Dynamics of 2-Day Equatorial Waves

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ABSTRACT

The dynamics of the 2-day wave, a type of convectively coupled disturbance that frequents the equatorial western Pacific, is examined using observations and a linear primitive equation model. A statistical composite of the wave’s kinematic and thermodynamic structure is presented. It is shown that 1) the wave’s wind and temperature perturbations can be modeled as linear responses to convective heating and cooling, and 2) the bulk of the wave’s dynamical and convective structure can be represented with two vertical modes. The observations and model results suggest that the 2-day wave is an $n=1$ westward-propagating inertia–gravity wave with a shallow equivalent depth (14 m) that results from the partial cancelation of adiabatic temperature changes due to vertical motion by convective heating and cooling.

1. Introduction

During the northern winter large-scale westward-propagating convective disturbances with periods of about 2 days frequent the equatorial western Pacific (Takayabu 1994b). These waves usually occur in series (e.g., Takayabu et al. 1996) and are most prevalent during the active phase of the Madden–Julian oscillation (MJO; Madden and Julian 1994; Nakazawa 1988; Hendon and Liebmann 1994; Schrage et al. 2001; Clayson et al. 2002). A number of studies have provided satellite images of individual and composite 2-day waves that show zonal wavelengths of 2000–4000 km and westward propagation of 10–30 m s$^{-1}$ (Hendon and Liebmann 1994; Takayabu 1994b; Chen et al. 1996; Takayabu et al. 1996; Haertel and Johnson 1998; Wheeler et al. 2000). However, not all 2-day high-cloudiness oscillations in the equatorial western Pacific are associated with westward-propagating waves (Schrage et al. 2001; Clayson et al. 2002). For this study, we use the term “2-day wave” to refer exclusively to westward propagating disturbances.

Two-day waves are part of a broad spectrum of convectively coupled equatorial waves (Matsumo 1966) whose dispersion characteristics resemble those of classical shallow-water equatorial waves with equivalent depths of 12 to 50 m (Takayabu 1994a; Wheeler and Kiladis 1999). The spectral signal of 2-day waves lies along the dispersion curve for the $n=1$ westward-propagating inertia–gravity (WIG) wave (Takayabu 1994b; Takayabu et al. 1996; Wheeler and Kiladis 1999). While a number of attempts have been made to explain the prevalence of 12- to 50- m implied equivalent depths for convectively coupled equatorial waves in general (Emanuel et al. 1994; Mapes 2000; Majda and Schefter 2001; Majda et al. 2004; Lindzen 2003), the proposed theories differ widely and the question remains open. Similarly, there is no consensus on the generation and dynamics of 2-day waves. In addition to being referred to as $n=1$ WIG waves they have been described as long-lived squall lines (Hendon and Liebmann 1994; Takayabu et al. 1996) and the result of the interaction of diurnal oscillations with gravity wave dynamics (Liebmann et al. 1997; Chen and Houze 1997).

The climatological point of maximum occurrence of 2-day waves was well sampled by the Tropical Ocean Global Atmosphere Coupled Ocean–Atmosphere Response Experiment (TOGA COARE; Webster and Lukas 1992; Takayabu 1994b; Wheeler et al. 2000). Analysis of COARE data produced many of the results discussed above, and also revealed the following characteristics of 2-day waves. 1) There are four stages in their life cycle: the shallow-convection, initial-tower, mature, and decay stages (Fig. 1, taken from Takayabu et al. 1996); 2) the 2-day waves comprise envelopes of cloud clusters rather than individual clusters (Chen et al. 1996).
Despite the wealth of insight about 2-day waves already gathered from COARE, there is still more to be gained from that dataset. Previous COARE studies of 2-day waves only considered a limited number of cases [e.g., eight in Takayabu et al. (1996) and six in Haertel and Johnson (1998)], and few attempts have been made to analyze the detailed dynamics of 2-day waves. In the present study our objectives are 1) to construct a statistical composite of all 2-day waves that occurred during COARE and 2) to test the null hypothesis that the wind and temperature perturbations that accompany 2-day waves can be modeled as linear responses to convective heating and cooling. Whether or not this hypothesis can be rejected, we hope to gain some insight into the nature of the coupling between the convection and the dynamics of the wave.

This paper is organized as follows: section 2 describes the data and methods used in this study; section 3 discusses observations of a composite 2-day wave; section 4 presents a simulation of the linear response to the composite 2-day wave’s convective heating and cooling; section 5 tests the ability of a two vertical mode dynamical system to represent 2-day waves; section 6 is a discussion; and section 7 is a summary.

