Thermodynamic Constraints on the Morphology of Simulated Midlatitude Squall Lines

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ABSTRACT

This study examines how environmental thermodynamics constrain the morphology of simulated idealized midlatitude squall lines (SLs). The thermodynamic soundings used for simulating various SLs are specified primarily by prescribed vertical profiles of the convective available potential energy (CAPE) and the level of free convection. This framework, which contemplates the latent instability properties of both low- and midtropospheric air, is considered to be convenient for investigating layer-lifting convective phenomena.

Results show that frequently used CAPE indices are unsuitable for diagnosing SL characteristics, while integrated CAPE (ICAPE) discriminates the amplitude of the storm-induced heating for a given value of environmental shear. The skill of ICAPE follows from its relation to the buoyancy attained by low- and midtropospheric parcels as they ascend over the cold pool under layer-lifting convection. Environmental kinematics also affect the storm-induced heating, with stronger low-level shear leading to a greater proportion of inflowing latent unstable air among total storm-relative inflow, thus producing higher temperatures aloft. The precipitable water accounts for much of the precipitation-rate variation for a given value of shear. The precipitation efficiency is lower in environments with weaker shear and dryer midtropospheric conditions.

Cold pool temperatures are slightly affected by environmental variations beneath the layer of minimum moist static energy, with drier midtropospheric conditions and weaker shear leading to warmer cold pools. SLs with a small vertical gradient of cold pool buoyancy propagate less rapidly and produce small surface wind speeds. Cold pool properties could be affected by a descending branch of the front-to-rear flow, which crosses over with the rear inflow jet.

1. Introduction

Observational studies (e.g., Bluestein and Jain 1985) have revealed that severe midlatitude squall-line (SL) environments are characterized by having relatively large latent instability (Schultz et al. 2000), as measured by the convective available potential energy (CAPE), as well as considerable low-to-midtropospheric vertical wind shear, perpendicular to which the convective line generally organizes. The importance of environmental wind shear for the organization and longevity of SLs has been elucidated through numerical simulations (Thorpe et al. 1982; Rotunno et al. 1988; Weisman 1992; Weisman and Rotunno 2004), but the specific role of CAPE in determining storm characteristics remains unclear. For instance, numerical studies by James et al. (2006) and Takemi (2007, 2010) found systematic morphological variations of idealized SLs when simulated using different environmental soundings with nearly identical CAPE. This dependence on the thermodynamic environment for a given CAPE is not well understood, and it may explain why statistical analyses of proximity soundings have not yielded significant correlations between SL characteristics and environmental wind shear or CAPE (Wyss and Emanuel 1988; Evans and Doswell 2001).

This work will investigate the mechanisms through which the thermodynamic environment constrains SL characteristics, focusing on the development of cold pools and upper-tropospheric heating (e.g., Pandya and Durran 1996). This will proceed by analyzing storms simulated by a cloud-resolving model (CRM) under different environmental conditions. CRMs provide detailed output data and allow simulations using idealized settings, rendering them suitable for the purpose of this study. But besides uncertainties caused by numerical techniques and parameterizations used in CRMs...
(Weisman et al. 1997; Bryan et al. 2003; Bryan 2005; Morrison et al. 2009; Bryan and Morrison 2012), this approach is complicated by the design of soundings representative of the spectrum of naturally occurring environments. In fact, Evans and Doswell (2001) note that SLs are observed in a wider variety of environments than those suggested by numerical investigations, a matter that is intimately tied to the formulation of soundings. Some related studies are briefly reviewed below.

Weisman (1993) used analytic profiles of relative humidity (RH) and potential temperature, the latter interpolating between prescribed potential temperatures at the surface and the tropopause. The study concluded that CAPE of at least 2000 J kg\(^{-1}\) is required to sustain severe long-lived bow echoes and observed that the intensity of surface winds tended to be greater in simulations with higher-CAPE environments. The same procedure for designing soundings was used by Takemi (2007, 2010), who suggested that, for a given value of CAPE and vertical wind shear, the average low-to-midtropospheric temperature lapse rate is the main feature modulating an SL’s intensity. McCaul and Weisman (2001) proposed a different technique for generating soundings, determining temperatures above the mixed layer via specification of the buoyancy profile of a pseudoadiabatically lifted surface parcel. This procedure was used by James et al. (2006), who concluded that low levels of midtropospheric specific humidity enhance cold pool development, thus favoring SL organization and intensity.

The aforementioned studies have provided valuable insights into the importance of different environmental features for modulating the structure of SLs, but they also leave relevant questions unanswered. For example, the threshold for severe bow echo development given by Weisman (1993) overestimates the CAPE values observed by Cohen et al. (2007), which might have resulted from the numerical framework [e.g., resolution and parameterizations (e.g., Bryan and Morrison 2012)], but it may also reflect the limitations of the method used for generating soundings. Regarding the latter possibility, it is worth noting that the two techniques described above determine soundings primarily through temperatures, with moisture specified through a relative humidity profile. Another matter worth discussing concerns the similarity between the sensitivity tests considered by James et al. (2006) and Takemi (2010), suggesting that both studies analyzed analogous intercase environmental variations, but for which they elaborated different interpretations. In fact, James and Markowski (2010) showed that dry midtropospheric conditions could be detrimental to SL intensity, in contrast with the conclusion of James et al. (2006). These apparent inconsistencies highlight the lack of a robust conceptual framework for interpreting the environmental modulation of SL characteristics.

This study proposes yet another sounding-generating technique, focusing on the vertical distributions of CAPE and of the level of free convection (LFC). In this context, CAPE and LFC are considered as functions of height, providing a correspondence between the level of origin of parcels on a sounding and their respective CAPE and LFC indices.\(^1\) The motivation follows from the geometry of SLs, which develop deep cold pools (Bryan et al. 2005) that displace layers of low- and midtropospheric air vertically to the upper troposphere, a process commonly referred to as layer-lifting convection (Bryan and Fritsch 2000; Mechem et al. 2002) or slantwise layer overturning (Moncrieff 2010). We hypothesize that latent heating of midtropospheric parcels ascending under layer-lifting convection may be relevant for determining the degree of upper-tropospheric warming, which, in turn, feeds back on the mesoscale circulations (Lafore and Moncrieff 1989; Weisman 1992; Pandya and Durran 1996). In line with this hypothesis, and in contrast with the traditional focus on indices derived from a single parcel, we analyze numerically simulated SLs in thermodynamic environments constrained by vertical distributions of CAPE and LFC, emphasizing the usefulness of integrated CAPE (ICAPE; Mapes 1993) as an environmental diagnostic for the potential upper-tropospheric heating.

This paper is organized as follows: The methodology used for simulating SLs is presented in section 2, including a description of the environments to be considered (section 2a) and CRM specifications (section 2b). Results from numerical simulations are presented in section 3, focusing on qualitative intercomparisons between simulated storms, presenting an analysis of along-line-averaged fields in moderately intercomparisons (section 3a), Lagrangian particle trajectories (section 3b), and extensions to weakly and strongly sheared environments (section 3c). A discussion follows in section 4, including a review of the previous literature in light of the results presented herein (section 4a) and a description of a schematic diagram to summarize the main results of this study (section 4b). Section 5 concludes this work. Mathematical details of the method used for designing soundings are presented in the appendix.

