’’ contributions. The resulting departure of $\tau_j^f(u)$ from an exponential implies the appearance of a spurious cooling contribution on use of (3) in place of (9) (Varghese 2003; Savijärvi 2006).

Figure 2 shows the bandwise flux discrepancy, (10), normalized by the correct reflected flux at the surface $(1-\epsilon_g)F_j^{\downarrow}(0)$, plotted against z for a water-vapor-laden tropical atmosphere. The plots are for both an opaque (1550–1650 cm^{-1}: the 6.3-μm vibration–rotation band) and a transparent band (720–800 cm^{-1}) with $\tau_j^f(u)$ given by (11) and the band parameters taken from Rodgers and Walshaw (1966). The discrepancy between the two reflected fluxes is smaller in the opaque band because the atmosphere is essentially infinite in extent in this interval [$\tau_j^f(u_t) \to 0$]. This is best seen by comparing the

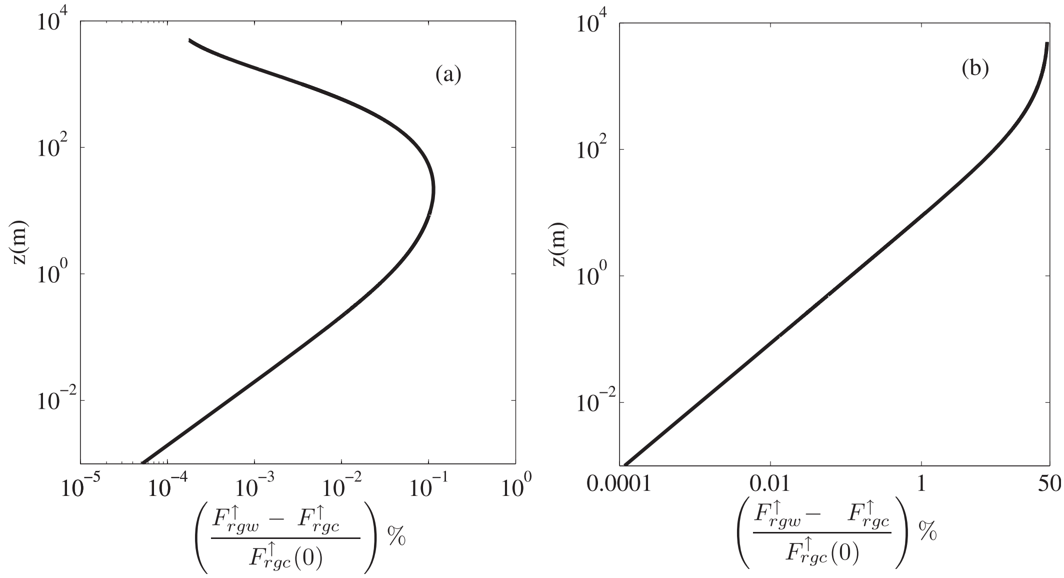

FIG. 2. Differences in the erroneous and correct reflected fluxes (normalized by the reflected flux at the surface in the particular band) for a tropical atmosphere and for the frequency ranges (a) 1550–1650 cm^{-1} (the 6.3-μm vibration–rotation band) and (b) 720–800 cm^{-1}; the band-averaged transmittance is given by (11) with the band parameters taken from Rodgers and Walshaw (1966).

actual transmittance for the bandwise reflected flux in an isothermal atmosphere $\tau_{rj}^{fi}(u)$ with $\tau_j^f(u)$. An expression for the former may be obtained directly by accounting for the photon pathlength before and after reflection; the total distance traversed is $(u + u_t)$, and the cumulative emission in the jth band is therefore given by $[1 - \tau_j^f(u + u_t)]B_j$. Subtracting the upward emission from a column of height u, given by $[1 - \tau_j^f(u)]B_j$, and normalizing by the surface flux $\{F_j^{\downarrow}(0) = B_j[1 - \tau_j^f(u_t)]\}$, one obtains the correct isothermal transmittance as

$$\tau_{rj}^{fi}(u) = \frac{\tau_j^f(u) - \tau_j^f(u + u_t)}{1 - \tau_j^f(u_t)}. \tag{12}$$

Clearly, $\tau_{rj}^{fi}(u)$ and $\tau_j^f(u)$ equal each other in the limit $\tau_j^f(u_t), \tau_j^f(u + u_t) \rightarrow 0$.

The mention of a near-surface spurious cooling error in the above discussion implicitly assumes that the reflected-flux transmittance $\tau_{rj}^{fi}(u)$ given by (12), is always smaller than $\tau_j^f(u)$. This assumption is reinforced by Figs. 1 and 2, where the erroneous reflected flux is always seen to be greater than the actual one. It is of interest to inquire if there is the possibility of a spurious heating. In other words, restricting our consideration to an isothermal atmosphere (a reasonable approximation for the small length scales that characterize the erroneous flux divergence), are there conditions where the actual transmittance $\tau_{rj}^{fi}(u)$ is greater than $\tau_j^{fi}(u)$? For this purpose, (12) may be rewritten in the following alternate form:

$$[\tau_j^{fi}(u) - \tau_{rj}^{fi}(u)][1 - \tau_j^{fi}(u_t)] = \tau_j^{fi}(u + u_t) - \tau_j^{fi}(u)\tau_j^{fi}(u_t). \tag{13}$$

The rhs of (13) must be negative for a spurious heating contribution. Writing $\tau_j^{fi}(u) = \exp[-f(u)]$, this translates to the condition $f(u + u_t) > f(u) + f(u_t)$. Now, with the equality sign, this relation is the Cauchy functional equation, satisfied by $f(u) \propto u$, which corresponds, of course, to a gray atmosphere. The above inequality is satisfied when $f(u)$ varies more rapidly than a linear function, say, $f(u) \propto u^x$ (with $x > 1$); a transmittance of the form $\exp(-u^x)$ would, therefore, lead to a spurious heating contribution. Although a mathematical possibility, such a functional form appears to be not relevant to tropospheric heat exchanges with pressure-broadened spectra. This is clearly evident from the form of the transmittance in the strong-line regime, in which case $f(u) \propto u^{1/2}$ and therefore $f(u + u_t) < f(u) + f(u_t)$, implying that $\tau_j^{fi}(u) > \tau_{rj}^{fi}(u)$.

