Large-Eddy Simulation of a Residual Layer: Low-Level Jet, Convective Rolls, and Kelvin–Helmholtz Instability

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ABSTRACT

Diurnal variations of an atmospheric boundary layer from 0900 LST on day 33 to 0600 LST on day 34 of the Wangara experiment are studied using a large-eddy simulation (LES) model that includes longwave radiation and baroclinicity. The present study directs its particular attention to phenomena in a residual layer (RL). As the surface heat flux decreases, an inertial oscillation is initiated and is accompanied by a low-level jet (LLJ) at a height of approximately 200 m. The maximum wind speed of the LLJ exceeds 12 m s$^{-1}$ at 0300 LST on day 34. After 2100 LST on day 33, the horizontal advection due to the LLJ under a large-scale horizontal gradient of temperature destabilizes the RL and consequently induces horizontal convective rolls, parallel to a vertical wind shear (VWS) vector, between heights of 400 and 1400 m. The VWS in the layer between the bottom of the convective rolls and the gradually growing LLJ maximum is intensified after midnight, and the gradient Richardson number falls below its critical value of 0.25 at a height of 400 m at 0130 LST on day 34. An empirical orthogonal function analysis demonstrates that Kelvin–Helmholtz (KH) vortices appear below the convective rolls and are coupled with them. This study suggests that horizontal convective rolls can occur in an RL because an LLJ often advects warmer air to the lower layer according to a large-scale gradient of temperature and that the rolls may coexist with KH vortices in a stable boundary layer because the LLJ gradually increases a VWS.

1. Introduction

A typical diurnal structure of the atmospheric boundary layer (ABL) in high pressure regions over land is classified into three major components: the convective boundary layer (CBL), stable boundary layer (SBL), and residual layer (RL) (Stull 1988). During the daytime, thermals rise from the ground surface heated by the solar radiation, and the well-mixed CBL develops. With the approach of sunset, the longwave radiative cooling exceeds the solar heating, and the SBL starts to grow from the ground surface. The RL is a remnant of the daytime CBL that is decoupled from the ground surface and has near-neutral stratification. The aim of the present study is to explore phenomena particularly occurring in an RL using large-eddy simulation (LES).

Because turbulent motions in the CBL have relatively large scales in space and time, they are more easily measured and thus have been examined by a large number of observational and numerical studies. For example, Caughey and Palmer (1979) showed universal characteristics of turbulent statistics based on observations, Schmidt and Schumann (1989) exhibited polygonal cells and their coherency using LES, and Weckwerth et al. (1997) suggested environmental conditions determining features of horizontal convective rolls based on observations and simulations. In the SBL, on the other hand, turbulence becomes weak or intermittent, and its dominant scales decrease. Also the presence of internal gravity
waves, low-level jets (LLJs), and shear instabilities complicates processes in the SBL. To clarify phenomena and their dynamics in the SBL, several field campaigns, such as the 1998 Stable Atmospheric Boundary Layer Experiment in Spain (SABLES-98) (e.g., Cuxart et al. 2000) and the 1999 Cooperative Atmosphere–Surface Exchange Study (CASES-99) (e.g., Poulos et al. 2002), have been conducted. Data collected in CASES-99 have been used by many researchers and have been increasing our understanding of the SBL. For example, based on data from Doppler lidar and in situ sensors, Sun et al. (2002) demonstrated the occurrence of both local thermal and shear instabilities associated with a density current and the resulting intermittent turbulence, Banta et al. (2006) assembled statistics of mean wind and continuous turbulence associated with LLJs according to three types of profile shape of the velocity variance, and Blumen et al. (2001) and Newsom and Banta (2003) detected Kelvin–Helmholtz (KH) waves appearing below an LLJ maximum and associated intermittent turbulence. To the authors’ knowledge, however, there are few studies focusing on the RL, the near-neutral stratification of which is likely to be thermally destabilized by some kind of forcing, such as radiative cooling and large-scale advection.

A field project named the Wangara experiment was conducted in a flat and near-homogeneous area of southeastern Australia during July and August 1967 (Clarke et al. 1971). In the nighttime on days 33–34 of this experiment, an RL was identified. Several researchers simulated the evolution of ABLs through these days: some used one-dimensional models based on LLJs; others used three-dimensional LES techniques (e.g., Basu et al. 2008; Shibuya et al. 2014). They, however, did not discuss phenomena in the RL. To capture three-dimensional structures of phenomena, the LES that explicitly simulates turbulent motions is desirable. Also major forcing factors that control the structure of the nocturnal ABL are the radiative cooling and geostrophic wind (e.g., Baas et al. 2009). Thus we will use an LES model that includes the longwave radiation and the geostrophic wind based on the observation. Simulated flows are composed of various scales of motions. To extract dominant coherent structures from the flows, we will use an empirical orthogonal function (EOF) analysis (e.g., Wilson 1996; Nakanishi and Niino 2012).

Because three-dimensional structures were not observed in the Wangara experiment, we cannot confirm the simulated three-dimensional phenomena based on the observation. However, the present study deepens our understanding of the RL and may also contribute to improvements of parameterizations in mesoscale numerical models.