\(^1\) The vertical distribution of CAPE as considered herein differs from that contemplated by Blanchard (1998), who focused on varying the level up to which buoyancy is integrated from the LFC of a single parcel.
2. Methodology

2a. Environmental thermodynamics and kinematics

Various thermodynamic environments were generated using a method described in the appendix. This technique allows the specification of soundings via vertical distributions of CAPE (J kg$^{-1}$) and LFC (hPa) [i.e., CAPE = CAPE($p$) and LFC = LFC($p$), where $p$ is the pressure level of origin of a parcel on the sounding] (Fig. 1 illustrates the lifting of parcels originating at various pressure levels on a schematic skew $T$–$\ln p$ chart). It is shown in the appendix that, once an upper-tropospheric sounding and a boundary layer lapse rate have been prescribed, temperature and water vapor mixing ratio profiles are uniquely determined throughout the low- and midtroposphere by specifying vertical distributions of CAPE and LFC. This technique is useful because the soundings are obtained by means of parameters that are physically meaningful in the context of moist convection, where CAPE measures the potential updraft intensity, while LFC determines the distance a parcel must be lifted to trigger the latent instability. The class of soundings under consideration have a conditionally unstable lapse rate below a certain pressure level (LFC*) being absolutely stable above, as Fig. 1 illustrates. This is required for the uniqueness of the sounding as determined by CAPE and LFC profiles.

The design of thermodynamic environments is based on perturbations to the CAPE and LFC profiles of a sounding that will be referred to as baseline (black solid lines in Figs. 2a and 2b, respectively; see also thick solid

FIG. 1. Schematic representation of the type of soundings under consideration as displayed on a skew $T$–$\ln p$ chart. The black and blue thick solid lines represent temperature and dewpoint temperature, respectively. The dashed lines correspond to pseudoadiabats of latent unstable parcels, which are those originating at pressure levels in the set $P$. Portions of dry adiabats and water vapor mixing ratio lines, corresponding to the green and purple lines, respectively, were added to visualize the displacement of parcels from different heights in $P$. Refer to the appendix for details on the notation.

FIG. 2. (a) Different CAPE profiles used for generating soundings are displayed, with the $y$ axis representing the pressure level of parcel origin. The line labeled as baseline corresponds to the environments’ baseline, base_sens, and stable. (b) LFC profiles used for generating soundings are displayed; the $y$ axis is as in (a). The line labeled as baseline corresponds to the environments’ baseline, stable, low, and high.
lines in Fig. 3a). This sounding is similar to those used in several numerical studies of idealized SLs (e.g., Rotunno et al. 1988; Weisman et al. 1988; Weisman 1992, 1993; Bryan et al. 2003; Weisman and Rotunno 2004; Bryan 2005; Bryan et al. 2006; Morrison et al. 2009; Seigel et al. 2013). Two perturbations having baseline’s CAPE profile will be considered. The base_sens sounding has a lower LFC than baseline above the boundary layer (gray line in Fig. 2b), which implies that parcels in the former environment must be lifted farther in pressure coordinates to attain neutral buoyancy. It is worth highlighting that this environment has an elevated nearly dry adiabatic layer (e.g., Carlson et al. 1983; Bryan et al. 2005) and a relatively dry midtroposphere. Both baseline and base_sens have identical boundary layers, being nearly dry adiabatic. The other sounding with baseline’s CAPE, herein referred to as stable (thin solid lines in Fig. 3a), has an LFC profile identical to baseline but a relatively moist and stable boundary layer. This case is meant to represent nocturnal systems (e.g., Parker 2008).

Two sensitivity tests were designed where the CAPE profile is modified. The low and high soundings (thin and thick lines, respectively, in Fig. 3b) are meant to test the impact of correspondingly decreasing and increasing baseline’s CAPE (dashed and dotted lines, respectively, in Fig. 2a) while holding LFC fixed. Thus, parcels originating at the same pressure level in low and high reach their LFC at identical pressure levels aloft, but having very different CAPE values. On the other hand, the cases shallow (dotted dewpoint line in Fig. 3c) and deep (dashed dewpoint line in Fig. 3c) were designed to study the impact of varying LFC (the dashed and dotted lines in Fig. 2b, respectively) while maintaining baseline’s temperature profile. Consequently, the CAPE profiles of deep (dark gray line in Fig. 2a) and shallow (light gray line in Fig. 2a) differ considerably, the latter having a shallower layer with most unstable CAPE (MUCAPE; i.e., maximum CAPE) than deep. The low, high, deep, and shallow environments have the same boundary layer lapse rate as baseline. The prescribed upper-tropospheric sounding was specified through the analytical profile of Weisman and Klemp (1982), being the same for all environments, except for base_sens, where a slightly different profile was used in order for its boundary layer to be identical to that of baseline.
The general characteristics of all soundings are listed in Table 1. Except in shallow, MUCAPE equals the mean-layer CAPE (MLCAPE; i.e., CAPE of a parcel with the average characteristics of the lowermost 100 hPa) in each sounding; in shallow, MLCAPE is approximately 3% lower than MUCAPE. The precipitable water (PW; kg m$^{-2}$) varies considerably between baseline, shallow, and deep, but MUCAPE is identical among these cases. Similarly, the stable, baseline, and base_sens environments differ in their PW, while having identical CAPE profiles.

A variable of interest is ICAPE (J m$^{-2}$; Mapes 1993), defined as

$$ICAPE = g^{-1} \int_P CAPE(p) \, dp,$$

where $g$ is the acceleration due to gravity, and $P$ is the set containing the pressure levels of parcels for which CAPE is defined (see Fig. 1). ICAPE, which measures the bulk latent instability throughout the atmospheric column, is considered herein as a potentially useful diagnostic in layer-lifting convection. Note that low and shallow have similar values of ICAPE, even though the former has much lower MUCAPE and MLCAPE than the latter. On the other hand, ICAPE differs considerably between shallow and deep, both of which have identical MUCAPE and very similar MLCAPE.

The initial wind profiles have static winds above 3.5 km, whereas below there is constant wind shear directed perpendicular to the line of convective initiation, which is discussed in the next subsection. This frame of reference differs from the more common specification of zero environmental surface wind speed, which was done in order to guarantee that the storms remain near the region of convective initiation (see next subsection). This does not affect the nature of the results, as the lack of surface interactions renders the systems horizontally Galilean invariant. Because of the importance of kinematics in determining SL morphology, we performed simulations with three different values of shear for each thermodynamic sounding: $\Delta U = 8$, 16, and 24 m s$^{-1}$, where $\Delta U$ is the difference between horizontal wind speed at 3.5 km and at the surface. These kinematic configurations are illustrated in Fig. 4. As reported in Evans and Doswell (2001) and Cohen et al. (2007), these wind shear values are commonly observed in SL environments. Simulations with different environmental shear will be distinguished by an underscore, followed by the corresponding $\Delta U$ value (e.g., baseline_24 and shallow_16).

