Does Wind Shear Cause Hydrometeor Size Sorting?

DANIEL T. DAWSON II
Center for Analysis and Prediction of Storms, Norman, Oklahoma

EDWARD R. MANSELL
NOAA/National Severe Storms Laboratory, Norman, Oklahoma

MATTHEW R. KUMJIAN
The Pennsylvania State University, University Park, Pennsylvania

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ABSTRACT

Several recent studies have implicated vertical wind shear in producing steady-state size sorting of a distribution of hydrometeors falling at their terminal velocity, which varies as a function of hydrometeor diameter. In particular, this mechanism has been invoked to explain both the strength and storm-relative orientation of the commonly observed differential reflectivity ($Z_{DR}$) arc in supercell thunderstorms. However, the actual role of the shear has not been fully clarified. In this study, a simple analytical model is used to show that the fundamental source of size sorting is the storm-relative wind field itself and, in particular, its mean taken over the depth of the sorting layer. Wind shear is only strictly required for producing sustained size sorting in the special but common case of a precipitation source having a motion that lies on the hodograph (such as with the environmental winds at the source level). In supercells, the precipitation source (the rotating updraft) does not necessarily move with the winds at any level. It is shown that this off-hodograph propagation and the associated storm-relative mean wind is responsible for the positive correlation of size-sorting observables (such as $Z_{DR}$) and storm-relative helicity that has been noted in previous work.

1. Introduction

Hydrometeor size sorting (HSS) is a ubiquitous feature of precipitation systems and has been the subject of several recent studies exploring its nature and implications for physical understanding of observed hydrometeor size distributions. HSS occurs because hydrometeors exhibit a general increase of terminal fall speed with diameter, so that trajectories for hydrometeors of different sizes will in general be different depending on the microphysical and dynamical nature of a given precipitation system. Herein, we are concerned only with steady-state HSS associated with horizontal winds and not the separate mechanisms of transient HSS associated with a finite impulse of precipitation or HSS by an updraft (Kumjian and Ryzhkov 2012, hereafter KR12).

Marshall (1953, hereafter M53) investigated the patterns produced by falling snow and pointed out that the parabolic “mare’s tail” trajectories of the falling aggregates would only occur in the presence of vertical shear of the horizontal wind in the sedimentation layer and that the largest, fastest-falling particles would be preferentially found nearer the leading edge of the precipitation shaft owing to their steeper fall trajectories. Gunn and Marshall (1955, hereafter GM55) explicitly discussed the role of wind shear in HSS for differently sized drops falling in various wind profiles. They point out that the sequence of particle sizes experienced at a fixed point on the ground depends on the ground-relative translation of the precipitation shaft as a whole, such that in some cases the largest particles may actually arrive after the smallest ones and not...
necessarily along the leading edge as in the case considered in M53.

More recently, HSS has been implicated in the development of polarimetric features of supercell thunderstorms, in particular the differential reflectivity ($Z_{DR}$) arc near the right (left) edge of the forward flank of right (left)-moving supercells (Kumjian and Ryzhkov 2008, hereafter KR08, and 2009, hereafter KR09), which results from a preponderance of relatively large drops. KR09 established a positive correlation between the magnitudes of $Z_{DR}$ in the ZDR arc and the magnitude of the storm-relative environmental helicity (SREH; hereafter the magnitude, not the signed quantity). Additionally, they noted that the orientation of the $Z_{DR}$ arc was roughly perpendicular to the low-level storm-relative winds. Dawson et al. (2014, hereafter D14) investigated the separate and combined contributions of rain and hail size sorting to the overall distribution of low-level polarimetric signatures. They found that, since hail typically falls over a greater depth than rain (which originates below the melting level from melting hail), the differing storm-relative wind profiles over the deep (shallow) layers of hail (rain) sedimentation led to different patterns of HSS for the two categories, which was at least partially described by different storm-relative mean wind patterns. They noted that the orientation of the $Z_{DR}$ arc was still described by the pattern of HSS for the two categories, which was at least partially described by different storm-relative mean wind vectors. The gradient of hail mean diameter tended to align with the mean storm-relative wind vector taken over a deep layer; the gradient of rain mean diameter depended both on the mean storm-relative wind taken over a shallow layer and on the pattern of hail mean diameter from which it melted (D14). D14 also discussed the impact on the size-sorting pattern of the storm-induced perturbations to the environmental wind profile, which can be substantial depending on the type of precipitation system. In supercells, the greatest modifications are found in the vicinity of the mesocyclone and updraft, with weaker effects farther out into the forward-flank precipitation region. Therefore, throughout the rest of the paper, we will simply use the words “wind shear” and storm-relative helicity (SRH) without the “environmental” modifier. For simplicity, we still assume horizontally homogeneous wind profiles in keeping with previous work (KR09; KR12; D14). We refer the reader to D14 for further discussion of this issue.

