The Microphysical Building Blocks of Low-Level Potential Vorticity Anomalies in an Idealized Extratropical Cyclone

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ABSTRACT

Diabatically generated low-level potential vorticity (PV) anomalies in extratropical cyclones enhance near-surface winds and influence the cyclone’s development. Positive and negative PV anomalies in the warm-frontal region of an extratropical cyclone, simulated with an idealized moist baroclinic channel model, are investigated to identify the microphysical processes that produce them. Using a novel method based on backward trajectories from the PV anomalies, the contribution of different microphysical processes to the formation of the anomalies is quantified. It is found that, for each anomaly, typically one specific microphysical process takes the leading role in its diabatic generation. A large but rather weak low- and midlevel positive anomaly is produced by depositional growth of ice and snow. Two smaller but stronger positive anomalies at lower levels are generated mainly by in-cloud condensational heating at the warm front and below-cloud rain evaporation and snow melting 200 km farther north. In addition, near-surface negative anomalies are produced by snow melting and snow sublimation. In summary, this idealized study reveals that (i) a variety of microphysical processes are involved in generating the complex mesoscale PV structures along the warm front; (ii) the model representation of these processes, some of them still insufficiently understood and parameterized, therefore matters for an accurate prediction of these features; (iii) below-cloud processes are also relevant for PV anomalies located in clouds, owing to accumulation of diabatic PV tendencies along ascending air parcels; and (iv) the diabatic history of the air parcels is essential in order to explain the observed PV pattern.

1. Introduction

Extratropical cyclones affect midlatitude weather through the accompanying winds and strong precipitation. The intensification of an extratropical cyclone can be understood through the mutual interaction between an upper-level potential vorticity (PV) anomaly of stratospheric origin, a low-level PV anomaly of diabatic origin, and a surface potential temperature anomaly (e.g., Davis and Emanuel 1991). This low-level positive PV anomaly not only impacts cyclone intensification but also enhances surface winds. A good understanding of the physical mechanisms that form low-level positive PV anomalies is therefore relevant. Several studies have shown the essential role played by condensational heating in establishing these low-level positive PV anomalies. Manabe (1956) was one of the first to recognize the importance of condensational heating in cyclone intensification. The studies of Hoskins and Berrisford (1988) and Whitaker et al. (1988) are early examples where the PV framework (Rossby 1939; Ertel 1942; Eliassen and Kleinschmidt 1957; Hoskins et al. 1985) was applied to case studies in order to understand the intensification of extratropical cyclones. Davis and Emanuel (1991) used the PV inversion technique and were able to quantify that the low-level positive PV produced by condensational heating explained about 40% of the total cyclonic circulation in the storm [see also Huo et al. (1999)]. Kuo et al. (1991) showed in a numerical case study the importance of latent heating for cyclone intensification and stressed the nonlinear relation between dry dynamics and latent heat release. Ahmadi-Givi et al. (2004) looked in detail how the different diabatic anomalies interact and showed in a case study that the upper-level PV anomaly was responsible for initiating cyclogenesis, but the low-level positive PV anomaly was crucial in the intensification phase of the cyclone. Using dropsonde and aircraft observations of a strong North Atlantic cyclone, Neiman et al. (1993) estimated low-level PV values up to 6 PV units (1 PVU = 10^{-6} K kg^{-1} m^{2} s^{-1}) locally along the bent-back front. Stoeblinga (1996) found...
that the low-level diabatic PV anomaly in the simulated cyclone contributed about 70% to the surface winds. In addition, he showed how this low-level anomaly could couple to the upper-level PV anomaly, thereby accelerating the propagation of the surface wave and slowing down the propagation of the upper-level wave. This coupling and the so-called formation of a PV tower has been studied in several other case studies (e.g., Reed et al. 1992; Rossa et al. 2000; Wernli et al. 2002).

In summary, both observational, modeling, and climatological studies have shown the diabatic origin of the low-level PV anomaly and its relevance for both surface wind fields and the evolution of the cyclone. However, most studies that looked into the origin of the low-level positive PV used relatively simple microphysical schemes, without representing ice microphysics and/or below-cloud cooling processes. Also, few studies so far looked in detail at the different microphysical contributions involved in the low-level diabatic PV modification.