### b. Model

The numerical simulations were performed with the system for atmospheric modeling (SAM), which is a nonhydrostatic CRM based on the anelastic fluid equations (Khairoutdinov and Randall 2003). Horizontal boundaries are doubly periodic; thus, a relatively long domain is required in the across-line direction to prevent storm-induced perturbations from traversing the domain and affecting the SL. The conserved thermodynamic variable in SAM is the liquid/ice water static...
energy; consequently, moist static energy in temperature units \( h \) is used instead of equivalent potential temperature. Subgrid-scale (SGS) processes were accounted for by a first-order Smagorinsky closure model, and no significant variations in the fields analyzed herein were observed when using a 1.5-order closure similar to the model of Deardorff (1980). A version of the double-moment microphysics parameterization of Morrison et al. (2005) was employed for cloud (water and ice) and precipitating (rain, snow, and graupel) phenomena. As in most idealized SL simulations, the lower boundary condition is free slip, with all surface fluxes, Coriolis effects, and radiative heat fluxes being neglected.

The numerical simulations were carried out for 7.5 h with a 2.5-s time step. The horizontal domain extends \( 128 \times 1536 \text{ km}^2 \) (along-line \( \times \) across-line directions, respectively), with 500-m grid spacing in both directions. The domain extends to 28.5 km in the vertical through 64 levels with variable spacing, this being 100 m near the surface and gradually increasing up to 500 m at 5000-m height, above which it stays constant at 500 m. Bryan et al. (2003) argued that SLs require greater resolution than that used here to resolve turbulent processes up to the inertial subrange; yet the same study showed that even simulations with 1-km grid spacing display the general characteristics of the storms, as revealed by results using higher resolution, justifying the expectation that the grid spacing used herein is sufficient for identifying systematic variations in the simulations under different environmental configurations.

Newtonian damping is applied to prognostic variables in the upper-third portion of the domain. We found that, by 4 h of simulation time, SGS processes produced noticeable vertical mixing of air far from the storm, resulting mainly from the discontinuity in the environmental vertical wind shear at 3.5-km height (Fig. 4). In this study, we intended for environmental conditions to remain constant throughout the simulation, which is why pointwise nudging to the initial kinematic and thermodynamic state was performed between 768 and 1536 km in the across-line direction. The environmental kinematic setup described above (i.e., with static ground-relative winds above 3.5 km) allowed the storms to remain distant from this portion of the domain. The frequency used for nudging the region between 968 and 1336 km was \( 1/3600 \text{ s}^{-1} \), linearly decreasing to \( 0 \text{ s}^{-1} \) between 968 and 768 km, and also between 1336 and 1536 km. The initialization procedure is analogous to that used by Morrison et al. (2015), consisting of horizontal wind convergence centered on \( x_c = 384 \text{ km} \) (across-line direction), with horizontal radius \( x_r = 10 \text{ km} \), maximum height \( z_r = 10 \text{ km} \), and defined as follows:

\[
\frac{\partial u}{\partial t} = \gamma \cos \left[ \frac{\pi (x - x_c)}{2x_r} \right] \left( \cosh \left( \frac{2.5z}{z_r} \right) \right)^{-2},
\]

where \( x \) is the across-line coordinate, \( z \) is height, and \( \gamma \) is a time-dependent variable equal to \( 0.1 \text{ m s}^{-2} \) between initiation and 3300 s, linearly decreasing to \( 0 \text{ m s}^{-2} \) at 3600 s. Random temperature perturbations of 0.8 K amplitude were added at initialization in the region of convergence forcing in order for the system to develop three-dimensionality.

We also use the Lagrangian particle tracking algorithm described in Yamaguchi and Randall (2012). The trajectories were computed every time step using third-order Lagrange interpolation in space and three half-time-step iterations using a second-order Runge–Kutta method. SGS effects were neglected in the Lagrangian particle trajectories, and further specifications will be described alongside the corresponding results.

### 3. Results

Figure 5 shows the time evolution of the domain-wide surface precipitation rate (PR; mm day\(^{-1}\)) for all \( \Delta U = 16 \) simulations. In all cases, PR remains near maximum values after 4 h of simulation time, an indication that the storms have reached a mature state (Houze et al. 1989). Given that this study is concerned with SLs at maturity, all results will correspond to fields at and after 4 h.

#### a. Simulations with \( \Delta U = 16 \)

Table 2 presents some diagnostics of the \( \Delta U = 16 \) simulations, which will be discussed alongside related results. Regardless of the CAPE distributions, PR can be ordered by PW (Fig. 5), with a couple of minor exceptions: PR in baseline_16 is barely distinguishable...
The three simulations in Fig. 6 produce similar upper tropospheric buoyancy fields, which hydrostatically lead to similar mesolows, corresponding to the midtropospheric mesoscale feature of negative pressure perturbations. Cold pools, on the other hand, differ considerably, with base_sens_16 producing a relatively deep cold pool, with a notable midtropospheric minimum located toward its upshear end, while stable_16 produces near-surface buoyancy values of relatively small amplitude. The remarkable similarity in within-storm soundings taken roughly through the center of the mesolow, depicted by Fig. 7a, suggests that these contrasting features result mainly from environmental temperatures (i.e., base_sens_16’s relatively warm midtroposphere and stable_16’s relatively cold low troposphere). Notwithstanding their contrasting low- and midtropospheric environments, these simulations produced similar cold pool intensities $c$ ($\text{m s}^{-1}$); see Table 2, defined as

$$c = \left[ -2 \int_{0}^{z'} \bar{b}(z) \, dz \right]^{1/2},$$

where $\bar{b}$ is the along-line-averaged buoyancy at the across-line location of the within-storm sounding in Fig. 7, while $z'$ is the height where neutral buoyancy is first attained [Rotunno et al. (1988) discusses the importance of $c$]. It is worth noting that we did not find significant differences between baseline_16 and base_sens_16 that may be directly attributable to the vertical distribution of LFC. Therefore the LFC, which may be more relevant to convective triggering, will not be further discussed in this study.

Regarding kinematics, stable_16 produced the deepest outflow, with maximum horizontal wind speeds remaining elevated near this storm’s cold pool edge (Fig. 6c), as opposed to maximum wind speeds descending to the surface in baseline_16 (Fig. 6a) and base_sens_16 (Fig. 6b). Weisman (1992) argued that, for a given environmental shear and $c$, these features,
which are relevant in terms of the potential for wind-induced damage, depend on the magnitude of the horizontal buoyancy gradient at the FTRF–RIJ boundary, with a stronger gradient producing more elevated winds at the cold outflow edge (see their Figs. 15 and 23). However, the three simulations under consideration produced very similar \( c \) (Table 2), while Figs. 6 and 7a show that there are no significant differences between the buoyancy fields above their cold pools. In fact, stable_16 is the only \( \Delta U = 16 \) simulation where maximum outflow winds do not spread to the surface, as manifested by its relatively low maximum surface wind speeds in Table 2.\(^2\) We interpret this behavior as a result of the smaller vertical buoyancy gradient in stable_16’s cold pool, which is consistent with results

\(^2\) For comparison with naturally occurring events or other simulations, the surface wind speed values in Table 2 must be adjusted to the appropriate horizontal frame of reference.
by Droegemeier and Wilhelmson (1987) showing that more weakly stratified outflows are deeper and propagate more slowly.