2. Data and methods
   a. Sounding data

The sounding data used for this study were collected during the intensive operating period (IOP) of TOGA COARE, which was from 1 November 1992 through 28 February 1993. For a detailed description of this experiment and a map of the sounding stations see Webster and Lukas (1992). Radiosonde, wind profiler, and surface data were objectively analyzed onto a grid (Ciesielski et al. 1997, 2003) with a vertical resolution of 25 hPa, a temporal resolution of 6 h, and a horizontal resolution of 1°. Many of the plots we present are average vertical profiles for grid points within the Intensive Flux Array (IFA), which was an array of sounding stations centered near 2°S, 155°E.

b. Satellite data

Gridded measurements of infrared brightness temperature ($T_B$) by the Geostationary Meteorological Satellite were obtained at a resolution of $1/10^\circ \times 1/10^\circ$ over 15°S–15°N, 130°E–180° and averaged in $1^\circ \times 1^\circ$ bins.

c. Compositing technique

To obtain the composite 2-day wave we simply temporally filter (Duchon 1979) the average $T_B$ over the IFA, retaining fluctuations between 1.5 and 3 days (Haertel and Johnson 1998, Fig. 5), and then project dependent variables (such as temperature, wind, and moisture) onto the filtered $T_B$ at a series of lags (e.g., Wheeler et al. 2000; Straub and Kiladis 2002). In this linear approach the amplitude of the independent variable is assigned an arbitrary value. In the examples used here the wave is scaled to a $-20^\circ$C perturbation in IFA $T_B$ at zero lag, a typical value during the peak phase of a strong 2-day wave (Takayabu et al. 1996, Fig. 4; Haertel and Johnson 1998, Fig. 6).
For the two mode simulation (section 5) we project the 2-day wave’s heating onto the first two linear vertical modes (Fulton and Schubert 1985) of an atmosphere with a rigid upper boundary at 150 hPa. The vertical structure functions for the modes are calculated for the average IFA sounding using the method described by Haertel and Johnson (1998).

3. The composite 2-day wave

This section discusses the cloudiness, rainfall, horizontal structure and propagation, and vertical structure of the composite 2-day wave. For each plot presented here, hour 0 corresponds to the time of the minimum $T_b$ over the IFA.

a. Local cloudiness and rain rate

The composite IFA $T_b$ has local minima at $-48$, 0, and 48 h (Fig. 2a). While one would expect such a pattern given the compositing technique, it might also reflect the tendency of 2-day waves to occur in series, which has been well documented in previous studies (e.g., Takayabu 1994b; Takayabu et al. 1996; Clayson et al. 2002). The $T_b$ signal appears to be a good proxy for moist convection; the budget-derived perturbation rain rate is roughly out of phase with $T_b$, and has a similar relative amplitude at all lags (Fig. 2b). The minimum $T_b$ lags the maximum rain rate slightly, which likely reflects the time it takes a cirrus anvil to develop from convective towers. The amplitude of the rain-rate perturbation is comparable to the mean rain rate for the IFA for the IOP (Johnson and Ciesielski 2000).

b. Horizontal structure and propagation

The composite wave has a scale of about 1000–1500 km (Figs. 3a–e). It originates more than 1000 km to the east of the IFA when a large-scale disturbance divides into northward- and westward-propagating components. The latter intensifies as it passes over the IFA and dissipates about a day later. The wave propagates at about 16 m s$^{-1}$. Upper-level wind anomalies diverge from the negative $T_b$ anomalies and converge into positive $T_b$ anomalies, supporting the interpretation of $T_b$ as a proxy for deep convection.

Throughout the wave’s life cycle the high cloudiness exhibits a weak signal of zonal periodicity with a wavelength of about 25$^\circ$ longitude (e.g., Figs. 3a,c). The wavelike structure propagates westward within a stationary amplitude envelope centered on the IFA (the base point for the composite). The horizontal structure of the composite is similar to that of the composite 2-day wave constructed by Takayabu (1994b), which is an average of 263 cases that occurred between 1980 and 1989 in the boreal winter in the same region. However, Takayabu’s composite lacks the northward-propagating disturbance in the vicinity of 170$^\circ$E, so it is possible that this portion of the composite represents a peculiarity of the 1992–93 season.

c. Vertical structure

The composite 2-day wave is accompanied by changes in temperature, moisture, heating, moistening, winds, and divergence through most of the troposphere over the IFA. These changes are detailed below.

1) Temperature and moisture

The highest amplitude temperature perturbations occur near the surface and near the tropopause (Fig. 4a). The near-surface air cools nearly 1 K during the heaviest rainfall, reaching a minimum temperature at 6 h. In the remainder of the troposphere lower levels are cool and upper levels are warm when the wave passes over; that is, $T_b$ is in phase (out of phase) with lower-tropospheric (upper tropospheric) temperature. In most of the free troposphere temperature perturbations rise with time except above 300 hPa where perturbations descend with time. Considering that 2-day waves propagate westward, this structure implies eastward (westward) tilts below (above) 300 hPa. Similar tilts and phase relationships between $T_b$ and tropospheric temperature have also been observed in other types of convectively coupled waves (e.g., Wheeler et al. 2000; Straub and Kiladis 2003b).