The arguments above show that the deviation of the band-averaged transmittance from an exponential is directly linked to the existence of a strong-line regime, and that the nature of this deviation is such as to lead to a spurious cooling contribution. It is therefore worth emphasizing that typical infrared spectra of atmospheric gases have pressure-broadened line widths [O(0.01–0.1 cm^{-1})] smaller than the smallest interline spacing [O(1 cm^{-1}) due to rotation transitions]; in other words, atmospheric radiative exchanges correspond largely to the strong-line regime (Goody 1964). Indeed, the importance of the atmospheric window implies that radiative cooling in the lower troposphere is dominated by wing contributions; the weak window attenuation, modeled as the water vapor continuum, is thought to arise from cumulative far-wing contributions (Bignell 1970; Clough et al. 1989); although, the additional role of water vapor dimers continues to be debated (Ptashnik et al. 2011). The significance of the strong-line regime is also evident in emissivity parameterizations used for NBL modeling (Siqueira and Katul 2010; Garratt and Brost 1981; Rodgers 1967); the emissivity, for small u, being expressed in terms of $u^{1/2}$, rather than u. Scaling approaches for an inhomogeneous atmosphere have again been based on the strong-line approximation (Cess 1974; Ramanathan 1976; Chou and Arking 1980). Clearly, one expects a spurious cooling contribution in typical narrowband formulations. To the extent that the frequency interval used in a narrowband formulation is arbitrary (Ramanathan and Downey 1986), a spurious cooling error is expected in any frequency-parameterized scheme with the parameterization applied to intervals larger than an elementary line width; the error, of course, arises only when such a scheme is applied to reflective ground. The calculations of André and Mahrt (1982), Schaller (1977), and more recently, Savijärvi (2006), are examples in this regard. Because of the spurious cooling error, the cooling rate profiles obtained by Savijärvi (2006), using a narrowband formulation, are very sensitive to a departure of ϵ_g from unity. Reducing ϵ_g from 1 to 0.8, for a midlatitude summer (MLS) atmosphere led to perceptible cooling rate differences at heights of up to a kilometer; the cooling rate at 0.1 m, in particular, changed from 3.8 K day^{-1} for $\epsilon_g = 1$ to 9.5 K day^{-1} for $\epsilon_g = 0.8$. In contrast, the actual cooling rate profiles for the (dry) lapse rate atmosphere have been shown to be fairly insensitive to ϵ_g (Ponnulakshmi et al. 2012). Morcrette's original narrowband calculation (Morcrette 1977), extended by André and Mahrt (1982) to model NBL over a nonblack ground, highlights the incorrect effect of ϵ_g on the cooling rates. In contrast to the expected enhancement with decreasing ϵ_g, expected for a nocturnal inversion (Lieske and A. Stroschein 1967; Edwards 2009), the warming layer is already lost for $\epsilon_g = 0.965$.

3. Conclusions

In this paper, we have highlighted a fundamental error in the prevailing narrowband formulation for reflective ground together with its consequences for atmospheric cooling rate profiles. A necessary and sufficient condition for the error, in the form of an intense near-surface cooling, to occur is the deviation of the appropriate frequency-averaged transmissivity function from a simple exponential decay; the deviation results from the multiplicity of photon pathlengths in the relevant frequency interval. The latter is almost always true for atmospheric radiative exchanges. The infrared spectra of most atmospheric gases are dominated by vibration–rotation bands, and the specificity of the underlying (discrete) transitions renders the photon mean free path an extremely sensitive function of frequency. Thus, any frequency-parameterized radiation scheme that does not resolve intervals comparable to or smaller than an elementary line width will suffer from a spurious cooling error. We have presented the corrected scheme that removes the error.

The spurious cooling error inherent in a frequency-parameterized radiation scheme highlights the superiority of the k-distribution method and its extension (the correlated-k method) to an inhomogeneous atmosphere (Liou 2002). The method is based on the grouping of absorption coefficient (k_ν) values. Thus, the frequency integrals in the expressions for the monochromatic fluxes are replaced by integrals over k weighted by the cumulative probability distribution of photon pathlengths $g(k)$. The smooth variation of g with k, in sharp contrast to the rapid variation of k with ν, leads to an immediate computational advantage. In the present context, forming contiguous intervals based on k_ν values naturally negates the error in the reflected flux, since the error arises due to the disparity in photon pathlengths over short frequency intervals. Finally, it is worth mentioning that several calculations have been based on an exponential-sum fitting of the narrowband transmittances (Liou and Sasamori 1975; Stephens 1978), this being a discrete representation of the k-space integral with each decaying exponential corresponding to a gray subband; importantly, the flux divergence must be calculated as a sum of subband contributions. Not conforming to this procedure will lead to an error due to the huge disparity in the subband photon pathlengths, within a single band, as is the case in Varghese et al. (2003). Similar to Savijärvi (2006), the predicted flux-divergence profiles (for an MLS atmosphere) remain sensitive to a small deviation of ϵ_g from unity even at heights on the order of a kilometer; the surface cooling rate changes from 4.5 K day^{-1} for $\epsilon_g = 1$ to 37.5 K day^{-1} for $\epsilon_g = 0.8$.

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