The Response of an Idealized Atmosphere to Orographic Forcing: Zonal versus Meridional Propagation

NICHOLAS J. LUTSKO
Program in Atmospheric and Oceanic Sciences, Princeton University, Princeton, New Jersey

ISAAC M. HELD
NOAA/Geophysical Fluid Dynamics Laboratory, Princeton, New Jersey

(Manuscript received 12 January 2016, in final form 8 June 2016)

ABSTRACT
A dry atmospheric general circulation model is forced with large-scale, Gaussian orography in an attempt to isolate a regime in which the model responds linearly to orographic forcing and then to study the departures from linearity as the orography is increased in amplitude. In contrast to previous results, which emphasized the meridional propagation of orographically forced stationary waves, using the standard Held–Suarez (H–S) control climate, it is found that the linear regime is characterized by a meridionally trapped, zonally propagating wave. Meridionally trapped waves of this kind have been seen in other contexts, where they have been termed “circumglobal waves.” As the height of the orography is increased, the circumglobal wave coexists with a meridionally propagating wave and for large-enough heights the meridionally propagating wave dominates the response. A barotropic model on a sphere reproduces this trapped wave in the linear regime and also reproduces the transition to meridional propagation with increasing amplitude. However, mean-flow modification by the stationary waves is very different in the two models, making it difficult to argue that the transitions have the same causes. When adding asymmetry across the equator to the H–S control climate and placing the orography in the cooler hemisphere, it becomes harder to generate trapped waves in the GCM and the trapping becomes sensitive to the shape of the orography. The barotropic model overestimates the trapping in this case. These results suggest that an improved understanding of the role of circumglobal waves will be needed to understand the stationary wave field and its sensitivity to the changes in the zonal-mean climate.

1. Introduction

The response of the atmosphere to orography is one of the oldest problems in atmospheric dynamics. In one of the earliest studies, Charney and Eliassen (1949) addressed the problem in a barotropic quasigeostrophic model in a β-plane channel, linearized about a uniform zonal-mean flow comparable to that in Northern Hemisphere wintertime. Many subsequent studies continued to use quasigeostrophic models on a β plane in a channel geometry to simulate both orographic and thermally forced waves, reducing the problem to variations in longitude x or to variations in the x–z plane (Smagorinsky 1953; Saltzman 1963, 1965; Kasahara 1966; Derome and Wiin-Nielsen 1971; Egger 1978; Tung and Lindzen 1979; Davey 1980; Trevisan and Buzzi 1980). In this channel geometry, in which only zonal and vertical propagation are allowed, resonances appear generically. Therefore, the impression emerged that the atmosphere’s response to orography is highly sensitive to changes in parameters such as the zonal-mean flow and the frictional time scale. These resonances can in some cases also lead to the existence of multiple equilibrium states when the coupling to the mean flow is included (Charney and DeVore 1979).

A series of papers by Hoskins et al. dramatically changed this intuition in the late 1970s and early 1980s (Hoskins et al. 1977; Grose and Hoskins 1979; Hoskins and Karoly 1981). These works emphasized the importance...
for stationary Rossby waves of meridional as well as zonal propagation; in the special case of a zonal-mean zonal flow that is proportional to the cosine of the latitude ray tracing using the barotropic dispersion relationship shows that stationary waves exactly follow great circles (Hoskins and Karoly 1981). Ray tracing on more realistic zonal flows typically retains some of this great circle flavor.

The realization that the meridional motion of Rossby waves is just as important as their zonal motion led to a focus on the fate of stationary Rossby waves as they propagate into the tropics (Killworth and McIntyre 1985; Held and Phillips 1990; Esler et al. 2000). While nonlinear critical-layer theory reveals the potential for reflection of these waves back toward midlatitudes, the fact that the climatological stationary wave momentum flux is directed from the tropics toward midlatitudes indicates that on average these waves are preferentially absorbed rather than reflected. The study of resonances and multiple equilibria in idealized model configurations continued (Kallen 1981, 1985; Holloway and Eert 1987; Mak 1989; Jin and Ghil 1990), but as Kallen (1985) emphasized, the meridional propagation of Rossby waves into the tropics and the existence of absorption there makes it much harder for resonances to exist and, hence, multiple equilibria and the associated sensitivity to the mean flow. While the reflection of Rossby waves by a critical layer has been seen in general circulation models (GCMs; Walker and Magnusdottir 2003), it seems difficult to create enough reflection from the tropics, in the presence of mixing by large-scale transients and of the Hadley cell, that a wave would be able to propagate around the entire globe.

However, in a parallel series of papers it was noticed that midlatitude jets can act as waveguides for Rossby waves generated by idealized vorticity sources (Branstator 1983; Hoskins and Ambrizzi 1993; Branstator 2002), resulting in the waves propagating over long zonal distances with little meridional motion. Branstator (2002) coined the term “circumglobal waves” to describe this phenomenon, and circumglobal waves have been seen in observations as well as in GCMs and barotropic models (Branstator 2002; Brandefelt and Kornich 2008; Branstator and Selten 2009; Manola et al. 2013; Petoukhov et al. 2013; Saeed et al. 2014). To lowest order, they can be thought of as edge waves propagating along the sharp potential vorticity gradient at the center of a westerly jet. These waves may also play an important role in the response of the stationary wave field to climate change (Selten et al. 2004; Brandefelt and Kornich 2008; Petoukhov et al. 2013). Using a linear barotropic model, Manola et al. (2013) found that the existence of a waveguide effective at trapping these circumglobal waves is sensitive to changes in the zonal-mean flow.

