Wave and Jet Maintenance in Different Flow Regimes

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(Manuscript received 27 October 2015, in final form 18 March 2016)

ABSTRACT

The wave spectrum and zonal-mean-flow maintenance in different flow regimes of the jet stream are studied using a two-layer modified quasigeostrophic (QG) model. As the wave energy is increased by varying the model parameters, the flow transitions from a subtropical jet regime to a merged jet regime and then to an eddy-driven jet regime. The subtropical jet is maintained at the Hadley cell edge by zonal-mean advection of momentum, while eddy heat flux and eddy momentum flux convergence (EMFC) are weak and concentrated far poleward. The merged jet is narrow and persistent and is maintained by EMFC from a narrow wave spectrum, dominated by zonal wavenumber 5. The eddy-driven jet is wide and fluctuating and is maintained by EMFC from a wide wave spectrum. The wave–mean flow feedback mechanisms that maintain each regime are explained qualitatively.

The regime transitions are related to transitions in the wave spectrum. An analysis of the wave energy spectrum budget and a comparison with a quasi-linear version of the model show that the balance maintaining the spectrum in the merged and subtropical jet regimes is mainly a quasi-linear balance, whereas in the eddy-driven jet regime nonlinear inverse energy cascade takes place. The amplitude and wavenumber of the dominant wave mode in the merged and subtropical jet regimes are determined by the constraint that this mode would produce the wave fluxes necessary for maintaining a mean flow that is close to neutrality. In contrast, the equilibrated mean flow in the eddy-driven jet regime is weakly unstable.

1. Introduction

The strength, shape, and latitude of the tropospheric jet stream vary spatially and temporally in a wide range of time scales, affecting the regional climate. The properties of the jet, including its short-time-scale variability, are affected by the surrounding conditions, such as the season (Lindzen and Hou 1988; Kim and Lee 2004), changes in the diabatic forcing (Son and Lee 2005), sea surface temperatures (Lu et al. 2010), and stratospheric conditions (Polvani and Kushner 2002). Recently, several studies (Barnes et al. 2010; Garfinkel and Waugh 2014; Lorenz 2014b) have shown that the internal variability of the jet changes significantly with the jet latitude, a property that is affected by external conditions. A significant change in the dynamical properties of the flow as the external conditions change gradually can be viewed as a regime transition. In this paper, we study the flow regimes of the tropospheric jet stream and construct a conceptual picture of their maintenance in terms of wave–mean flow interactions.

The regime of the jet stream is strongly related to the relationship between the two types of jets observed in the atmosphere: the subtropical and midlatitude jets. Besides their different latitudinal locations, these two types of jets differ in their vertical structure, their variability properties, and their driving force (Lee and Kim 2003). The subtropical jet is concentrated close to the subtropical edge of the Hadley cell, it is relatively

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DOI: 10.1175/JAS-D-15-0321.1

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persistent, and it is driven, to a large extent, by advection of absolute angular momentum by the mean meridional circulation (MMC). The midlatitude jet is located inside a Ferrel cell and is associated with the midlatitude storm tracks. This jet is driven mainly by convergence of eddy momentum flux from baroclinic waves. Both jets exhibit a vertical wind shear that is associated with lower-level meridional temperature gradients, while the vertical shear and temperature gradient are stronger for the subtropical jet. The two types of jets may merge into a single jet or split into two separate interacting jets, giving rise to complex behavior (Son and Lee 2005; O’Rourke and Vallis 2013).

In the atmosphere, a strong jet at the upper troposphere near the subtropical edge of the Hadley cell is observed in winter in both hemispheres. In the Southern Hemisphere (SH) winter a second weaker and more barotropic (i.e., extending vertically to the surface) jet is observed in the midlatitudes. In the Northern Hemisphere (NH) winter the jet has a pronounced longitudinal structure, with two jets above the Atlantic Ocean and a single jet above Asia and the Pacific Ocean. In the SH summer the zonal wind maximum is inside the Ferrel cell and extends to the surface, indicating that eddy momentum flux convergence (EMFC) is the dominant driver of the jet. A double-jet state, where the subtropical and eddy-driven jets are well separated, is seen in the climatology of the zonal-mean wind in the SH autumn and spring.

The terminology used in the literature to describe the different types of jets is based on two categorizations: according to the latitude of the jet—“subtropical jet” and “midlatitude jet”—or according to the driving mechanism—“thermally driven” and “eddy driven,” where the thermal driving refers to the driving of the subtropical jet by the thermally induced Hadley circulation and the eddy driving refers to the driving of the midlatitude jet by EMFC. A comparison between the eddy fluxes in the SH winter and summer, performed by Kim and Lee (2004), indicates that the winter subtropical jet is indeed more thermally driven, while the summer midlatitude jet is more eddy driven. However, the NH winter Pacific jet, which is often referred to as “subtropical,” because of its latitudinal location, is driven partly by eddies and can more correctly be viewed as a merged (thermally driven and eddy driven) jet (Eichelberger and Hartmann 2007; Li and Wettstein 2012).

Several numerical studies have been conducted to study the regimes of the jet stream. A transition between a single-jet and double-jet regime was found as the external thermal driving was varied (Son and Lee 2005; Lu et al. 2010; Michel and Riviere 2014). The double-jet regime was found to exhibit stronger short-time-scale variability of the jet latitude, relative to the single-jet regime (Son and Lee 2006; Son et al. 2008; Eichelberger and Hartmann 2007; Gerber and Vallis 2007), consistent with observations, showing stronger persistence of the negative phase of the North Atlantic Oscillation, when the two jets above the NH Atlantic become closer or merge (Barnes and Hartmann 2010; Harnik et al. 2014). A different categorization of the jet regimes was proposed by Robinson (2006), who found that as the pole-to-equator temperature difference is increased in a two-layer model of the general circulation, an abrupt shift occurs from a state with very weak eddies and a subtropical jet to a state with strong eddies and a midlatitude persistent jet. As the pole-to-equator temperature difference is increased further, the flow becomes turbulent, and the jet is no longer persistent. He referred to the persistent midlatitude jet as a “self-maintaining jet” and showed that it is maintained because of the reinforcement of the zonal wind vertical shear at the latitude of maximum baroclinic growth by the wave-driven Ferrel circulation.

Despite the amount of evidence for the existence of different jet regimes, there is still no coherent picture that covers all the possible regimes and explains the dynamical mechanisms maintaining them. There is not yet an understanding of the role of the wave spectrum in the transition between the single- and double-jet regimes, and most studies of the single-jet regime focus on a state where the single jet is inside the Ferrel cell, indicating that it is a merged jet, rather than a subtropical thermally driven jet. Lachmy and Harnik (2014) studied the maintenance of the subtropical jet regime and showed that it can be maintained only when the waves are very weak. This is supported by the fact that the subtropical jets in both winters are concentrated at longitudinal sectors with relatively weak wave fluxes.

This study aims at mapping all the observed regimes under one framework in order to analyze and compare their maintenance mechanisms and to obtain a conceptual picture of the dynamics of each regime. We expand our analysis from Lachmy and Harnik (2014) to cover a wider range of flows and to study the maintenance of the wave spectrum as well as the mean flow in the different regimes. The numerical model used here is the two-layer spherical modified quasi-geostrophic (QG) model described in Lachmy and Harnik (2014). This model was designed to keep the simplest possible framework, while capturing the essential dynamics of the system. This allows us to cover a wide parameter space so that all dynamical regimes occur under one study.
In our analysis we identify three flow regimes, according to several properties of the flow, as shown in section 3. We chose to use a terminology of “sub-tropical,” “merged,” and “eddy driven” jet regimes to describe the three regimes. This terminology was chosen in order to relate to previous studies; however, it does not describe all aspects of the dynamics, which are quite complex.