There is plenty of evidence that below-cloud processes can be important for the evolution of a cyclone and its accompanying fronts. Clough and Franks (1991), using a two-dimensional model, showed that snow sublimation is essential in keeping mesoscale frontal downdrafts saturated, thereby further enhancing descent through diabatic cooling. Parker and Thorpe (1995) found that snow sublimation can significantly enhance the cross-frontal circulation. Clough et al. (2000) highlighted the relevance of cooling through snow sublimation for the mesoscale structure of the cyclone and suggested that the microphysical parameterizations are important for the detail of the cyclone structure. Forbes and Hogan (2006) found that an underestimation of the cooling due to snow sublimation in fronts can lead to significant forecast errors. Further, Szeto et al. (1988a,b) showed that snow melting needs to be incorporated to understand mesoscale dynamics in both winter and summer storms. The snow melting induced circulations have the potential to influence surface wind speeds by several meters per second. Barth and Parsons (1996) showed that cooling due to both snow sublimation and snow melting deepens the cold air mass behind the cold front and thereby enhances the cold frontal circulation. Huang and Emanuel (1991) investigated how cooling due to rain evaporation strengthens the frontal circulation to the north of the warm front and leads to a faster collapse of the surface gradients. For air parcels constituting the sting jet, Martínez-Alvarado et al. (2010) quantified in a case study the amount of evaporative cooling. Whereas the abovementioned studies looked in detail at the dynamical effects of a single microphysical process, Joos and Wernli (2012) quantified the importance of several microphysical processes on the evolution of the PV along and below a warm conveyor belt in a North Atlantic cyclone. They found that besides condensational heating, also depositional growth of ice and snow contributed significantly to the PV maximum along the warm conveyor belt. In addition, they indicated that sublimating or evaporating hydrometeors below the warm conveyor belt lead to the formation of distinct negative PV anomalies. Igel and van den Heever (2014) looked at the contributions of different microphysical processes to warm frontogenesis. They found that condensational heating tends to reduce the slope of the warm-frontal isentropic surface and, thereby, to first order weakens the warm front. Dearden et al. (2016) showed that ice depositional growth contributed significantly to the deepening of two summertime cyclones and how this process helps the formation of a PV tower.

Summarizing the above, there are several studies showing the relevance of various microphysical processes for the frontal dynamics or the warm conveyor belt in an extratropical cyclone. However (i) most of these studies looked at only one single microphysical process and (ii) they focused only at a part of the cyclone, and not at the cyclone as a whole. Therefore, in this study, we use the historically fruitful method of idealized extratropical cyclone modeling (e.g., Simmons and Hoskins 1979; Davies et al. 1991; Thorncroft et al. 1993) to quantify the importance of different microphysical processes for the diabatic formation of low-level PV anomalies in an extratropical cyclone. Specifically we want to investigate (i) the microphysical processes contributing to the dynamically important low-level positive PV anomalies, (ii) the role played by in-cloud versus below-cloud processes for the low-level positive and negative PV anomalies, and (iii) the different scales at which the processes are important. For answering these questions we designed a detailed and novel Lagrangian PV budget based on integrating diabatic PV rates along backward trajectories.

The structure of the paper is as follows. The next section explains the methods: first the idealized channel model, and then the construction of the cloud diabatic PV budget. Section 3 briefly describes the cyclone development with a focus on the microphysics. The main results are presented in section 4, and the paper concludes with a summary and discussion in section 5.

2. Methods

a. Idealized baroclinic channel model

The cyclone to be studied is simulated with the Consortium for Small-Scale Modeling (COSMO) model (Steppeler et al. 2003) in the idealized baroclinic
channel model setup developed by Schemm et al. (2013). The basic state (shown in Fig. 1) consists of a baroclinic atmosphere in thermal wind balance, which is perturbed through the addition of a PV anomaly at the tropopause level. In contrast to Schemm et al. (2013), we extended the domain in the vertical up to a height of 16 km and slightly increased the baroclinicity by setting the parameters $B_1$ and $B_2$ [see Eq. (4) in Schemm et al. (2013)] to 13 and 11 K, respectively. The model setup used is highly idealized: there are no surface fluxes of heat and moisture, there is no friction, and the radiation, convection, and turbulence schemes are switched off.