The aim of the current study is to clarify the role of the wind shear and SRH on HSS using rain as an example. We adopt the simple analytical approach of M53 and GM55 to explain the positive correlation of $Z_{DR}$ and SRH and the observed orientation of the $Z_{DR}$ arc found in KR09 as well as the orientation of the mean diameter gradients in D14. We find that the fundamental explanation behind these phenomena is not the wind shear but rather the storm-relative wind profile, which may or may not exhibit shear. Specifically, HSS of this type occurs because of hydrometers of different sizes falling at different terminal velocities through a given storm-relative wind profile. The differing residence times in turn result in hydrometers of different sizes being advected different distances by the same wind profile along the direction of the mean storm-relative wind. We show that the earlier attributions of HSS to the shear assumed a special case where the “generating source” moves with the environmental wind at that level (M53; GM55). In this special case, wind shear below the generating source is indeed required to produce net HSS at the bottom of the layer. This assumption is inappropriate for generating sources whose motion may depart significantly from the environmental wind (i.e., off-hodograph motion), such as with supercells and mesoscale convective systems ( MCSs) driven by cold pool propagation superimposed on a background wind profile.

2. Analytical sedimentation model

The horizontal equation of motion for a raindrop falling through an arbitrary horizontally homogeneous vertical wind profile can be written simply (M53) as

$$\frac{dx}{dt} = u_x(z),$$

where $x$ is the horizontal position vector of the drop relative to its initial location and $u_x(z)$ is the vertically varying horizontal wind relative to the drop’s motion at its origin. Throughout the rest of this paper, we will refer to this simply as the storm-relative wind. M53 and GM55 referred to the drop’s horizontal motion at its origin as the generating-source motion and assumed that it was equal to the environmental wind at its level. Crucially, we relax this assumption in this study. Equation (1) assumes that the horizontal motion of the drop responds instantaneously to the horizontal wind at a given level once leaving the environs of its generating source, which is a reasonable approximation for our purposes (e.g., Stackpole 1961).

Assuming that the drop falls at a constant terminal velocity $dz/dt = v_t(D)$, where $D$ is the drop equivolume diameter, we can readily express the horizontal change in position of the drop in terms of depth $z$. We have for a given drop (M53)

$$\frac{dx}{dz} = \frac{u_x(z)}{v_t(D)}.$$

In this study, we use the empirically derived velocity–diameter relationship for rain of Brandes et al. (2002):
Integrating (2) over total depth $Z$ (where $Z$ increases downward toward the ground from zero at the generating source), we have

$$v_t(D) = -0.1021 + 4.932D - 0.9551D^2 + 0.07934D^3 - 0.002362D^4. \quad (3)$$

for any depth $Z$, and $\bar{u}_t$ is the mean storm-relative wind over the layer spanning the generating source to $Z$. Thus, the horizontal displacement $x$ of the drop depends only on its terminal velocity, the total depth over which it falls, and the mean storm-relative wind in that layer (GM55). Thus, two different wind profiles with the same $\bar{u}_t$ will yield identical HSS patterns at the bottom of the layer (Fig. 1).