For conceptual simplicity, we consider the zonal mean flow and the waves separately and break the question of the flow maintenance into two questions: (i) How is the mean flow maintained in statistically steady state in the presence of waves? (ii) How is the wave spectrum maintained in statistically steady state with a specific mean-flow profile?

The maintenance of the wave spectrum is examined in the context of the existing theories of baroclinic adjustment and geostrophic turbulence. Baroclinic adjustment theory is based on the idea that the atmosphere adjusts to a neutral state so that in the time mean there is no net growth or decay of baroclinic waves (Stone 1978). If nonlinear interactions are neglected, it is expected that the most unstable baroclinic wave would dominate the spectrum and contribute most to the wave fluxes that maintain the neutral mean flow. Baroclinic adjustment theory was a significant step forward in the understanding of the observed mean flow; however, it was found to be incomplete because of the role of nonlinear wave–wave interactions. Salmon (1980) introduced the theory of geostrophic turbulence, which predicts that in steady state the energy would be transferred from the mean flow to the baroclinically unstable waves at the Rossby radius of deformation and then transferred nonlinearly upscale, where it would be dissipated by boundary layer friction, and downscale, where it would dissipate by turbulent diffusion. The resulting spectrum peaks at a scale that is larger than the scale of the most unstable mode and depends on the halting mechanism of the inverse (i.e., upscale) energy cascade. While baroclinic adjustment is more applicable when nonlinear interactions are very weak, geostrophic turbulence is relevant for cases where there is a wide enough separation between the Rossby deformation radius and the halting scale of the inverse cascade, which leads to strong nonlinear interactions.

There were a few attempts to build a theory that is applicable for both weak and strong nonlinearities. Cehelsky and Tung (1991) and Welch and Tung (1998) developed a nonlinear baroclinic adjustment theory, which accounts for the effect of the inverse energy cascade. This theory suggests that the flow would equilibrate in a state that is supercritical for the most unstable mode but nearly neutral for the energy containing mode. Held and Larichev (1996) developed a scaling theory for homogeneous baroclinically unstable flows on a β plane, based on geostrophic turbulence and diffusive scaling. Their theory shows that the degree of instability, the degree of nonlinearity, and the extent of the inverse energy cascade all scale by the same nondimensional parameter, which they called “supercriticality.” Zurita-Gotor (2007) showed that the Held and Larichev (1996) scaling is applicable for a wide range of supercriticalities, including low values, when nonlinear interactions are weak. They further showed that the supercriticality of the equilibrated mean flow is not associated with a stability threshold of a certain linear mode, in contrast to the predictions of nonlinear baroclinic adjustment.

An alternative approach, in the frame of stochastic structural stability theory (SSST; Farrell and Ioannou 2003, 2009), assumes that all waves are excited by nonlinear interactions and models this source as stochastic forcing. This approach leads to a set of equations for the mean flow and wave statistics in the form of a covariance matrix so that a stable solution for the coupled system can be found, with no need to solve the full time-dependent equations explicitly. It was found to be useful for predicting the equilibrated mean flow and wave fluxes but requires a tuning of the stochastic forcing and is valid only when the flow is sufficiently turbulent (DelSole 2004).

Research on flow regimes of the jet stream has not yet related to these different theories for the equilibration of the wave spectrum. We examine the possibility that the different flow regimes are related to the degree of nonlinearity of the wave spectrum and explore the equilibration mechanism of the waves by analyzing the linear stability and the inverse energy cascade in each regime.

The paper is organized as follows: The numerical model and the setup of the simulations are described in section 2. The flow regimes in the model parameter sweep and their properties are described in section 3. The maintenance of the mean flow and wave spectrum in each regime is analyzed in sections 4 and 5, respectively. Discussion and conclusions are given in sections 6 and 7, respectively.

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1 The term “eddy-driven jet regime” describes a state where there is an eddy-driven jet at relatively high latitudes, which is dynamically distinguishable from a weaker subtropical jet at the Hadley cell edge. This state may sometimes be viewed as a “double-jet regime,” but we chose not to use this term, since in some cases we found no upper-layer zonal wind minimum that separates between the two jets, as shown for example in Fig. 4a below.
2. Experiment design

All the results presented in this study are from simulations with the idealized model described in Lachmy and Harnik (2014). The model has two layers in the vertical direction, representing the upper and lower troposphere. The model equations are a version of the fully nonlinear OG equations on a sphere, but without neglecting the ageostrophic term representing the advection of zonal-mean momentum by the MMC. The flow is forced by Newtonian relaxation to a radiative equilibrium profile and by surface friction. Numerical hyperdiffusion is added to simulate dissipation in the subgrid scales. The model equations are given in appendix A of Lachmy and Harnik (2014).

This modified OG model provides a simple framework where the mutual interactions between the mean flow, the waves, and the MMC can be studied. The model includes the baroclinic and barotropic interactions between the different components, as well as nonlinear wave–wave interactions. The inclusion of advection of zonal-mean momentum by the MMC, which is usually neglected in OG models, is essential for reproducing the momentum balance of the subtropical jet (Held and Hou 1980). The model is capable of qualitatively reproducing the balances maintaining the jet and the midlatitude eddies, and its simplicity allows for a detailed analysis of these balances. We discuss the limitations of the model in section 6.

In the text below, the upper and lower layers are denoted by subscripts 1 and 2, respectively. The barotropic and baroclinic components for any arbitrary variable $P$ are defined as $P_M = (1/2)(P_1 + P_2)$ and $P_T = (1/2)(P_1 - P_2)$, respectively. We use the following notation: $\mu = \sin \phi$, $U = u \cos \phi$, and $V = v \cos \phi$, where $\phi$ is the latitude, and $u$ and $v$ are the zonal and meridional winds, respectively. A bar denotes zonal averaging and a prime denotes deviation from the zonal average: that is, wave component.

The model setup, radiative equilibrium profile, and fixed parameter values are the same as in Lachmy and Harnik (2014), except for a slight difference in the scheme used for the mean-flow numerical diffusion (see appendix A). The surface friction and radiative relaxation time scales are $\tau_f = 3.9$ days and $\tau_r = 11.6$ days, respectively. These time scales are held fixed, and we expect the results to be qualitatively robust to small changes in these parameters. Temperature is represented in the model in terms of the baroclinic component of the zonal wind $\overline{u_T}$ and enters the equations through the diagnostic equation for $\overline{u_T}$, which is derived from the thermal wind relations. The radiative equilibrium profile of $\overline{u_T}$ [(\overline{u_T})_E; see appendix B] simulates winter conditions in the SH. We focus in this study on winter conditions, where we obtain the widest range of flow regimes (see section 6).

To examine the role of the wave amplitude in determining the regime of the jet, we chose the control parameters to be a layer thickness parameter $H$ and a wave damping parameter $r$. The parameter $H$ appears in the model equations where the vertical derivatives are truncated to two layers so that vertical derivatives are proportional to $1/H$, which has the effect of reducing the importance of baroclinic processes, relative to barotropic processes as $H$ is increased. The parameter $H$ also appears in the radiative relaxation term. Since $(\overline{u_T})_E$ is the same for all simulations, the vertical shear in radiative equilibrium, which is equal to $(2/H)(\overline{u_T})_E$, becomes weaker as $H$ is increased, and this reduces the degree of baroclinic instability of the radiative equilibrium state. The wave damping parameter $r$ is a dimensionless parameter that multiplies the waves’ surface friction and radiative relaxation parameters, $1/(\tau_f)_{WV}$ and $1/(\tau_r)_{WV}$, and is equal to 1 when the mean flow and the waves have the same damping rates. Note that larger values of $r$ correspond to stronger wave damping.

Changing both $H$ and $r$ allows for a wide range of flow regimes to be produced and allows for the examination of both effects of controlling the baroclinicity by changing $H$ and controlling the wave amplitude directly by changing $r$.