In respect to cases with varying CAPE, Fig. 8 displays the along-line-averaged fields of low_16, high_16, deep_16, and shallow_16. Despite marked differences in upper-tropospheric buoyancy fields, all simulations depicted in Fig. 8 developed maximum outflow winds that spread to the surface, lending support to our contention on the role of the cold pool buoyancy profile for determining this particular feature. Clearly, high_16 (Fig. 8b) and deep_16 (Fig. 8d) produced perturbation fields of greater amplitude than low_16 (Fig. 8a) and shallow_16 (Fig. 8c), as is also revealed by PS, PR, and the maximum surface wind speeds in Table 2. The contrasting appearances of shallow_16 and deep_16 are noteworthy, since both environments have identical temperature soundings, the same MUCAPE, and very similar MLCAPE. In this regard, it is also worth highlighting that low_16 produces a more intense storm than shallow_16 by several measures (i.e., PS, PR, and maximum surface wind speeds), despite the former environment’s lower MUCAPE, MLCAPE, and low-to-midtropospheric temperature lapse rate. These results manifest the limitations of some commonly used parcel indices and temperature lapse rates (e.g., Takemi 2010) for capturing relevant dynamical features of SLs.

Within-storm soundings in Fig. 9 reveal interesting characteristics of simulated cold pools. For instance, shallow_16 stands out for producing a relatively warm (Fig. 9a) and dry (Fig. 9b) cold outflow, as is the case of base_sens_16 in Figs. 7a and 7b, thus linking the low PE in these storms to their relatively dry midtropospheric environments. These intercase differences contrast with the similarity in surface cold pool temperatures among all simulations shown in Figs. 7a and 9a. This result indicates that surface cold pool temperatures are highly constrained by the mid- and upper-tropospheric portion of the sounding, as all the environments have nearly identical soundings above 600 hPa. This observation is not surprising under the model of saturated pseudoadiabatic descent of air from the level of minimum h (e.g., Fawbush and Miller 1954), but it is not trivial, because turbulent mixing and subsaturated conditions prevail in downdrafts (Gilmore and Wicker 1998). It remains to be tested whether this statement.

Fig. 7. Soundings passing roughly through the center of the mesolow at 4 h of simulation time. (a) The temperature soundings of baseline_16 (solid black), base_sens_16 (dashed), and stable_16 (dotted) are shown alongside stable_16’s dewpoint temperatures (gray), with the other two moisture profiles omitted for readability. (b) The RHs corresponding to the soundings in (a).
holds for different values of shear, a matter that will be addressed in section 3c. However, as a consequence of the aforementioned results on cold pool properties, and consistent with observations by Bryan et al. (2005), it follows that $c$ cannot be estimated exclusively through surface temperature measurements, as suggested by Evans and Doswell (2001) and Stensrud et al. (2005).
In contrast to results from simulations with identical CAPE profiles, temperatures above the melting line in Fig. 9a differ considerably among the simulations, low_16 and shallow_16 being considerably colder therein than high_16 and deep_16. Interestingly, the upper-tropospheric temperatures in the $\Delta U = 16$ simulations can be ordered by ICAPE, which is shown by the soundings in Fig. 10. This is of great dynamical importance, given that positive buoyancy values above the cold pool drive horizontal motions at the RIJ via the associated mesolow, while also leading to the descent of midtropospheric air into the cold outflow through vorticity generation and cooling by microphysical processes (e.g., Lafore and Moncrieff 1989; Weisman 1992). Pandya and Durran (1996) showed the importance of the heating and cooling profiles for driving the mesoscale circulations in SLs through gravity waves, while Fig. 10 suggests that ICAPE constrains the amplitude of the upper-tropospheric heating. To aid the interpretation of this result, the next subsection will analyze the simulated convective process through particle trajectories.

b. Particle trajectories

To shed more light on the convective processes in the present simulations, Lagrangian particles were placed at 4 h of simulation time with initial positions at various heights, about 6 and 15 km ahead of the cold pool edge, and with 1-km spacing in the along-line direction. As an example, Fig. 11 displays trajectories relative to the cold pool edge traced by 40 particles projected onto the across-line–height plane in the baseline_16 simulation. The data used for this plot are from output at 5-min intervals of trajectories computed every time step, with the paths colored according to the particle’s temperature with respect to the initial environment. The along-line-averaged $h$ field at 6 h is contoured in the background for reference.

One of the most salient characteristics of the convective process portrayed in Fig. 11 is that all low- and midtropospheric particles cross the deep convective line, reflecting the propagation of the simulated SL with respect to environmental winds at those heights. This is the essence of layer-lifting convection, where deep layers of environmental air constitute the core of the convective process. The highly turbulent conditions within the deep convective line are demonstrated in Fig. 11 by the variety of trajectories followed by particles with identical initial thermodynamic conditions (i.e., originating at the same height). Such convective processes differ fundamentally from models where air from a single layer constitutes the updraft core, as is
implicitly assumed in parcel theory or in the stochastic mixing model of Raymond and Blyth (1986), which James and Markowski (2010) applied to SLs. To characterize convection in the present simulations, it is necessary to contemplate the thermodynamic properties of air from all layers of storm-relative inflow, which mix turbulently at the deep convective line, in fractions dependent on the storm-relative inflow rate.

The relevance of ICAPE as a diagnostic in layer-lifting convection is evinced in the trajectories displayed in Fig. 11, where all low- and midtropospheric parcels ascend, mix at the deep convective line, and traverse the storm nearly horizontally at a level determined by a buoyancy sorting process (Raymond and Blyth 1986). The histograms of particle heights at 5-h simulation time, displayed in Fig. 12, further suggest that ICAPE modulates the convective processes, showing that more low- and midtropospheric particles reach above 8 km in simulations having greater ICAPE. On the other hand, there is no clear indication that convection from any single layer is dependent on its respective CAPE.

For example, the number of low-tropospheric particles reaching above 8 km is greater in low_16 than in shallow_16, notwithstanding similar $c$ values and larger low-tropospheric CAPE characterizing the latter environment. Furthermore, in each simulation the number of particles that reach above 8 km is maximum when the level of origin is 1800 m, despite having lower CAPE than the particles originating at 300 m.

