Understanding the Influence of Measurement Uncertainty on the Atmospheric Transition in Rainfall and Column Water Vapor

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ABSTRACT

Measurement uncertainty plays a key role in understanding physical relationships. This is particularly the case near phase transitions where order parameters undergo fast changes and display large variability. Here the proposed atmospheric continuous phase transition is examined by analyzing uncertainty in rain-rate and column water vapor measurements from the Tropical Rainfall Measuring Mission and through an idealized error analysis. It is shown through both of these approaches that microwave rain-rate retrievals can mimic a continuous phase transition. This occurs because microwave retrievals of instantaneous rain rates have a suppressed range. This work also suggests that column water vapor noise may provide part of the plateau seen in the observational relationship. Using updated measurements, this work indicates that the atmosphere is unlikely to undergo a continuous phase transition in rain rate but, instead, contains much larger variability in rain rates at extreme column water vapor values than previously thought. This implies that the atmosphere transitions from a low-variance nonraining state to a high-variance raining state at extreme column water vapor values.

1. Introduction

Rainfall and water vapor are central parts of Earth’s hydrological cycle and understanding their relationship remains an important challenge (Raymond 2000; Held and Soden 2006; Allan and Soden 2008). One approach to this problem is to examine oceanic rainfall $P$ as a function of column water vapor $w$ (Bretherton et al. 2004). The resulting curve, called the $P$–$w$ relationship, probes the conversion of water vapor to rainfall, where $w$ acts both as a source and sink. The curve depends on condensation and convective physics, and the $P$–$w$ shape provides information about these processes.

In early work, Peters and Neelin (2006, 2009) examined the $P$–$w$ relationship using instantaneous rain rates and column water vapor retrievals from the Tropical Rainfall Measuring Mission (TRMM) using the TRMM Microwave Imager (TMI) (Kummerow et al. 1998). They used Remote Sensing Systems (RSS) retrievals at approximately 50-km resolution, with the column water vapor retrievals being conducted in both rainy and nonrainy scenes (Wentz 1997; Wentz and Spencer 1998). They found an upswing past a threshold $w_*$ and then a plateau in the mean rain rate for high column water vapor values. This curve had the form

$$P \sim (w - w_*)^\alpha,$$

where $\alpha$ is a constant. The variance also displayed a peak near $w_*$ such that

$$\sigma^2 \sim 1/|w - w_*|^{-\beta},$$

where $\beta$ is another constant. Further work showed that the curves coalescence when separated by column-averaged atmospheric temperature (Neelin et al. 2009) and apparent clustering was detected (Peters et al. 2009). These observed features seemed to provide evidence for a continuous phase transition in the $P$–$w$ system (Neelin et al. 2008). Here “continuous” means that the average $P$–$w$ relationship does not contain a jump or gap as $w$ is increased. The proposed continuous phase transition is between atmospheric states rather than between water phases.

In general, the defining characteristics of a continuous phase transition are a fast smooth jump in the order parameter (here the mean rain rate $P$) leading to
a plateau or slope change [Eq. (1)] and a variance (or susceptibility) peak at this jump [Eq. (2)] (Yeomans 1992). Noting these features in combination means that the system has changed its stable thermodynamic state across the continuous phase transition. A continuous phase transition can be identified when both a jump in the order parameter and a corresponding peak in the susceptibility are present. The exact location at which this occurs is called a critical point, with Eqs. (1) and (2) providing the definition of the critical point \( w_* \) in a continuous phase transition (Stanley 1987; Yeomans 1992). For the \( P-w \) transition suggested by Peters and Neelin (2006), this would signal an abrupt change from nonraining to raining as the total column water vapor is increased past \( w_* \), with the system showing large fluctuations near \( w_* \). If a continuous phase transition were present, then above \( w_* \), the system would be producing extreme surface rainfall with certainty because the variance reduces. Critical phenomena, such as “universality,” would also be present near the critical point (Yeomans 1992). Understanding the true behavior of the \( P-w \) curve past the strong upswing is therefore important for understanding rainfall extremes.

Phase transitions are traditionally examined in the laboratory, where noise sources are minimized, as, for example, in liquid \( ^4 \)He measurements (Rittner and Reppy 2006). However, for satellite measurements of the climate system, this is not always possible. Routinely, one has to use observations and reconstructions that contain significant measurement uncertainty, which is sometimes poorly understood (e.g., Ricciardulli and Wentz 2004; Tian et al. 2013). Here, “measurement uncertainty” refers to additive or multiplicative noise, saturation, or bias introduced through a retrieval algorithm. Other types of uncertainty exist in satellite measurements, such as natural variability about a relationship. The influence of measurement uncertainty on natural variability will be examined here for the \( P-w \) relationship.

It is known that the satellite retrievals of instantaneous oceanic rain rate and column water vapor from TMI used to build the \( P-w \) relationship in Peters and Neelin (2006, 2009) do contain uncertainty (Wentz 1997; Wentz and Spencer 1998). For microwave observations, this is because many sources aggregate into a combined brightness temperature measurement, which then have to be deconstructed. The effect of uncertainty on the \( P-w \) relationship has not been examined in previous studies and is currently unclear.