The model simulations presented in Lachmy and Harnik (2014) showed separate one-dimensional parameter sweeps over $H$ and $r$, which captured mostly the merged and subtropical jet regimes. Here we show a two-dimensional sweep where $H = 7, 7.5, 8, 8.5, 9, 9.5$, and $10\,\text{km}$ and $r = 0.5, 1, 1.5, 2$. This gives a total of 28 model simulations that qualitatively capture the subtropical, merged, and eddy-driven jet regimes (see Table 1 and details in section 3).

The initial conditions are the same for all the simulations presented in this study. The initial $\overline{U_T}$ has a Gaussian profile, concentrated at $30^\circ\text{S}$ with a maximum value of $50\,\text{m s}^{-1}$. Initial lower-layer winds are zero. The initial potential vorticity (PV) of the waves is a wavenumber-1 disturbance, concentrated at latitude $45^\circ\text{S}$. Each experiment is run for 1500 model days, where the last 1000 days represent the statistically steady state. A few tests showed that the statistically steady state is not sensitive to the choice of initial conditions.

3. Regimes in the model parameter sweep

The time-averaged properties of the flow in the statistically steady state are examined in the parameter sweep over $H$ and $r$. The analysis below indicates a
qualitative change in the flow properties in certain regions of the parameter space, which we identify as regime transitions. Figure 1 shows the latitude of maximum $u_1$ in the SH and the eddy kinetic energy (EKE) averaged over the SH of the model for all the simulations as a function of $H$ and $r$. As $r$ or $H$ is increased, the EKE reduces and the jet shifts equatorward, but the rate of these changes is not uniform over the parameter space. The different simulations in Fig. 1 are marked according to the regime categorization, which will be justified below, after presenting several properties of the flow.

The rate of change of EKE with model parameters is lowest in the merged jet regime and highest in the eddy-driven jet regime. The latitude of maximum $u_1$ is around 50° throughout the eddy-driven jet regime; then it decreases slowly to around 45° in the merged jet regime; and then the jet latitude shifts abruptly to around 30° in the subtropical jet regime.

We examine the dependence of the flow properties on the strength of the eddies by sorting all the simulations by their time- and space-averaged EKE in the SH, which is a measure of eddy strength that varies monotonically with $r$ and almost monotonically with $H$ (Fig. 1b).

### Table 1. Model simulations sorted according to the EKE ($m^2 s^{-2}$) averaged over the SH of the model, in descending order. The dimensionless wave damping parameter is $r$ and $H$ is the layer thickness (km). EDJ, MJ, and STJ denote the eddy-driven jet regime, merged jet regime, and subtropical jet regime simulations, respectively. Dashes denote simulations that are not categorized in any regime because they are a mixture of the eddy-driven and merged jet regimes.

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**FIG. 1.** (a) The latitude of maximum $u_1$ in the SH and (b) the EKE averaged over the SH of the model, for all the simulations as a function of $H$ and $r$. White, black, and gray circles mark simulations in the eddy-driven, merged, and subtropical jet regimes, respectively. The × markers denote simulations that are a mixture of the eddy-driven and merged jet regimes, in the sense that either the simulations vacillate between the two regimes or the characteristics of some variables are typical of the merged jet regime and some of the eddy-driven jet regime. Larger markers are for the three example simulations (see text in section 4).
summarizes the run number, the values of the parameters $r$ and $H$, the EKE, and the regime categorization of each simulation, where the run number is the index of the simulation, after the simulations are sorted by their EKE in descending order.

Figure 2 shows the following time-mean variables as a function of the run number, sorted by the EKE: the latitudes of maximum $\overline{u_1}$ and $\overline{u_2}$ in the SH, determined using a parabolic fit; the maximal values of the time-averaged $\overline{u_1}$ and $\overline{u_2}$ in the SH; the standard deviation of the jet latitude, defined as the standard deviation of the time series of the latitude of maximum $\overline{u_1}$ in the SH; and the width of the polar cell, defined as the distance in degrees from the pole to the latitude where $\overline{u_1}$ changes sign, which is determined using a linear fit.

Figure 2a shows that the jet latitude (i.e., the latitude of maximum $\overline{u_1}$) shifts equatorward as the EKE decreases, with an abrupt shift at the transition from the merged to the subtropical jet regime. Surface westerlies (i.e., the maxima of $\overline{u_2}$) are poleward of the jet and shift equatorward with decreasing EKE, except in the subtropical jet regime, where they shift poleward. In the merged jet regime the latitudes of the jet and the surface westerlies are very close (around $1^\circ$–$3^\circ$ apart), while in the subtropical jet regime they are well separated (around $13^\circ$ apart in run 28). Figure 2b shows that the upper-layer jet strengthens in the transitions from the eddy-driven to the merged jet regime and from the merged to the subtropical jet regime, with no significant tendency within the eddy-driven and merged jet regimes as the EKE decreases and a weakening tendency within the subtropical jet regime. The surface westerlies weaken with decreasing EKE, except within the merged jet regime, where there is no clear tendency. The maximum vertical shear ($2\overline{u_T} = \overline{u_1} - \overline{u_2}$) is highest in the subtropical jet regime and lowest in the eddy-driven jet regime (not shown; note that the values of $\overline{u_1}$ and $\overline{u_2}$ in Fig. 2b are not at the same latitude).

Figure 2c shows that the variability of the jet latitude tends to decrease with decreasing EKE, with significantly smaller standard deviations in the merged and subtropical jet regimes, compared to the eddy-driven jet regime. Figure 2d shows that the polar cell is practically absent in the eddy-driven jet regime and...
appears abruptly at the transition to the merged jet regime. The polar cell width increases with decreasing EKE in the merged and subtropical jet regimes. The existence of a polar cell plays a role in the maintenance of the merged jet, as explained in section 4b below. The three flow regimes can be discerned also by looking at the properties related to the waves shown in Fig. 3.

Figure 3 shows the following time-mean variables as a function of the run number: the spatial-averaged EKE in the SH; the spatial-averaged positive EMFC in the SH; the spatial-averaged poleward heat flux in the SH, defined here as $-\mu(V^2\psi/\gamma)$, where $\psi$ is the wave streamfunction and the minus sign is added to get the poleward heat flux in the SH; and the dominant zonal wavenumber, defined as $\left[\sum_m [mE(m)]/\sum_m [E(m)]\right]$, where $m$ is the zonal wavenumber and $E(m)$ is its energy averaged over time and space in the SH.

The EMFC and poleward eddy heat flux decrease according to the EKE as $H$ and $r$ are increased (cf. Figs. 3a–c, 1b), except within the merged jet regime, where there is no clear tendency of the EMFC and the heat flux is almost constant. There is a significant drop in the EMFC at the transition from the merged to the subtropical jet regime. The dominant zonal wavenumber increases at the transition from the eddy-driven to the merged jet regime from wavenumbers 3–4 to wavenumbers 4–5 (Fig. 3d), except for run 23 in the merged jet regime, where wavenumber 6 is the dominant wavenumber. In the subtropical jet regime, the dominant wavenumber drops back to around 4. An examination of the energy spectrum as a function of wavenumber (Fig. 8 in section 5a below) shows a significant narrowing at the transition from the eddy-driven to the merged jet regime, where wavenumber 5 becomes considerably stronger than all other wavenumbers.

The above analysis implies that in this parameter sweep there are three flow regimes and that the transitions between the regimes occur as the equilibrated EKE decreases. The nature of the three regimes and the terminology we chose for each regime are better understood after the following analysis of the flow maintenance.