Before proceeding to the next subsection, it is worth highlighting that particles ending up beneath 4 km after crossing the deep convective line in Fig. 11 cool and slowly descend as they traverse the SL. These particles, which formally constitute the upper portion of the cold pool, take part in a relatively intense downdraft at a crossover zone between this descending FTRF branch and the RIJ, where mixing is likely to occur. The descending FTRF is separated from its ascending counterpart by the melting line, and the cooling experienced by slowly descending particles results mainly from melting of ice (not shown) (the midtropospheric buoyancy minimum at the rear of base_sens_16’s cold pool is related to this descending FTRF). It is therefore plausible that neglecting ice microphysics in early SL simulations may be the reason why this mesoscale flow was not highlighted before, which may also explain why cold pools observed by Bryan et al. (2005) were deeper than those reproduced by many numerical simulations.

c. Simulations with $\Delta U = 8$ and $\Delta U = 24$

The simulated SLs varied systematically with changes in $\Delta U$, with stronger shear leading to storms with a more upright appearance and deeper reaching updrafts, in agreement with past studies (e.g., Rotunno et al. 1988). It is also worth noting that strongly sheared environments did not produce surface spreading winds within the cold outflow, which is consistent with results by Weisman (1992). And although all simulations display a descending FTRF branch, this circulation extends farther below the melting line with decreasing $\Delta U$, probably as a result of deeper cold outflows being produced in more strongly sheared environments. The morphological variations brought about by changes in $\Delta U$ have been the subject of many past studies, and will not be described in further detail throughout this subsection. The focus here will be on showing that the aforementioned systematic morphological variations of SLs in different thermodynamic environments are also present in the $\Delta U = 8$ and $\Delta U = 24$ simulations.

Diagnostics of SLs simulated in $\Delta U = 8$ and $\Delta U = 24$ environments are shown in Tables 3 and 4, respectively. It is important to note that shallow_24 did not reach a mature state, which can be seen in the PR oscillation between 1 and 4 h in Fig. 13a, after which the PR remains.

3 This fact does not result from cold pool–shear optimality (Rotunno et al. 1988), as the most optimal storms among those under consideration are shallow_16 and low_16.

4 All simulations considered herein produced layer-lifting convection, in contrast with tropical SLs simulated by Mechem et al. (2002), where dry midtropospheric conditions inhibited layer lifting.
low relative to its peak value. The discrimination of PR by PW is evident in Figs. 13a and 13b, and Tables 3 and 4 show that the environments with the driest midtropospheric conditions (i.e., shallow and base_sens) have the lowest PE, regardless of $\Delta U$. Also, in Figs. 5 and 13 the systematic increase in PR as $\Delta U$ becomes stronger is apparent, which has been reported in many past studies (e.g., Rotunno et al. 1988; Weisman et al. 1988; Weisman and Rotunno 2004; Bryan et al. 2006). Fovell and Ogura (1989) attributed this behavior mainly to the WVIR, but Tables 2–4 show that PE varies considerably with $\Delta U$, which might be related to the fraction of latent unstable air among the total storm-relative inflow, as explained below.

Regarding upper-tropospheric thermodynamics within the storms, the ordering of temperatures by ICAPE still holds for the strongly and weakly sheared simulations. To illustrate this, a scatterplot depicting all simulations on an ICAPE–$B$ plane is shown in Fig. 14, where $B$ measures the upper-tropospheric warming caused by the storm, defined as

$$B = \left(2 \int_{4 \text{ km}}^{12 \text{ km}} \bar{b} \, dz \right)^{1/2},$$

with $\bar{b}$ being the along-line-averaged buoyancy at an across-line location near the center of the mesolow. The lower limit of integration for $B$ is at 4 km because it is near the line of neutral buoyancy in all simulations, and the upper limit at 12 km was chosen because it is near the tropopause. For each environmental shear, a nearly linear relation between ICAPE and $B$ is evident in Fig. 14, resulting from the layer-lifting process revealed by Fig. 11. In addition, Fig. 14 shows that simulations with more strongly
sheared environments produce greater $B$, consistent with findings by Fovell and Ogura (1989) and Weisman (1992). The latter noted that the equivalent potential temperature of Lagrangian particles was more nearly conserved in more optimal configurations [i.e., cases with $c/\Delta U$ closer to 1 (e.g., Weisman and Rotunno 2004)], which led Weisman (1992) to conclude that greater $B$ resulted from less turbulent mixing experienced by more rapidly ascending particles. The validity of this interpretation by itself is not clear, given that greater vertical velocities could produce enhanced mixing because of greater stress at the updraft boundaries. In fact, the $h$ of low-tropospheric particles is more nearly conserved in deep_16 than in shallow_16 (not shown), despite identical low-tropospheric CAPE and a less optimal cold pool–shear configuration in the former simulation. This distinction in terms of $h$ conservation could result from differences in the surroundings through which particles ascend under layer-lifting convection, rather than cold pool–shear optimality. It is thus worth considering if the strength of the environmental shear can affect the surroundings through which particles ascend.

We suggest that the dependence of $B$ on $\Delta U$ for a given sounding results from the modulation of storm-relative environmental winds by the kinematic environment (i.e., $\Delta U$ in the simulations considered herein). To illustrate this, Fig. 15 presents a scatterplot where the simulations are represented according to their PSs and their shear-layer inflow fractions (SLIFs), the SLIF being the storm-relative mass flux of air from the environmental shear layer as a fraction of the total storm-relative mass flux. The results in Fig. 15 are consistent with those of Fovell and Ogura (1989), who found higher SLIF in more strongly sheared cases. The relevance of the SLIF in the environments considered herein follows from the fact that most of the latent unstable layer (i.e., the layer through which CAPE is defined) is found

**TABLE 3.** As in Table 2, but for the $\Delta U = 8$ simulations.

<table>
<thead>
<tr>
<th>Case</th>
<th>PR</th>
<th>$C$</th>
<th>PS</th>
<th>max($u_0$)</th>
<th>max($\tau_0$)</th>
<th>PE</th>
</tr>
</thead>
<tbody>
<tr>
<td>baseline</td>
<td>11</td>
<td>30</td>
<td>11</td>
<td>38.6</td>
<td>26.8</td>
<td>29</td>
</tr>
<tr>
<td>base_sens</td>
<td>9</td>
<td>29</td>
<td>10.5</td>
<td>37.9</td>
<td>17.4</td>
<td>25</td>
</tr>
<tr>
<td>stable</td>
<td>13</td>
<td>29</td>
<td>9.7</td>
<td>32.2</td>
<td>16.4</td>
<td>36</td>
</tr>
<tr>
<td>low</td>
<td>8</td>
<td>27</td>
<td>9.7</td>
<td>33.4</td>
<td>17.4</td>
<td>24</td>
</tr>
<tr>
<td>high</td>
<td>16</td>
<td>35</td>
<td>13.5</td>
<td>39.6</td>
<td>22.8</td>
<td>35</td>
</tr>
<tr>
<td>shallow</td>
<td>5</td>
<td>26</td>
<td>9.2</td>
<td>32.7</td>
<td>15.9</td>
<td>16</td>
</tr>
<tr>
<td>deep</td>
<td>15</td>
<td>32</td>
<td>11.7</td>
<td>40</td>
<td>22.2</td>
<td>36</td>
</tr>
</tbody>
</table>

**TABLE 4.** As in Table 2, but for the $\Delta U = 24$ simulations. The shallow_24 data are omitted because the storm did not reach a mature state (see text).