The baroclinic channel measures 16 800 km × 8400 km in the zonal and meridional directions, respectively. The grid spacing is 21 km in the horizontal direction and 200 m in the vertical. The Coriolis parameter is kept constant at its value at 45°N; that is, $f = 1.03 \times 10^{-4} \text{s}^{-1}$. The initial state is strongly baroclinic with a jet stream peak velocity of about 60 m s$^{-1}$ (see Fig. 1). Adding an elliptically shaped PV anomaly with an amplitude of 2 PVU at the tropopause level in the initial state triggers the formation of a cyclone, followed by upstream and downstream development. In our study we focus on the downstream cyclone, which develops according to the Shapiro–Keyser cyclone model (Shapiro and Keyser 1990).

b. Constructing a cloud diabatic PV budget

1) DIABATIC HEATING RATES AND PV MODIFICATION

The different cloud and precipitation processes are represented in the model through a one-moment microphysical scheme (Doms et al. 2011). The scheme includes prognostic equations for cloud water, cloud ice, rain, and snow. The transfer of water between the different categories leads to diabatic heating or cooling. As in Joos and Wernli (2012, hereafter JW2012) the total diabatic heating rate (DHR) can be calculated as follows:

$$\text{DHR} = \frac{L_V}{c_{pd}} (S_c + S_r) + \frac{L_S}{c_{pd}} (S_i + S_s),$$

where $L_V$ and $L_S$ represent the latent heat of vaporization and of sublimation, respectively, and the $S$ terms represent the sources and sinks for cloud water $c$, rain $r$, cloud ice $i$, and snow $s$. This total DHR can be split into the different microphysical processes as in JW2012. However, we add some complexity by distinguishing between in-cloud and below-cloud processes. A diabatic heating or cooling process is categorized as in cloud when there is cloud water or cloud ice at the grid point and as below cloud when there is no cloud water nor cloud ice at the grid point. To simplify things a bit, some microphysical processes are grouped. For example, the depositional growth of cloud ice, depositional growth of snow, and the growth of cloud ice into the snow category are considered as one process. See Table 1 for an overview of the different process categories. We neglect the term $S_{\text{rime}}$, because diabatic PV changes due to this process were negligible. Note that the term “cloud diabatic processes” is used for both in-cloud and below-cloud diabatic processes. All below-cloud processes are cooling processes, whereas most in-cloud processes are heating processes. Snow melting is an exception; it is a cooling process that can occur in as well as below clouds.

The modification of PV due to a specific microphysical process $i$, referred to as cloud diabatic PV rate (CDPVR$_i$), can be calculated with the following equation [after Hoskins et al. (1985)]:

$$\text{CDPVR}_i = \frac{1}{\rho} \boldsymbol{\eta} \cdot \nabla (\text{DHR}_i),$$

where DHR$_i$ denotes the diabatic heating rate due to process $i$, as listed in Table 1; $\boldsymbol{\eta}$ denotes the absolute vorticity vector and $\rho$ is the air density.
2) LAGRANGIAN PV MODIFICATION

Whereas this instantaneous PV rate tells how an air parcel’s PV is modified at a certain moment, to explain positive or negative PV anomalies in certain regions of a cyclone one needs to take into account the history of the air parcels and their PV modification. The total effect of the different cloud diabatic processes on a parcel’s PV can be calculated by integrating the cloud diabatic PV rates [Eq. (2)] along backward trajectories until the time \( t_0 \) before any clouds have formed in the cyclone. The method of integrating diabatic PV rates along the flow was mentioned by Stoelinga [1996, see his Eq. (6)]. However, he rewrote this equation in an Eulerian form and solved it on the model grid. We will adopt the Lagrangian perspective, with one of the advantages being the possibility to analyze when, where, and through which process a certain set of air parcels gained or lost their PV. We will refer to the along-flow integrated cloud diabatic PV rates as the cloud diabatic PV (CDPV), again separately for every process \( i \):

\[
\text{CDPV}_i[x(t)] = \int_{t_0}^t \text{CDPVR}_i[x(\tau)] \, d\tau,
\]

where \( x(\tau) \) is the air parcel's trajectory. For calculating the trajectories we make use of the Lagrangian Analysis Tool (LAGRANTO) trajectory model (Sprenger and Wernli 2015). Since we are interested in all cloud diabatic PV anomalies in the vicinity of the cyclone, the backward trajectories are started from every model grid point inside a box around the cyclone center measuring \( 2940 \text{ km} \times 2940 \text{ km} \times 12 \text{ km} \). Using Eq. (3) we obtain different CDPV values for each grid point within this box. The PV of an air parcel at time and position \( x(t) \) can then be expressed as follows:

\[
\text{PV}[x(t)] = \text{PV}[x(t_0)] + \sum_i \text{CDPV}_i[x(t)] + \text{RES};
\]

that is, the PV at time \( t \) is equal to PV at time \( t_0 \) (before clouds formed) plus the sum of all cloud diabatic PV contributions along the air parcel’s trajectory, and RES is a residual. In addition we introduce the “total Lagrangian PV anomaly” of an air parcel at the position \( x(t) \) as follows:

\[
\text{PV}^\ast_{\text{Lag}}[x(t)] = \text{PV}[x(t)] - \text{PV}[x(t_0)].
\]

A comparison of the total Lagrangian PV anomaly with the total CDPV [the second term on the rhs of Eq. (4)] gives us insight in the size of the residual and the accuracy of our Lagrangian budget approach.