In the following work, a connection is made between circumglobal waves and the atmosphere’s response to orography. Specifically, we find that in an idealized, dry GCM the response to a small Gaussian mountain resembles a circumglobal wave. Our original motivation for carrying out these simulations was to revisit the problem of finding a regime in which the atmosphere’s response is a linear function of the height of the orography. If such a regime exists, then a link can be made between linear stationary wave models (e.g., Ting and Held 1990) and GCMs, and the breakdown of the linear response as the mountain height is increased can also be investigated. This problem was previously studied in a low-resolution (R15), moist GCM by Cook and Held (1992), who varied the height of idealized Gaussian topography from 700 m to 4 km and found a “near linear” regime between 700 m and 2 km. Here we have extended the range of heights from 250 m to 5 km and use a dry GCM with T42 and T85 resolutions, in which the climate is zonally symmetric in the absence of the orographic perturbations. (The orography we study has a half-width of 15° latitude and longitude; despite this large size, we refer to this localized orography as a “mountain” in the following.)

We find that for small-enough mountains the response is linear, with the spatial structure independent of the height of the mountain, the amplitude of the response proportional to the height, and the mountain torque on the surface quadratic in the height. In our control setup this linear response is dominated by a circumglobal wave and so the response is potentially sensitive to the mean flow. The possibility of resonances is also reintroduced, although in the case examined the forced wave loses amplitude too rapidly to create significant constructive or destructive interference. However, the circumglobal wave coexists with a meridionally propagating wave and in the nonlinear regime the meridional wave increasingly dominates the response. In addition to the different horizontal directions of propagation, these waves have noticeably different vertical structures close to the mountain.

We also modify the GCM, changing the equilibrium temperature structure by adding asymmetry across the equator to create a cooler and a warmer hemisphere. Topography is then added in the cooler hemisphere to examine the robustness of the zonal propagation in the linear regime.

To better understand the GCM’s behavior, we seek the simplest model, which can be used as an analog for the GCM. We find that a barotropic model on a sphere can qualitatively reproduce the linear response in the
control setting, but not in the case with north–south asymmetry in the equilibrium temperatures. In the latter case the GCM’s linear response is not trapped but the barotropic model continues to predict trapping. Consistently, the barotropic index of refraction continues to suggest the potential for trapping. The barotropic model does simulate a transition from zonal to meridional propagation as the topographic amplitude is increased, but the underlying dynamics in the two models appear to be substantially different.

In the following section we describe the GCM model and experiments, before the GCM results are presented in sections 3 and 4. The barotropic model is described in section 5, where it is used to understand the linear response of the GCM and also compared with the index of refraction. In section 6 the nonlinear response of the barotropic model is compared with the GCM’s nonlinear response before we end with our conclusions in section 7.

2. Primitive equation model and experiments

The primitive equation simulations were carried out using the GFDL spectral dynamical core forced by zonally symmetric Newtonian relaxation to a prescribed equilibrium temperature field and damped by Rayleigh friction near the surface. The parameter settings are the standard Held and Suarez (1994) parameters with forcing symmetric about the equator. This setup produces an equinoctial climate similar to that of the real atmosphere, though the north–south temperature gradient is somewhat larger than observed and temperature is constant in the stratosphere.

The model was run at T42 resolution with 30 evenly spaced sigma levels and data were sampled once per day. The control simulation consisted of a 40 000-day integration, with the first 2000 days discarded to ensure the model had spun up. We have confirmed the qualitative character of the transition from zonal to meridional propagation at T85 resolution (section 3b). While there are undoubtedly some quantitative differences, and integrations with higher horizontal and vertical resolutions are certainly possible, we focus here on generating long enough integrations to isolate the linear regime within this particular model.

To perturb the model Gaussian mountains were added, of the form

\[ h(\phi, \lambda) = H \exp \left\{ - \left[ \frac{(\phi - \phi_0)^2}{\alpha^2} + \frac{(\lambda - \lambda_0)^2}{\beta^2} \right] \right\}, \]  

(1)

where \( H \) is the maximum height of the mountain (m); \( \phi \) and \( \lambda \) are latitude and longitude, respectively; and \( \alpha \) and \( \beta \) are half-widths, both set to 15° in the initial perturbation experiments. The values of \( \phi_0 \) and \( \lambda_0 \) were set to 90°E and 45°N, respectively. This is the same form as the mountains used by Cook and Held (1992).

We varied \( H \) from 250 to 5000 m. For heights less than 250 m, the responses were not clearly separable from the noise. Cases with \( H \) less than 1 km were run for 40 000 days and the responses were obtained by discarding the first 2000 days of each perturbation experiment and averaging over the rest of the integration. Cases with larger mountains equilibrated more quickly and so were only run for 20 000 days. The length of all T85 simulations was 20 000 days.

For ease of presentation, we will focus on latitude–longitude cross sections of the eddy streamfunction responses at 350 hPa and pressure–longitude cross sections of the eddy zonal wind responses at 46°N. However, the important features and trends we describe can be seen at other latitudes and pressures and using other fields.