2. Model description

Our LES model is nearly the same as that of Nakanishi (2000), except that the condensation process is neglected and the large-scale horizontal gradient of temperature is considered. We use the subgrid-scale model of Sullivan et al. (1994), which consists of isotropic and anisotropic parts below the middle of the CBL or SBL and turns to the traditional isotropic model above it, including throughout the RL. As the surface is approached, the mean vertical shear increases, anisotropy is induced in turbulent flows, and the scales of turbulent motions decrease. Sullivan et al. (1994) consider that, for a finite grid size, transition from LES to RANS is needed near the surface and parameterize the anisotropic part based on Prandtl’s mixing length hypothesis. Although they solved a prognostic equation for the subgrid-scale turbulent kinetic energy (TKE), we obtain it diagnostically (Smagorinsky 1963; Lilly 1966). Several minor changes from Nakanishi (2000) will be described below.

a. Governing equations

In this study, an incompressible Boussinesq flow is assumed, and the governing equations for velocity components \( u, v, w \), potential temperature \( \theta \), and specific humidity \( q_v \) are schematically written as

\[
\begin{align*}
\frac{\partial \Phi}{\partial t} + \frac{\partial \Phi}{\partial x} + \frac{\partial \Phi}{\partial y} + \frac{\partial \Phi}{\partial z} &= \frac{\partial U \Phi}{\partial x} - \frac{\partial V \Phi}{\partial y} - \frac{\partial W \Phi}{\partial z} - \frac{\partial w' \Phi'}{\partial z} + E_\Phi, \\
where \Phi represents u, v, w, \theta, or q_v; E_\Phi is a forcing; an overbar denotes a resolved-scale variable; and a prime denotes a subgrid-scale variable. Each forcing is given by

\[
E_u = -\frac{\partial p}{\partial x} + f(\nu - V_G),
\]

\[
E_v = -\frac{\partial p}{\partial y} - f(\nu - U_G),
\]

\[
E_w = -\frac{\partial p}{\partial z} + \frac{g}{\Theta_0} (\bar{\theta}_y - \langle \bar{\theta}_y \rangle),
\]

\[
E_\theta = \frac{\bar{\theta}}{T} \frac{1}{\rho c_p} \frac{\partial F}{\partial z} - f \Theta_0 \left( \nu \frac{\partial V_G}{\partial z} - \nu \frac{\partial U_G}{\partial z} \right), \quad \text{and}
\]

\[
E_{q_v} = 0,
\]

\]
where $p$ is the kinematic temperature, $\theta_v = \theta(1 + 0.61q_v)$ is the virtual potential temperature, $T$ is the absolute temperature, $\rho$ is the air density, $F$ is the net upward radiative flux, $U_G$ and $V_G$ are the geostrophic velocity components, $f$ is the Coriolis parameter, $\Theta_0$ is the reference temperature (283.15 K), $g$ is the acceleration due to gravity (9.81 m s$^{-2}$), $c_p$ is the specific heat of dry air at constant pressure (1004 J K$^{-1}$ kg$^{-1}$), and the angle brackets denote a horizontal average. The first term on the right-hand side (rhs) of Eq. (5) indicates the radiative heating, and the second term indicates the horizontal advection associated with the large-scale horizontal gradient of temperature, obtained from the thermal wind relations for the dry atmosphere (e.g., Yamada and Mellor 1975; Sorbjan 2004).

b. Radiation scheme

Because we particularly focus on the RL and will give observed temperature as the lower boundary condition, we consider only the longwave radiation. Following Katayama (1972), the net upward longwave radiative flux $F$ is given by

$$F(z) = \tau(z, z_c)\left[\pi B(T_g) - \pi B(T_s)\right] + \int_{z_c}^{z} \tau(z, z') \frac{d\pi B(T_{z'})}{dz'} dz' + \tau(z, z_c)\left[\pi B(T_g) - \pi B(T_s)\right] + \tilde{\tau}(z, z_c)\pi B(T_s), \quad (7)$$

where $\pi B$ is the blackbody radiative flux; $\tau(z, z')$ and $\tilde{\tau}(z, z')$ are two types of weighted-mean transmission function of a mixture of water vapor and carbon dioxide between $z$ and $z'$; and the subscripts $g$, $s$, $t$, and $c$ denote the ground level, screen level (generally 1.5–2 m), top of the computational domain, and critical level (height of $T = 220$ K), respectively. Heating or cooling of the air near the ground surface is known to be sensitive to the temperature difference between the ground and screen levels (Ha and Mahrt 2003), which are selected to be the roughness length for heat and 1.5 m, respectively. We compute $F$ using horizontally averaged $\theta$ and $q_v$ with a time step of 30 s to save CPU time.

c. Boundary conditions

The lower boundary conditions for $u$, $v$, $\theta$, and $q_v$ are given in the form of subgrid-scale fluxes based on the Monin–Obukhov similarity theory, with a roughness length for momentum $z_{0m} = 0.01$ m, the value of which was used by Yamada and Mellor (1975) and Basu et al. (2008). If temperature and humidity at $z = 1.2$ m are used, the friction velocity $u_*, \text{temperature scale } \theta_*, \text{and humidity scale } q_*$ are written as

$$u_* = \frac{kM(z_1 + z_{0m})}{\ln[(z_1 + z_{0m})/z_{0m}] - \psi_m[(z_1 + z_{0m})/L] + \psi_m(z_{0m}/L)}, \quad (8)$$

$$\theta_* = \frac{k[\Theta(z_1 + z_{0m}) - \Theta_0]}{\ln[(z_1 + z_{0m})/1.2] - \psi_h[(z_1 + z_{0m})/L] + \psi_h(1.2/L)} \quad \text{and}$$

$$q_* = \frac{\beta k[Q(z_1 + z_{0m}) - Q_s]}{\ln[(z_1 + z_{0m})/1.2] - \psi_h[(z_1 + z_{0m})/L] + \psi_h(1.2/L)}, \quad (10)$$

where $M$, $\Theta$, and $Q$ are the horizontal mean wind speed, potential temperature, and specific humidity at the lowest grid level $z_1 + z_{0m}$, respectively; $\Theta_s$ and $Q_s$ are the mean potential temperature and saturation specific humidity at $z = 1.2$ m, respectively; and $k$ is the von Kármán constant, $\beta$ is the evaporation efficiency, $L = \Theta_0 u_*/\rho g (\theta_* + 0.61\Theta_0 q_*)$ is the Obukhov length, and