3) EULERIAN PV ANOMALY

Our approach of calculating Lagrangian anomalies through integrating cloud diabatic PV rates along backward trajectories is different from the conventional way of defining an Eulerian PV anomaly, which usually corresponds to the deviation from a certain reference state. Depending on the focus of the study this reference is taken to be the PV at a certain reference time or the mean PV over a certain time period (e.g., a climatology). Our aim is to compare Eulerian anomalies with Lagrangian anomalies at \( t = 36 \text{ h} \); therefore, we will take as a reference PV the field at \( t_0 = 0 \text{ h} \). Thus, the Eulerian PV anomaly is calculated as follows:

\[
\text{PV}^\ast_{\text{Eul}}(t) = \text{PV}(t) - \text{PV}(t_0).
\]

3. Cyclone development

A brief overview of the cyclone development and the accompanying clouds and precipitation is presented. The upper-level positive PV anomaly located at the center of the baroclinic zone leads to the development of the primary cyclone, which propagates eastward as can be seen in Fig. 2. After 3 days (Fig. 2a) the primary cyclone has a central sea level pressure below 990 hPa. The first signatures of both upstream and downstream development are visible to the west and east of the primary cyclone, respectively. After 5 days (Fig. 2b) the primary cyclone has developed to a mature system, whereas the downstream cyclone just starts to develop. Figure 3 shows the development of the downstream cyclone over the next 36 h in more detail. From now on we will refer
to day 5 of the simulation as $t = 0$, which corresponds to the cloud-free incipient stage of the downstream cyclone. Within 36 h the sea level pressure drops by more than 35 hPa, and clouds and precipitation develop (green contours in Fig. 3) and modify the PV field. For a detailed description of the development of the cyclone and its fronts, the reader is referred to Schemm et al. (2013). Here, the focus will be on the development of clouds and precipitation (Fig. 4a) and the associated latent heating and cooling (Fig. 4b) in the downstream cyclone. At about $t = 13$ h the first ice clouds and snow form in the cyclone. It is from this moment that cloud diabatic processes modify the PV. Note that it takes about 9 h before the first snow reaches the surface. Apparently, the early snow sublimates before reaching the surface, which is also indicated by the latent cooling shown in Fig. 4b. At about $t = 21$–24 h the first water clouds and rain appear. Between $t = 24$ and 36 h the cyclone develops a bent-back warm front and its structure (Fig. 3d) corresponds to the T-bone stage in the Shapiro–Keyser model, even though in our simulation there are no clouds at the cold front. Note that during this time period the in-cloud heating processes increase strongly, whereas the below-cloud cooling processes increase much less (Fig. 4b). Snowmelt (occurring both in and below the cloud) is particularly weak. At $t = 36$ h the total latent heating is about 4–5 times stronger compared to the total latent cooling. The dominant hydrometeor is snow and surface rainfall has strongly increased compared to 12 h earlier (Fig. 4a). How the different cloud diabatic processes influence the PV will be investigated in the next section.

4. Cloud diabatic PV budgets

a. Total cloud diabatic PV across the warm front

Figure 5a presents the total CDPV in a cross section perpendicular to the warm front extending from the cyclone center into the cold air (see Fig. 3d). Before analyzing in detail this structure and how it is shaped by the different CDPV contributions in section 4b, we first compare the total CDPV (Fig. 5a) to the total Lagrangian anomaly (Fig. 5b) defined in Eq. (5). The total CDPV agrees very well with the total Lagrangian anomaly. On the one hand, this can be expected,
because all other PV nonconserving processes are switched off in this simulation. On the other hand, it is also a proof of the reliability of the rather involved calculation of CDPV. In certain regions there is a small residual as shown in Fig. 5c. This is due to either numerical PV nonconservation (Cooper et al. 1992) or errors, either in the trajectory positions, the calculation of the diabatic PV rates, or the interpolation of these rates to the trajectory positions.