3. Primitive equation model results

a. The small mountains

We begin by considering the cases with \( H < 1 \) km. The eddy streamfunction responses at 350 hPa for each of these experiments are shown in Fig. 1 and the eddy zonal wind responses at 46°N are shown in Fig. 2. In both figures the fields have been multiplied by 1000 m/H, and we refer to these as normalized responses in the following.

In the latitude–longitude cross sections the responses are very similar in all cases and the normalization shows that the responses also have roughly the same magnitude. For each mountain, the strongest response is the low directly to the southeast of the mountain and then consists of a zonal wave, which extends east almost around the entire latitude circle. There is also a weak high in the response on the northwest flank of the mountain. Similarly, the responses closely resemble each other in the vertical cross sections, consisting of an equivalent barotropic wave centered near 300 hPa with wavenumber 5. The response is strongest directly to the east of the mountain and then the external Rossby wave weakens gradually as it extends around the entire globe.

The zonal waves seen here resemble the “circum-global” waves described by Branstator (2002), which suggests that the stationary wave produced by the orography is being trapped by the jet. If the flow is sufficiently slowly varying, an index of refraction can be used to determine whether a waveguide exists that can trap stationary external Rossby waves. There are a number of reasons why this diagnostic may not be quantitatively useful, beginning with the fact that the
flow may not be sufficiently slowly varying. But it is safe to assume that a waveguide in the index of refraction (a region of propagation surrounded by regions of evanescence) is a necessary condition for the trapping of stationary waves. Therefore, we speak of the index of refraction as indicating the “potential” for trapping.

One can also have a situation in which temporal variations in the flow smooth the long-term-mean potential vorticity field, reducing the potential for trapping that exists over shorter time averages. But once again one can still hope for qualitative guidance from the index of refraction regarding the potential for trapping.

By transforming to Mercator coordinates, the index of refraction \( n_s^2 \) for barotropic waves can be defined as (Hoskins and Karoly 1981)

\[
n_s^2 = a \left\{ \cos^2 \phi \left( \beta + \frac{1}{a} \frac{\partial \xi}{\partial \phi} \right) \right\} / |u|.
\]

(2)
where $a$ is Earth’s radius, $\zeta$ is the relative vorticity, $u$ is the zonal wind, and square brackets indicate zonal averages. For baroclinic models, Held et al. (1985) showed that there is a meridionally varying pressure $p_e(\phi)$ at which external Rossby waves obey the dispersion relation of barotropic waves; hence, the correct $[u]$ to use in Eq. (2) is $[u(p_e)]$. Held et al. (1985) found that for Northern Hemisphere winter conditions $p_e$ is between 450 and 400 hPa and so rather than calculate $p_e$ explicitly, in Fig. 3 we simply show the time-averaged zonal-mean zonal winds from the control simulation for a range of pressures near 400 hPa as well as the corresponding profiles of $n^2_s$. In the time mean wavenumber-5 waves are trapped at 300 hPa, though at other levels the jet is also close to being a waveguide, and so this index of refraction suggests that trapping is plausible, possibly with nonnegligible leakage.

**FIG. 2.** Pressure–longitude cross sections taken at 46° latitude of the eddy zonal wind response of the primitive equation model to idealized topography with $H =$ (top left) 250, (top right) 333, (middle left) 400, (middle right) 500, and (bottom left) 700 m. The contour interval is 0.5 m s$^{-1}$, the zero contour is shown by a thicker line, and areas of negative winds are shaded in gray. The responses have been normalized by 1000 m/$H$.

![Figure 2](http://example.com/fig2.png)

**FIG. 3.** (left) The time-averaged zonal-mean zonal wind profile from a control simulation of the primitive equation model at 300, 350, 400, and 450 hPa (solid lines) and the zonal wind profile at 350 hPa from an unperturbed experiment with $\varepsilon = 10$ K (dashed line). (right) As in the left panel, but for the index of refraction $n^2_s$. Zonal wavenumbers to the left of the curves are able to propagate while those to the right are evanescent.

![Figure 3](http://example.com/fig3.png)
Indeed, even at these small mountain heights there is some leakage from the waveguide in the form of meridional propagation that seems to begin farthest downstream at the smallest values of $H$ and then progressively becomes apparent closer to the mountain as $H$ increases. The leakage or damping is sufficient in these cases that the amplitude is reduced to small values before returning to the forced region.

**b. The large mountains**

The responses for the rest of the mountains are shown in Fig. 4, which plots the normalized eddy streamfunction responses to the larger mountains at 350 hPa; Fig. 5, which shows the unnormalized eddy streamfunction response for $H = 4$ and 5 km; and Fig. 6, which shows the unnormalized eddy zonal velocity responses to the largest mountains at 46°.

For $H = 1$ km the response resembles those of the smaller cases, but close to the mountain the wave propagates more strongly into the tropics, while also becoming dominated by a smaller zonal wavenumber. There is still some evidence of the zonal wave farther from the mountain, while the anticyclone on the northern
flank of the mountain becomes relatively stronger. As $H$ is further increased, the zonal wave continues to become relatively weaker and the response east of the mountain is then dominated by a wave that propagates meridionally into the subtropics. We note, however, that the circumglobal wave is still present in these simulations; it is just much weaker than the meridional wave, as can be seen from Fig. 5. To the west of the mountain, the anticyclone continues to strengthen and move closer to the center of the mountain; for $H = 5$ km, it is almost directly over the mountain (bottom panel of Fig. 5). In this respect the response in the vicinity of the mountain for large $H$ resembles the linear $f$-plane response.