<table>
<thead>
<tr>
<th>Case</th>
<th>PR</th>
<th>$C$</th>
<th>PS</th>
<th>max($u_0$)</th>
<th>max($\tau_0$)</th>
<th>PE</th>
</tr>
</thead>
<tbody>
<tr>
<td>baseline</td>
<td>22</td>
<td>35</td>
<td>1.8</td>
<td>24.1</td>
<td>4.8</td>
<td>62</td>
</tr>
<tr>
<td>base_sens</td>
<td>20</td>
<td>33</td>
<td>2.4</td>
<td>23</td>
<td>4.6</td>
<td>55</td>
</tr>
<tr>
<td>stable</td>
<td>22</td>
<td>34</td>
<td>0.9</td>
<td>28.2</td>
<td>0.5</td>
<td>64</td>
</tr>
<tr>
<td>low</td>
<td>20</td>
<td>31</td>
<td>1.3</td>
<td>25.3</td>
<td>3.3</td>
<td>61</td>
</tr>
<tr>
<td>high</td>
<td>27</td>
<td>39</td>
<td>3.6</td>
<td>28.4</td>
<td>9.9</td>
<td>64</td>
</tr>
<tr>
<td>shallow</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>deep</td>
<td>25</td>
<td>37</td>
<td>2.6</td>
<td>26.9</td>
<td>6.2</td>
<td>65</td>
</tr>
</tbody>
</table>
within the sheared layer. The latent unstable parcels are precisely those that may cause upper-tropospheric warming when lifted, with CAPE providing a measure for buoyancy under parcel ascent; thus, ICAPE becomes the relevant parameter under layer lifting. On the other hand, the inflowing air above the sheared layer mixes with latent unstable parcels at the deep convective line, potentially cooling by rain evaporation, ice sublimation, or by subsaturated ascent. Hence, in environments where shear and latent instability are concentrated at low-tropospheric levels, a greater SLIF will tend to produce higher upper-tropospheric temperatures and, by the same reasoning, a greater PE.

With respect to surface cold pool temperatures, Fig. 16 shows that, with the exception of shallow_8 and base_sens_8, the intercase variations among the previously described environments are small. The outlying behavior of shallow_8 and base_sens_8 is probably a consequence of their relatively dry midtropospheric environments, to which surface cold pool temperatures in weakly sheared environments become highly sensitive (notice the small PEs of these storms in Table 3, as well as their low SLIFs in Fig. 15). These two cases aside, the intercase similarity in temperatures displayed by

FIG. 13. As in Fig. 5, except representing cases with (a) \( \Delta U = 8 \) and (b) \( \Delta U = 24 \).

FIG. 14. Scatterplot representing all simulations on an ICAPE–B plane (see text for further details).

FIG. 15. Scatterplot representing all simulations on a PS–shear-layer inflow fraction plane (see text for further details).

FIG. 16. Along-line-averaged cold pool temperatures at the lowest height of model output (50 m), with the across-line location selected as in Fig. 7.
Fig. 16 warrants further consideration of the origins of surface cold pool air. To this end, Fig. 17 shows a histogram of the height reached by backward trajectories of surface cold pool air in baseline_16 at 4 h, computed back to the initial simulation time. It shows that the major source of cold pool air is the 3–4-km layer, where 37% of trajectories are found at initiation; however, 55% of trajectories correspond to air with origins above 4 km, which contains the layer with minimum \( h \), and wherein there are barely any intercase environmental variations. These results are consistent with our contention that the similarity in surface cold pool temperatures among the different simulations results from their nearly identical mid- and upper-tropospheric environmental thermodynamics.

Results shown in Fig. 16 suggest the possibility of deriving accurate estimates of surface cold pool temperatures from the mid- and upper-tropospheric portion of the sounding, at least in environments characterized by either high midtropospheric RH or moderate-to-strong shear at low levels. To test the degree to which surface cold pool temperatures change because of variations in the upper portion of the sounding, two additional simulations with \( \Delta U = 16 \) were performed, referred to as dry and warm. Both have the same CAPE, LFC, and boundary layer lapse rate as baseline, but with different prescribed upper-tropospheric soundings. The dry sounding has identical upper-tropospheric temperatures, but with 80% of baseline’s RH above the latent unstable layer; warm is 3 K warmer than baseline but has the same RHs throughout the upper troposphere. As portrayed in Fig. 16, dry produces surface cold pool temperatures similar to those in previously described simulations, while warm produces much higher temperatures therein. The lower-than-3-K difference in surface cold pool temperatures between warm and baseline_16 results in part from decreasing pseudoadiabatic lapse rates with increasing temperature. These results show that, for the environments considered herein, mid- and upper-tropospheric temperatures provide strong constraints on the surface cold pool temperatures. In addition, it is worth noting that dry midtropospheric conditions do not necessarily lead to lower surface cold pool temperatures, nor to stronger cold pools, in agreement with results by James and Markowski (2010).

4. Discussion

a. Previous studies in light of the present results

Most of the previous literature on the dependence of SLs on the thermodynamic environment has focused on CAPE (e.g., Weisman 1992, 1993; James et al. 2006; Takemi 2007, 2010; James and Markowski 2010). It is interesting that the main conclusions reached by such studies can be interpreted within the framework presented herein. For instance, the soundings considered by Weisman (1992, 1993) are such that cases with greater CAPE also have greater ICAPE, as is shown in Fig. 18, thus explaining the systematic behavior of storms simulated in different CAPE environments. In fact, the environmental variations considered by Weisman are similar to the ones between high and low (i.e., changing the temperature lapse rate) the main difference being...
that Weisman (1992, 1993) specified identical surface temperatures among different environments.

Similar observations apply to results by James et al. (2006) and Takemi (2007, 2010), in which soundings with greater temperature lapse rates and lower levels of specific humidity are the ones with greater ICAPE [see Fig. 15 in Takemi (2010)]. In fact, Takemi (2010) mentions the importance of the vertical distribution of CAPE for modulating the intensity of simulated SLs but does not provide arguments relating this feature to the upper-tropospheric heating. Instead, the vertical distribution of CAPE was proposed as a physical explanation for why the temperature lapse rate modulates storm intensity, as deeper layers of MUCAPE were found in soundings with steeper lapse rates. However, results presented herein show the importance of the latent instability properties of air above the layer with MUCAPE, as exemplified by the simulations in low and shallow environments, the latter producing more intense storms despite having a smaller temperature lapse rate and lower CAPE values throughout the low troposphere. Furthermore, the latent heating of ascending air with origins above the latent unstable layer is also important for determining the upper-tropospheric warming, as evinced by the deep-reaching trajectories of particles originating at 4800 m in Fig. 11d. We think that this may partly explain why James and Markowski (2010) found dry midtropospheric conditions to be detrimental to the intensity of updrafts in SLs simulated in environments having similar ICAPE.

In regard to the cold pool, it is worth mentioning that we find no clear link between SL morphology and downdraft CAPE (DCAPE), an index that has been associated with the downdraft strength and cold pool properties (e.g., Evans and Doswell 2001; Cohen et al. 2007). Consistent with results by Gilmore and Wicker (1998), subsaturated conditions were present throughout the RIJ of simulated storms (not shown), suggesting that the downdraft is constrained by the rate of latent cooling. Nonetheless, the finding pertaining to the lack of sensitivity in surface cold pool temperatures among the simulations with similar mid- and upper-tropospheric soundings may be relevant for forecasting and parameterizing cold pool properties. To our knowledge, this fact has not been reported in previous numerical studies, probably because sounding variations have been mainly accomplished through changes to mid- and upper-tropospheric temperatures.