In common with previous work (M53; GM55; KR09; KR12; D14), we make a few other simplifying assumptions in our analyses. We assume no change in $v_t$ with air density. We assume that the hydrometeors do not change appreciably in size (i.e., they do not evaporate, coalesce, or break up). We assume that the transition of a hydrometeor from the generating source to being influenced by the surrounding wind profile (i.e., the ejection of the hydrometeor from the updraft into its surroundings) is instantaneous, when in reality there will be a transition period in which the hydrometeor moves from being primarily influenced by the updraft circulation associated with the generating source to being primarily influenced by the surrounding horizontal winds. Additionally, we are examining the case of quasi-steady-state HSS, in which the precipitation source is continuous in time and space, whereas real-world precipitation systems clearly exhibit substantial spatiotemporal variability. Finally, we choose a single level for the precipitation source in our analysis, when in real precipitation systems, particularly deep convective storms, hydrometeors of many different sizes will be “generated” at many different levels. Our analysis is thus concerned with what happens to a given distribution of hydrometeors (which may already be influenced by size sorting in the general case) as it falls through the layer below this level. Regardless, the aforementioned previous work showed that the patterns of HSS predicted using these simplifying assumptions compare qualitatively well to the observed patterns, increasing our confidence that the essential physics of the phenomenon are being captured.

3. Size sorting by the storm-relative winds

a. Demonstration for idealized and real-world hodographs

We first apply the analytical model to the case of a straight hodograph with shear of $20 \text{ m s}^{-1}$ over the lowest 3 km for three different storm motions (Figs. 2a,c,e).
Trajectories of raindrops with diameters of 8, 1, and 0.5 mm are also shown. The trajectory endpoints (green crosses in Fig. 2) form a line that is exactly parallel to the storm-relative mean wind vector for each profile. The horizontal spread of the trajectory endpoints can be used to quantify the amount of size sorting that has occurred in the layer. Specifically, we define the magnitude of HSS as $HSS_{\text{mag}} = |\Delta x|/Z$, where $\Delta x = x_f - x_o$, and the subscripts $s$ and $l$ refer to small (0.5 mm) and large (8 mm) drops, respectively.

To visualize the low-level HSS pattern, we perform the following procedure, which is reminiscent of that employed by KR09. To represent a spatially extended precipitation source, a circular region of "generating points" with a radius of 10 km is specified at 3 km AGL, with points every 100 m in the horizontal directions (not shown). For each of these generating points, the surface horizontal displacement of drops with diameters ranging from 0.25 to 8.0 mm, at intervals of 0.05 mm, is computed using (4), and the number density of each drop bin is weighted by a Marshall and Palmer (1948)–type exponential distribution. The distribution for each generating point is computed such that the mean volume diameter $D_m$ is a constant 2 mm, and the liquid water content $W$ varies with a cosine-squared function from a maximum of 3 g kg$^{-1}$ at the center of the circular region to 0 g kg$^{-1}$ at the edge. Other distributions and parameters could be chosen but do not affect the qualitative picture. For each drop size bin at each generating point, we then compute $Z$ and $Z_{\text{DR}}$ for an S-band radar using the T-matrix (Waterman 1969; Vivekanandan et al. 1991; Mishchenko 2000) approach described in Jung et al. (2008, 2010). We then gather the surface termination points of each drop size into 500$\times$500 m$^2$ spatial bins. For each spatial bin, we then compute the total $Z$ and $Z_{\text{DR}}$ (Figs. 2b,d,f) from the relative contributions of all trajectory termination points within that bin.