When the near-surface air is relatively cool around
6 h, it is also relatively dry (Fig. 4b). This is consistent with the idea that convective downdrafts transport low-entropy air from mid levels into the boundary layer. In contrast, between 650 and 900 hPa the opposite relationship exists between temperature and moisture perturbations; positive moisture anomalies accompany negative temperature perturbations (Figs. 4a–b).

The near-surface and lower-tropospheric temperature and moisture changes are believed to modulate the wave’s convection. Chen and Houze (1997) noted that the near-surface cooling and drying that accompanies 2-day waves reduces the moist static energy available for convection. Haertel and Johnson (1998) showed that the lower-tropospheric temperature changes above the mixed layer alter the level of free convection, with negative (positive) perturbations at this level making convection more (less) favorable. Takayabu et al. (1996) also noted that low-level warming stabilizes the lower troposphere following the wave’s passage.

2) Heating and moistening

The technique of Yanai et al. (1973) was used to calculate the apparent heating and moistening associated with the composite wave. The heating perturbation exceeds 6 K day$^{-1}$ around 400 hPa at 0 h (Fig. 4c). The heating is confined to the lower troposphere around $-18$ h, spans most of the troposphere from $-12$ to 0 h, and is confined to the upper troposphere around 6 h. This heating pattern is consistent with the convective cycle identified by Takayabu et al. (1996), which includes shallow convection followed by deep convection and then stratiform precipitation (Fig. 1). Around $-24$ and $+24$ h there is cooling with a similar vertical struc-
Fig. 4. Composite time–pressure series over the IFA. The contour interval is given at the top of each plot. Regions with an absolute value greater than the contour interval are shaded, dark (light) for positive (negative) values. Zero contours are omitted. (a) Perturbation temperature, (b) perturbation specific humidity, (c) perturbation heating, (d) perturbation drying, (e) perturbation zonal wind, (f) perturbation divergence, and (g) perturbation pressure velocity.
tured and a maximum amplitude of just over 5 K day$^{-1}$. It is emphasized that this is perturbation cooling; the basic state includes an average heating with a maximum amplitude of nearly 4 K day$^{-1}$, so the total cooling peaks at just over 1 K day$^{-1}$ at this time, and it likely represents the effects of radiation in the absence of deep convection.

Like the heating anomalies, the moistening anomalies rise with time (Fig. 4d). However, in contrast to heating the near-surface and free-tropospheric moistening are in phase. Around $-6$ h, when the precipitation is most intense, the entire troposphere has a convective drying tendency of up to 2 g kg$^{-1}$ day$^{-1}$.

The heating and moistening are able to directly account for the near-surface temperature and moisture changes (Figs. 4a–b). However, the temperature perturbations in the free troposphere are opposite to what one would expect from examining only the heating profile; the temperature generally falls (rises) when there is heating (cooling). The reason for this is that large-scale upward (downward) vertical motion accompanies heating (cooling), and the effects of vertical advection more than offset the heating and cooling (refer to section 5 for a quantitative analysis of this point).

### 3) WINDS AND DIVERGENCE

Below 200 hPa composite zonal wind anomalies rise with time and above 200 hPa they descend with time (Fig. 4c). The maximum amplitude exceeds 3 m s$^{-1}$ around 200 hPa, but amplitudes remain below 1 m s$^{-1}$ in the middle and lower troposphere. The meridional wind perturbations are in general much weaker, and appear to be nothing more than filtered noise (not shown). Takayabu et al. (1996) and Haertel and Johnson (1998) also examined the vertical structure of wind perturbations for 2-day wave composites that comprised eight and six cases, respectively. Both studies noted a similar structure for the zonal-wind time series, although their fields were more noisy because the composites included fewer cases.

Prior to the arrival of the wave (around $-12$ h) perturbation low-level convergence peaks near the surface (Fig. 4f). As the wave passes over the IFA, the level of maximum convergence rises, reaching 500 hPa by 6 h. Just after the heaviest rainfall at 0 h there is intense upper-level divergence exceeding $10^{-3}$ s$^{-1}$ at 200 hPa. The structure of the $\omega$ (pressure velocity) field is very similar to that of heating (Fig. 4c); perturbations rise with time and maximum amplitudes occur in the mid to upper troposphere (Fig. 4g). The adiabatic cooling (warming) associated with the upward (downward) motion is very nearly in balance with the heating (cooling). However, the vertical motion more than compensates for the heating producing the weak temperature perturbations depicted in Fig. 4a. The divergence and $\omega$ fields are consistent with the morphology of convection observed by Takayabu et al. (1996) (Fig. 1).

### 4) Simulation

In this section we explore the null hypothesis that the wind and temperature perturbations associated with 2-day waves can be modeled as linear responses to convective heating and cooling. A linear numerical model is used to simulate the atmospheric response to an idealization of the composite wave’s perturbation heating, and the simulation is compared to the composite analysis.