**b. Conceptual model of convection in SLs**

To summarize the results presented throughout this study, Fig. 19 shows a schematic diagram depicting the kinematic and thermodynamic structure of a mature mid-latitude SL with trailing stratiform precipitation. Environmental winds are represented on a frame of reference with a static cold pool edge, with latent instability shown in green (the brighter the tone, the higher CAPE is in that layer). The highly turbulent deep convective line, represented by the area in red, mixes the air from all levels of storm-relative inflow, followed by a buoyancy sorting of air parcels, as described by Raymond and Blyth (1986). The positively buoyant and slowly ascending FTRF (AFTRF) is found above the melting line, being near saturation and with temperatures determined by the storm-relative flux of
environmental latent instability at all heights. Beneath the melting line, the descending FTRF branch (DFTRF) is shown, which maintains negative buoyancy as it slowly descends, mainly as a result of melting of ice particles throughout the stratiform region. The RIJ encounters the DFTRF at a crossover zone where mixing is likely to occur, possibly affecting the cold outflow air. Most of the DFTRF air enters the rear exit flow, where it joins RIJ air that does not feed the cold outflow. Temperatures within the cold outflow are modulated by the thermodynamic properties of RIJ air, while its buoyancy structure is determined by low- and midtropospheric environmental temperatures.

The novel features in Fig. 19 are the inclusion of the DRTRF and its associated crossover zone with RIJ, the distinction between the cold pool and the cold outflow, and the emphasis on both the system-relative winds and the latent instability of air from all heights. The conceptual model of tropical SLs proposed by Zipser (1977) (their Fig. 13) also contemplates storm-relative inflow at all levels, but, in their model, only low-tropospheric air takes part in mesoscale ascent, after having crossed over low- h air. Thus, the conceptualization by Zipser (1977) implicitly assumes a lower degree of turbulent mixing at the leading line than that reflected by the trajectories in Fig. 11. One explanation for the highly turbulent conditions at the deep convective line is the presence of moist absolutely unstable layers (Bryan and Fritsch 2000) in the simulations considered herein (not shown). Yet, it is beyond the scope of this work to provide an in-depth description of turbulence as represented by the present simulations.

Even though turbulent mixing is recognized as a defining characteristic of the deep convective line, we caution against the stochastic mixing framework of Raymond and Blyth (1986) to conceptualize SL convection, as proposed by James and Markowski (2010). Our analyses suggest two reasons why stochastic mixing in its traditional form may not be appropriate for the type of storms simulated herein. First, air from all inflowing layers takes part in SL convection in proportions determined by the structure of storm-relative airmass flux, contrary to the assumption that near-surface air constitutes the core of the updraft, which is implicit in the stochastic mixing framework. Second, although parcels do seem to be vertically distributed by a buoyancy sorting process, their final height is not determined by the environmental sounding itself, as within-storm temperatures differ greatly from environmental values. Nonetheless, we think that the stochastic mixing model could provide a useful conceptual framework if mixing between air samples from all heights is incorporated, with the storm-relative inflow structure determining the mixing fractions and the buoyancy sorting being dependent on within-storm thermodynamics.

5. Conclusions

In this study, we analyzed numerical simulations of midlatitude squall lines conducted in thermodynamic environments carefully designed via vertical distributions of CAPE and LFC. For a given value of environmental wind shear, we found that ICAPE discriminates simulated storms in terms of upper-tropospheric heating, a feature that is known to drive the mesoscale circulations in squall lines. This result was interpreted through layer-lifting convection, in which all layers of storm-relative inflow take part in the convective process, while ICAPE is argued to provide a bulk metric of buoyancy under the vertical displacement of mid- and low-tropospheric air. However, by itself, ICAPE cannot fully account for the thermodynamic characteristics of updrafts produced by squall lines, as the modulation of the storm-relative inflow of air must also be considered. In this regard, environmental shear was found to be of great importance for modulating the fraction of inflowing latent unstable air, providing an explanation for the increase in within-storm heating associated with storms in more strongly sheared environments.

Low-tropospheric environmental temperatures have noticeable impacts on SL characteristics, with a smaller lapse rate leading to slower propagation, a deeper cold outflow, and diminished surface wind speeds. These differences are attributed to the vertical distribution of buoyancy within the cold pool, but a more extensive and detailed investigation is needed to gain a deep understanding of these features. We also found that, among the simulations considered herein, with the exception of cases in weakly sheared and dry midtropospheric environments, surface cold pool temperatures are insensitive to the strength of the low-tropospheric shear, as well as to changes in mid- and low-tropospheric environmental thermodynamics. But noticeable differences in cold pool temperatures above the surface are found among the simulated storms, with drier midtropospheric environments producing relatively warm outflows. In addition, the thermodynamic properties of the cold outflow could be substantially affected by a subsiding FTRF branch, a flow that crosses over with the RIJ.

Results reported herein suggest the potential usefulness of the precipitable water as an environmental diagnostic for the surface precipitation rate, regardless of the vertical distribution of CAPE. We found that, for a given value of low-tropospheric wind shear, the rate of water vapor processed by squall lines is the main factor affecting surface precipitation rates. However, the precipitation efficiency decreases as the environments have weaker shear and lower midtropospheric
relative humidity. A relevant feature in terms of flood potential is the relatively small propagation speed produced under environments with smaller low-tropospheric temperature lapse rates, but these environments also appear to be the least conducive to wind-induced damage.

We have to note that the range of environments considered in this study is by no means exhaustive. For instance, we did not explore the rather wide range of CAPE profiles that are attainable for a given ICAPE. In addition, variations to upper-tropospheric temperatures for a particular profile of CAPE have not been fully explored. The associated changes to the height of the melting line and pseudoadiabatic lapse rates could impact substantially on squall-line morphology. Analyses contemplating these types of environments could be particularly relevant in the context of future climate change scenarios. In addition, the class of soundings under consideration excludes cases with strong temperature inversions, a common feature of midlatitude squall-line environments.

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APPENDIX

Technique for Generating Soundings

In this appendix it will be shown that, given a low-tropospheric lapse rate (LTLR) and an upper-tropospheric sounding (UTS), the low- and midtropospheric portion of the sounding can be uniquely determined by prescribing CAPE and LFC profiles. Virtual effects are neglected for simplicity, but their inclusion is straightforward. Detailed information on the thermodynamic relations used herein can be found in Emanuel (1994).