To see if atmospheric models can capture the \( P-w \) curve, recent attempts have been made to compare the observed \( P-w \) curve with model-derived curves. Sahany et al. (2012) compared the simulated \( P-w \) relationship found in CAM3.5 at approximately 50-km resolution with the RSS TMI observations. They found that CAM3.5 captured the location of the upswing \( w_* \) (the transition to strong rainfall) but did not show any plateau. Also, Yano et al. (2012) undertook an analysis of model runs at cloud-permitting 4-km resolution and found an upswing but no plateau. These studies suggested the \( P-w \) curve from TMI was only partially reproduced by models across a range of length scales. The variance of the \( P-w \) curve was not examined in these studies.

In the presence of measurement uncertainty, three important questions arise:

(i) How is the \( P-w \) relationship influenced by this uncertainty?
(ii) Is the continuous phase transition interpretation robust against uncertainty?
(iii) Can the disagreement between observed and model \( P-w \) relationships be reconciled?

These questions are investigated here, taking two approaches. The first approach examines the mean and variance of the observational \( P-w \) curve through both TMI and Precipitation Radar (PR) retrievals from TRMM. These retrievals are discussed next. The second approach incorporates measurement uncertainty in idealized \( P-w \) relationships to understand the influence of this uncertainty. This theoretical approach, which is commonly used to assess relationship robustness (e.g., Krauss and Romanelli 1990), allows a more extensive exploration of the effect of measurement uncertainty on the \( P-w \) curve than does observational retrievals alone.

2. Observations

The TRMM satellite contains two instruments that can retrieve rain rate: TMI and the PR. Initial studies of the \( P-w \) relationship were limited to precipitation and column-integrated water vapor from the RSS TMI data. Here, additional independent precipitation retrievals are examined using data from PR and TMI. This will allow the proposed \( P-w \) continuous phase transition interpretation to be assessed using multiple instantaneous rain-rate retrievals, which has not been carried out to date.

This study examines the \( P-w \) relationship over the tropical oceans and uses four different rain-rate algorithms and one column water vapor retrieval algorithm. The rainfall retrieval algorithms are the RSS version 4 (Wentz and Spencer 1998; Wentz 2006; Hilburn and Wentz 2008), GPROF (2A12 version 7) (Kummerow et al. 2001, 2011), 2A25 version 7 using PR alone (Iguchi et al. 2000), and the combined PR and TMI (PR+TMI)
algorithm 2B31 version 7 (Haddad et al. 1997). The RSS and GPROF algorithms use TMI data to retrieve rain rates. The RSS algorithm diagnoses cloud water and scales this to a rain rate and GPROF uses an inversion procedure based on an observationally derived database to match TMI measurements with a best hydrometeor (and rain-rate) scene. For 2A25 and 2B31, the PR observations (which are attenuated radar echoes backscattered through the rain column) are used to estimate the rain rate. For 2A25, a rain rate is estimated using assumed k–Z and Z–R relationships (Iguchi et al. 2000). For 2B31, observations from TMI are additionally used to constrain the rain-rate retrieval. The only column water vapor retrieval algorithm used is that of RSS (Wentz and Spencer 1998) because this is the only algorithm validated in rainy scenes. Measurement uncertainty of these retrievals has been estimated and this is now discussed.

Validation of TMI products can be either be undertaken for instantaneous properties of the retrieval or for properties averaged over much longer periods. For instantaneous validation, ground-based efforts by Wolff and Fisher (2008, 2009) have estimated the measurement characteristics and uncertainties of the instantaneous rain-rate retrievals algorithms discussed above using TMI and PR data over oceans. Validation of instantaneous rain rates shows microwave-only retrievals underestimate true rain rates, especially for extreme rain rates (Wentz and Spencer 1998; Wolff and Fisher 2008). This is because they have a limited rain-rate retrieval range that only reaches 10–20 mm h\(^{-1}\). Recent analysis over land by Tang et al. (2014) also showed that microwave-only retrievals have a narrow dynamical range. Cecil and Wingo (2009) showed that for rainfall, differences between microwave-only and radar retrievals could be as large as a factor of 10 for extreme instantaneous rain rates in cyclones. Recent analysis by Zagrodnik and Jiang (2013) showed that the GPROF TMI retrieval underestimates moderate to heavy rainfall (>15 mm h\(^{-1}\)) when compared to rainfall algorithms derived from PR data, particularly in hurricanes. These studies indicate that for PR-based retrievals, the dynamic range is 75%–100% of the true range, whereas for the TMI-only algorithms (RSSv4 and 2A25), the dynamic range is 10%–30% of the true range. This reduced dynamic range of TMI-based algorithms is shown in Fig. 1 for Tropical Storm Dolores. The validation of rain rates indicates that multiplicative uncertainty or instrument saturation is the dominant form of measurement uncertainty for instantaneous microwave-based TMI rain-rate retrievals.