#### a. The numerical model

The numerical model solves the inviscid primitive equations on a $\beta$ plane linearized about a basic state of rest:

\[
\frac{\partial u}{\partial t} - \beta vy + \frac{\partial \phi}{\partial x} = 0 \tag{1}
\]

\[
\frac{\partial v}{\partial t} + \beta uy + \frac{\partial \phi}{\partial y} = 0 \tag{2}
\]

\[
\frac{\partial \phi}{\partial \rho} = -\frac{RT}{\rho} \tag{3}
\]

\[
\frac{\partial T}{\partial t} - S\omega = Q \tag{4}
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial \rho} = 0, \tag{5}
\]

where $x$ and $y$ represent horizontal displacement, $p$ is pressure, $t$ is time, $u$ is perturbation zonal velocity, $v$ is perturbation meridional velocity, $\beta = \partial f/\partial y$ (where $f$ is the Coriolis parameter), $T$ is perturbation temperature, $R$ is the gas constant for dry air, $\phi$ is perturbation geopotential, $\omega$ is perturbation vertical velocity, $S = -\bar{T} (\partial \ln \rho/\partial \rho)$ (where $\theta$ is potential temperature and the overbars denote basic-state variables that are functions of $p$ only), and $Q$ is perturbation heating. This system of equations was selected because its solutions are algebraically equivalent to those of classical equatorial wave theory (Matsumo 1966; Lindzen 1967; Fulton and Schubert 1985), which previously has been used to interpret the observed horizontal structures and dispersion characteristics of convectively coupled tropical waves (Takayabu 1994a,b; Wheeler and Kiladis 1999; Wheeler et al. 2000).

Equations (1)–(5) are approximated using fourth-order spatial differencing with $\Delta x = \Delta y = 100$ km, second-order vertical differencing with $\Delta \rho = 6.25$ hPa, and leapfrog time differencing with $\Delta t = 180$ s. Vertical velocity is set to zero at both the lower (1000 hPa) and upper boundaries (6.25 hPa). The high location of the upper boundary allows vertical propagation of gravity waves with minimal reflection into the domain of interest. Horizontal boundaries are also placed sufficiently far from domain of interest (at $x = \pm 2000$ km and $y = \pm 5000$ km) that they have a negligible impact on
the solution. The external mode is filtered from the solution to allow the use of a long time step.

Analysis of average absolute vorticity fields for the COARE IOP suggest that through most of the troposphere the dynamical equator lies between 1° and 2°S (similar to what Takayabu 1994b observed), which is very close to the latitude of the center of the IFA (2°S).

Hence, simulated vertical profiles over the model origin (x = 0, y = 0) are compared to observed profiles over the IFA.

c. Comparing the simulation to observations

The time–pressure series of heating over the origin of the model domain (Fig. 6a) closely resembles that observed above the IFA (Fig. 4c), indicating that (6)–(9) adequately represent the observed heating. The simulated temperature profile (Fig. 6b) shows most of the structure that appears in the composite temperature analysis (Fig. 4a): lower-tropospheric temperature perturbations are out of phase with upper-tropospheric temperature perturbations, amplitudes are a few tenths of a degree Celsius, and oscillations are weak or absent around 500 and 900 hPa. There are local minima/maxima near the surface and just below the melting level (~575 hPa). Tilted structures indicate vertical wave propagation near the tropopause. The degree of similarity between the observed and simulated temperature perturbations is remarkable and is strong evidence in support of our null hypothesis.
The simulated time–pressure series of zonal wind (Fig. 6c) also exhibits most of the structure that appears in the composite zonal-wind analysis (Fig. 4e); each field contains wavelike perturbations that rise with time through most of the troposphere and descend with time near the tropopause. For both cases the oscillations have local amplitude maxima near the surface, at midlevels, and near 200 hPa. Simulated and observed amplitudes are similar at mid and upper levels, but near the surface the simulated wind field is 2–3 times more intense than the observed field. This difference is likely a consequence of the fact that the simulation is inviscid whereas boundary layer turbulence damps near-surface winds in nature. At most pressure levels the phasing of the wind perturbations in the simulation is similar to that in the observations except for just above and just below 200 hPa, and near 700 hPa. Since $Q$ is symmetric about $y = 0$, along $y = 0$ (and over the model origin) there is no meridional wind perturbation in the simulation.

The horizontal structure of the simulated wind field is also similar to that of the observed field. At 0 h the simulated winds diverge from the center of the wave and turn toward the right (left) to the northeast and southwest (northwest and southeast) of the wave (Fig. 7a). The divergence of winds and turning is similar in the observed wind field (Fig. 3c). At 12 h the simulated winds diverge from the decaying wave (centered near $x = -700$ km, $y = 0$) and arc around to converge into a suppressed region to the east of the wave (Fig. 7b). Once again the simulated wind structure is similar to that observed (Fig. 3d). For both times the amplitudes of the simulated winds are comparable to those of the observed winds (Figs. 3c,d; 7a–b).

The simulation reproduces most of the qualitative structure of the wind and temperature perturbations.
Figure 8. A vertical mode decomposition of the observed perturbation heating for (a) the 49 m s\(^{-1}\) mode and (b) the 23 m s\(^{-1}\) mode. Each plot is shaded and contoured following the convection described in Fig. 4.