Consistently, in the vertical cross sections the strong negative zonal wind anomaly over the east flank of the mountain broadens to the west as $H$ is increased, so that for $H = 5$ km it is centered directly over the mountain. Conversely, the positive wind anomaly east of this weakens significantly relative to the negative anomaly and shifts upward as $H$ is increased. Away from the mountain, the wavenumber-5 external wave continues to be present even in the cases with the largest mountains.

Finally, Fig. 7 plots the normalized eddy streamfunction responses at 350 hPa for $H = 333$ m, 700 m, and 4 km in simulations at T85 resolution, demonstrating that our results are robust to doubling the horizontal resolution of the model.

c. Mountain torque

Another metric that can be used to assess the linearity of the response to orography is the eastward torque exerted by the mountain on the atmosphere, which according to linear theory should be proportional to $H^2$. Figure 8 shows the normalized zonally averaged mountain torque per degree of latitude which the surface exerts on the atmosphere

$$T = -a^2 \cos \phi \left[ \frac{\partial h}{\partial \lambda} \right] \times \frac{H_{\text{ref}}^2}{H^2},$$

where $a$ is Earth’s radius, $p_s$ is the surface pressure, $H_{\text{ref}} = 1000$ m, the square brackets denote a zonal mean, and the overbar indicates a time-averaged quantity. As a crude measure of the sampling error, the shading envelopes the region between $T$ calculated using the first and second halves of the simulation. (The sampling error is very small for $H \geq 1$ km.)

For $H$ between 333 m and 1 km the normalized torques are similar to each other, with a strong minimum over the mountain and then a weaker maximum near 54°N. For $H = 2$ km the negative (westward) torque on the atmosphere over the mountain is still close to the linear limit but the small positive (eastward) torque to the north is roughly a quarter as large as for the smaller cases after normalization and is also shifted farther north by about 7°. So using the mountain torque as a metric of linearity in the vicinity of the mountain, the dominant transition from linear to nonlinear takes place between 1 and 2 km, although as we have seen, the departures from the linear limit are substantial farther downstream by $H = 1$ km. As $H$ is increased from 2 km, the normalized torques continue to weaken but retain the same shape. This reduction in the normalized torque corresponds to the gradual movement of the high pressure region over the mountain and, hence, $p_s$ and $\partial h/\partial \lambda$ move out of phase.

Fig. 5. The unnormalized eddy streamfunction responses at 350 hPa for $H =$ (top) 4 and (bottom) 5 km. The contour interval is 10 m$^2$ s$^{-1}$ between $-100$ and $100$ m$^2$ s$^{-1}$ and then the interval is 50 m$^2$ s$^{-1}$ between $\pm 100$ and $\pm 400$ m$^2$ s$^{-1}$. Black contours have been added for clarity.
For a barotropic flow linearized about a zonally symmetric basic state, the Plumb (1985) activity flux is given by

$$A = \frac{1}{2a^2} \left[ \frac{1}{\cos \phi} \left( \frac{\partial \psi^2}{\partial \lambda} - \psi \frac{\partial^2 \psi}{\partial \lambda^2} \right) \right] \cdot$$

Figure 9 plots the Plumb (1985) wave activity flux vectors at 350 hPa (arrows; normalized by $H^2$), the divergence of the Plumb flux (shading; also normalized by $H^2$), the critical line ($u = 0$; red), and the turning latitudes (where $n_s = 5$; green) for $H = 333$ m, 1 km, and 2 km. Plots of the horizontal component of the full baroclinic Plumb flux are qualitatively similar.

The Plumb flux allows us to examine where the rectified effect of the orographic stationary wave affects the flow locally. The zonal-mean winds do not vary significantly between the different cases, suggesting that it is nonlinearities close to the mountain that cause the increased meridional propagation. In the 333-m case the flux close to the mountain is directed to the southeast but downstream of the mountain the flux is purely zonal. For $H = 1$ and 2 km the zonal flux is much weaker relative to the meridional flux close to the mountain.

This relative weakening of the zonal flux can be used to quantify the transition to nonlinearity. In Fig. 10 we plot the ratio of the meridional Plumb flux across 30°N to the zonal Plumb flux across the 160° latitude circle. There is a clear jump from a ratio of about 2 to a ratio of more than 7 between $H = 700$ m and 1 km, further demonstrating the transition from zonal to meridional propagation. This behavior is qualitatively similar when other latitudes and longitudes are used.

There is a broad region to the southeast of the mountains in which the convergence of the Plumb flux acts to decelerate the flow (Fig. 9). This region is north of the critical line and examination of the potential vorticity field shows that there is no significant wave breaking in the time-mean field, suggesting that potential vorticity mixing due to transient eddies effectively damps the wave before it can break near the critical line. However, the deceleration is in the vicinity of the equatorward turning latitude and the waveguide is progressively eroded as $H$ is increased. Comparing the terms in Eq. (2), this erosion appears to be mostly due to changes in the absolute vorticity induced by the deceleration. In the 2-km experiment the region of deceleration is broad enough to distort the critical line, which is pulled a few degrees northward near 120°.