$$\psi_{m,h}(x) = \int_{0}^{x} \frac{1 - \phi_{m,h}(x')}{x'} dx', \quad (11)$$

with the similarity functions $\phi_m$ and $\phi_h$ for momentum and heat, respectively (Dyer 1974). The layer below $z = z_{0m}$ is beyond the scope of the Monin–Obukhov similarity theory, so that it should not be included in the computational domain. The lowest grid level of $z_1 + z_{0m}$ means that the domain is moved vertically by $z_{0m}$. In terms of $u_*$, $\theta_*$, and $q_*$, the subgrid-scale fluxes at the surface are given by

$$\overline{u'w'(z_{0m})} = -u_*^2 \frac{\pi(z_1 + z_{0m})}{M(z_1 + z_{0m})}, \quad (12)$$
The air temperature at \( z = 1.2 \) m is given by linearly interpolating the hourly observations during the Wangara experiment (Clarke et al. 1971). The saturation specific humidity is estimated using the Tetens formula for the saturation water vapor pressure \( e_s = 611 \times \exp[17.27 (T - 273.15)/(T - 35.85)] \) Pa). The evaporation efficiency is selected to be 0.02, so that simulated specific humidity within the CBL approximately agrees with the observed humidity. The roughness length for heat, which is equal to \( z_{0m}/7.4 \) (Garratt 1992), is used to obtain the ground-level temperature for the radiation scheme.

The upper boundary is a stress-free rigid lid: \( \bar{w} \), \( \partial u/\partial z \), \( \partial v/\partial z \), and \( \partial q/\partial z \) vanish, whereas \( \partial \theta/\partial z \) is fixed to the initial value of 0.0075 K m\(^{-1}\). Periodic conditions are applied to the lateral boundaries.

d. Large-scale forcing and numerical conditions

During the Wangara experiment, the geostrophic wind at the surface was estimated every 3 h, and the thermal wind (i.e., a difference of geostrophic winds at different heights) from the surface to 1 km and from 1 to 2 km were estimated every 12 h (Clarke et al. 1971). Following Yamada and Mellor (1975) and Basu et al. (2008), we interpolate these data and prescribe the geostrophic wind that varies both with height and with time. The magnitude of the geostrophic wind tends to increase gradually with time at almost all heights. The Coriolis parameter is set to \( f = -8.26 \times 10^{-5} \) s\(^{-1}\), corresponding to latitude 34.5°S.

The vertical distributions of mean quantities at the initial time are given by the observations at 0900 LST on day 33 of the Wangara experiment (Clarke et al. 1971). The LES is started by initially adding random perturbations with a maximum amplitude of 0.5 m s\(^{-1}\) to the mean velocity components below \( z = 50 \) m, which is considered to be the depth of an evolving CBL at 0900 LST on day 33 and is performed until 0600 LST on day 34.

The size of the computational domain is 5 km \( \times \) 5 km horizontally and 2 km vertically. Note that it is not easy to extend the vertical domain, because all the data, including the thermal winds, during the Wangara experiment were collected below 2 km. The domain is divided into 250 \( \times \) 250 \( \times \) 100 grid boxes, with a uniform grid spacing of 20 m in all directions. Spatial derivatives are approximated by a centered difference of second-order accuracy in a staggered-grid system. Time integration of the governing equations is done by using the second-order Adams–Bashforth scheme with a time step of 0.2 s. Because the computational domain has only a depth of 2 km near to the ABL depth, a damping layer to avoid reflection of gravity waves from the upper boundary is not placed. A minor influence of the reflection of gravity waves will be suggested by weak energy in the upper layer in plots of TKE and EOF modes.

3. Results and discussions

In section 3a, results of the LES are compared with the Wangara data. In sections 3b and 3c, notable three-dimensional structures derived from the LES are examined with an EOF analysis.

a. Comparisons with the observation

1) SURFACE HEAT FLUX AND FRICTION VELOCITY

Temporal variations of sensible heat flux at the surface and friction velocity are shown in Fig. 1. They are not recorded in the Wangara data (Clarke et al. 1971) but are estimated by Hicks (1981), who used the measured wind and temperature differences between \( z = 1 \) and 4 m, and the similarity functions of Dyer (1974) for unstable conditions and Hicks (1976) for statically stable conditions. The analyzed sensible heat flux during the daytime shows variations like a sine curve with a maximum at approximately 1300 LST, whereas the heat flux during the nighttime maintains a nearly constant value of approximately \(-0.03 \) K m s\(^{-1}\) (Fig. 1a). Compared with the analyzed heat flux, the simulated heat flux is small before 1500 LST but nearly the same after 2100 LST.

The analyzed friction velocity continues to increase until the evening (Fig. 1b). However, it decreases suddenly to approximately 0.06 m s\(^{-1}\) during the evening transition and keeps this magnitude until the morning transition. The simulated friction velocity nearly agrees with the analysis before 1600 LST. Although the LES also reproduces the sudden decrease during the evening transition reasonably well, the simulated friction velocity during the nighttime is 2–3 times larger than the analysis. Similar results were reported by Yamada and Mellor (1975) and Basu et al. (2008). The latter authors, who used a dynamic subgrid-scale model with a vertical resolution of 12.5 m, stated that this discrepancy results from the contemporary surface layer schemes that are unable to correctly predict the magnitude of momentum fluxes under stable conditions.
2) POTENTIAL TEMPERATURE

Figure 2 shows vertical profiles of horizontally averaged potential temperature obtained from the LES (Figs. 2b and 2d) together with those of observed potential temperature (Figs. 2a and 2c). During the daytime, a CBL that has nearly uniform potential temperature grows vertically with time (Fig. 2a). The simulated CBL also develops with time, and the convection clearly penetrates into the overlying stable layer (Fig. 2b). Such penetrative convection does not seem to occur in the observation after 1500 LST (Fig. 2a), however. This difference is most likely the result of subsidence in a high pressure system (Deardorff 1974; Yamada and Mellor 1975), which is neglected in our LES. As also reported in previous studies (Yamada and Mellor 1975; Basu et al. 2008), the simulated temperature in the CBL is slightly lower than the observation, which may again result from the lack of subsidence in our LES.