5. The two vertical mode model

The results of the previous section demonstrate that the primitive equations linearized about a basic state of rest capture the basic dynamics of 2-day waves (if the heating is given). However, the theories that attempt to explain the observed dispersion of convectively coupled equatorial waves are based on even more simplified dynamical systems that rely on what might be perceived as unrealistic assumptions. In particular, the models of Mapes (2000) and Majda and Schefter (2001) include only two vertical modes and a rigid upper boundary. In this section we examine to what extent these approximations distort the dynamics of 2-day waves. We repeat the simulation presented in the previous section using a dynamical system with just two vertical modes.

a. Vertical mode decomposition

A solution to the primitive equations linearized about a basic state of rest with a rigid upper boundary may be decomposed into a superposition of vertical modes, each of which is algebraically equivalent to a solution of the linearized shallow-water equations (e.g., Fulton and Schubert 1985). For a given vertical mode, profiles of geopotential and winds are proportional to a vertical structure function that is an eigenfunction of a differential operator. Typical vertical structure functions resemble deformed cosine functions, and their exact shapes depend on the temperature profile of the basic state. Each vertical mode is associated with an equivalent depth, or gravity wave speed, that identifies the mode with the shallow-water system (having that depth or gravity wave speed) to which the mode is algebraically equivalent.

In an unbounded atmosphere there is a continuum of vertical modes (and equivalent depths). The practice of approximating the dynamics of such an atmosphere using vertical modes for a bounded atmosphere is analogous to approximating a continuous Fourier transform with a discrete transform. Each mode of the resulting discrete system represents a spectral band of vertical modes of the continuous system, and solutions of the discrete system may be regarded as approximations of solutions of the continuous system (e.g., Mapes 1998). The accuracy of the approximation depends on the positioning of the upper boundary, with a higher upper boundary leading to a greater accuracy. However, in some cases, reasonable solutions may be obtained by placing the upper boundary at the lowest possible position, which is immediately above the prescribed forcing. For example, Pandya et al. (1993) show that for such a positioning of the upper boundary above a forcing characteristic of a mesoscale convective system, the buoyancy and wind response of the bounded atmosphere are quite similar to those of the unbounded atmosphere. One advantage of using such a maximal discretization (or minimal decomposition) is that it simplifies the solution and its interpretation (e.g., Nicholls et al. 1991; Haertel and Johnson 2000).

The simulated 2-day wave’s perturbation heating is significantly nonzero only below 150 hPa (Fig. 4c). Therefore, we can obtain a minimal decomposition of the heating by projecting it onto the vertical modes for an atmosphere having the COARE mean temperature profile and an upper boundary at 150 hPa (Fulton and Schubert 1985). The heating projects strongly onto just two modes. One, which is often referred to as the first baroclinic mode, has a gravity wave speed of 49 m s\(^{-1}\) and is associated with deep convective heating of the same sign throughout the troposphere peaking at mid levels (Fig. 8a). The second baroclinic mode (Fig. 8b) has a gravity wave speed of 23 m s\(^{-1}\) and is interpreted as stratiform heating of opposite sign in the lower and
upper troposphere (Mapes and Houze 1995). Note that the heating associated with stratiform precipitation is much weaker than the first mode and in the upper troposphere lags deep convective heating by 6 h. When the two-mode heating is subtracted from the observed perturbation heating (Fig. 4c), the residual is very small in amplitude except for a shallow region slightly exceeding 1°C day⁻¹ near the surface (not shown).

b. The response of a bounded atmosphere to the two-mode heating

We now present a simulation in which the heating function described in section 4b is projected onto the two vertical modes described above and applied to an atmosphere having a rigid upper boundary at 150 hPa. The two-mode simulation reproduces most of the structure of tropospheric wind and temperature perturbations present in the composite analysis. This result supports the use of two vertical mode models for analyzing the dynamics of convectively coupled equatorial waves.

1) Comparison to observations

The vertical profile of heating in the center of the model domain (Fig. 9a) has a similar amplitude, vertical structure, and time dependence to the observed profile (Fig. 4c) except that it lacks the near-surface warming and cooling. The simulated temperature profile (Fig. 9b) also shows most of the structure that appears in the composite temperature analysis (Fig. 4a): lower-tropospheric temperature perturbations are out of phase with upper-tropospheric temperature perturbations, amplitudes are a few tenths of a degree kelvin, and oscillations are weak or absent around 200, 500, and 900 hPa. There are two main differences, however. First, near-surface temperature perturbations are not present in the two-mode simulation (which is consistent with the lack of near-surface heating and cooling). Second, the two-mode system is unable to represent the vertical wave propagation near the tropopause. Nevertheless, considering the simplicity of the two-mode system, the degree of similarity between the observed and simulated temperature perturbations is remarkable.