The erosion of the waveguide is one possible cause of the transition from predominantly zonal to meridional propagation but there are several other possibilities. As shown in Swanson et al. (1997), barotropic jets have a finite carrying capacity for wave activity and so if the wave activity generated by the mountain exceeds this limit it would lead to nonlinear behavior. However, we have not found clear evidence for this mechanism here. Another possible cause of the transition is rotation of the vertical motion along the topography; we return to this in section 5c.

---

**FIG. 6.** As in Fig. 2, but for $H = (top \ left) 1$, (top right) 2, (middle left) 3, (middle right) 4, and (bottom left) 5 km, and the responses have not been normalized. The contour interval is 1 m s$^{-1}$, the zero contour is shown by a thick line, and areas of negative winds are shaded in gray. Where visible, the mountains are shaded in black.
4. Other model configurations

The previous section emphasized the zonal nature of the linear response, in contrast to the many studies that have focused on the meridional propagation of orographically forced stationary waves. The implication is that there is sensitivity to the mean flow, with some flows resulting in better waveguides. To confirm this is the case, we have repeated some of the experiments from the previous section with an additional term added to the reference temperature, equal to \( \epsilon \sin(\phi) \), with \( \epsilon = 10 \text{ K} \) and with the orography in the cooler hemisphere. The dashed curves in Fig. 3 show the zonal wind and index of refraction profiles for this cooler hemisphere. Wavenumber-5 waves are no longer trapped but there is now a region between about 35° and 54° in which wavenumber-4 waves could be trapped.

In the top panel of Fig. 11, we show the Plumb flux at 350 hPa for the new setup with \( H = 400 \text{ m} \), which we have verified is in the linear regime. The response close to the mountain is directed to the southeast and is stronger than in the corresponding \( \epsilon = 0 \text{ K} \) experiment. Using finer scales for the arrows and contours shows that away from the mountain there is a weak zonal wave, and inspection of the eddy streamfunction shows that this wave has wavenumber 4, as expected. Since the trapping region is further poleward in this setup we have run experiments with the mountain centered farther north, but this does not increase zonal propagation.

The bottom panel of Fig. 11 shows results from a case in which \( H = 400 \text{ m} \) and the latitudinal half-width [\( \alpha \) in Eq. (1)] of the mountain is decreased to 7.5°. Stationary forcing potentially excites all waves with the appropriate total stationary number, \( k^2 + \ell^2 \), but the relative forcing of the different \((k, \ell)\) pairs depends on the shape of the forcing. Ridgelike forcing should project more strongly on larger \( k \) (small \( \ell \)) compared to more isotropic topography, favoring zonal propagation since the ratio of zonal to meridional propagation is \( k/\ell \). Therefore, a more zonal response is induced, which propagates almost around the entire latitude circle. However, since the linear response is roughly proportional to the total volume of the mountain, the response close to the mountain is weaker than for the original mountain. The response for larger mountain heights resembles the nonlinear response when \( \epsilon = 0 \text{ K} \) with both the Gaussian mountain and the ridge.

These results demonstrate the limitations of the index of refraction as a diagnostic of trapping for the GCM. While it correctly predicts which waves will be trapped in the two climates, it does not predict that trapping in the \( \epsilon = 10 \text{ K} \) case is weaker; nor does it in itself predict that the ridgelike mountain produces a more strongly trapped response without further consideration of the relative excitation of different ray paths.

5. Understanding the linear response
a. Barotropic model formulation

The barotropic model is formulated as a spectral model that solves the barotropic vorticity equation on a sphere, with Newtonian relaxation to a prescribed zonal-mean vorticity field \( \zeta_\ell \) included in order...
to generate zonal-mean zonal winds which match the primitive equation model

$$\frac{\partial \zeta}{\partial t} = -J(\psi, f + \zeta + h) + r[\zeta_E(\phi) - \zeta(\lambda, \phi, t)] - \nu \nabla^8 \zeta,$$

(5)

where $\psi$ is the streamfunction, $\zeta$ is the vorticity ($\nabla^2 \psi$), $J(\cdot, \cdot)$ is a two-dimensional Jacobian operator, and $\nu$ is a hyperdiffusion constant (we use $\nu = 10^{-4}$ m$^4$ s$^{-1}$). Also, $r = 10 \text{ day}^{-1}$ in all experiments shown here, though we have varied $r$ by a factor of 2 to ensure that our results are not sensitive to this choice, and $\zeta_E$ is calculated from the zonal-mean zonal wind profile we wish the model to relax to $u_E$, which is discussed below.

Mountains are introduced into the model by a perturbation to the vorticity $h = f_0 H_b / H_s$, with $f_0 = 10^{-4}$ s$^{-1}$, $H_b$ representing the orography, and $H_s$ representing the depth of the layer. The orography is given the same Gaussian form as in the primitive equation model. This setup is very similar to the model used for the “direct numerical simulations” of Wang and Kushner (2010) and is also similar to the shallow water experiments of Polvani et al. (1999).