For column water vapor retrievals, the best current instantaneous validations indicate the uncertainty is additive rather than multiplicative. Column water vapor retrievals importantly use different algorithms in nonraining and rainy scenes, which have different errors. Microwave retrievals in nonraining scenes are quite accurate with a root-mean-square (rms) error of \(\approx 1.2 \text{ mm} \) (Wentz 1997). In rainy scenes, however, the algorithm must be modified because the rain column attenuates and adds noise to the measurement. For the RSS retrieval of \(w\) from the TMI dataset, the rainy-scene rms error is approximately 5.0 mm, with about half of this coming from ground-based and satellite observation spatial–temporal mismatches of radiosondes, implying a \(w\) retrieval error of about 3.0 mm (Wentz and Spencer 1998). Work by Holloway and Neelin (2009, 2010), in which validation was performed in combined rainy and nonrainy scenes, also found similar levels of \(w\) uncertainty.

In addition to the instantaneous validation discussed above, validation over longer periods provides important information about the average \(P–w\) relationships that will be discussed below. Berg et al. (2006) examined retrieval bias as a function of column water vapor using both TMI- and PR-derived rain rates and version 3 of the RSS column water vapor retrieval. They found that TMI and PR bias systematically shifted as column water vapor increased, with average PR-derived rain rate decreasing relative to TMI-derived rain rates at a given \(w\).

Based on the validation discussed above, the PR+TMI rain-rate algorithm (2B31 version 7) along with the RSS version 4 column water vapor algorithm are taken as the reference retrievals in this work. In general, it is difficult to say definitively which algorithm is best in all situations (Masunaga et al. 2002; Ikai and Nakamura 2003; Berg et al. 2006). However, given that the \(P–w\) relationship comprises a mean and variance curve, the need for an accurate variance estimation implies that the best choice retrieval is the one with the most realistic instantaneous dynamic range in rain rate. PR+TMI is chosen here and although it may suffer from small levels of saturation at very high rain rates (where there are also fewer counts), its substantially larger dynamic range than TMI-derived products makes it the best reference choice.

### 3. \(P–w\) relationships from TRMM

Previous work examined the \(P–w\) relationship using mostly microwave rainfall-rate retrievals (Bretherton et al. 2004) and the original interpretation of a continuous

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FIG. 1. Demonstration of TRMM retrieval properties for Tropical Storm Dolores in the northeastern Pacific at 0.5° resolution at 2344 UTC 15 Jul 2009. The circle indicates the location of Dolores (18°N, 117.5°W). Four different rain-rate retrievals (and their version) are shown: (a) RSS, (b) GPROF, (c) PR, and (d) PR+TMI. (e) Column water vapor from RSS. (a) RSS and (b) GPROF (TMI-based retrievals) saturate at 10–20 mm h$^{-1}$, whereas (c),(d) PR-based algorithms have a much larger range of rain rates, reaching ≥50 mm h$^{-1}$. Comparison with (e) shows that these rain-rate differences occur at high $w$ values (≥55 – 60 mm). White indicates no data in all panels and zero rain rate is light gray in (a)–(d).
phase transition by Peters and Neelin (2006) used microwave retrievals from RSS. More recently Peters et al. (2009) used rainfall estimates from PR observations to examine the average $P-w$ relationship. Now that more satellite data is available to build a variance curve with good statistics, this quantity is examined. Given the importance of the variance peak to the continuous phase transition interpretation, confirmation of the peak from rainfall derived from PR observations would provide evidence for the continuous phase transition, whereas absence of the peak would cast doubt on such an interpretation. This question is important because a confirmed continuous phase transition would imply “universality” in interactions near the critical point (Yeomans 1992), which models would have to reproduce.

In Fig. 2, the $P-w$ relationship derived from TRMM using four different rain-rate retrieval algorithms is shown. Figures 2a and 2b use TMI-only algorithms: RSS version 4 (Wentz and Spencer 1998) and GPROF (2A12 version 7) (Kummerow et al. 2001), respectively. Figure 2c uses the PR observations only (2A25 version 7) (Iguchi et al. 2000), and Fig. 2d an algorithm using PR and TMI observations (2B31 version 7) (Haddad et al. 1997). All panels use the RSS $w$ retrieval. Each panel shows level-2 swath data from 2006 to 2011. These were selected for high-quality retrievals where possible, averaged to 0.5° × 0.5° with at least 80% areal coverage, and masked so that all datasets contained the same measurements at nadir. A resolution of 0.25° × 0.25° was also considered with similar results. Finally, these were then summed from 20°S to 20°N and averaged. To assess the convergence of the derived $P-w$ relationships estimates of their convergence error are shown. Here the curves are sampled by year-to-year variability (equivalent to dropping about 15% of points from each sample) by both omitting full years and part years from 2006 to 2011.