The simulated vertical profile of zonal wind (Fig. 9c) also exhibits most of the structure that appears in the composite zonal-wind analysis (Fig. 4e). Each field contains wavelike perturbations that rise with time through the troposphere. For both cases, the oscillations have local amplitude maxima near the surface, between 400 and 600 hPa, and near 200 hPa. Of course, the two-mode system is unable to represent the descending perturbations above 200 hPa associated with vertical wave propagation.

2) Contributions from modes

A fairly complete picture of the dynamical structure of 2-day waves is provided by Fig. 10a, which is an east–west cross section of the simulated wave along \( y = 0 \) at \(-6\) h (the time of peak rainfall). Near the center of the wave strong upward motion (\( \sim 100 \) hPa day⁻¹) accompanies strong perturbation heating in the mid troposphere. The heating tilts toward the east, and is flanked to each side by similarly tilted perturbation subsidence and cooling. The lower troposphere is cool in the region of heating, and the strongest upward motion occurs just to the west of the cool anomaly. Near-surface easterlies slightly trail the cool anomaly, with upper-

\[ Q(1 \text{C/day}) \]

\[ T(0.1 \text{C}) \]

\[ u(0.25 \text{m/s}) \]
Fig. 10. East–west vertical cross section of the simulated wave at −6 h for (a) both modes, (b) the 49 m s⁻¹ mode, and (c) the 23 m s⁻¹ mode. Perturbation temperature is contoured with a 0.1-K contour interval, solid (dashed) contours are positive (negative), and the zero contour is omitted. Regions with perturbation heating amplitudes greater than 1 K day⁻¹ are shaded, dark (light) for positive (negative) values. Vectors illustrate zonal and vertical winds.

tropospheric winds diverging above the heating. The structure of the wind and temperature perturbations in Fig. 10a is quite similar to those in the composite \( n = 1 \) westward inertio–gravity wave presented by Wheeler et al. (2000, see their Fig. 23).

One advantage of the two vertical mode simulation of 2-day waves is that it represents the complex structure displayed in Fig. 10a as the superposition of two relatively simple waves, whose vertical structures are independent of horizontal position (Figs. 10b–c). Both of these waves (or vertical modes) make significant contributions to the heating and velocity fields (Figs. 10b–c). The first mode contributes only weak temperature perturbations in the troposphere (Fig. 10b), but the second mode contributes most of the temperature structure: the out-of-phase lower- and upper-tropospheric temper-
ature anomalies (Fig. 10c). The first mode’s temperature perturbations are in quadrature with the heating (Fig. 10b), and the second mode’s temperature perturbations are nearly in quadrature with the heating (Fig. 10c) but shifted slightly toward having a positive correlation with heating. Vertical tilts in Fig. 10a can be interpreted as resulting from superposition of the vertical motion, heating, and temperature fields between the two modes in Figs. 10b and 10c. This effect is built into the models of Mapes (2000) and Majda and Shefter (2001) in which peak stratiform precipitation lags deep convection by a few hours.

An analysis of the balance between the heating perturbation \( Q \) and the adiabatic temperature change due to vertical motion \( S \omega \) for each vertical mode illustrates why the second mode contributes most of the temperature structure. For the first mode \( Q \) and \( S \omega \) are out of phase and nearly equal in amplitude (Fig. 11a). The temperature tendency \( \partial T/\partial t \), which is the sum of \( Q \) and \( S \omega \) [Eq. (4)], is roughly 1/20 of what would result from either the heating or the vertical motion alone and is a consequence of \( |S \omega| \) being slightly greater than \( |Q| \). For the second mode \( S \omega \) and \( Q \) are also nearly out of phase, but \( |Q| \) is significantly weaker than \( |S \omega| \) resulting in a temperature tendency with a peak amplitude of about 1 K day\(^{-1}\). Because the temperature pattern in the composite 2-day wave (Fig. 4a) is quite similar to that in this simulation, we expect that the conclusions that we reach here about balance between \( S \omega \) and \( Q \) also apply to that wave as well.

6. Discussion

a. Testing the hypothesis

One of the objectives of this study is to test the null hypothesis that the wind and temperature perturbations associated with 2-day waves can be modeled as linear responses to convective heating perturbations. To the extent that Figs. 4 and 6 display reasonable agreement, given both observational error and limitations of the technique used to isolate the observed signals, we conclude that this hypothesis cannot be rejected based on our test.