For linear external Rossby waves, the orography must be reduced by a factor $\alpha = \psi_{surf}(p_\alpha) / \psi(p_s)$ in order to match the primitive equation model, where $\psi_{surf}$ is the external mode streamfunction at the surface and $\psi(p_s)$ is the external mode streamfunction at a latitudinally varying equivalent barotropic level (Held et al. 1985). We have subsumed this factor in the definition of the layer depth by setting $H_s = H_s^* / \alpha$, where $H_s^*$ is the actual depth of the barotropic layer. Rather than calculating $\alpha$ explicitly we simply take a representative value of 0.5 and let $H_s^* = 5$ km so that $H_s = 10$ km. While one could try to mimic the linear response of the GCM in a barotropic model if the flow is dominated by external Rossby waves, there is no obvious way of quantitatively capturing the GCM’s nonlinear response in a barotropic model. So we focus here on qualitative rather than quantitative comparisons between the GCM and barotropic solutions.

The barotropic model was run at T85 resolution and data were again recorded once per model day. For each choice of $u_E$ the model was integrated with $H_b$ varying from 10 to 100 m, in increments of 10 m, and then with $H_b$ varying from 200 to 4000 m, in increments of 100 m—making 49 experiments in total. In each experiment the model was integrated for 1000 days, with averages taken over the second half of the simulation.

b. Mimicking the responses of the GCM

To qualitatively reproduce the results from the primitive equation model the zonal winds in the barotropic model were relaxed to the time-mean, zonal-mean zonal wind profile at 350 hPa from the control simulation of the primitive equation model at T42 resolution; that is, $u_E = \bar{u}_{\text{control,350hPa}}$. The computations in Held et al. (1985) suggest a somewhat lower equivalent barotropic level, given the dispersion relation of the external Rossby wave, and we have verified that our results with the barotropic model are robust to choosing a lower level.
We show results using the flow at 350 hPa in order to make a more direct comparison with the figures in sections 3 and 4.

The streamfunction responses to three different mountain heights in this simulation are shown in the left panels of Fig. 12. For $H_b = 200$ m a zonal wave is seen that resembles the circumglobal wave from the primitive equation model except that it extends farther zonally and has no response to the north of the orography. By comparing the responses for different heights, we have found that the transition to nonlinearity takes place between $H_b = 300$ and 500 m. The middle-left panel of Fig. 12 shows that when $H_b = 500$ m the zonal wave is still present, though it is relatively weaker, and there is a new wave train propagating toward the equator. For $H_b = 1$ km the meridional wave appears to have merged with the zonal wave to the southeast of the orography (bottom-left panel), while downstream the zonal wave is still present.

The transition from zonal to meridional propagation can also be seen in the Plumb fluxes (Fig. 13), as the meridional flux is larger and the zonal flux significantly weaker for $H_b = 500$ m than for $H_b = 200$ m. Just as for the primitive equation model then, the linear response consists of a circumglobal wave, while the transition to nonlinearity is characterized by the development of a meridionally propagating wave.

However, when $u_E$ is set to the profile generated by the GCM in the $\varepsilon = 10$-K case, the barotropic response consists of a zonally trapped wavenumber-4 wave, and
forcing with a more ridgelike mountain does not (qualitatively) affect the response (not shown). Hence trapping is favored in the barotropic model compared with the GCM, and the barotropic model cannot capture the sensitivity of the GCM to the zonal-mean wind profile and the shape of the orography.

c. The index of refraction and the barotropic model

It is striking how different the linear regime in our GCM simulation is from the classic result in Hoskins and Karoly (1981) and the corresponding barotropic result in Grose and Hoskins (1979). This difference can be understood qualitatively from the barotropic index of refraction. We have examined the linear responses in the barotropic model in which $u_E$ is a linear combination of the 350-hPa wind profile from the control experiment of the GCM and the 300-hPa zonal wind profile used in Hoskins and Karoly (1981) [the average Northern Hemisphere wintertime flow from Oort and Rasmusson (1971)] $[u]_{300,HK}$. That is, $u_e$ is given by

$$ u_e = c \times [u]_{350,GCM} + (1 - c) \times [u]_{300,HK} \quad (6) $$

where $c$ is a constant. A third-order $B$ spline was used to interpolate $[u]_{300,HK}$ onto the latitude grid used in our barotropic model. As can be seen from Fig. 14, this zonal wind profile does not act as a waveguide near the mountain.

![Fig. 10. The ratio of the meridional component of the Plumb flux integrated across the 30°N latitude circle to the zonal component of the Plumb flux integrated across the 160° longitude circle plotted as a function of $H$. Note the logarithmic scale on the $x$ axis.](image1)

![Fig. 11. As in Fig. 9, but for experiments with $e = 10$ K. (top) The response to a 400-m Gaussian mountain and (bottom) the response to a 400-m ridgelike mountain. The contour interval is now $4 \times 10^{-4}$ m s$^{-1}$ and the green lines show the $n_e = 4$ contours.](image2)
When $c = 0$ the response is what would be expected from Hoskins and Karoly (1981), consisting of two waves traveling along great circles: one propagating poleward away from the orography and the other propagating equatorward (top-left panel of Fig. 15). For $c = 0.4$ the response is similar, though there is less poleward propagation, but when $c$ is increased to 0.8 the poleward-propagating wave is mostly inhibited. This is due to the development of a turning latitude for long waves at high latitudes (see corresponding line in right panel of Fig. 14). Finally, when $c = 1$ the waveguide forms and the zonally trapped wave is clearly present (bottom-right panel of Fig. 15). From this, we conclude that the response of the barotropic model can be mostly explained by the index of refraction and that for understanding the linear regime of the GCM, at least, the barotropic model does not add to what can be learned from the index of refraction.