We also apply the model to two real-world supercell hodographs: the 8 May 2003 Oklahoma City, Oklahoma, tornado supercell and the 1 June 2008 central Oklahoma nontornadic supercell (Figs. 3a and 3b,
respectively). The former has been the subject of several observational and modeling studies (e.g., Romine et al. 2008; Dowell et al. 2011; Dawson et al. 2013; Yussouf et al. 2013), while the latter was the subject of the observational and simulation studies of Kumjian et al. (2010) and D14, respectively. The resulting pattern of simulated $Z_{\text{DR}}$ for the 1 June 2008 case (Fig. 3d) is similar to the much more complex numerical simulations of D14, and in both this and the 8 May case (Fig. 3c) the orientation of the simulated $Z_{\text{DR}}$ gradient compares favorably to the overall orientation of the observed $Z_{\text{DR}}$ arcs (Figs. 3e,f), as indicated by the magenta lines and arrows in (Figs. 3c–f). However, it is important to recognize that, as shown by D14, the magnitude and orientation of the $Z_{\text{DR}}$ arc are also influenced substantially by the sorting of hail, which falls through
a deeper layer before melting into rain below the melting level. Therefore, more generally, the orientation of the $Z_{\text{DR}}$ gradient will be a function of the combination of the storm-relative mean wind taken through the deeper layer of hail sorting and the shallower layer (i.e., below the melting level) of rain sorting.

Additionally, the melting hail will be expected to produce a rain distribution characterized by a relative abundance of large (6–8 mm) drops (e.g., Rasmussen and Heymsfield 1987; Ryzhkov et al. 2013), yielding very high $Z_{\text{DR}}$ on the order of 5 dB (KR08; D14). In the simple sedimentation model shown here, the progenitor distribution is assumed to be exponential, such that smaller drops vastly outnumber these larger drops. Since drops above 4 mm in diameter fall at roughly the same terminal velocity (Fig. 4, black line), little sorting occurs among these sizes and they have similar termination points. The simulated $Z_{\text{DR}}$ in these regions (i.e., near the southern edge of the precipitation pattern in Figs. 3c and 3d) is thus weighted toward somewhat lower values (3–4 dB, Fig. 4, blue line) owing to the greater abundance of these medium-sized drops. In short, the presence of the very high values of $Z_{\text{DR}}$ in the observed $Z_{\text{DR}}$ arcs (Figs. 3e,f) can be taken as evidence of a distribution of raindrops (or even water coated ice cores) heavily skewed toward large diameters, owing to their derivation from melting hail. Further discussion of these points can be found in D14.

b. HSS, wind shear, and SRH

As demonstrated in the previous section, the HSS pattern obtained at the bottom of a layer will have
a different magnitude and orientation even for the same wind profile if the storm motion is different. For the wind profile and storm motions shown in Figs. 2a, 2c, and 2e, the 0–3-km SRH is 100 m² s⁻² in each case. Moreover, it turns out that one can easily construct a wind profile and storm motion with substantial SRH but no net HSS and vice versa. Examples of both “pathological” cases are shown in Fig. 5. In Figs. 5a and 5b, the example is of a circular hodograph centered on the origin with an assumed storm motion of zero (e.g., pure Beltrami flow). The 0–3-km SRH is 628 m² s⁻². We assume zero storm motion for illustration purposes. The hydrometeor trajectories are circles with beginning and end points at the origin such that no net HSS occurs. In Figs. 5c and 5d, The hodograph is one of zero ground-relative wind from 0 to 3 km and an assumed storm motion of 10 m s⁻¹ to the southeast. A plausible real-world approximation to the latter case is that of a cold pool–driven squall line propagating into a quiescent atmosphere. This case has both zero SRH and zero 0–3-km wind shear, and yet substantial HSS still occurs (Figs. 5c,d). Thus, we see that neither SRH nor wind shear is fundamental to HSS but rather only the magnitude of the storm-relative mean wind. We also see by examining the constant-wind case that, in the case of a generating source moving on the hodograph, shear...
would be required for a nonzero storm-relative mean wind and, thus, HSS.