Comparing the CDPV anomaly (Fig. 5a) to the Eulerian anomaly (Fig. 5d), it becomes evident that the two anomalies are almost identical in the lower troposphere (0–3 km); there are small deviations in the midtroposphere (3–6 km), whereas in the upper troposphere (6–8 km) the differences are large. The small differences between the two fields in the midtroposphere can most probably be explained by the effect of cross-isentropic PV transport whereas the large differences in the upper troposphere are due to isentropic advection due to strong isentropic PV gradients close to the tropopause level. We underline that for our main region of interest (0–6 km) the differences between the two fields are small. Also from now on, when we refer to a PV anomaly, we refer to an anomaly of CDPV (Fig. 5a).

b. Detailed structure across the warm front

The aim of this section is to understand the complex structure of the CDPV field in the warm-frontal cross section. It will be shown how the strength but also the structure of the anomalies is determined by the different in-cloud and below-cloud diabatic PV contributions. Note how we distinguish between the terms “anomaly” and “contribution.” Anomalies refer to the total CDPV field, whereas contributions refer to CDPV contributions from the individual in-cloud and below-cloud processes (CDPV).

Every panel in Fig. 6 shows the total CDPV field in the background. As can be seen in Fig. 6a, the warm front is
characterized by a strong low-level positive PV anomaly, which extends from the surface up to about 4 km and its maximum of about 1.5 PVU is approximately 1 km above the surface. The strongest winds at the surface of more than 20 m s\(^{-1}\) are found at and just north of the front (i.e., north of this positive PV anomaly). About 250 km north of the warm front, at the surface, there is another positive PV anomaly that tilts southward with height, it crosses the melting layer (gray line), and has its maximum of about 1.5 PVU just above the melting layer. Between 0- and 1-km heights, this narrow positive anomaly is flanked by two rather small negative anomalies. At about 5–6-km altitude, a larger-size negative PV anomaly can be identified.

Figure 6b shows the structure of clouds and precipitation in the same cross section. At the warm front, there is a deep cloud structure extending from the surface up to 7 km. The melting layer slopes from the surface up to 2 km and snow melting leads to strong rain at the surface near the surface front. About 100 km farther south rainwater contents are much lower and decreasing from the melting layer toward the surface. In this region the air is subsaturated (not shown) and rain evaporates before reaching the surface. Toward the
cold side of the front, the 0°C isotherm is close to the surface and snow reaches the surface. Most intense snowfall occurs about 200 km north of the warm front, whereas farther north snow does not reach the surface due to sublimation.

Figures 6c and 6d show the diabatic PV contributions (CDPV) of selected in-cloud and below-cloud processes, respectively. Note again that these are not instantaneous values but, rather, integrated along backward trajectories. For the in-cloud processes (Fig. 6c), the largest positive contributions are from condensation (green contours) and ice and snow deposition (orange contours) both contributing locally about 0.5 PVU. The former is strongest near \( z = 1 \) km whereas the latter has maxima near 4 km. In between there are small positive contributions from snow melting (yellow contours). Comparing this to the total CDPV shown in shading reveals that in certain regions positive and negative contributions from two different processes cancel. Strong negative contributions are found between 3- and 6-km heights in a region approximately following the cloud tops (see Fig. 6b). These regions are characterized by low PV since they lost up to 1 PVU as a result of ice and snow deposition. Contributions of the below-cloud processes (Fig. 6d) are generally weaker than those from in-cloud processes. Whereas the in-cloud processes have the strongest negative contributions above 3 km, below-cloud processes contribute strongly to negative anomalies near the surface. The negative anomaly just below the melting layer seems to be related to snow melting (Fig. 6d, between 100 and 200 km). Farther north, sublimation of snow or ice leads to a negative anomaly. The below-cloud processes do not only lead to negative anomalies; they also contribute to the positive low-level PV anomalies. Rain evaporation contributes positively to both low-level strong positive PV anomalies, whereas snow melting only contributes positively to the northern positive PV anomaly (Fig. 6d). When comparing the below-cloud PV anomalies with the cloud pattern in Fig. 6b, it becomes evident that below-cloud processes are important also for the PV of air parcels that are at this moment located within the cloud. Also, there are regions outside the cloud, where PV has been modified before as a result of in-cloud processes. This is an important result, which indicates that (i) at a certain time the PV field cannot be understood from instantaneous diabatic
tendencies and (ii) the Lagrangian (i.e., time integrated) perspective is essential for understanding the PV structure in an extratropical cyclone.