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2 In all the simulations, the EMFC is positive around the mid-latitudes, where the eddy-driven jet is formed. We take the spatial average of the positive EMFC as a measure of the jet driving by the waves.
4. Mean-flow maintenance

The different maintenance mechanisms of the subtropical, merged, and eddy-driven jets can be appreciated by considering the zonal-mean momentum balance and the eddy fluxes in steady state. Equations for the upper- and lower-layer momentum balance can be obtained from the model equations for $\overline{U}_1$ and $\overline{U}_2$ with zero time derivatives, while neglecting the small terms of numerical diffusion, lower-layer EMFC, and lower-layer mean-flow advection [see Eqs. (A7a) and (A7b) in Lachmy and Harnik (2014)]:

$$2\Omega \mu - \frac{1}{a} \frac{\partial \overline{U}_1}{\partial \mu} \mathbf{V}_1 = \frac{1}{a} \frac{\partial (\overline{U}_1 \mathbf{V})}{\partial \mu}$$ (1)

$$2\Omega \mu \mathbf{V}_2 = \overline{U}_2 / \tau_f$$ (2)

where $\Omega$ is Earth’s rotation rate and $a$ is Earth’s radius. These balances are consistent with the observed balances in the upper and lower troposphere (Dima et al. 2005). Since in the extratropical atmosphere, as well as in our model, $|2\Omega \mu| > |(1/a)(\partial U/\partial \mu)|$, it follows from Eq. (1) that the EMFC (equal to minus the right-hand side) has to be positive inside the Ferrel cell, where $\nu$ and $\mu$ have opposite signs, and negative inside the Hadley and polar cells, where $\nu$ and $\mu$ have the same sign, except for the part of the winter Hadley cell that penetrates into the summer hemisphere. This means that waves act to accelerate the zonal wind inside the Ferrel cell and decelerate it inside the Hadley cell (outside the ascending branch in the summer hemisphere) and polar cell. A jet at the Hadley cell edge or inside the Hadley cell can therefore never be driven by eddies.

Eddy heat flux and the associated baroclinic growth are located inside the Ferrel cell (Vallis 2006). Therefore, a subtropical jet at the Hadley cell edge, despite its strong vertical shear, cannot act as a source for baroclinic growth at the jet latitude. Lachmy and Harnik (2014) showed that the subtropical jet can be maintained at the Hadley cell edge in the presence of baroclinic waves because of stabilization of the waves at the tropics and sub-tropics by the $\beta$ effect, which restricts the baroclinic growth to higher latitudes. The maintenance of a jet inside the Ferrel cell is essentially different, since it is located where the EMFC is positive. The different vertical structures of the subtropical jet and a jet inside the Ferrel cell are an outcome of the lower-troposphere momentum balance [Eq. (2)], which requires that $\overline{\eta}$ be negative inside the Hadley and polar cells and positive inside the Ferrel cell so that the subtropical jet is located above the zero surface westerlies and a jet inside the Ferrel cell is located above the positive surface westerlies.

The structure of the eddy heat flux is qualitatively explained in terms of the linear instability properties of the zonal-mean flow. Baroclinic instability requires that the meridional PV gradient be positive at the upper layer and negative at the lower layer, with stronger absolute values of the PV gradient leading to higher growth rates. The latitude of maximum EMFC is expected to be close to the latitude of maximum poleward eddy heat flux, according to the Eliassen–Palm relations (Eliassen and Palm 1961).

Taking all the above considerations into account allows us to construct a qualitative picture of the mean-flow maintenance from looking at the time-averaged mean flow and wave fluxes. For this purpose, we analyze in detail a representative simulation from each of the three regimes: run 1, with $r = 0.5$ and $H = 7$ km, for the eddy-driven jet; run 20, with $r = 1.5$ and $H = 9$ km, for the merged jet; and run 28, with $r = 2$ and $H = 10$ km, for the subtropical jet (marked by larger circles in Figs. 1, 2, 3, 8, 9).

a. Eddy-driven jet regime

Figure 4 shows the following time-averaged variables as a function of latitude for the eddy-driven jet example (run 1): $\overline{\eta}$, $\overline{\eta}$, the difference between them, which is twice the baroclinic zonal wind, $2\overline{\eta}$, together with the radiative equilibrium profile, $2(\overline{\eta})_E$ (Fig. 4a); the meridional gradient of the zonal-mean PV in the upper and lower layers (Fig. 4b); the zonal-mean meridional wind in the upper layer (Fig. 4c); and the EMFC and poleward eddy heat flux (Fig. 4d). The latitudes of maximum $\overline{\eta}$ and $\overline{\eta}$ are marked for reference.

In this simulation the maximum $\overline{\eta}$, which represents an eddy-driven jet at $52^\circ$S, is well separated from the maximum $\overline{\eta}$, which represents a weak subtropical jet at $32^\circ$S, close to the Hadley cell edge, where the meridional wind is zero (Figs. 4a,c). At the latitude of maximum $\overline{\eta}$ the vertical shear is very close to its radiative equilibrium value, indicating that the eddy-driven jet is an almost barotropic jet superimposed on the radiative equilibrium shear. The upper-layer jet has a wide structure, and the surface westerlies cover a wide latitudinal band, consistent with the wide Ferrel cell, indicated by the positive values of $\overline{\eta}$ between latitude $35.5^\circ$S and the pole (Fig. 4c) and with the condition for momentum balance in the lower layer [Eq. (2)].

The positive upper-layer and negative lower-layer PV gradients are both wide, with a maximum close to the jet latitude (Fig. 4b). The poleward heat flux is positive throughout the SH, with a maximum collocated with the most negative lower-layer PV gradient and a wide structure as a result of the structure of the PV gradient in both layers. The EMFC is positive inside the Ferrel cell.
and negative inside the Hadley cell, consistent with the condition for momentum balance in the upper layer [Eq. (1)]. The maximum EMFC is around 5° poleward of the jet and collocated with the maximum surface westerlies.

We note that these profiles represent the time-mean balance, while the time variability in this regime is quite large, as implied by the large standard deviation of the jet latitude (Fig. 2c). The more complex time-dependent behavior of the flow will be discussed in a separate paper.

Further analysis of the time-mean picture in terms of the maintenance of the wave spectrum is given in section 5.

b. Merged jet regime

Figure 5 shows the same time-averaged variables as in Fig. 4, but for the merged jet example (run 20). In this simulation the maxima of the upper- and lower-layer zonal winds and the maximum vertical shear are nearly collocated, around latitude 45°S (Fig. 5a). The vertical shear of the zonal wind is maintained above its radiative equilibrium value at the jet latitude. The jet is maintained by strong EMFC, which peaks slightly poleward of the jet latitude (Fig. 5d).

The negative lower-layer zonal wind is collocated with the latitudes where the upper-layer zonal wind has a convex shape. This convex shape leads to two local minima of the upper-layer PV gradient, with near-zero values at latitudes 26° and 65°S (Fig. 5b). These are also latitudes of negative EMFC (Fig. 5d), indicating that momentum flux divergence due to wave breaking at both flanks of the jet leads to the formation of two local minima of the upper-layer PV gradient. In addition, these negative EMFC regions enable the maintenance of the polar cell, contribute to the Hadley cell maintenance, and lead to surface easterlies at low and high latitudes through the conditions for upper- and lower-layer momentum balance [Eqs. (1) and (2)].

The existence of a polar cell causes the Ferrel cell to be narrow, which in turn enables the adiabatic heating and cooling associated with the Ferrel cell to maintain a strong temperature gradient, above radiative equilibrium values, as we find in an analysis of the different terms in the heat equation (not shown). The sharpness of the jet causes the upper-layer PV gradient to have a sharp peak at the jet latitude (Fig. 5b) between the two local minima. The lower-layer PV gradient is negative all through the extratropics, with the most negative values poleward of the jet. The PV gradient structure causes the heat flux to have significant values only between the two local minima of the upper-layer PV gradient, with a maximum close to the jet latitude (Fig. 5d).
The merged jet maintenance mechanism that comes out of this analysis is similar to that of the self-maintaining jet described by Robinson (2006), where the MMC induced by the waves is responsible for restoring the vertical shear of the mean flow, which in turn enables the maintenance of the baroclinic waves. The structure of the wave fluxes and their feedback with the mean flow are related to the sharp spectrum of the waves in this regime, as shown below in section 5.