During the nighttime, a strong SBL forms near the surface, and its depth increases to approximately 500 m by 3000 LST (=0600 LST on day 34) (Fig. 2c), where the SBL depth is defined by the height at which the vertical gradient of potential temperature changes sharply. The simulated SBL agrees well with the observation until 2400 LST, but its depth increases more slowly than the observation after 2400 LST, only reaching approximately 300 m at 3000 LST (Fig. 2d). This discrepancy was also shown by Basu et al. (2008). One of its causes could be a mesoscale horizontal advection that is not included in Eq. (5), although further examinations are needed. In an RL with nearly uniform potential temperature (approximately between $z = 200$ and 1400 m), the vertical gradient of the simulated potential temperature after 2400 LST becomes smaller than that at 2100 LST (Fig. 2d).

3) HORIZONTAL WIND SPEED

Figure 3 shows vertical profiles of horizontal wind speed. The wind speed is 5 m s$^{-1}$ or less during the daytime (Fig. 3a), whereas that in the ABL increases to 10 m s$^{-1}$ or so during the nighttime (Fig. 3c). After 2400 LST, an LLJ occurs near $z = 200$ m, which is at or slightly above the top of the SBL at 2400 LST (Fig. 2c). Although the top of the SBL continues to increase until 3000 LST, the height of the LLJ maximum increases very little (e.g., Mahrt et al. 1979). The maximum wind speed of the LLJ exceeds 13 m s$^{-1}$ at 2700 LST. The LES reproduces these features reasonably well (Figs. 3b and 3d), except that the simulated layer with nearly uniform wind speed is deeper and the simulated maximum wind speed of the LLJ is slightly smaller. This deeper uniform layer is caused by the highly penetrative convection as was observed for the potential temperature (Fig. 2b).

Both in the observation and in the simulation (Figs. 3c and 3d), the vertical gradient of the wind speed in the RL is negative at 2400 LST. After 2400 LST, however, it nearly vanishes again, which may be caused by some forcing that diminishes the vertical gradient of potential temperature in the RL (Fig. 2d) and by an inertial oscillation (e.g., Van de Wiel et al. 2010). The LLJ then starts to have a sharp maximum (referred to herein as a “nose”).

4) SPECIFIC HUMIDITY

Figure 4 shows vertical profiles of specific humidity. The specific humidity is also nearly uniform in the CBL (Fig. 4a). In the lower part of the CBL, it tends to decrease with time because of only a small amount of evaporation from the surface. The LES reproduces the distributions of the observed specific humidity.
humidity well (Fig. 4b), except for large specific humidity near the top of the CBL after 1500 LST, which is also caused by the highly penetrative convection (Fig. 2b).

During the nighttime, very large differences between the observation and simulation are found (Figs. 4c and 4d). The simulated specific humidity changes little with time (Fig. 4d). No dewfall to the surface occurs in the LES, because $3 \text{g kg}^{-1}$ in the SBL is smaller than the saturation specific humidity for the temperature at $z = 1.2 \text{m}$. In the observation (Fig. 4c), on the other hand, the specific humidity in the upper part of the RL decreases from above with time after 2400 LST, suggesting an effect of the subsidence. The specific humidity below $z = 500 \text{m}$, however, increases and reaches a maximum at 2700 LST. We note that the shape of its distribution is fairly similar to that of the LLJ (Fig. 3c). The equation for the specific humidity [Eq. (6)] does not include its large-scale horizontal gradient, the term of which depends on the distribution of the LLJ [see Eq. (5)]. The above shape similarity (Figs. 3c and 4c) suggests the need to consider an effect of the large-scale horizontal gradient also for the specific humidity. On the other hand, the difference of the distributions between the potential temperature (Fig. 2c) and the others (Figs. 3c and 4c) implies the occurrence of turbulent motions to eliminate a thermally unstable state above the LLJ nose.

From 2100 to 2400 LST, the observed specific humidity increases throughout the ABL (Fig. 4c). This question may be also partly answered by considering the large-scale horizontal gradient of specific humidity, as will be described in section 3b(2).
b. Horizontal convective rolls

1) TURBULENT KINETIC ENERGY

The TKE, defined by half the sum of the three velocity variances, was not measured in the observation. Figure 5 shows its vertical distributions for the LES. The TKE increases with time until 1500 LST and then decreases rapidly throughout the CBL (Fig. 5a), because it is mainly produced by buoyancy (cf. Fig. 1b). At 2100 LST, the TKE becomes very small (Fig. 5b), except near the surface and the top of the RL, where the vertical wind shear (VWS) is large (Fig. 3d). After 2100 LST, however, it starts to increase in the RL, implying that the stratification in the RL becomes unstable and turbulent motions are reactivated. The TKE has a minimum value around the LLJ nose because of the small VWS, as also reported by Mahrt et al. (1979) and Banta et al. (2006).

2) HEATING RATES

To confirm the destabilization of the RL during the nighttime (Fig. 5b), vertical profiles of simulated heating rates due to turbulence $T$, radiation $R$, and horizontal advection associated with the large-scale horizontal gradient of temperature $A$ at 2100 and 2530 LST are shown in Fig. 6, where $T$, $R$, and $A$ are computed by horizontally averaging the third term on the rhs of Eq. (1) and the first and second terms on the rhs of Eq. (5), respectively. The heating $R$, both at 2100 and at 2530 LST (Figs. 6a and 6b), has a minimum near the top of the SBL (Fig. 2d) and then broadly increases with height, showing a contribution to the stabilization of the atmospheric layer. At 2100 LST (Fig. 6a), $A$ is small throughout the simulated layer, particularly in the RL, and $T$ is also small in the RL.