With Fig. 6 we only investigated one specific section across the three-dimensional anomalies. To be more systematic we now identify the different anomalies as connected three-dimensional regions, where the absolute value of the total CDPV is larger than a threshold value of 0.2 PVU. Small anomalies covering less than 100 grid points are neglected. For each anomaly, we analyze the individual CDPV contributions.

One large positive anomaly is centered along the (bent back) warm front. This anomaly stretches from the surface up to about 5 km and is approximately 400 km wide and 2000 km long. Within this positive anomaly, there are two regions with particularly high CDPV values. Therefore, an additional threshold of 0.75 PVU was applied to separately analyze these stronger parts of the positive PV anomaly. As can be seen in Fig. 7a, the positive anomaly is split into three distinct anomalies: a relatively large but weak anomaly, POS_MID; a smaller but stronger anomaly located approximately at the warm front, POS_SOUTH; and another strong anomaly located approximately 150 km farther north, POS_NORTH. In addition to these three positive anomalies, we identify three negative anomalies (see Fig. 7a): a large anomaly in the upper troposphere, NEG_UP; a small anomaly, NEG_MID, in between the two strong positive anomalies at low levels; and another low-level negative anomaly, NEG_NORTH, located north of POS_NORTH. Figure 7b indicates that many of these anomalies are zonally extended along the (bent back) warm front. Their extent ranges from about 1500 km for POS_SOUTH and POS_NORTH to about 3000 km for NEG_UP.

### c. Microphysical processes contributing to the individual anomalies

In Fig. 8, the contributions from the different cloud diabatic processes to the CDPV are shown separately for each of the previously defined PV anomalies. Most of the negative anomalies exist mainly because of a single process, which is different for each anomaly. The NEG_UP anomaly (Fig. 8a) exists because of PV destruction caused by depositional growth of ice and snow with a strength of about 0.5 PVU. The other processes do not contribute significantly to the formation of NEG_UP. PV destruction due to snow melting in and below clouds fully explains the existence of NEG_MID (Fig. 8b). For this anomaly, the negative contributions are partially offset by positive contributions from condensation and rain evaporation, effectively reducing the strength of this anomaly. NEG_NORTH (Fig. 8c) is a rather weak anomaly of about 0.3 PVU, which is the result of below-cloud snow sublimation.

For the positive PV anomalies, the picture is more complex. The anomaly POS_MID (Fig. 8d) has a net positive contribution of about 0.25 PVU from depositional growth of ice and snow. The net contributions from the other in- and below-cloud processes are close to zero, owing to both positive and negative contributions within the anomaly. POS_SOUTH (Fig. 8e) has strong contributions of 0.8 PVU from in-cloud condensation but also small contributions of 0.2 PVU from below-cloud rain evaporation. Both processes influence the whole area of the PV anomaly. For the existence of

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**Fig. 7.** Selected positive (red) and negative (blue) CDPV anomalies at $t = 36$ h (a) in the same vertical section (A–B) across the warm front as shown in Fig. 5 and (b) as seen from above. For the dark red anomalies a threshold of 0.75 PVU was chosen; all other anomalies have a threshold of 0.2 PVU. Also shown in (b) are the horizontal extents of the anomalies at $z = 1200$ m for POS_SOUTH and POS_NORTH (dark red) and NEG_MID and NEG_NORTH (dark blue), at $z = 3000$ m for POS_MID (light red), and at $z = 5000$ m for NEG_UP (light blue).
POS_NORTH (Fig. 8f), below-cloud processes are crucial. This anomaly owes its strength to snow melting and rain evaporation, both contributing about 0.4 PVU. Whereas the negative anomalies are mainly formed by a single cloud diabatic process, the positive anomaly as a whole (POS_MID, POS_NORTH, and POS_SOUTH) exists as a result of contributions from many different processes. Depositional growth of ice and snow acts over a large region; however, the associated PV modification is small in magnitude. The PV modification due to condensation, below-cloud rain evaporation, and snow melting are much stronger but act at much smaller scales. Below-cloud processes contribute with about 20% to POS_SOUTH. For POS_NORTH most contributions are from below-cloud processes. Note that this anomaly, existing due to below-cloud processes, is located partially within the cloud (see Fig. 6). This analysis shows that the complex pattern of PV anomalies can only be understood by (i) taking into account the contributions from all cloud diabatic processes to the PV modification and (ii) by applying a Lagrangian perspective where the PV modifications can be traced along the pathway of the air parcels that end up in the anomalies.
In the next section we will explore in more detail how the two strong positive anomalies and the northern negative anomaly have developed over time.