The term “merged jet regime” was chosen for this regime because the main jet shares properties of an eddy-driven jet and a subtropical jet. While the main jet is driven by EMFC inside the Ferrel cell, there is no separate jet at the Hadley cell edge, and much of the momentum of the subtropical jet is transferred poleward into the midlatitude jet, leaving a weak local maximum of the upper-layer zonal wind inside the Hadley cell, around latitude 19°S (Fig. 5a), equatorward of the most negative EMFC (Fig. 5d). Unlike in the eddy-driven jet regime, the vertical shear of the zonal wind is maximal at the latitude of the main jet and not at the Hadley cell edge. The main jet is closer to the Hadley cell edge and more persistent than in the eddy-driven jet regime (Figs. 2a,d).

c. Subtropical jet regime

The maintenance mechanism of the subtropical jet regime was analyzed in Lachmy and Harnik (2014). We review it again here for comparison with the eddy-driven and merged jet regimes. Figure 6 shows the same time-averaged variables as in Fig. 4, but for the subtropical jet example (run 28).

In this simulation the upper-layer zonal wind is strong and maximal at latitude 30°S, close to the subtropical edge of the Hadley cell (Figs. 6a,c). The lower-layer zonal wind is very weak, with the maximum westerlies 15° poleward of the jet and easterlies at low and high latitudes, in balance with the Hadley and polar cells (Fig. 6c). The structure of this jet is substantially different from that of the eddy-driven and merged jets in that it is located at the Hadley cell edge, above the zero surface westerlies where the EMFC is zero (Fig. 6d), indicating that it is driven by advection of momentum by the Hadley circulation and not by EMFC. The positive EMFC at the midlatitudes drives a very weak barotropic jet, expressed as a shoulder in the upper-layer zonal wind profile, around latitude 45°S, above the surface westerlies. A very weak polar jet around latitude 80°S is maintained by the Coriolis force associated with the polar cell against the effect of the negative EMFC at high latitudes.

The upper-layer PV gradient is maximal near the jet latitude, with two additional local maxima at the latitudes of the weak midlatitude eddy-driven jet and high-latitude polar jet. The lower-layer PV gradient is negative only poleward of latitude 28°S, with near-zero values at the jet latitude. As explained in Lachmy and
Harnik (2014), the positive lower-layer PV gradient at the tropics and subtropics, as a result of the β effect, stabilizes the waves at these latitudes. When the jet is farther poleward, as in the eddy-driven and merged jet regimes (Figs. 4b, 5b), baroclinic growth can occur at the jet latitude, but when the jet is close to the region of positive lower-layer PV gradient, as in the subtropical jet regime, baroclinic growth occurs far poleward of the jet. This is indeed seen in the heat flux profile (Fig. 5d), which is maximal 19° poleward of the jet. This stabilizing mechanism explains the abruptness of the transition from the merged to the subtropical jet regime as the model parameters are varied (Figs. 1, 2, 3), as explained in Lachmy and Harnik (2014).

5. Equilibration of the wave spectrum

a. Wave spectrum and energy budget

The structures of the eddy momentum and heat fluxes are strongly affected by the spectral properties of the waves, including their zonal wavenumber and phase speed. Figure 7 shows the contribution of the different zonal wavenumbers to the EMFC and eddy heat flux for the same example simulations analyzed in section 4. It is seen from Figs. 7a,b that the wide structure of the EMFC and heat flux in the eddy-driven jet regime is a result of both the wide structure of the fluxes from each wavenumber and the spread between the contributions from different wavenumbers, where longer waves tend to be concentrated at higher latitudes. In the merged jet regime (Figs. 7c,d), the narrow structure of the wave fluxes, which was shown in section 4b to play a major role in the maintenance of the jet, results from the sharp spectrum, dominated by a zonal wavenumber-5 mode. In the subtropical jet regime (Figs. 7e,f), the reduction in the strength of the wave fluxes relative to the merged jet regime (note the different color coding) is mainly due to the decay of the zonal wavenumber-5 mode, while longer waves contribute to the fluxes at higher latitudes, leaving the subtropical jet affected only very weakly by the waves.

In this section we examine the maintenance of the wave spectrum that produces these fluxes. We consider only zonal wavenumbers 1–8, while wavenumbers 9–42 are very weak and play a qualitatively similar role as wavenumber 8.

The top panel of Fig. 8 shows the wave energy spectrum as a function of the run number, sorted by the domain-averaged EKE, in descending order (see Table 1). The regimes of the simulations are denoted by markers on the x axis, with larger markers for the three example simulations. The eddy-driven jet regime is characterized by a wide spectrum with a maximum at wavenumber 3 or 4 and a secondary local maximum at wavenumber 1. The merged jet regime is characterized by a very sharp spectrum, dominated by wavenumber 5, with the exception of run 23, where wavenumber 6
dominates the spectrum. In the subtropical jet regime the maximum at wavenumber 5 becomes weaker than in the merged jet regime, while the low wavenumbers stay roughly the same.

To examine the role of linear instability we look at the linear growth rate of the most unstable mode for each zonal wavenumber in each simulation (lower panel of Fig. 8). The growth rates were calculated by integrating a linear version of the model, in which wave–wave interactions were set to zero, with the same parameters as in the original nonlinear simulation and with the mean flow held fixed at the statistically steady-state profile of the corresponding nonlinear simulation. The integration was initiated with a very weak white noise perturbation to allow for all wavenumbers to develop and continued to day 277, by which time linear growth or decay was observed for all wavenumbers. A linear fit was made for the log of the wave energy for each wavenumber over the period of linear growth in order to determine the linear growth rate.

In the eddy-driven jet regime the growth rate is positive for wavenumbers 3–6 and for some simulations also for wavenumbers 2 or 7, with the maximal growth rate increasing as the EKE increases. In the merged jet regime medium-scale waves (wavenumbers 4–6) are stable or close to neutrality, while the linear growth rate of wavenumbers 2 and 3 is positive. It should be noted that these unstable modes are concentrated at high
latitudes and have a very different structure from the wavenumber-2 and wavenumber-3 waves in the non-linear merged jet regime simulations. In the subtropical jet regime, the pattern is similar to that of the merged jet regime, but with the medium-scale waves becoming even more stable. Comparing the two panels of Fig. 8, it is clear that the most energetic mode in the statistically steady state is not related to the linear most unstable mode of the time-mean flow. While the energy spectrum shifts from low to medium wavenumbers at the transition from the eddy-driven jet regime to the merged jet regime, the linear growth rate shifts from medium to low wavenumbers.

It is important to note that the largest growth rate from all the parameter sweep (at run 1, $m = 4$) is around 1/7 day$^{-1}$ and the largest growth rate in the merged jet regime is around 1/20 day$^{-1}$. The time scales associated with these growth rates are longer than the time scales of the eddy life cycle of the corresponding model simulations. The above results imply that the time-averaged mean flow is too close to neutrality to allow for linear wave growth when taking the damping terms into account. This result is consistent with the concept of baroclinic adjustment, though not in its classical form. Nonlinear interactions can act to balance the growth of waves at the scales that gain energy from interaction with the mean flow so that the equilibrated mean flow can be unstable with respect to these waves (Cehelsky and Tung 1991; Welch and Tung 1998). The degree of instability of the mean flow is related to the degree of nonlinearity of the wave spectrum (Held and Larichev 1996; Zurita-Gotor 2007). The relatively higher linear growth rates in the eddy-driven jet regime indicate that nonlinear wave–wave interactions are more significant in that regime.