The mean curves show generally good agreement among the retrievals. Differences are seen in the “plateau region,” with the microwave-only retrievals in Figs. 2a and 2b showing higher slopes in this region than the retrievals using PR observations in Figs. 2c and 2d, which are flatter. The difference in the plateau region appears consistent with the work of Berg et al. (2006), where it was shown that average TMI-derived rain rates are higher than average PR-derived rain rates as column water vapor increases. Based on Fig. 2, a turning over of the upswing is present for all rain-rate retrievals, with a clear flattening present (suggesting a true plateau) in Figs. 2a, 2c, and 2d and an obvious slope change in Fig. 2b. This indicates the plateau region is likely not a calibration artifact of the TMI rain-rate retrieval, although some ambiguity exists about its true slope given
the dispersion seen here. All four retrievals agree about the location of the upswing in the $P-w$ curve, with $w_e \approx 66$ mm. If one assumes a power-law fit is appropriate, fitting $P \sim (w-w_e)^a$ past $w_e$ gives an exponent of less than 1, although there is considerable uncertainty in the $a$ value. Curves in the major oceanic areas along with isolated regions have been examined and similar agreement amongst algorithms was found.

In contrast to the mean response, the rain-rate variance in Fig. 2 shows distinctly different behavior among the retrievals. The GPROF retrieval in Fig. 2b shows similar behavior to RSS with a decrease after $w_e$. The retrievals based on PR and PR+TMI observations in Figs. 2c and 2d do not show a peak in the variance but remain approximately flat as $w$ is increased past $w_e$. This clearly disagrees with the RSS microwave retrieval in Fig. 2a, which shows a peak at $w_e$ and then a decrease. The same behavior is found irrespective of the oceanic area examined.

The reason for the difference in the variance can be traced to limitations in the TMI retrievals. As discussed earlier, examination of instantaneous rainfall rates from TRMM has shown that TMI retrievals can saturate at rain rates from 10 to 20 mm h$^{-1}$ (Wentz and Spencer 1998; Wolff and Fisher 2008, 2009), particularly for the RSS retrieval. This saturation substantially reduces the range of the retrieval at high rain rates, with a decrease of up to a factor of 10 when compared to retrievals using PR observations (Cecil and Wingo 2009). Validation against instantaneous rainfall rates estimated from ground-based measurements indicates retrievals using PR observations retain realistic dynamic range, with the PR+TMI-based retrieval (2B31) being the most accurate at high rain rates (Wolff and Fisher 2008). The contribution to the variance at $0.5^\circ \times 0.5^\circ$ resolution from scatter about the retrieval, rather than a reduced dynamic range, can be estimated from Wolff and Fisher (2009) and Islam et al. (2012). These studies take place at approximately 150 and 16 km$^2$, respectively, and scaling their reported scatters to $0.5^\circ \times 0.5^\circ$ leads to a variance contribution estimate between 0.2 and 3.0 (mm h$^{-1}$)$^2$ depending on the scaling assumed (from area$^{-1}$ to area$^{-0.5}$). This estimate indicates that retrieval scatter is not a major contributor to the variance at high $w$ at $0.5^\circ \times 0.5^\circ$ resolution, but rather the dynamic range is the dominant factor.

How does the suppressed range of the RSS TMI retrieval at extreme rain rates influence the $P-w$ curves? Figure 2 shows that the mean response is not strongly altered. However, the $P-w$ variance—one of the key indicators of a continuous phase transition—will be reduced if there is a suppressed dynamic range of extreme rain rates. Since this reduced dynamic range in the RSS product occurs for extreme rain rates forming the upswing and plateau—that is, above $w_e$ in the $P-w$ curve—this will lead to a decrease in the variance past this point, as seen in Fig. 2a.

Given that the PR-derived products are considered the best from ground validation, the variance peak in the $P-w$ curve, which is vital for the continuous phase transition interpretation of the $P-w$ relationship, is absent from observational retrievals with a realistic dynamic range. Since the variance is approximately flat for the PR-based retrievals past $w_e$, this means large variability exists in extreme rain rates at extreme column water vapor values. This large variability does not support a continuous phase transition interpretation since if such a transition was present, then the variance should reduce. As such, a critical point does not seem to be present. It should also be noted that a discontinuous jump does not appear in the mean $P-w$ relationships for any retrieval in Fig. 2, and this indicates that a first-order phase transition is unlikely to be present in this system.

4. Understanding $P$ and $w$ uncertainty

To understand why the variance curve has changed substantially between retrievals and to explore measurement uncertainty further, different amounts of uncertainty is now convolved with idealized $P-w$ curves. This idealized analysis will use a Monte Carlo method to incorporate uncertainty (Robert and Casella 2004) and allows scenarios to be tested in a controlled manner. This is a common method to assess the influence of measurement uncertainty and is used in physics and the climate sciences (e.g., Taylor 1996; Krauss and Romanelli 1990; Kuczera and Parent 1998; Binder and Heermann 2010) and also generalizes to other noise sources or physics. It is also an appropriate method here because each $0.5^\circ \times 0.5^\circ$ pixel will have a different $w$ value in the observational retrieval. The analysis here is not seeking to conduct an inversion to obtain a true $P-w$ relationship but rather to explore the influence of measurement uncertainty on assumed underlying $P-w$ relationships.