**d. Temporal evolution of diabatic PV rates of selected anomalies**

The Lagrangian perspective enables not only attribution of particular PV anomalies to processes but also quantification of the time scales of the buildup of these anomalies. As can be seen in Figs. 9a and 9b the anomaly POS_SOUTH is produced during the last approximately 6 h because of below-cloud rain evaporation and in-cloud condensation. During this time, the median PV value of the air parcels rises from about 0.3 to about 1.3 PVU, whereas they ascend from about 200 to 700 m with half of the total height gained in the last 4 h. In contrast to POS_SOUTH, the northern strong positive anomaly, POS_NORTH (see Figs. 9c,d), is built up over a longer time period of approximately 18 h. Air parcels forming this anomaly ascend only about 350 m in 36 h. They gain their PV mainly as a result of below-cloud rain evaporation and snow melting. These processes occur when hydrometeors fall through the air parcels as they travel westward just to the north of the bent-back front (not shown). The negative anomaly, NEG_NORTH (Figs. 9e,f), forms as a result of snow sublimation in a very slowly rising airstream during a time of approximately 6 h. Note that whereas several earlier studies (e.g., Huang and Emanuel 1991; Marecal and Lemaitre 1995; Parker and Thorpe 1995) showed the relevance for snow melting and rain evaporation in downdraft regions, we find that these cooling processes can actually increase the PV of rising parcels when the cooling processes are located below these parcels. Our analysis further reveals that the
formation of the different anomalies is characterized by different time scales and trajectory pathways.

5. Summary and discussion

A novel Lagrangian method was developed to provide insight into the microphysical building blocks of low- and midlevel diabatically produced PV anomalies in an idealized extratropical cyclone. The positive anomaly as a whole extends about 1500 km along the (bent back) warm front. The cross-frontal structure is complex. It results from an overlap of different patterns that can—to first order—be interpreted as dipoles as PV is produced below and destroyed above the maximum of diabatic heating (e.g., condensation) and monopoles when the heating/cooling process takes place close to the surface (e.g., rain evaporation). Each pattern corresponds to a certain microphysical process, and the individual patterns might interfere constructively or destructively. The overlap of these patterns results in three different positive PV anomalies with different amplitudes. A rather large but weak positive anomaly is formed because of depositional growth of cloud ice and snow. A smaller but strong positive anomaly at lower levels is formed mainly as a result of condensation heating. About 200 km to the north, there exists another small but strong positive anomaly, which owes its existence mainly to below-cloud rain evaporation and snow melting. Whereas the positive anomalies are produced by a complex combination of different processes, the generation of the negative anomalies is characterized by a single microphysical process for each anomaly. The large upper-tropospheric negative anomaly is formed as a result of PV destruction caused by the depositional growth of cloud ice and snow. For the low-level negative anomaly north of the surface warm front, snow melting is the dominant process, and for the low-level negative anomaly farther north, PV is decreased mainly because of the cooling induced by snow sublimation. Furthermore, the microphysical processes contributing to both the positive and negative anomalies act on different scales. Depositional growth of cloud ice and snow is relevant over a large region (about 300 km in the cross-frontal direction)—however, rather small in magnitude (typically 0.2–0.4 PVU). On the other hand, cloud condensation and below-cloud processes are rather strong in magnitude (typically 0.3–1.0 PVU) but act at smaller scales (about 50 km in the cross-frontal direction). It was found that whereas below-cloud processes contribute only slightly to the strong southern positive PV anomaly, the northern positive PV anomaly consists almost fully of below-cloud contributions from snow melting and rain evaporation, highlighting the importance of below-cloud processes for the positive PV anomaly as a whole. The existence of PV anomalies located in the cloud, though created by below-cloud processes, can only be understood by considering the history of the air parcels in these anomalies. Air parcels experience PV modification due to below-cloud processes as long as they are located below the cloud. Later, when ascending into the cloud, they carry the signal of below-cloud PV modification into the region where in-cloud processes can further modify their PV. It was shown that the strong low-level southern positive PV anomaly builds up in a relatively short time of 4 h whereas the northern anomaly is created within about 18 h.