While there are limitations to this test that might provide a bias toward not rejecting the hypothesis (e.g., the heating and temperature/wind fields are not independent datasets), there are several ways the simulation could have yielded significantly different temperature and wind perturbations that apparently did not occur. First, budget-derived profiles of heating are easily contaminated by convective and mesoscale noise (Mapes et al. 2003). An erroneous heating profile would likely have produced wind and temperature perturbations that did not agree with observations. Second, if 2-day waves were forced by preexisting dry dynamics (e.g., Straub and Kiladis 2003a), only a portion of their wind and temperature would be reproducible as a response to heating function. Third, if nonlinear dynamics or advection by basic-state winds were fundamental to the generation of the wave’s kinematic and thermodynamic structure, the dynamical system that we chose likely would not have reproduced the observations. Hence, while the results presented in section 4 are not a foolproof test of the hypothesis, they do strongly support the hypothesis and instill confidence that the observed heating profile is accurate.

b. The two vertical mode model of 2-day waves

The simulation presented in section 5 shows that the two-mode model captures the basic dynamics of 2-day waves. It represents their tilted heating, wind, and temperature perturbations as the superposition of two simple waves (Fig. 10). This model also sheds light on the origin of the temperature structure (Fig. 4a): deep tropospheric heating and cooling are nearly balanced by adiabatic temperature changes due to vertical motion (Figs. 10b, 11a), but the second vertical mode is more out of balance, resulting in the out of phase lower- and upper-tropospheric anomalies (Figs. 10c, 11b).

Although the vertical modes are dry-dynamical constructs, they appear to represent the moist physical processes of deep convection, shallow convection, and stratiform precipitation. Mapes (2000) associated the
first vertical mode’s positive heating with deep convection and the second vertical mode’s upper-tropospheric heating and lower-tropospheric cooling with stratiform precipitation. However, we emphasize that it is also appropriate to associate the second mode’s heating of the opposite sign, upper-tropospheric cooling and lower-tropospheric warming, with shallow convection. The 2-day wave schematic of Takayabu et al. (1996) (Fig. 1) shows shallow convection on the leading edge of the wave that is accompanied by upper-tropospheric subsidence (and presumably radiational cooling). The cloud structure in this schematic is based on, among other things, a histogram of cloud-top brightness temperatures. On the leading edge of the simulated 2-day wave the second vertical mode contributes lower-tropospheric warming and upper-tropospheric cooling that are accompanied by rising motion and subsidence respectively (Fig. 10c), and which match up with the region of shallow convection in Fig. 1. Hence, the two vertical mode model of 2-day waves not only captures the wave’s basic dynamics, but it also represents the wave’s convective structure as well.

c. An \( n = 1 \) westward-propagating inertio–gravity wave?

As we mention in section 1, one of the theories of 2-day waves is that they are \( n = 1 \) westward propagating inertio–gravity (WIG) waves. The results presented in this paper are generally consistent with this idea. First, the simulations demonstrate that a dynamical system linearized about a basic state, such as that from which equatorial wave theory is derived, can capture the dynamics of 2-day waves. Second, the horizontal structure of the composite 2-day wave resembles the theoretical structure for an \( n = 1 \) inertio–gravity wave. The composite wave has a zonal wavelength of about 2800 km and a phase speed of about 16 m s\(^{-1}\) (Fig. 3). The forced \( n = 1 \) WIG wave (Lindzen 1967) with these characteristics has an equivalent depth of 14.3 m. Its horizontal structure is depicted in Fig. 12 (adapted from Matsuno 1966). Velocity vectors diverge from centers of downward vertical motion and arc around and converge into centers of upward vertical motion. Its geopotential height (not shown) and vertical velocity perturbations are symmetric about the equator. The composite 2-day wave has a similar horizontal structure (e.g., Fig. 3d).

Of course, the theoretical wave and composite wave differ in that the former has an infinite zonal extent, and the latter is a “wave packet” a few thousand kilometers across, but it appears that their dynamical essence is the same. Finally, the dynamical structure of the 2-day wave is very similar to that of the composite convectively coupled wave constructed by Wheeler et al. (2000, hereafter WKW) that was obtained by filtering for spectral components that lie along the dispersion curve for the \( n = 1 \) WIG wave. The patterns of tropospheric temperature, zonal wind, and vertical motion perturbations are nearly identical in the two waves (compare Fig. 10a with Fig. 23 of WKW). The phase speed of the 2-day wave is much slower than the 30 m s\(^{-1}\) phase speed found in WKW, but there are several factors that may be contributing to this discrepancy: 1) The composite WIG wave in WKW has a larger zonal wavelength, so based on the dispersion diagram for the \( n = 1 \) WIG waves one would expect it to propagate more rapidly; 2) during COARE the mean flow at low levels was westerly, which would have slowed the 2-day waves; 3) the COARE waves may have been phase locked to the diurnal cycle over the warm pool (Takayabu et al. 1996); and 4) even for a fixed horizontal wavenumber the \( n = 1 \) WIG waves analyzed by WKW covered a range of frequencies (see their Fig. 2b), that is, had a range of phase speeds, and one would expect that a composite constructed from a handful of cases might have some differing properties from a composite constructed from more than a decade’s worth of data.