6. Understanding the nonlinear response

a. Vertical motion along the mountain surface

Since the barotropic model’s response undergoes a similar transition from zonal to meridional propagation, it is natural to ask whether the barotropic model can be
used to understand the nonlinear response of the GCM. We start by considering how the shape of the vortex stretching in both models changes as the orographic forcing is strengthened. In both the primitive equation model and the barotropic model the vertical motion at the surface of the mountain is given by

$$F_{nl} = \nabla \cdot (\nabla h),$$

where $v$ is the flow at the lowest model level in the primitive equation model. At small heights $F_{nl}$ is dominated by its linear approximation $F_l$, while at larger heights the nonlinear terms are more important. In both the GCM and the barotropic model $F_l$ consists of the winds flowing up the west flank of the mountain and down the east flank of the mountain, while the nonlinear terms cause $F_{nl}$ to rotate clockwise. Consistent with the responses transitioning from being more zonal to more meridional, the forcing goes from being aligned east–west to being more northwest–southeast. This progression is shown for the barotropic model in Fig. 16 (the rotation is weaker in the GCM).

As was mentioned in section 3d, this rotation is a possible cause of the transitions in the two models. In particular, Ringler and Cook (1997) suggest that the difference between $F_l$ and $F_{nl}$ is a useful measure of the nonlinearity of the response. To test the importance of this rotation of the vortex stretching, we have run an experiment with the barotropic model in which the orographic forcing is replaced by a fixed vorticity source:

$$\frac{\partial \zeta(\lambda, \phi, t)}{\partial t} = -J(\psi, f + \zeta) + c F_l(\lambda, \phi) + \ldots,$$

where $c$ is a constant factor and $F_l$ is the linear forcing,

$$F_l = -f_0 \frac{\partial h}{\partial x}.$$
This keeps the shape of the forcing fixed while its amplitude can be varied, eliminating the rotation of the forcing by the nonlinear terms. The responses in this experiment are qualitatively unchanged from the responses to the orographic forcing (not shown) and so the rotation of the forcing with increasing amplitude is not responsible for the transition in the barotropic model. The suggestion is that this result likely carries over to the primitive equation model.

b. Plumb fluxes in the barotropic model

Returning to Fig. 13, even for $H_b = 200\text{ m}$ there is a small amount of leakage from the waveguide, which propagates all the way to the equatorward critical line, where the wave breaks. For $H_b = 500\text{ m}$ the zonal flux is relatively weaker and the meridional flux is strengthened, as seen in the GCM, but the leakage from the waveguide has caused a significant northward deformation of the critical line near $120^\circ$. When $H_b = 1\text{ km}$ the critical line is highly deformed and the wave reflects off the critical line. Inspection of the potential vorticity field shows the presence of a Kelvin cat’s eye, confirming that a reflecting critical layer has been created. For $H_b \approx 2700\text{ m}$ the response actually propagates around the entire latitude circle by reflecting off of critical layers to the north and south.

The transition to meridional propagation takes place before the development of the reflecting critical layer, but we believe that the deformation of the critical line is still responsible for the transition in the barotropic model. For instance, when the critical layer is close enough to the waveguide, it could absorb wave activity from the waveguide, promoting meridional propagation. To test whether this is the case, we have repeated the same experiment but with $r$ linearly increasing between $30^\circ$ and $15^\circ\text{N}$ and between $30^\circ$ and $15^\circ\text{S}$ from 10 to $1\text{ day}^{-1}$. The value of $r$ is $1\text{ day}^{-1}$ between $15^\circ\text{N}$ and $15^\circ\text{S}$. This setup does not allow the leakage from the waveguide to propagate as far into the tropics and hence stops the deformation of the critical layer. The results (right panels of Fig. 12) show that this delays the transition to nonlinearity, as even at $H_b = 1\text{ km}$ the response is in the linear regime. At large-enough $H_b$ the response does become nonlinear, though not because of a deformation of the critical line, and the response is very different to the nonlinear responses of either the barotropic model or the GCM.

The barotropic model is thus able to qualitatively reproduce the transition from a meridionally trapped wave to a meridionally propagating wave seen in the GCM, but this transition takes place for different reasons in the two models. Reasons for the inability of the barotropic model to mimic the behavior of the primitive equation model may include the lack of transient eddies and the way the mean flow is restored in the absence of a Hadley cell. These differences mean that the barotropic model as formulated here cannot be used to investigate the breakdown of the linear response in the primitive equation model quantitatively, although in both models there is leakage from the waveguide at even the smallest mountain heights, and this leakage seems to precede the transition to a response with more meridional propagation.
7. Conclusions

In this study we have added Gaussian orography to an idealized, dry GCM forced exactly as in Held and Suarez (1994), searching for a parameter regime in which the model’s response to the orography is a linear function of the height of the orography. For the standard Held–Suarez setup the smallest mountains considered ($H < 1 \text{ km}$) appear to be in a linear regime for which the response consists of a zonal wave with wavenumber 5 that is trapped in the midlatitude jet, resembling the circumglobal wave of Branstator (2002). For experiments with larger $H$ the response is increasingly dominated by a wave that propagates meridionally into the tropics and the response normalized by the height of the mountain also becomes relatively weaker.