The previous section showed that $P$ uncertainty can change the $P-w$ curve, but another important source of uncertainty exists in column water vapor. In laboratory experiments, noise in the independent variable would be small; however, for column water vapor retrievals, it can be large, with an rms of 1.2 and about 3.0 mm in nonrainy and rainy scenes, respectively (i.e., 5% at $w \sim 60$ mm in rainy scenes) (Wentz and Spencer 1998). Given that there are two sources of measurement uncertainty, an interesting question arises: what can we learn about the influence of measurement uncertainty from the study of an idealized $P-w$ relationship?
The results presented in Fig. 2 show that the plateau or a slope change at \( w > w_0 \) was an apparently robust feature of the \( P-w \) relationship. Analysis of model runs by Sahany et al. (2012) and Yano et al. (2012) have shown that an extensive plateau was seemingly absent at high \( w \) (their Figs. 7 and. 4a, respectively), although there was a strong upswing in rain rate past \( w_0 \). Since the observations shown in Fig. 2 already contain uncertainty in \( w \), it is therefore interesting to ask the following: can noise in the observational \( w \) measurement provide a partial explanation for the absence of a plateau in the model-derived \( P-w \) relationships?

To understand the influence of \( w \) noise on the \( P-w \) curves, a Monte Carlo approach is used. This is required for the variance calculations because measurement error must be propagated through derived relationships. Consideration is given to 1) errors in rainy scenes being larger than in nonrainy scenes because the algorithms are different and 2) the number of counts at high \( w \) decreasing exponentially. These two factors mean that, near and above the rain-rate upswing \( w_0 \), errors at slightly lower \( w \) have a much larger effect than one would think and the influence of \( w \) noise is systematically shifted to higher \( w \) values.

To test the influence of measurement uncertainty, an idealized \( P-w \) relationship is employed. This idealized approach allows qualitatively different scenarios to be tested, such as those observed in Figs. 2a and 2d. This approach also allows us to develop recommendations for the size of retrieval errors that could be tolerated without substantially changing the \( P-w \) relationship. To ensure a consistent mean relationship between possible scenarios, the underlying rain-rate upswing is assumed to have the form

\[
P = P_0 \left[ 1 + \tanh \left( \frac{w - w_0}{\Delta w} \right) \right]/2, \tag{3}
\]

where \( P_0 \) is a normalization, \( w_0 \) is a location parameter, \( \Delta w \) controls the upswing shape, and there is a potential cutoff above \( w_{\text{cut}} \). The values \( P_0 = 10\, \text{mm h}^{-1} \), \( w_0 = 65\, \text{mm} \), and \( \Delta w = 2\, \text{mm} \) are used in what follows. There are three reasons for choosing this form: 1) it captures the observations from Fig. 2, 2) it can be used to model the \( P-w \) curves from both Sahany et al. (2012) and Yano et al. (2012) by introducing a cutoff, and 3) continuous phase transitional behavior can often be approximated by this form (Yeomans 1992). This means that Eq. (3) provides a well-chosen base form from which to investigate the influence of uncertainty. This approach is also not restricted by model or observationally derived curves, which may contain systematic errors. In what follows, column water vapor is sampled to reproduce observed number counts. The results presented below are robust against realistic changes in \( P_0 \), \( w_0 \), and \( \Delta w \). Variance characteristics in the precipitation observation (full dynamic range for the PR or limited range retrievals for TMI) are added to Eq. (3) as discussed below. This means the final idealized \( P-w \) relationships are not one to one, but retain the mean response given by Eq. (3).

To model different dynamic ranges in the rain-rate retrievals in a simple manner at large \( w \) values, the following multiplicative form is used in the Monte Carlo calculation to sample the underlying rain rate to an observed rain rate \( P_{\text{obs}} \) such that

\[
P_{\text{obs}} = P \left[ 1 + \delta P(\sigma_P)s(w) \right], \tag{4}
\]

where \( \delta P \) is a Gaussian distribution with standard deviation \( \sigma_P \) and \( s(w) \) modulates the variance in the underlying relationship as explained below. For the rain-rate values, explicit retrieval saturation and extra additive Gaussian noise were considered, all with similar results. High- and low-dynamic-range rain-rate retrievals are considered using \( \sigma_P = 0.1 \) and \( \sigma_P = 0.5 \), respectively. This gives maximum rain rates of approximately 13 and 30\,\text{mm h}^{-1}, respectively, consistent with the properties of the RSS TMI and PR-based precipitation retrievals. It models the limited-rain-rate range of microwave compared to radar-derived retrievals. Only the mean and variance curves derived from the Monte Carlo method are shown in what follows for clarity.

For measurement uncertainty in column water vapor, additive noise in \( w \) is considered, with the observed column water vapor \( w_{\text{obs}} \) given by

\[
w_{\text{obs}} = w + \delta w(\sigma_w), \tag{5}
\]

where \( \delta w \) is a Gaussian noise source with standard deviation \( \sigma_w \). Different amounts of \( w \) noise ranging from \( \sigma_w = 1 \) to 5\,\text{mm} are considered. Biased \( w \) retrievals were considered; however, these did not have a substantial influence owing to the small retrieval bias in the current best validations.