At 2530 LST (Fig. 6b), however, $A$ has a maximum near the LLJ nose (Fig. 3d) and then decreases with
height. Its magnitude at the bottom of the RL (\(z \sim 200\) m) is approximately \(0.1\) K h\(^{-1}\) larger than the magnitude at the top of the RL (\(z \sim 1400\) m). The heating \(T\) varies in response to \(A\) and is negative (positive) in the lower (upper) part of the RL, implying that the major turbulent motions consist of thermal convection. In the present case, \(A\) mainly destabilizes the RL. The increase of \(A\) after 2100 LST may suggest that, if the large-scale gradient term analogous to the second term on the rhs of Eq. (5) is considered for the specific humidity, the increase of specific humidity from 2100 to 2400 LST (Fig. 4c) is explained.

**3) DIRECTION OF VERTICAL WIND SHEAR**

Because the VWS exists in a thermally unstable RL (Figs. 3c and 3d), convection, if it occurs, would have a roll structure that is nearly aligned with the VWS vector (e.g., Asai 1970, 1972). Figure 7 shows the direction of the VWS vector \(\left(\frac{\partial \overline{V}}{\partial z}, \frac{\partial \overline{V}}{\partial z}\right)\). The observed VWS vector at 2400 LST points to the west near the surface and turns counterclockwise to approximately \(-30^\circ\) at \(z = 300\) m (Fig. 7a). The VWS vector above \(z = 300\) m seems to have an average direction of approximately \(-30^\circ\), although it considerably fluctuates. The simulated VWS vector shows similar characteristics (Fig. 7b). It temporally changes only in the lower part of the RL. At 2530 LST, its direction is approximately \(-30^\circ\) throughout the RL.

**4) HORIZONTAL VELOCITY FLUCTUATION**

Figure 8 shows horizontal distribution of simulated velocity fluctuations in the \(x\) direction at 2530 LST at \(z = 710\) m near the middle of the RL. Three stripes are aligned diagonally from the upper left to lower right.
with a transverse wavelength of approximately 2000 m. Their orientation of approximately $-30^\circ$ corresponds to the direction of the VWS vector in the RL (Fig. 7).

5) EOF MODE

To demonstrate that the stripes in Fig. 8 have a coherent roll structure, we make an EOF analysis, which is an eigenanalysis of correlation functions. The correlation functions are calculated from the LES data collected with a time interval of 30 s from 2530 to 2600 LST. Each eigenmode has energy corresponding to its eigenvalue, and its flow structure is obtained from its eigenfunction. Figure 9 shows vertical distribution of along-roll, cross-roll, and vertical velocity fluctuations for EOF mode (1, 2) that has the largest eigenvalue with a contribution of 6.9% to all modes. The horizontal wavelength and orientation of rolls for this mode is the same as those of the stripes in Fig. 8. The rolls have their amplitude over a depth of 1000 m between $z = 400$ and 1400 m. The aspect ratio of the rolls, defined by the ratio of their horizontal wavelength to their depth, is estimated to be approximately 2.2, which is consistent with that of the fastest-growing
mode of thermal convection (e.g., Asai 1970). The along-roll velocity fluctuations are positive (negative) in the downdraft (updraft) regions.

To complement the source of the rolls, the production of TKE in the vertical plane perpendicular to the roll axis is examined. The equations for TKE in the cross-roll and vertical directions, $e_{\text{cross}}$ and $e_{\text{vert}}$, are schematically written as

$$\frac{\partial e_{\text{cross}}(k_x, k_y, n)}{\partial t} = -E[U_{\text{cross}}(k_x, k_y, n)] \frac{\partial E[U_{\text{cross}}]}{\partial z} + E \left[ \overline{U(k_x, k_y)} \overline{W_{\text{cross}}(k_x, k_y, n)} \right] + D_{\text{cross}} \quad \text{and (16)}$$

$$\frac{\partial e_{\text{vert}}(k_x, k_y, n)}{\partial t} = \frac{g}{\Theta_0} E[U_{\text{vert}}(k_x, k_y)] + E \left[ \overline{U(k_x, k_y)} \overline{W_{\text{vert}}(k_x, k_y, n)} \right] + D_{\text{vert}} \quad \text{and (17)}$$

where $(k_x, k_y)$ are the horizontal wavenumbers, $n$ is the eigenmode, and $E[\cdot]$ represents the time average between 2530 and 2600 LST (Wilson and Wyngaard 1996; Nakanishi and Niino 2012). The first term on the rhs of Eq. (16) is the shear production in the cross-roll direction $S_{\text{cross}}$; that of Eq. (17) is the buoyancy production $B$; the second terms of Eqs. (16) and (17) are the energy redistributions $R_{\text{cross}}$ and $R_{\text{vert}}$, respectively ($R_{\text{cross}} + R_{\text{vert}} = 0$, because the EOF mode is two dimensional); and their third terms $D_{\text{cross}}$ and $D_{\text{vert}}$ are the sum of the Coriolis force effect, turbulent transport, interscale transfer, and dissipation. Note that, in our EOF analysis, $\theta_V$ is not decomposed into eigenmodes [cf. appendix C of Nakanishi and Niino (2012)]. Figure 10 shows that $S_{\text{cross}}$ is small and $B$ is large in a large part of the RL, illustrating that the cross-roll direction coincides with the direction of the small VWS, and the TKE of the rolls

![Figure 7](http://journals.ametsoc.org/doi/abs/10.1175/JAS-D-13-0402.1)

**FIG. 7.** Vertical profiles of the direction of vertical wind shear vector obtained from (a) the Wangara experiment at 2400 LST and (b) the LES at 2400, 2530, and 2700 LST on day 33. The direction is measured counterclockwise from the east. The Wangara data at 2700 LST have much larger fluctuations than at 2400 LST, so they are not plotted. The insert in (b) shows simulated mean wind vectors at $z = 90, 150, 250, 450,$ and $950$ m at 2530 LST.