To examine the relative role of wave–mean flow interactions and wave–wave interactions in the maintenance of the wave energy spectrum, we look at the time-averaged energy tendency terms that come from wave–mean flow and wave–wave interactions for each zonal wavenumber in each simulation (Fig. 9). The top panel of Fig. 9 shows that the growth by wave–mean flow interactions is highest for wavenumbers 3–6 and becomes concentrated at higher wavenumbers as the total EKE decreases. The contribution of nonlinear wave–wave interactions to the energy tendency (bottom panel of Fig. 9) shows that in all the simulations energy is transferred from medium-scale waves (wavenumbers 3–6) to long waves (wavenumbers 1 and 2) and short waves (wavenumbers 7 and higher). The nonlinear terms are most significant in the eddy-driven jet regime. The sum of the two terms is balanced by damping terms, which are negative and approximately proportional to the energy for all wavenumbers (not shown).

A comparison of the wave–mean flow and wave–wave interaction terms (top and bottom panels of Fig. 9, respectively) with the energy spectrum (top panel of Fig. 8) shows that, while in the merged and subtropical jet regimes the dominant modes are those that contribute most to the energy growth by wave–mean flow interactions, in the eddy-driven jet regime the energy that accumulates in the long waves (wavenumbers 1–3) is gained via nonlinear interactions with medium-scale waves (mostly wavenumber 4), which gain their energy from the mean flow.

The picture that emerges from the above analysis is that the wave spectrum in the merged and subtropical jet regimes is maintained mainly by a balance between the energy gain from wave–mean flow interactions and the energy loss from the linear damping terms. Nonlinear wave–wave interactions play a minor role in these regimes, and the waves are in statistically steady state because of the mean flow being close to neutrality when taking the damping terms into account in the stability calculation. In the eddy-driven jet regime, nonlinear wave–wave interactions are responsible for the transfer of energy from medium-scale waves to long waves, leading to a much wider spectrum. The mean flow in the eddy-driven jet regime is weakly unstable for medium-scale waves.

A quantitative way of estimating the degree of nonlinearity in a two-layer model is to look at the supercriticality (Held and Larichev 1996), which is defined as $\xi = U/\beta \lambda^2$, where $U$ is the thermal wind ($2\pi\nu$ in our
notation), \( \lambda = NH/f \) is the Rossby deformation radius, \( N \) is the Brunt–Väisälä frequency, and \( f \) is the Coriolis parameter. In the homogeneous two-layer \( \beta \)-plane framework of Held and Larichev (1996) the supercriticality is greater (less) than 1 for a linearly unstable (stable) mean flow. The scaling theory of Held and Larichev argues that the ratio between the length scale of the energy containing eddy and the Rossby deformation radius and the ratio between the root-mean-square eddy velocity and the thermal wind, are both proportional to the supercriticality.

In our calculation, we take the supercriticality to be

\[
\zeta = \left[ \frac{\epsilon}{2a^2} \left( \frac{\mu_2}{1 - \mu_2} \right) \frac{U}{U_{\tau}} \right] \left( \frac{2\Omega}{a^2} - \frac{1}{a^2} \frac{\partial^2 U}{\partial \mu^2} \right),
\]

where \( \epsilon = 8(a\Omega/NH)^2 \). This definition follows from the original one, where \( U/\lambda^2 \) is replaced by minus the baroclinic component of the lower-layer PV gradient [see Eq. (A12) in Lachmy and Harnik (2014)] and \( \beta \) is replaced by a sum of the meridional gradient of the Coriolis parameter and the curvature term so that \( \zeta > 1 \) when the lower-layer PV gradient is negative, which is a necessary condition for baroclinic instability. The inclusion of the curvature term follows from the expansion of the theory of Held and Larichev for nonhomogeneous flows by Zurita-Gotor (2007).

Figure 10 shows the supercriticality at the jet latitude (the latitude of maximum \( \tau \); see Fig. 2a) as a function of run number, sorted by the EKE. The supercriticality is in the ranges of 1.7 < \( \zeta < 2.15 \), 1.3 < \( \zeta < 1.9 \), and 1.25 < \( \zeta < 1.65 \) for the eddy-driven, merged, and subtropical jet regime simulations, respectively. There is a clear tendency toward lower supercriticalities as the EKE decreases throughout the parameter sweep, except within the merged jet regime.

The supercriticality values imply that the eddy-driven jet regime has a higher degree of nonlinearity than the merged and subtropical jet regimes, and the tendency toward higher values for the more energetic simulations within the eddy-driven jet regime implies that this regime behaves consistently with turbulent scaling theories. However, the lack of any clear tendency of the supercriticality within the merged jet regime indicates that turbulent theories do not describe the dynamics of the flow equilibration in this regime, as it is controlled by quasi-linear wave–mean flow dynamics. To further examine the quasi-linear equilibration of the flow we repeated the above experiments with a quasi-linear version of the model, as described next.

b. Comparison with quasi-linear equilibration

A quasi-linear version of the model, which does not include wave–wave interactions, was integrated with the same parameters as in the three nonlinear simulations described in section 4 and for an additional simulation of the subtropical jet regime, as explained below. The initial conditions for the mean flow are the same as in the nonlinear simulations, and the initial PV of the waves is a weak white noise disturbance, concentrated at latitude 45°S. The time-mean variables are averages over the last 1500 days of the 3000 days integrations, when the system is in statistically steady state.
The quasi-linear simulation with \( H = 10 \text{ km} \) and \( r = 2 \), corresponding to the subtropical jet regime example, showed a transition to the merged jet regime. We leave the examination of this transition outside the scope of this work and focus on understanding the quasi-linear equilibration within each regime. For this purpose, we chose to examine a quasi-linear simulation that reproduces the subtropical jet regime. This was achieved with the parameter values \( H = 11 \text{ km} \) and \( r = 2.5 \). We label this simulation STJ2.

Figure 11 shows the time and zonal-mean upper-layer zonal wind and EMFC and the energy spectrum for zonal wavenumbers 1–8 for the nonlinear model simulations and the corresponding quasi-linear simulations. In all three examples the upper-layer zonal wind and EMFC in the quasi-linear model are very similar to those in the nonlinear model, except for a few local differences.

In the eddy-driven jet example there is an increase in the zonal wind at high latitudes (Fig. 11a), which is composed of an increase in the lower-layer zonal wind (not shown), because of the slightly stronger EMFC at these latitudes (Fig. 11d), and of an increase in \( \mathbf{u}_T \) at high latitudes, because of a narrower eddy heat flux, which maintains a stronger temperature gradient at its flanks (not shown). These small modifications of the wave fluxes and the resulting mean flow are produced by waves with a significantly different energy spectrum. The energy of wavenumbers 3–5 is higher in the quasi-linear simulation, with the energy of wavenumber 5 being twice that in the nonlinear simulation, while wavenumbers 1–2 and 6–10 almost disappear (Fig. 11g). The long and short waves that disappear in the quasi-linear version are those that gain energy by nonlinear interactions in the nonlinear version (Fig. 9).

In the merged jet example the local minima of \( \mathbf{u}_1 \) and the EMFC are sharper in the quasi-linear simulation (Figs. 11b,e), which can be attributed to the absence of nonlinear interactions that tend to spread out the wave breaking region. The spectrum in the quasi-linear simulation is dominated by wavenumber 5, as in the nonlinear simulation, but with the energy about 3 times higher and with a larger amplitude of wavenumber 2. Medium-scale waves, which dominate the eddy fluxes in the eddy-driven and merged jet examples (Fig. 7), have higher amplitudes in the quasi-linear simulations than in the nonlinear simulations, yet they lead to similar wave fluxes. This occurs because in the quasi-linear
simulations wave fluxes change sign during the life cycle (not shown), so higher amplitudes are needed to produce the same fluxes.