To understand the influence of measurement uncertainty, Eqs. (4) and (5) are used to build different variance forms to explore via the Monte Carlo method. A Monte Carlo method automatically propagates measurement error and this allows us to understand the influence of uncertainty on different underlying \( P-w \) relationships. A probability integral could also be used if error propagation is correctly incorporated. The first case considered, shown in Fig. 3, is a continuous phase transition that includes a plateau in the mean response and a decrease in the variance past \( w_0 \). Second, Fig. 4 shows the case of similar mean and variance responses where the curves both increase and contain a plateau. Figure 5 shows an identical case to Fig. 3; however,
When a cutoff is used, meaning that no plateau is present in the underlying relationships.

The different variances in Figs. 3 and 4 are achieved by assuming different forms for \( s(w) \), which modulates the variance with \( w \). Using different \( s(w) \) modulations means that the underlying \( P-w \) relationship has a different variance form, irrespective of the \( P \) dynamic range or the \( w \) retrieval noise. This allows the influence of the Monte Carlo calculation on different types of natural variability to be explored. The specific modulations used for \( s(w) \) are \( 3\{\tanh[(w-61)/5]+1\}\{\tanh[(61-w)/5]+1\} \) and \( 2\{\tanh[(w-65)/2]+1\}/5 \), respectively. Although these forms may seem complex, their role is to qualitatively produce a variance peak in Fig. 3b (emulating a continuous phase transition) and a variance increase and plateau in Fig. 4b. The results presented below are robust to changes in the form of \( s(w) \) as long as the qualitative features are preserved (e.g., equivalent splines could also be used).

Figure 3 shows the Monte Carlo calculation over uncertainty in \( w \) for the \( P-w \) relationship using both a low- and high-dynamic-range retrieval in the presence of a continuous phase transition. The low dynamic range uses \( \sigma_P = 0.1 \) and high dynamic range uses \( \sigma_P = 0.5 \). These two dynamic ranges model the saturation of the
RSS TMI product and the much larger dynamic range of the PR+TMI retrieval algorithm. Figures 3a and 3b show the $P$–$w$ relationship before any $w$ noise is added, and in both cases, a strong upswing in the mean response is present along with a variance peak for the high range retrieval. The remaining panels show $w$ uncertainty incorporated into the $P$–$w$ curves. Figures 3c,d, 3e,f, and 3g,h show increasing amounts of $w$ noise with $\sigma_w = 1$, 3, and 5 mm, respectively. Noise in nonrainy scenes is scaled by the RSS rms retrieval ratio. It is immediately clear from Fig. 3e that introducing $w$ uncertainty in the low-dynamic-range retrieval can change the variance curve.

The cases where there is no underlying continuous phase transition are now considered. This means either a variance peak is absent and/or a plateau is absent. Figure 4 shows the case of no variance peak. Here both the mean and variance increase and contain a plateau, using the variance modification discussed above. Note that there is no apparent continuous phase transition in the underlying relationship (Fig. 4b). The configuration is otherwise the same as Fig. 3. This interesting case shows that measurement uncertainty can change the variance substantially. For example, Fig. 4e shows that a peak in the variance is now seen when $\sigma_w = 3$ mm for low-dynamic-range retrievals. This shows, very simply, that $w$ measurement uncertainty can produce a variance peak for low-dynamic-range retrievals. This is interesting because the physical relationship with no $w$ noise from Fig. 4a did not contain a peak in the variance. Figures 4g and h shows that, for $\sigma_w = 5$ mm, there is no peak in the variance around the underlying upswing, implying that only a limited range in $\sigma_w$ produces a peak in the variance at reasonable $w$ values. In this case, $w$ noise extends the already present plateau.

![Fig. 4. As in Fig. 3, but for a $P$–$w$ relationship in which the variance increases past the transition to strong rain rates and then plateaus along with the mean response.](image-url)
Finally, Fig. 5 shows an identical case to Fig. 3, except this time a cutoff of $w_{\text{cut}} \leq 68$ mm is employed, meaning that there is no plateau in the underlying $P-w$ curves. The cutoff represents a point where there are no further intrinsic $w$ observations. The case of a cutoff provides an idealized representation of the curves in Sahany et al. (2012) and Yano et al. (2012) and is particularly relevant to Fig. 7 of Sahany et al. (2012) where a cutoff of observed but the observational $P-w$ relationship still contains a plateau beyond the cutoff. When $w$ noise is added there are obvious changes in the mean and variance. First, for $\sigma_w = 1$ and 3 mm, both the low- and high-dynamic-range mean responses develop plateaus that were not present in the underlying $P-w$ relationship in Figs. 5a and 5b. This occurs because adding $w$ noise preferentially moves extreme $w$ values (with large mean $P$ responses) to even higher values, thereby creating a plateau. This happens easily because of the exponential decline in the number of counts at high $w$. For $\sigma_w = 5$ mm, the plateau is extended beyond the expected range in $w$, indicating the effect is only relevant for a small range in $\sigma_w$. This shows that uncertainty in $w$ can partly emulate a plateau in certain cases even if the underlying physical relationship does not contain one.