Modifying the GCM’s climate by introducing an interhemispheric asymmetry in the radiative equilibrium temperature significantly reduces the trapping when the Gaussian topography is placed in the cooler hemisphere. The stationary wave response is thus able to propagate farther meridionally, even for small mountains, as would be expected from previous studies (e.g., Hoskins and Karoly 1981; Cook and Held 1992). However, if a more ridgelike mountain is used then zonal waves are more strongly excited and zonal propagation plays a more prominent role in the response. The index of refraction shows that in both climates there is the potential for trapping, but it cannot by itself determine whether the waveguides will be excited by a given orographic forcing.

As a first step toward understanding the transition, we have added orographic forcing to a barotropic model on a sphere. Whether the mean flow is relaxed to the flow from the simulation forced with the standard Held–Suarez settings or to the flow from the simulation with the interhemispheric asymmetry, this model’s response to weak, Gaussian forcing consists of a meridionally trapped, zonally propagating wave. This is consistent with what is expected from the barotropic index of refraction and is similar to the results of Manola et al. (2013). For strong-enough forcing, a meridional wave develops, which increasingly dominates the response.

In this sense the barotropic model can successfully reproduce the GCM’s transition from zonal to meridional

![Fig. 15. The Plumb fluxes (arrows), critical lines (red), and $n_s = 5$ contours (green) for $H_b = 100 \text{ m}$ and $c = (\text{top left}) 0$, (top right) $0.4$, (bottom left) $0.8$, and (bottom right) $1$. When $c = 0 u_E$ is the 300-hPa zonal-mean wind profile from Hoskins and Karoly (1981) and when $c = 1 u_E$ is the zonal-mean wind profile at 350 hPa in the control simulation of the GCM under equinoctial conditions. The scale of the arrows is the same in all panels.](http://journals.ametsoc.org/jas/article-pdf/73/9/3701/3853108/jas-d-16-0021_1.pdf)

**Fig. 15.** The Plumb fluxes (arrows), critical lines (red), and $n_s = 5$ contours (green) for $H_b = 100 \text{ m}$ and $c = (\text{top left}) 0$, (top right) $0.4$, (bottom left) $0.8$, and (bottom right) $1$. When $c = 0 u_E$ is the 300-hPa zonal-mean wind profile from Hoskins and Karoly (1981) and when $c = 1 u_E$ is the zonal-mean wind profile at 350 hPa in the control simulation of the GCM under equinoctial conditions. The scale of the arrows is the same in all panels.
proportion; however, the mechanisms responsible for these transitions appear to be quite different in the two models. In the barotropic model leakage from the waveguide is able to propagate to the equatorward critical line, deforming it and pulling it poleward. We believe that the critical line is responsible for the transition when it is close enough to the waveguide, perhaps by absorbing wave activity from the waveguide. For stronger forcings, a nonlinear, reflecting critical layer develops. If the wave activity is unable to reach the critical line (e.g., if the mean flow is restored more strongly at low latitudes), then the transition to nonlinearity is significantly delayed and the nonlinear response is very different.

We have been unable to confirm what causes the regime transition in the GCM, though in this model most of the leakage from the waveguide is mixed by transient eddies before reaching the critical line and so deformation of the critical line cannot be responsible for the increased meridional propagation. Instead, some possible causes of the transition in the GCM are nonlinearities near the source preventing the waveguide from being excited or nonlinear distortion of the waveguide itself. We believe that in both models the rotation of the vortex stretching as the orographic forcing is increased plays a minor role in the transition.

So although much of the theory of stationary waves has been developed in barotropic settings, barotropic dynamics are not sufficient for understanding the transition in the GCM. One-layer systems that more closely mimic the upper-tropospheric dynamics of the GCM will be needed if simple models are to be useful in analyzing the GCM results quantitatively.

The orography is the only zonal asymmetry present in our experiments and so the orographic waves are free to propagate without interacting with other stationary waves, which is another way for linear dynamics to break down in a more realistic setting. We have also not explored how the results are affected by changing the latitude of the orography, apart from briefly in section 4.

By isolating a regime in which a dry GCM responds linearly to orography, this study reminds us that the responses to orography of realistic amplitude, even to isolated orography imposed on an otherwise zonally symmetric climate, may not resemble the linear responses, even qualitatively. Our results also add to the growing body of literature concerning circumglobal waves, which may be of interest to oceanographers studying the interaction of ocean ridges and the Antarctic Circumpolar Current (e.g., Thompson 2010). Finally, these results bring the study of stationary waves back to the earlier emphasis on zonal propagation and the potential sensitivity of stationary waves to the mean flow.

Acknowledgments. We thank Steve Garner, Yi Ming, and Brian Hoskins for helpful comments and discussions and three anonymous reviewers for careful readings of the manuscript. Nicholas Lutsko was supported by NSF Grant DGE 1148900.

REFERENCES


Fig. 16. The full vertical motion forced by the topography $F_{\text{og}}$ for the barotropic experiment when $H_b = \text{(top) 200 m and (bottom) 1 km}$. The plots have been normalized by $H_b/1000$ m and the contour interval is 250 m day$^{-1}$, with positive contours solid and negative contours dashed. The thin dashed lines indicate the latitude of the peak orography and the thick gray lines connect the centers of vertical motion.