a. Notation

Let \( T(p) \), \( q_s(p) \), and \( s_{ps}(p) \), respectively, denote the temperature, water vapor mixing ratio, and pseudoequilibrium lapse rate describing a sounding, with each assumed to be a smooth function of pressure \( p \).\(^{A1}\) The temperature of a parcel with pseudoequilibrium \( s_{ps} \) when at pressure level \( p' \) and with 100\% RH is denoted by \( T_{ps}(s_{ps}, p') \). Hence, the set \( P = \{ p \mid T(p') < T_{ps}(s_{ps}(p), p') \text{ for some } p' < p \} \) contains the pressure levels where the sounding has latent unstable parcels; that is, parcels for which CAPE is defined and positive, as shown in Fig. 1 (some notation in Fig. 1 is presented below). The vertical distribution of LFC is given by

\[
LFC(p) = \max \{ p' \mid T(p') < T_{ps}(s_{ps}(p), p') \text{ for some } p' < p \},
\]

and similarly for the equilibrium level (EL),

\[
EL(p) = \min \{ p' \mid T(p') < T_{ps}(s_{ps}(p), p') \text{ for some } p' < p \},
\]

while CAPE is defined as,

\[
\text{CAPE}(p) = R_d \int_{EL(p)}^{LFC(p)} \{ T_{ps}(s_{ps}(p), p') - T(p') \} d \ln p',
\]

where \( R_d \) is the dry-air gas constant. These are expressed as functions of the levels of origin of latent unstable parcels and thus have \( P \) as their domain. Finally, let \( \Gamma(p) \), \( \Gamma_{\text{d}}(p) \), and \( \Gamma_{\text{ps}}(p) \), respectively, represent the environmental, dry adiabatic, and pseudoadiabatic lapse rates in pressure coordinates, the latter two evaluated at the point \( T(p) \).

b. Class of soundings under consideration

For reasons discussed below, soundings are assumed to satisfy the following restrictions on \( \Gamma \), as illustrated in Fig. 1:

1) \( \Gamma_{\text{ps}}(p) < \Gamma(p) < \Gamma_{\text{d}}(p) \) (i.e., \( \Gamma \) is conditionally unstable) for \( LFC(p^*) = LFC^* < p < p_0 \), where \( p_0 \) is the surface pressure and \( p^* = \inf(P) \).

2) \( \Gamma(p) < \Gamma_{\text{ps}}(p) \) (i.e., \( \Gamma \) is absolutely stable) for \( p < LFC^* \).

3) \( EL \) is finite.

Restrictions 1 and 2 and the smoothness of \( T \) imply that there exists a unique point of tangency between \( T \) and pseudoadiabats, found at height \( LFC^* \). The key

\[^{A1}\] Subscripts ‘‘ps’’ refer to variables related to pseudoadiabats.
observation for linking the CAPE–LFC and the $T$–$q_v$
profiles is that each point $T(p)$ for LFC* $< p \leq$ MULFC $= \max_p[LFC(p)]$ corresponds to the point
where some latent unstable parcel attains its LFC. This follows from condition 1, which guarantees that each
pseudoadiabat given by $T_p[p, T_p(p), p']$ for $p \in P$ will cross $T$ only once below LFC* (i.e., $T$ is nowhere parallel
to pseudoadiabats below LFC*); and analogous obser-
vations apply to EL(p) above LFC* as a result of condi-
tions 2 and 3 (see Fig. 1). Therefore, LFCs[s_p(p)] $= LFC(p)$ and ELs[s_p(p)] $= EL(p)$ are well defined
functions, relating the values in the ranges of LFC and
EL, respectively, to the values in the set $S = \{s_p(p) | p \in P\}$. Consequently,

$$\text{CAPE}_s(s_p) = R_d \int_{\text{ELs}(s_p)}^{\text{LFCs}(s_p)} \left[ T_p(s_p, p') - T(p') \right] \ln p' dp'$$

(A1)

defines the values in the range of CAPE as a function of
the values in S. These functional relations allow the re-
cov ery of $T$–$q_v$ from CAPE–LFC profiles via $s_p$, as
described below.

$$s_p(p) - s_p(p^*) = \int_{p^*}^p \left( \frac{\text{dCAPE}}{dp} \right) d p' \left\{ R_d \int_{\text{ELs}(p')}^{\text{LFCs}(p')} \frac{\partial T_p}[s_p, p]}{\partial s_p} d \ln p' \right\}^{-1} dp',$$

where $(\text{dCAPE}/dp)_{p'}$ is the derivative of CAPE evalu-
ated at $p'$.

Therefore, starting with the value $s_p(p^*)$ implicit in
the UTS’s $T[LFC^*]$, $s_p(p)$ can be recovered from
CAPE, LFC, and EL, the latter given by levels where
pseudoadiabats cross the UTS. Then, $T$ can be deter-
mined between LFC* and MULFC by the temperature
of a saturated parcel originating at $p$ attains at LFC (p),
given by $T_p[p, LFC(p)]$ for $p \in P$. Below MULFC, $T$ values follow from the prescribed LTLR.
Once $T(p)$ is known, $q_v(p)$ can be deduced for $p \in P$ by
requiring that the sounding have pseudoentropy $s_p(p)$
at level $p$.\textsuperscript{A2}

d. Generating soundings in practice

This method is envisioned primarily as a tool for de-
signing sensitivity tests for studying the dependence of numerically simulated storms on thermodynamic
variations in the environment. Hence, a proposed
starting point is to choose a baseline sounding satisfying

\textsuperscript{A2} The term $q_v(p)$ must be prescribed explicitly: $\forall p \in P$.

c. Constraining soundings by CAPE and LFC

This sounding-generating technique requires prior
specification of $T$ below MULFC (the LTLR) as well as
$T$–$q_v$ above LFC* (the UTS). The former is needed so
that low-tropospheric $T$–$q_v$ can be uniquely determined,
while the latter implicitly determines EL(p), with $s_p(p)$
playing an important role in both cases, as will be shown
below. To find $T$–$q_v$ below LFC*, the chain rule is used to relate the derivatives of CAPE and CAPEs:

$$\frac{d\text{CAPE}}{dp} = \left( \frac{d\text{CAPE}_s}{ds_p} \right) \left( \frac{ds_p}{dp} \right),$$

which, by applying the Leibniz integral rule to (A1), gives

$$\frac{d\text{CAPE}}{dp} = \left[ R_d \int_{\text{ELs}(p')}^{\text{LFCs}(p')} \frac{\partial T_p}[s_p, p]}{\partial s_p} d \ln p' \right] \left( \frac{ds_p}{dp} \right),$$

(A2)

additional terms being zero because $T[LFCs(s_p)] =
T_p[s_p, LFCs(s_p)]$ and $T[ELs(s_p)] = T_p[ELs(s_p)]$. Given that $d\text{CAPE}/dp$ is known and $0 < \partial T_p[s_p, p']/\partial s_p$
everywhere, $ds_p/dp$ can be integrated in (A2) to give

conditions 1 and 2, compute its associated CAPE(p) and
LFC(p) profiles, and then generate soundings with
variations from baseline’s CAPE(p), LFC(p), UTS, and
LTLR. The following points must be considered when
using this method:

- Uniqueness of $q_v(p)$ for $p \in P$ is guaranteed if CAPE is
  strictly positive therein.
- For consistency with the type of soundings under
  consideration, the relation CAPE(p) $\leftrightarrow$ LFC(p) for
  $p \in P$ must be bijective.
- CAPE(p*) = 0, 0 is CAPE, and LFC(p) $< p$ must hold.
- LFC* must be identical to the value derived from
  UTS.
- To guarantee dry static stability, $(d\text{CAPE}/dp)/(dLFC/dp)$
must be below a certain threshold at every height.
- LTLR may be lower than $\Gamma_{ps}$, but care is needed to avoid saturation.

\textbf{REFERENCES}