There is one remaining $P-w$ form to consider. This is where no plateau is present in the intrinsic $P-w$ relationship and $P$ and its variance keeps increasing over the full $w$ range. This case is qualitatively equivalent to Fig. 5, except one ignores everything beyond the cutoff in all panels. When $w$ noise is incorporated, a plateau would not be observed in the mean response. The variance would also generally increase with no obvious peak, except for the low dynamic range with $\sigma_w = 1$ mm where a small peak would be seen. For this scenario, the
plateau and variance peak would be pushed into unrealistic \( w \) values.

Figures 3–5 all show a reduction in mean precipitation for high \( w \) as \( \sigma_w \) increases. This occurs for similar reasons to the plateau appearance in Fig. 5. Since mean precipitation increases monotonically with \( w \), but the number counts decrease exponentially as \( w \) increases past \( \sigma_w \), lower \( w \) values and therefore lower \( P \) values (when noise is convolved) receive greater weight in the mean precipitation calculation. This causes the precipitation curve to reduce as \( \sigma_w \) increases.

Figure 5 also shows that the variance can change substantially when measurement uncertainty is incorporated. For the low-dynamic-range retrieval in Fig. 5e with \( w \) noise similar to the RSS product, \( \sigma_w = 3 \text{ mm} \), a peak in the variance is now visible. Further, the high-dynamic-range retrieval in Fig. 5f with \( \sigma_w = 3 \text{ mm} \) shows an approximately flat variance after \( \sigma_w \), consistent with the PR+TMI retrieval algorithm in Fig. 2d. This indicates that incorporating measurement uncertainty in \( P \) and \( w \) can potentially create aspects of a plateau in the mean response and a variance peak. The large changes in variance occur because there are large slope changes occurring in the upswing region of the mean \( P-w \) curve.

Based on the results in Figs. 3e, 4e, and 5e, where a variance peak in the low range retrieval can be seen irrespective of whether one exists in the underlying high range retrieval relationship (Figs. 3b, 4b, and 5b), measurement uncertainty can mimic a continuous phase transition in this system even where one does not exist. Further, Figs. 3–5 strongly indicate that measurement uncertainty must be accounted for when undertaking the comparison in the plateau region. This is because uncertainty obscures the physical interpretation of the \( P-w \) curves. This can hide other factors, such as resolution or convective parameterizations, which may influence the representation of the \( P-w \) relationship.

### 5. Models of the \( P-w \) curve

Current literature models that attempt to capture the \( P-w \) relationship are now discussed in light of the new observational results presented here, since these new data favor different \( P-w \) variance curves with an updated physical interpretation. The new observational benchmark for comparison with models (which has been convolved with instrument uncertainty) is Fig. 2d, because there are excellent indications that the PR+TMI retrieval algorithm (2B31) has a realistic instantaneous dynamic range from ground-based validation. This new benchmark has a different variance curve than the previous benchmark in Fig. 2a, although the location of the upswing in \( P \) is similar between retrievals.

Two types of modeling approaches have been used in \( P-w \) comparisons. The first approach is atmospheric models (Sahany et al. 2012; Yano et al. 2012). These have been successful in capturing the location and upswing shape of the transition. However, these models have been less successful at capturing the plateau, and their variance behavior has not been explored. In principle, atmospheric models should reproduce Fig. 2d when convolved with measurement uncertainty. Currently, the exact conditions that lead to atmospheric models reproducing this form, in particular a potential plateau and the variance curve, are unknown. Improvements in the modeling of rainfall, either through improved convective parameterizations (Frenkel et al. 2012), explicit simulation at high resolution (Bryan et al. 2003), or greater understanding of microphysics parameterizations (Kim et al. 2013), are likely to be important factors.

The other types of models are conceptual models. Muller et al. (2009) used a stability approach where all water vapor in the atmospheric column is converted to surface rainfall above a threshold in \( w \), although the model was agnostic about the continuous phase transition interpretation. More recently, Stechmann and Neelin (2011, 2014) suggested a stochastic Fokker–Planck model that reproduced the TMI observations (along with other statistics). Based on the TMI+PR retrieval in Fig. 2d, both of these models are now lacking in variance at high \( w \) values. This implies that the model assumptions about high \( w \) variability in the rain-rate response no longer agree with the best observations.

Although there is disagreement between the observations and conceptual models, the simple nature of these models means that they can be adapted. For example, incorporating extra variability at high \( w \) could be achieved in Muller et al. (2009) by modifying their Eq. (1)

\[
P = (w_{upper} + w_{lower})H(w_{lower} - w_s) \tag{6}
\]

such that

\[
H(w_{lower} - w_s) \to f(w_{lower} - w_s), \tag{7}
\]

where instead of a Heavyside function \( H \), which has no variability at high \( w \) above the 0–1 threshold, the function \( f \) is stochastic. This would add extra variability in the \( P-w \) curve at high \( w \). The model of Stechmann and Neelin (2011, 2014) could be similarly modified.