![Figure 8](http://journals.ametsoc.org/doi/abs/10.1175/JAS-D-13-0402.1)

**FIG. 8.** Horizontal distribution of simulated velocity fluctuations in the $x$ direction at $z = 710$ m at 2530 LST on day 33. The fluctuations are differences from the averaged velocity over the horizontal plane.
is produced by the buoyancy (Asai 1972). According to Eqs. (16) and (17), $e_{vert}$ is first produced by $B$, and then is partly distributed to $e_{cross}$ through $R_{vert}$ and $R_{cross}$.

The results in Figs. 5–10 demonstrate that the horizontal advection due to the LLJ in the presence of the large-scale horizontal gradient of temperature destabilizes the RL, and the horizontal convective rolls parallel to the VWS vector are generated.

c. Low-level jet and Kelvin–Helmholtz instability

1) INERTIAL OSCILLATION AND LOW-LEVEL JET

To examine why the LLJ occurs near $z = 200$ m, hodographs of horizontal wind at three heights around the LLJ nose (Figs. 3c and 3d) from 1200 to 3000 LST are shown in Fig. 11. In the daytime (before 1800 LST) when convection is active, the horizontal wind vectors at the three heights show nearly similar variations (Fig. 11a). After 1800 LST, however, they start to rotate counterclockwise on different arcs having different centers.

Blackadar (1957) stated that nocturnal winds exhibit an inertial oscillation around the geostrophic wind with a period of $2\pi/|f|$ and an amplitude equal to the ageostrophic wind speed at sunset. This finding does not explain that the center of the observed oscillations varies with height (Fig. 11a). Recently, Van de Vel et al. (2010) proposed a new model for inertial oscillations. One-dimensional equations of motion are given by

$$\frac{\partial U}{\partial t} = f(V - V_G) + \frac{\partial \tau_x}{\partial z} = f(V - V_{eq}) \quad \text{and} \quad (18)$$

$$\frac{\partial V}{\partial t} = -f(U - U_G) + \frac{\partial \tau_y}{\partial z} = -f(U - U_{eq}), \quad (19)$$

where $(U, V), (U_G, V_G)$, and $(U_{eq}, V_{eq})$ are the mean horizontal, geostrophic, and equilibrium velocity components, respectively, and $(\tau_x, \tau_y)$ are the turbulent stresses. By analogy with the TKE in Fig. 5b, the turbulent stresses are relatively large in the SBL (below $z \sim 200$ m) throughout the night. If the equilibrium wind is independent of time, Eqs. (18) and (19) describe an oscillation around the equilibrium wind. Although the geostrophic and equilibrium winds in our LES vary with time (see section 2d), their variations are slower than those of the oscillation.

Figure 11b shows hodographs of simulated horizontal wind together with the equilibrium wind represented by an average of its values at 2100 and 2400 LST, which are times between the initiation of the oscillation (1800 LST) and the appearance of the maximum LLJ (2700 LST). The LES reproduces the observed oscillations with different arcs and centers at the three heights reasonably well. The wind vector at each height rotates approximately $180^\circ$ around the corresponding equilibrium wind.
vector between 1800 and 3000 LST. This period is in fair agreement with that of the inertial oscillation $2\pi/|f| \approx 21$ h. The point of the equilibrium wind vector (i.e., the center of the oscillation) and the magnitude of its vector difference from the wind vector at 1800 LST (i.e., the amplitude of the oscillation) demonstrate that the LLJ nose appears near $z = 190$ m.

2) RICHARDSON NUMBER AND INFLECTION POINT

The LLJ generates strong VWS below its nose (e.g., Mahrt et al. 1979; Banta et al. 2006). To examine a possibility of shear instability, vertical distributions of the gradient Richardson number $R_i$, defined by

$$R_i = \frac{g}{\Theta_0} \frac{\partial (\Theta V)}{\partial z} \left[ \left( \frac{\partial (u)}{\partial z} \right)^2 + \left( \frac{\partial (v)}{\partial z} \right)^2 \right]^{-1},$$

are plotted in Fig. 12. The observed $R_i$ at 2400 LST has maxima near the LLJ nose and at $z = 500$ m (Fig. 12a). The layer in which $R_i$ is smaller than its critical value of 0.25 is between $z = 500$ and 900 m. Although the VWS increases below the LLJ nose (Fig. 3c), the large

![Figure 11](image1.png)

**Fig. 11.** Hodographs of horizontal wind near the LLJ nose from 1200 to 3000 LST on day 33: (a) the Wangara experiment ($z = 100, 200, and 400$ m) and (b) the LES ($z = 90, 190, and 390$ m). An open circle designates the wind at 1200 LST, and filled circles designate those for every 3 h. The cross, star, and diamond in (b) represent the equilibrium winds [Eqs. (18) and (19)] at $z = 90, 190, and 390$ m, respectively.

![Figure 12](image2.png)

**Fig. 12.** As in Fig. 7, but for the gradient Richardson number. The vertical thin line represents the critical Richardson number 0.25.

![Figure 13](image3.png)
temperature gradient there (Fig. 2c) keeps Ri larger than 0.25. The simulated Ri at 2400 LST also has similar characteristics (Fig. 12b). Ri, 0.25 above z = 500 m results from the near-neutral stratification in the RL (Fig. 2d). At 2530 LST, Ri decreases between z = 200 and 500 m and falls below 0.25 at z = 400 m, because the VWS there increases with gradually growing LLJ nose after 2400 LST (Figs. 3c and 3d). Ri at z = 400 m, however, exceeds 0.25 at 2540 LST and, at 2700 LST, the layer below z = 700 m results in having Ri > 0.25. These changes may suggest that, between 2400 and 2700 LST, shear instability is triggered by the strong VWS above the LLJ nose, and then the dynamically unstable state is eliminated. Note that the Ozmidov scale, defined by \( \frac{\varepsilon^{1/2}}{N^{3/2}} \) with the energy dissipation rate \( \varepsilon \) and the buoyancy frequency \( N \), near \( z = 400 \) m, is at least 2 times larger than the vertical grid spacing \( \Delta z = 20 \) m, although it is smaller than 10 m, within the SBL.