One question that remains open for 2-day waves and other convectively coupled equatorial waves is the cause of their shallow equivalent depth. As we mention above, if the composite 2-day wave is an \( n = 1 \) WIG wave, it has an equivalent depth of about 14 m, which is significantly different from the “dry” equivalent depths of the first two vertical modes, which are 245 m and 54 m, respectively. Analysis of the balance between \( Q \) and \( S_\theta \) in the simulation (Fig. 11) suggests that the former partially cancels the latter, effectively resulting in a reduced static stability. For each mode the degree of cancelation is roughly consistent with the reduction of equivalent depth. For the first vertical mode \( Q \) cancels all but about 1/20 of \( S_\theta \) (Fig. 11a), and the equivalent depth is reduced by a factor of almost 20. For the second vertical mode the degree of cancelation varies, but on average \( Q \) cancels all but 1/4 of \( S_\theta \) (Fig. 11b), and the equivalent depth is reduced by a factor of about 4. Since the simulated temperature perturbations closely resemble the
observed temperature perturbations, we expect that a similar degree of cancelation exists for 2-day waves in nature. But a key question is: what determines the degree to which $Q$ cancels $S_o$? In the simulation the degree of cancelation between $Q$ and $S_o$ depends on the extent to which the forcing resonates with the waves it excites, which depends on the prescribed phase speed of the forcing (e.g., Haertel and Johnson 2000). However, in nature the opposite causality might exist; that is, physical processes could determine the degree of cancelation between $Q$ and $S_o$, which in turn could determine the speed of the convectively coupled wave. Theories have been proposed that quantify the reduction of static stability by deep convection (e.g., Neelin and Held 1987; Emanuel et al. 1994) and it has been simulated in models (e.g., Sobel and Bretherton 2003), but we are unaware of any such theories for shallow convection and stratiform precipitation. Of course, it is possible that one of the two vertical modes is driving the propagation of the system and that the other amounts to a forced response. It is also possible that the reduction of equivalent depth results from the the interaction of the two vertical modes (Mapes 2000; Majda and Shefter 2001; Majda et al. 2004). In any event, the results presented here support approaching this problem from the context of a two vertical mode system, which seems to do an excellent job of both capturing the free tropospheric dynamics of the wave and representing its moist convective processes.

d. The coupling between the wave and convection

The simulations presented here are unrealistic in the sense that the interaction between the wave circulation and the convection is one way; that is, a complete picture of the dynamics of 2-day waves will be obtained only once the feedback of the wave circulation on the convective heating is quantified as well. We are currently studying this problem and discuss only preliminary results in this paper. Two points worthy of mention are 1) the rain rate (Fig. 2b) is highly correlated with the column-integrated moist static energy and 2) changes to the latter are dominated by the horizontal divergence associated with the second vertical mode. In particular, on the leading edge of the wave (e.g., near $x = -800$ km in Fig. 10c), the second vertical mode converges near-surface air with high moist static energy, diverges midlevel air with low moist static energy, and converges upper-tropospheric air with high moist static energy. Perturbations to the moist static energy profile, which closely follow perturbations to the specific humidity profile (Fig. 4b), are most significant in the lower troposphere.

We are exploring the idea that much of the change in the rain rate can be attributed to the change in the gross moist static stability (Neelin and Held 1987) of the first vertical mode. In other words, when the lower troposphere is moistened, the low-level convergence and upper-level divergence associated with the first vertical mode becomes less efficient at “cooling” the column, a localized positive temperature perturbation develops, and the convergence and divergence increase in magnitude in response to the associated changes in the Laplacian of the hydrostatic pressure. Since the divergence circulation of the first vertical mode dominates the moisture budget, the rain rate increases as well. Of course, this theory assumes that the moist static energy converged by the first vertical mode is quickly converted to heat, and much of that converged by the second vertical mode is stored as a water vapor perturbation. While this theory is generally consistent with the observations presented here, a careful analysis of the moist static energy budget of 2-day waves is necessary to test its validity.

7. Summary

In this paper we present a statistical composite of the 2-day equatorial waves that occurred during the Tropical Ocean Global Atmosphere Coupled Ocean–Atmosphere Response Experiment. The composite wave is accompanied by a negative (positive) temperature perturbation in the lower (upper) troposphere and by zonal wind perturbations that rise with time through most of the troposphere but descend with time near the tropopause.

The hypothesis that the wave’s wind and temperature perturbations can be modeled as linear responses to its convective heating and cooling is tested. A linear model is used to simulate the atmospheric response to an idealized version of the composite wave’s heating. The model produces wind and temperature perturbations that are very similar to the observed perturbations.

The ability of a two vertical mode system to represent the wave’s dynamics is also tested. The heating is projected onto the first two vertical modes of an atmosphere having a rigid boundary at 150 hPa. The two-mode heating is also able to generate the kinematic and thermodynamic structure of the wave in the free troposphere. The two modes of heating are associated with the moist physical processes of deep convection and shallow convection/stratiform precipitation, respectively.

The horizontal and vertical structures of the composite 2-day wave are shown to resemble those of the $n = 1$ westward-propagating inertia–gravity wave, but with an equivalent depth much more shallow than dry dynamics would predict. It is suggested that the shallow equivalent depth results from the partial cancelation of the adiabatic temperature changes due to vertical motion by convective heating and cooling, with varying degrees of cancelation for the two vertical modes.

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REFERENCES