In the subtropical jet example (Figs. 11c,f) the main difference is at high latitudes, where there is stronger EMFC in the quasi-linear simulation. The quasi-linear subtropical jet spectrum (Fig. 11i) is almost identical to that of the nonlinear subtropical jet simulation, with wavenumber 3 dominating at approximately the same amplitude.

To better understand the conditions for the equilibration of the wave spectrum, we examine the separate effects of the mean-flow profile and the model parameters on the linear stability of the flow by running the linear version of the model, described in section 5a, with different mean-flow profiles and model parameters. We take the six mean-flow profiles from the simulations shown in Fig. 11 (the time-averaged zonal-mean winds from the nonlinear and quasi-linear simulations) and for each calculate the linear growth rates, but using the parameter sets of the eddy-driven ($H=7\text{ km, } r=0.5$), merged ($H=9\text{ km, } r=1.5$), and subtropical ($H=11\text{ km, } r=2.5$) jet simulations. The three panels in Fig. 12 show the linear growth rates for each mean-flow profile as a function of the zonal wavenumber for the eddy-driven, merged, and subtropical jet parameter sets.

The parameters affect the stability properties directly by stabilizing the waves as $H$ and $r$ are increased. This is evident from the fact that the growth rates decrease for all wavenumbers and mean-flow profiles when moving from the eddy-driven jet parameters (Fig. 12a) to the merged jet parameters (Fig. 12b) and then to the subtropical jet parameters (Fig. 12c). Note the differences in the scaling of the $y$ axis. There are several interesting points to observe regarding the stability properties of the different mean-flow profiles:

1) Comparing the linear growth rates for the different mean-flow profiles, it is seen that for all the parameter sets (Figs. 12a,b,c) and for almost all wavenumbers, the linear growth rates are highest when the mean flow is taken from the subtropical jet simulations (thick solid curves) and lowest when the mean flow is from the eddy-driven simulations (thin solid curves). This means that the subtropical jet mean flow is more unstable and the eddy-driven jet mean flow is more stable for a given $H$.

2) The equilibrated mean flows that are closest to neutrality within each set of parameters (Figs. 12a,b,c) are those taken from the simulations with the same parameters as in the corresponding linear simulation. For example, for the parameters $H=7\text{ km and } r=0.5$ (Fig. 12a), the jet profiles taken from the nonlinear and quasi-linear simulations with $H=7\text{ km and } r=0.5$ are closest to being neutral.

3) Comparing the linear growth rates for the mean-flow profiles from the quasi-linear simulations (red curves) with those from the corresponding nonlinear simulations (blue curves), it is seen that the quasi-linear mean flows are more stable with respect to medium-scale waves (wavenumbers 3–5), with the largest difference in the eddy-driven jet simulations (Fig. 12a, thin curves).
The comparison of the quasi-linear and nonlinear model results suggests that wave–mean flow interactions play a major role in determining the equilibrated wave spectrum by imposing a constraint on wave fluxes. The equilibrated wave spectrum has to be such as to produce the fluxes necessary for maintaining a specific mean-flow profile, which in turn maintains the wave spectrum in statistically steady state. For regimes that are nearly quasi-linear, such as the merged and subtropical jet regimes, the equilibrated mean flow is close to neutrality. As the degree of nonlinearity increases the equilibrated mean-flow profile becomes more unstable.

The change in the wave energy as the model parameters are varied is explained as follows: Decreasing the parameters $H$ and $r$ destabilizes the waves so that a more stable mean flow, for a given set of parameters, is needed in order to keep the wave in steady state, as observed in the transition from the subtropical to the merged and then to the eddy-driven jet regime. The more stable mean flow is maintained by stronger wave fluxes, and so the wave energy increases.

6. Discussion

The model simulations presented in this paper produced the eddy-driven, merged, and subtropical jet flow regimes, which show qualitative similarity to the type of flows observed in more complex numerical models and in the atmosphere. To assess the applicability of the conclusions drawn from this study, we compare a few aspects of the dynamics with those found in other models and in observations and discuss the limitations of the model assumptions and setup.

In our analysis of the three flow regimes, we found that the wave energy, which in our parameter sweep was controlled by the layer thickness $H$ and the wave damping parameter $r$, plays a key role in determining the flow regime, with the most energetic simulations being in the eddy-driven jet regime and the least energetic ones in the subtropical jet regime. Though we find no direct analog for the parameters $H$ and $r$ in the real atmosphere, we believe that any factor affecting the wave energy could potentially induce similar regime transitions. Several numerical studies have found a transition from a single- to a double-jet regime, when the diabatic heating is varied to increase the midlatitude temperature gradient (Son and Lee 2005, 2006; Michel and Rivière 2014). The single- and double-jet regimes in these studies bare a resemblance to the merged and eddy-driven jet regimes, respectively, both in the time-mean and variability properties of the mean flow.

The eddy feedbacks associated with a poleward shift of the jet were the subject of many studies in the past two decades, motivated by predictions of the atmospheric circulation response to increased greenhouse gas emissions. In some of these studies, changes in the wave properties in response to an imposed poleward shift of the jet are consistent with the changes we find between the merged and eddy-driven jet regime, and also within the eddy-driven jet regime, as the wave energy is increased and the jet is shifted poleward. These include an increase in the eddy length scale (Rivière 2011; Barnes and Hartmann 2011), predominant equatorward wave propagation (Kidston and Vallis 2012; Lorenz 2014a), and a decrease in cyclonic wave breaking frequency at the poleward flanks of the jet (Rivière 2011; Garfinkel and Waugh 2014). The latter two are expressed in our model by the lack of eddy momentum flux divergence on the poleward side of the jet in the eddy-driven jet regime and, consequently, by the absence of a polar cell (Figs. 2d, 4d). In addition to the changes in the eddy properties, many studies have found the jet latitude to vary more strongly and on shorter time scales as the jet shifts poleward (Son and Lee 2006; Gerber and Vallis 2007; Barnes et al. 2010), as seen in our study (Fig. 2c), and this will be examined more closely in a subsequent paper. While in the abovementioned studies the jet latitude shift is imposed, in our simulations it results from the change in the dynamics as the eddy energy increases. In this study, rather than isolating a specific feedback mechanism, we attempt to build an overall picture of the regimes, in which different feedback mechanisms act to maintain the flow in each regime.

The aspect of the jet regime transitions that has received the least attention in the literature is the equilibration of the wave spectrum. In our idealized model we find the wave spectrum to play a major role in the regime transition. We find that in the eddy-driven jet regime nonlinear wave–wave interactions are important and an inverse energy cascade takes place, as predicted by geostrophic turbulence theories. The supercriticality increases with the wave energy, consistent with the scaling theory of Held and Larichev (1996) and Zurita-Gotor (2007). This increase in supercriticality could also be related to the poleward shift of the jet as we move to the more energetic simulations, consistent with the findings of Chemke and Kaspi (2015) and Theiss (2004) in numerical simulations of fast rotating planets and oceanlike flows.

Turbulent theories are less relevant for the merged jet regime, where the spectrum is very narrow and there is no significant inverse energy cascade. We find that the dominant mode in this regime is the mode that is most efficient at maintaining the mean flow in the same position and structure needed for the maintenance of the mode itself. A useful approach for explaining the equilibrated state in this regime would be to consider the
conditions for equilibration of the coupled wave–mean flow system. SSST (Farrell and Ioannou 2003; DelSole 2004) seems most appropriate for this case, as it finds stable solutions for the wave–mean flow system, and the wave statistics are an outcome of these solutions. Further investigation is needed to assess the relevance of SSST for the merged jet regime.