Figure 13 shows vertical profiles of horizontal velocity components. An inflection point for the \( u \)-component profile in the observation at 2400 LST is located above the LLJ nose (Fig. 13a), and that in the simulation at 2530 LST is also at nearly the same location (Fig. 13b). The strong VWS near this inflection point is responsible for Ri < 0.25 at 2530 LST (Fig. 12b). Although the \( v \) component varies nearly monotonically with height (Figs. 13a and 13b), observed and simulated inflection points for its profile exist at z = 75 and 140 m, respectively.

3) POTENTIAL TEMPERATURE AND VELOCITIES

If a roll structure is generated by shear instability, the profiles of Ri (Fig. 12) and the \( u \) component (Fig. 13) would predict that it is aligned roughly in the y direction. However, horizontal distributions of simulated potential temperature around \( z = 200 \) m between 2530 and 2600 LST illustrate that two to four stripes are aligned in the \( x \) direction (not shown). In fact, their orientation of \( 0^\circ \) is perpendicular to the VWS vector near \( z = 200 \) m (Fig. 7).

In view of the orientation of the stripes, Fig. 14 shows vertical distributions in the \( y-z \) plane of simulated potential temperature and velocities averaged over the \( x \) direction at 2550 LST. Two pairs of vortices with a maximum amplitude between \( z = 100 \) and 200 m are visible (Fig. 14a).

Figure 15 shows temporal variations of amplitude and phase of waves with mode (0, 2), computed using the Fourier transform. The amplitude of the waves increases until 2600 LST while oscillating. It maintains a nearly constant magnitude between 2600 and 2630 LST and then decreases gradually (not shown). This variation demonstrates that the period between 2530 and 2600 LST is the developing stage of the waves. The phase of the waves constantly moves to south, and its velocity is estimated to be approximately \( \pm 5.6 \) m s\(^{-1} \). This velocity corresponds to the mean wind velocity in the \( y \) direction between \( z = 400 \) and 1400 m, where the horizontal convective rolls reside (Fig. 9).

To return to Fig. 14b, a horizontal velocity is obtained by subtracting the phase velocity of the waves from the \( v \) component. The undulation of the isentropic surface synchronizes with the flow.

4) EOF MODE

Figure 16 shows vertical distribution of along-roll, cross-roll, and vertical velocity fluctuations for EOF mode (0, 2), which corresponds to the vortices in Fig. 14.
and has the second largest eigenvalue with a contribution of 4.6% to all modes. Rolls similar to those in Fig. 9 exist in the RL. The orientation of these rolls is 0° and is slightly to the left of the orientation of the VWS vector in the RL (approximately -30°; Fig. 7). Figure 17 shows TKE productions for EOF mode (0, 2). In the RL, $S_{\text{cross}}$ is small and $B$ is dominant. These similarities in the RL between Figs. 9 and 16 and between Figs. 10 and 17 suggest that the rolls above $z = 400$ m in Fig. 16 are also part of the horizontal convective rolls.

The rolls in the RL are coupled with different rolls that have their center near $z = 200$ m (Fig. 16). The latter have different characteristics from the former: the orientation of the rolls (0°) is perpendicular to the VWS vector near their center (approximately -90°; Fig. 7), and the along-roll velocity fluctuations are positive (negative) where the cross-roll velocity fluctuations are negative (positive). Figure 17 shows that, at $z = 180$ m near the inflection point for the $\nu$-component profile (Fig. 13b), $S_{\text{cross}}$ has a maximum and correlates negatively with $B$. This negative correlation means that the TKE is transferred from kinetic energy of the mean flow and is partly passed to potential energy (Klaassen and Peltier 1985). The magnitude of $B$ appears to be large compared with that of $S_{\text{cross}}$, possibly because $\theta_\nu$ is not decomposed into eigenmodes [Eq. (17)]. The TKE production (Fig. 17), together with Ri (Fig. 12b), may suggest the occurrence of a KH-like instability in the lower layer, where the term “like” is added because $\Delta z$ is larger than the Ozmidov scale in the SBL. If the rolls in the lower layer are caused by the KH instability, an aspect ratio of their horizontal wavelength to the depth of the shear layer is one of important parameters. It is not easy, however, to determine the depth of the shear layer for the shear flow that turns its direction with height (Fig. 7). If the depth of the shear layer is defined by the boundary between the two types of roll, it is estimated to be approximately 400 m from Figs. 9 and 16. On the other hand, if it is defined by the height at which $S_{\text{cross}}$ vanishes, it is estimated to be approximately 600 m from Fig. 17. Because the roll wavelength is 2500 m (Fig. 16), these estimations give the aspect ratio of 4.2–6.3. This aspect ratio is reasonably consistent with
a theoretical value for the KH instability (e.g., Davis and Peltier 1976).

At 2530 LST, the location of $R_i < 0.25$ (Fig. 12b) is nearly the same as that of the inflection point for the $u$-component profile (Fig. 13b) and is different from that of the inflection point for the $v$-component profile (Fig. 13b) and that of the maximum $S_{cross}$ (Fig. 17). To explain the reason for this discrepancy, the energy transfer from a mode with a wavenumber vector parallel to the $x$ direction to that parallel to the $y$ direction needs to be examined. Although $D_{cross}$ and $D_{vert}$ constitute the energy transfer between different modes, they are computationally prohibitive to calculate (Wilson and Wyngaard 1996). Instead, we examine the energy redistributions for the original LES data that are not decomposed (Fig. 18). The TKE production in the $x$ direction $S_u$ has a maximum near $z = 400 \text{ m}$ at which $R_i < 0.25$ at 2530 LST (Fig. 12b). Around this height, the energy redistribution $R_y$ is negative, whereas $R_u$ and $R_w$ are positive. Thus, although the TKE in the $x$ direction is produced in the dynamically unstable layer with $R_i < 0.25$, a large part of the energy is transferred to the TKE in the other directions. This transfer is possibly because the horizontal convective rolls with larger energy prevent a structure from having an axis perpendicular to their axis.