In summary, the key findings of this study are as follows:

- The below-cloud processes of rain evaporation and snow melting contribute significantly to the formation of the low-level positive PV anomaly.
- A complex combination of microphysical processes dictates the shape and strength of the low-level positive PV anomaly.
- The snow melting and sublimation processes each produce a low-level negative anomaly.
- The cross-frontal scale at which different processes act ranges from ~50 km for the below-cloud processes to ~300 km for ice and snow deposition.
- A Lagrangian perspective is needed for understanding the formation of the complex low-level PV structures in extratropical cyclones.

In the final paragraphs, we put our results into the context of the existing literature. Parker and Thorpe (1995) found that the cooling induced by snow sublimation was mainly of local influence. This is consistent with our finding that only a small negative anomaly is related to snow sublimation. Clough and Franks (1991) argued that snow sublimation is more relevant in influencing the mesoscale dynamics than rain evaporation. Our findings slightly contradict this statement, since we find a rather weak positive anomaly due to snow sublimation and rather strong contributions from rain evaporation, mainly to the northern positive PV anomaly.

The relevance of snow melting, mentioned already by Sztó et al. (1988a), is confirmed in our study as snow melting is highly relevant for the formation of both a negative and a positive low-level PV anomaly. On the other hand, Dearden et al. (2016) found no significant influence of snow melting and sublimation on the mesoscale dynamics at the fronts of two summertime cyclones. Huang and Emanuel (1991) showed how cooling due to rain evaporation strengthens the frontal circulation to the north of the warm front and leads to a faster collapse of the surface gradients. This finding is consistent with the significant contributions of rain evaporation to the northern
positive PV anomaly. JW2012 calculated instantaneous diabatic PV rates in a section across the warm conveyor belt. For rain evaporation, they found a maximum of almost 1 PVU h\(^{-1}\) close to the surface. Our findings show that the signal as seen by JW2012 does not stay below the clouds but is transported upward and thereby contributes to a positive PV anomaly located within the cloud.

Igel and van den Heever (2014) looked at the diabatic contributions of different microphysical processes to warm frontogenesis, through analysis of the frontal slope. Since the perspective used in their study is different from the PV perspective used in our study, we should view our results as complementary to their work. For example, whereas Igel and van den Heever (2014) found that depositional growth of cloud ice and snow does not contribute significantly to the frontogenesis, we find that this process is responsible for the large but weak positive PV anomaly. Further, Igel and van den Heever (2014) found that diabatic processes are the main reason for the high static stability found along the frontal surface. This is consistent with our finding of the strong relatively shallow positive PV anomalies, since it can be shown that PV anomalies of limited vertical extent are related to strong static stability rather than cyclonic flow anomalies (Hoskins et al. 1985).

Our finding about the different scales of influence of depositional growth of ice and snow (important for the large-size positive PV anomaly) compared to snow sublimation (important for the small-size negative PV anomaly) is consistent with Forbes and Clark (2003). They stated that the former affects the large-scale synoptic evolution whereas the impact of the latter is restricted to the frontal scales. Coronel et al. (2015) showed that an idealized extratropical cyclone simulated with full microphysics (including ice deposition and below-cloud cooling processes) crosses the jet earlier when compared to a simulation with only condensational heating.

Although our method presents a detailed view on the different microphysical contributions, we cannot make any statement about the relevance of cloud diabatic processes compared to other nonconserving processes like radiation (Chagnon et al. 2013), friction, turbulence, or convection. Our method could be extended to include these effects; however, it is believed that the complexity of the microphysical contributions themselves is best presented in an isolated manner (i.e., without other nonconservative processes impacting their structure). In addition, other studies have assessed the impact of cloud diabatic processes as a whole in comparison to other PV nonconservative processes (e.g., Chagnon et al. 2013; Chagnon and Gray 2015). The diagnostic decomposition of the low-level PV anomalies into their microphysical building blocks is novel. A more common approach to assess the relevance of a certain process is to perform different sensitivity studies with and without latent heating or cooling due to the considered process. In contrast, our combined Lagrangian and PV-based approach gave us a way to assess the relevance of the different processes in a single simulation. The results might guide flight planning during field experiments in order to obtain observations in regions that are sensitive to a certain microphysical process. Also testing the dynamical impact of different microphysical schemes can be a useful application of our novel method. The question to what extent the structure and composition of the low-level PV anomalies in this idealized cyclone are representative for real cases is the subject of ongoing research.

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