KR09 showed that the magnitude of $Z_{\text{DR}}$ in their idealized simulations was positively correlated with SRH and suggested that changes in the magnitude and orientation of the $Z_{\text{DR}}$ arc could be a useful qualitative indicator of near-storm low-level wind shear changes. To examine this correlation in the context of this study, we take each of the hodographs examined in the previous sections and compute the magnitude of HSS resulting from a sample of 50 random storm motions in each case (Fig. 6). The storm motions are randomly chosen within a 10 m s$^{-1}$ radius of the 3-km ground-relative wind vector in the straight-hodograph case (Fig. 6a), the observed storm motions for the 8 May and 1 June supercell cases (Figs. 6b, c), and of the origin for the circular and zero-wind-hodograph cases (Figs. 6d, e). We then plot HSS$_{\text{mag}}$ as a function of the magnitude of SRH in Fig. 7 for each of the hodographs. We see a clear positive relationship for the 8 May and 1 June cases (green and blue dots in Fig. 7) reminiscent of Fig. 11 of KR09. This relationship is obtained because SRH is itself highly correlated with the magnitude of the storm-relative mean wind in these cases, owing to the mostly off-hodograph-right storm motions (recall that the SRH is proportional to the area swept out by the storm-relative winds over the depth of the layer in question, so provided that this area is not zero, a larger storm-relative mean wind is associated with a larger SRH). However, for a curved hodograph, one can obtain nonzero SRH even if the storm motion lies on the hodograph, so off-hodograph motion is not a strict requirement for this correlation.

The 1 June case shows more HSS for a given SRH compared to the 8 May case because of the overall stronger storm-relative mean winds. In contrast, a much weaker positive relationship is apparent for the straight-hodograph case (magenta dots in Fig. 7) owing to the clustering of storm motions more evenly around the hodograph (to reflect the expected even incidence of left and right movers for a straight hodograph; Fig. 6a). In particular, for those storm motions that lie roughly parallel to the hodograph, SRH will be minimized while HSS$_{\text{mag}}$ will be large or small depending on the magnitude of the storm-relative mean wind. The extreme example of this behavior occurs for the zero-wind-hodograph case, where SRH is zero for all storm motions, but HSS$_{\text{mag}}$ varies between 0 and 4 for the storm motions considered (turquoise dots in Fig. 7). Finally, for the circular-hodograph case (red dots in Fig. 7), SRH has a constant value of 628 m$^2$ s$^{-2}$ (all storm motions are contained within the hodograph), while HSS$_{\text{mag}}$ again varies between 0 and 4.

4. Discussion and conclusions

In this study, we questioned the role of wind shear in HSS and demonstrated using a simple analytical model that the fundamental cause of HSS over a given layer is the presence of nonzero storm-relative mean wind in that layer, irrespective of the presence of shear. We applied the model to idealized and real-world wind profiles to demonstrate the impact of varying storm motions on the pattern of HSS at the bottom of the layer, showing that the magnitude and direction of the gradient of HSS are uniquely determined by the storm-relative mean wind vector. We further demonstrate that the correlation of SRH with $Z_{\text{DR}}$ (a proxy for the amount of HSS of rain) found by KR09 is not fundamental and is itself due to the correlation of SRH and the magnitude of the storm-relative mean wind, both of which typically increase for increasingly off-hodograph storm motions that are common for supercells. In particular, HSS can occur even in the absence of wind shear or SRH in a layer and, conversely, little or no HSS can occur even in the presence of substantial SRH.

It is important to reiterate that the model of HSS examined in this study is highly simplified. In addition to the other simplifying assumptions already described, we have deliberately ignored the contributions to HSS from transient effects and from the action of updrafts (KR12). We have restricted our analysis in this manner in order to demonstrate in the simplest possible terms the true causative mechanism for HSS within the context of this model. Despite its simple nature, the explanatory power
of this model has been demonstrated through comparisons with observed hydrometeor distributions in supercells using the proxy of Z_{DR} in this and prior studies (e.g., KR08; KR09; KR12; D14).

As discussed by KR09 and D14, near-storm horizontal variation of the wind profile likely also contributes substantially to the unique patterns of HSS seen in supercells. Future work may examine this issue, as well as the general validity of other simplifying assumptions, in more detail.

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