The results from Figs. 11–18 suggest that the intensification of the VWS in the layer between the bottom of the convective rolls in the RL and the gradually growing LLJ nose lowers $R_i$ at $z = 400 \text{ m}$ below 0.25, the KH-like instability is induced, and the KH-like vortices are coupled with the convective rolls. Although the maximum amplitude of the vortices is located near the inflection point for the $v$-component profile below the LLJ nose, the trigger of the instability is considered to lie in the layer with $R_i < 0.25$ above the LLJ nose.

4. Summary and conclusions

An LES model is used to study diurnal variations of an ABL from 0900 LST on day 33 to 0600 LST on day 34 of

![Fig. 16. As in Fig. 9, but for EOF mode (0, 2). The horizontal wavelength of the roll is 2500 m, and the direction of the roll axis is eastward.](image1)

![Fig. 17. As in Fig. 10, but for EOF mode (0, 2).](image2)

![Fig. 18. Vertical profiles of shear production in the $x$ direction $S_u$ and energy redistributions in the $x$, $y$, and $z$ directions ($R_u$, $R_v$, $R_w$), averaged from 2530 to 2600 LST on day 33 for the LES data, where $R_u + R_v + R_w = 0$.](image3)
the Wangara experiment. It considers the longwave radiation based on a broadband transmittance scheme and the geostrophic wind based on the observation, both of which control the structure of the nocturnal ABL. The friction velocity and surface heat flux are calculated from the Monin–Obukhov similarity theory with a roughness length of 0.01 m for momentum, where the “surface” temperature is given by the air temperature observed at $z = 1.2 \text{ m}$ instead of the ground-surface temperature. The LES reproduces one-dimensional features of the observed boundary layer and gives useful insights into three-dimensional features that were not observed. The results are summarized as follows.

a. Comparisons with the observation

The LES reproduces the observed evolution of the ABL well, although several discrepancies are found, probably because of neglecting subsidence in a high pressure system and large-scale horizontal gradient of specific humidity. Improving the prediction of the friction velocity under stable conditions remains a challenge.

b. LLJ

Nocturnal winds exhibit an inertial oscillation around an equilibrium wind that varies with height (e.g., Van de Wiel et al. 2010), leading to the occurrence of an LLJ. The LLJ nose appears near $z = 200 \text{ m}$, which depends on the equilibrium wind and the wind at sunset, and initially nearly corresponds to the top of the SBL. The maximum wind speed of the LLJ exceeds $12 \text{ m s}^{-1}$ after midnight.

c. Horizontal convective rolls

Because an RL has near-neutral stratification, it can be destabilized thermally by external forcing. The horizontal advection due to the LLJ under the large-scale horizontal gradient of temperature warms up the lower part of the RL more strongly than its upper part, thus destabilizing the RL. Consequently, horizontal convective rolls parallel to the VWS vector in the RL are generated between $z = 400$ and $1400 \text{ m}$.

d. KH instability

As the LLJ nose stands out, the VWS is intensified between the bottom of the convective rolls in the RL and the LLJ nose, and the layer with $\text{Ri} < 0.25$ appears there. The resulting KH-like vortices occur below the convective rolls and are coupled with them. Although...
the maximum amplitude of the vortices is located below the LLJ nose, the trigger of the instability lies in the layer above the LLJ nose.

The present study shows that large-scale horizontal gradient of temperature is important for understanding the evolution of RLs. In this LES, the specific humidity was very small. A large amount of specific humidity and an appearance of cloud could also destabilize the RL through radiative cooling. We plan to examine such an effect of radiative cooling on the RL evolution in a future study.

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APPENDIX A

Daytime Flow Structure

For readers’ reference, a flow structure during the daytime is shown in Fig. A1. A striped pattern is not found for the velocity fluctuations in the x direction at \( z = 710 \) m, and a polygonal cellular pattern appears for the vertical velocities at \( z = 210 \) m, which corresponds to approximately 0.2 times the CBL depth.

APPENDIX B

Effects of the Domain Size

To examine the effects of the domain size on the development of roll structures, we have made the same experiment with a larger horizontal domain size of 10 km \( \times \) 10 km. The resulting horizontal distribution of velocity fluctuations in Fig. B1 is nearly the same as that in Fig. 8. An EOF analysis for the larger domain size, however, increases the number of eigenmodes, decreases the difference of each eigenvalue, and makes the selection of dominant modes arbitrary. For example, EOF mode (2, 4) that corresponds to EOF mode (1, 2) for the smaller domain size of 5 km \( \times \) 5 km is subdivided into EOF modes (2, 3), (2, 4), and (2, 5), each of which has only a contribution of 1%–2% to all modes. Thus we showed the results for the smaller domain size.

APPENDIX C

Effects of Subsidence

We have also made an experiment to examine the effects of subsidence by adding \(-W_a(\Phi)\alpha z\) to the rhs of Eq. (1). The large-scale vertical velocity \( W \) is assumed to vary linearly with height and to be independent of time:

\[
W(\text{m s}^{-1}) = \begin{cases} 
0, & z \geq 1800 \text{ m}, \\
-6 \times 10^{-3}, & z = 1500 \text{ m}, \\
0, & z = 0 \text{ m}.
\end{cases}
\]
Figure C1 shows part of the results. The subsidence controls the penetrative convection in the upper part of the CBL effectively. It also tends to make the stratification in the RL more stable and reduces the TKE of roll structures (Fig. C1b). Except for these changes, however, the results such as the location and speed of the LLJ are fairly similar to those without subsidence (Fig. C1a).

REFERENCES