The subtropical jet regime, in which the wave energy is extremely low, is the least realistically captured by our idealized model. This regime rarely appears in numerical models, where the jet is almost always located inside the Ferrel cell. As discussed in Lachmy and Harnik (2014), we suspect this has to do with the zonal symmetry of the model time-mean flow and forcing. In the atmosphere, the subtropical jet is found at longitudes where the eddy fluxes are weak, whereas the strong fluxes are located at other longitudes, where the eddy-driven jet dominates, resulting in a zonal-mean state with a strong jet at the Hadley cell edge alongside strong midlatitude eddies. In our model, the subtropical jet regime shows sensitivity to the radiative equilibrium profile. By changing the latitude of maximum radiative equilibrium temperature, we found that the subtropical jet is produced only with winter forcing conditions and not with summer or equinox conditions (not shown). This may be related to the Hadley cell edge being shifted equatorward in winter, which enables the maintenance of the jet at the Hadley cell edge at latitudes where the waves are stabilized by the positive lower-layer PV gradient [see section 4c and Lachmy and Harnik (2014)].

There are a few aspects of the dynamics that are misrepresented by the model because of its simplifications. Tropical dynamics are far from realistic, since the equations are based on QG assumptions and because tropical waves and moist convection are not included; dynamical feedbacks related to the vertical temperature profile, which may play a role in the mean-flow equilibration (Schneider 2004), are not simulated in this model, since the static stability is a constant model parameter; moist processes are not included; and the vertical truncation to two layers may introduce an exaggerated sensitivity to geometric details of the flow. However, the qualitative nature of extratropical wave–mean flow interactions and their role in determining the flow regime seem to be captured well by the model.

We note that we have not examined systematically the sensitivity of our results to variation of the radiative equilibrium profile and to the time scales of mean-flow surface friction and radiative relaxation. A broader perspective of the jet regimes could be obtained by enforcing these sensitivity tests, and is left for future study.

7 Conclusions

A transition from an eddy driven to a merged and then to a subtropical jet regime is found in the two-layer model described in Lachmy and Harnik (2014) as the layer thickness $H$ or wave damping parameter $r$ is increased and the flow equilibrates with weaker wave energy. The characteristics and dynamical maintenance of the three regimes are described as follows:

(i) The eddy-driven jet regime is characterized by a wide jet at relatively high latitudes with strong surface westerlies below the jet. The meridional circulation consists of a Hadley cell and a wide Ferrel cell, with no polar cell. The eddy kinetic energy is high and the flow is turbulent with a significant inverse energy cascade and strong temporal variations of the jet latitude. The time-averaged mean flow is weakly unstable for medium scale waves, and stable for long and short waves.

The maintenance of the jet is explained by the wide EMFC produced by the turbulent spectrum. The wide structure of the jet in turn enables baroclinic growth throughout the middle and high latitudes. The absence of negative EMFC on the poleward flanks of the jet explains the absence of a polar cell, through the condition for momentum balance.

(ii) The merged jet regime is characterized by a sharp jet inside a narrow Ferrel cell with surface westerlies below. There is very little variability of the jet latitude with time. The wave energy spectrum is sharp and dominated by a single zonal wavenumber, usually wavenumber 5.

The dominant mode produces positive EMFC at the jet latitude and negative EMFC at the jet flanks, where two local minima of the zonal wind are maintained. The high latitude negative EMFC leads to the existence of a polar cell. The jet structure enables baroclinic growth at a narrow latitudinal band. The narrow Ferrel cell maintains the vertical shear of the zonal wind above radiative equilibrium values, through the adiabatic heating and cooling. The maintenance mechanism of the merged jet is similar to that of the self-maintaining jet of Robinson (2006), as explained in section 4.2.

In this regime quasi-linear wave–mean flow interactions control the wave dynamics, while wave–wave interactions are relatively weak. The equilibrated spectrum is explained by the efficient feedback between the mean flow and the dominant mode.

(iii) The subtropical jet regime is characterized by a strong jet at the subtropical edge of the Hadley cell in the upper layer, maintained by advection of
absolute angular momentum by the Hadley circulation. The EMFC and poleward eddy heat flux are much weaker than in the merged jet regime and located far poleward of the jet maximum. The jet latitude varies weakly with time and the spectrum is maintained mainly by quasi-linear dynamics. Baroclinic growth does not occur at the jet latitude, due to the stabilizing effect of the positive PV gradient in the lower layer near the jet latitude, as explained in Lachmy and Harnik (2014).

While it is of high importance to understand the response of the flow to external forcing, especially in the context of climate change and climate variability, we have not dealt with this question directly here. The parameter sweep presented in this study offers a mapping of the flow regimes, which could serve as a reference for interpreting the response to perturbations in the thermal forcing, taking into consideration that the response depends on the flow regime of the unperturbed state and could induce a regime transition.

Finally, this study focuses on the time-mean balance of the mean flow and wave spectrum. The time dependent behavior of the system also varies greatly between the regimes. In a subsequent paper we will examine how the different wave spectra in each regime are associated with different wave–mean flow feedbacks and correspondingly different internal variability characteristics.

Acknowledgments. This article forms a part of the Ph.D. thesis of OL at Tel Aviv University. This work was funded by the Israeli Science Foundation Grants 1370/08 and 1537/12, and by the Binational Science Foundation Grant 2008/436. Part of the work of NH was done during her sabbatical at Stockholm University, where she was supported by a Rossby Visiting Fellowship from the International Meteorological Institute at Stockholm University. OL also thanks the Israeli Ministry of Science and Technology for a woman scientist’s scholarship. The authors thank Ori Adam for his assistance with the numerical tools used for this study, Steven Feldstein for his ideas and discussions regarding the equilibration of the wave spectrum, and three anonymous reviewers for their valuable comments on the manuscript and on the interpretation of the results.

APPENDIX A

Numerical Diffusion

The model used for this study is the same as described in Lachmy and Harnik (2014), but with a different scheme for the numerical diffusion of the mean flow. Here we replaced the term $-\left(\nu_{\text{MF}}/a^4\right)\partial^2 U/\partial t^4$ in the equation for $\partial U/\partial t$ used in Lachmy and Harnik (2014) with $-\left(\nu_{\text{MF}}/a^4\right)\partial^2 U/\partial \phi^4$, where $\nu_{\text{MF}}$ is the mean-flow diffusion coefficient. This version of the numerical diffusion is a closer approximation to the $-\nu \nabla^4$ operator. After this modification the diffusion coefficients were chosen to have the minimal values needed to prevent the accumulation of energy at the smallest scales. The diffusion coefficients for the mean flow and the waves are $\nu_{\text{MF}} = 5 \times 10^{15}$ m$^4$ s$^{-1}$ and $\nu_{\text{NWV}} = 2 \times 10^{16}$ m$^4$ s$^{-1}$, respectively.

This modification served to make the model computationally stable in a wider range of parameter values. It leads to some changes in the location of the regime transition in the parameter space, but the qualitative results are unchanged.

APPENDIX B

Radiative Equilibrium

The radiative equilibrium profile of the thermal wind, $(\bar{u_T})_E(\phi)$, was the same in all model integrations. It is the following profile smoothed by a running average filter:

\[
(\bar{u_T})_E(\phi) = \begin{cases} 
\cot(\sin\phi - \sin\phi_0) & \text{for } |\phi| > \phi_0, \\
0 & \text{for } |\phi| \leq \phi_0,
\end{cases}
\]

where $(u_T)_0 = 15$ m s$^{-1}$ and $\phi_0 = 10^\circ$. The wind difference profile $2(\bar{u_T})_E$ is plotted in Figs. 4, 5, and 6. This profile is based on the radiative equilibrium temperature profile of Lindzen and Hou (1988), which satisfies $\theta_E \approx (\sin\phi - \sin\phi_0)^2$. Using the thermal wind relations, $\sin(\bar{u_T}/\bar{z}) \approx \theta/\theta_0$ gives $u_T \approx \cot(\sin\phi - \sin\phi_0)$. This profile causes the Hadley cell ascending branch to be concentrated around $\phi_0$ and is meant to simulate winter conditions for the Southern Hemisphere.

REFERENCES