### 6. Conclusions

This work has shown that the characteristics of the \( P-w \) relationship can be substantially changed by measurement uncertainty. This result was found by considering...
both observational signatures and uncertainty analysis of idealized $P$--$w$ curves. On the observational side, when PR observations are used as the basis of the rain-rate retrieval, the variance peak, which is found for microwave retrievals and is paramount for the continuous phase transition interpretation of the observational $P$--$w$ curve, is not found. This occurs because PR-based retrievals have a much larger and more realistic range than the RSS TMI retrieval at extreme instantaneous rain rates. This means that the apparent continuous phase transition in the $P$--$w$ curve is due to the limits of microwave retrievals rather than the underlying physics.

The idealized error analysis showed that $w$ noise can 1) change the plateau shape (Figs. 3 and 4) or create part of a plateau (Fig. 5) depending on the level of $w$ noise and 2) create a peak in the variance for the low-dynamic-range retrieval, even when there is no variance peak in the underlying relationship (Figs. 4b and 5b). These two features are the signatures of a continuous phase transition and imply that uncertainty can produce an apparent continuous phase transition even when the underlying $P$--$w$ relationship does not contain one. We conclude that the proposed atmospheric continuous phase transition in $P$--$w$ is unlikely to be robust against measurement uncertainty. The implications of this result to the property of self-organized criticality are unclear, but it appears a critical point is unlikely to be present in the $P$--$w$ system.

The results presented here showed that the upswing location is not strongly influenced by column water vapor retrieval noise or by the type of rain-rate retrieval (as seen in Figs. 2–5). The upswing location is therefore a good measure for evaluating the transition to strong rainfall [as was undertaken in Suhany et al. (2012)]. However, the other properties of the underlying $P$--$w$ curve—the plateau and the variance—can be strongly modified by uncertainty. Since the plateau can be created or extended, or have its shape changed in the presence of $w$ measurement uncertainty, this implies $w$ uncertainty provides one source of the current disagreement between $P$--$w$ curves from models and those from observations. The poor representation of convection and convective organization in atmospheric models (Stephens et al. 2010; Rossow et al. 2013) may provide another possible source of disagreement. Nonetheless, the disagreement between models and observations has not been completely resolved here for the $P$--$w$ curve.

This work shows that future model–observation comparisons should incorporate an uncertainty analysis when working beyond the upswing location $w_0$ because $w$ noise can create larger $w$ values than expected. This is of paramount importance because it is unlikely that retrieval errors of column water vapor in rainy scenes will reduce below $\sigma_w \sim 3$ mm in the near future. Incorporating such an analysis would help avoid measurement uncertainty (such as additive or multiplicative noise, saturation, or bias) being misinterpreted as changes in natural variability.

This work shows that the “plateau region” could be modified by measurement uncertainty. The observational data indicated that, for the two PR-based retrievals, a plateau was present in the noise-convolved observational $P$--$w$ relationships. Although there is uncertainty regarding the plateau region, the potential presence of the plateau in the $P$--$w$ relationship over the tropics means that it could be used to inform the parameterization of convection. There are indications that high-resolution numerical weather models may capture aspects of this region (Bechtold 2009).

Before this study, it was believed the models of Muller et al. (2009) and Stechmann and Neelin (2011, 2014) did capture the plateau and the variance curve. With the updated PR-based observations presented here, it seems that these models no longer agree with the $P$--$w$ variance, although they do appear to capture the mean response. This is implied by the more extensive validation of the TRMM rainfall products in Wolff and Fisher (2008, 2009) and Cecil and Wingo (2009), which showed the PR-based algorithms have a more realistic dynamic range when retrieving rain rates.

Given that the PR-based rain-rate retrievals do not show a peak in the variance but flatten and remain high above $w_0$, the continuous phase transition suggested in the $P$--$w$ relationship by Peters and Neelin (2006) is not well supported by these new data. The observed curve from the PR-based retrievals instead implies that large variability remains at high $w$. This new analysis, along with the convolution of uncertainty in the idealized curves, indicates that the transition is not from one stable state to another stable state. Instead, the observed behavior suggests that the $P$--$w$ system moves from a non-raining state to a raining state and retains large variability in the raining state over Earth’s tropical region.

The two most important factors that experimental studies control in model–observation comparisons of the $P$--$w$ curve are the uncertainty in rain-rate and column water vapor retrievals. To prevent misidentification of the $P$--$w$ relationship, this work shows that a high-dynamic-range retrieval in $P$ was needed. Further, to avoid measurement uncertainty producing aspects of a plateau in the mean response, a low standard deviation of $\sigma_w < 1$ mm is required for rainy-scene column water vapor retrieval. Under these conditions, a continuous phase transition is not emulated in the noise-convolved $P$--$w$ curves. The Global Precipitation Measurement
mission (Hou et al. 2014), which is flying with a next-generation microwave imager and a dual-frequency radar, is well placed to improve our understanding of extreme hydrological processes, including the $P$–$w$ relationship, by reducing the retrieval uncertainty in instantaneous measurements.

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