Dynamics of Synoptic Eddy and Low-Frequency Flow Interaction. Part I: A Linear Closure

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ABSTRACT

The interaction between synoptic eddy and low-frequency flow (SELF) has been recognized for decades to play an important role in the dynamics of the low-frequency variability of the atmospheric circulation. In this three-part study a linear framework with a stochastic basic flow capturing both the climatological mean flow and climatological measures of the synoptic eddy flow is proposed. Based on this linear framework, a set of linear dynamic equations is derived for the ensemble-mean eddy forcing that is generated by anomalous time-mean flows. By assuming that such dynamically determined eddy-forcing anomalies approximately represent the time-mean anomalies of the synoptic eddy forcing and by using a quasi-equilibrium approximation, an analytical nonlocal dynamical closure is obtained for the two-way SELF feedback. This linear closure, directly relating time-mean anomalies of the synoptic eddy forcing to the anomalous time-mean flow, becomes an internal part of a new linear dynamic system for anomalous time-mean flow that is referred to as the low-frequency variability of the atmospheric circulation in this paper.

In Part I, the basic approach for the SELF closure is illustrated using a barotropic model. The SELF closure is tested through the comparison of the observed eddy-forcing patterns associated with the leading low-frequency modes with those derived using the SELF feedback closure. Examples are also given to illustrate an important role played by the SELF feedback in regulating the atmospheric responses to remote forcing. Further applications of the closure for understanding the dynamics of low-frequency modes as well as the extension of the closure to a multilevel primitive equation model will be given in Parts II and III, respectively.

1. Introduction

Over the middle and high latitudes of the Northern Hemisphere (NH) and Southern Hemisphere (SH), especially during the cold season months, the day-to-day atmospheric circulation and associated weather are highly chaotic and rich with transient baroclinic eddies with a typical lifetime of only a few days. Amidst this turbulent atmospheric circulation, there are preferred recurrent patterns that can often persist for much longer time. Several significant and recurrent patterns have been identified in the literature: the North Atlantic Oscillation (NAO), which has recently been suggested as a part of the global-scale pattern known as the Arctic Oscillation (AO) or Northern Annular Mode (NAM), the SH counterpart called the Antarctic Oscillation (AAO), or Southern Annular Mode (SAM; cf. Thompson and Wallace 2000), and the well-known Pacific–North American (PNA) pattern. These modes dominate the monthly mean anomalous flow or the low-frequency variability of time scales beyond the lifetime of synoptic eddies.

The mechanisms of the low-frequency variability were attributed to external forcing, energy sources drawing from the time–mean basic flow, and the so-called transient eddy forcing (e.g., Hoskins and Karoly 1981; Simmons et al. 1983; Held et al. 1989; Ting and

However, the transient eddy forcing results from internal dynamical interactions that partly depend on the low-frequency flow. For instance, on one hand, the monthly mean flow anomalies associated with NAO mode are known to accompany systematic changes in the variance fields of transient synoptic (cf. Hurrell and van Loon 1997). On the other hand, these monthly mean anomalies in the synoptic eddy forcing associated with systematic changes in the storm track regions, in turn, feed back onto the monthly mean flow anomalies (e.g., Lau 1988; Lau and Nath 1991). Cai and Mak (1990), Qin and Robinson (1992), and Branstator (1995), among others, demonstrated that there is a dynamic two-way interaction between the synoptic eddy and low-frequency flow. This two-way interaction, referred to as the SELF feedback in this paper, plays an important role in the formation of the leading patterns of the low-frequency variability. For instance, even without external sources, internal low-frequency variability, such as zonal index cycles, has been found in many simple quasigeostrophic and primitive equation models (Robinson 1991; James and James 1992; Yu and Hartmann 1993; Feldstein and Lee 1996; Akahori and Yoden 1997; Koo et al. 2002; Kravtsov et al. 2003). A number of atmospheric general circulation model simulations driven by climatological sea surface temperature (SST) produce modes similar to the observed AO and AAO (e.g., Yamazaki and Shinya 1999). Furthermore, recent observational evidence also points to the essential role that the SELF feedback plays in generating low-frequency modes such as the AO and AAO (DeWeaver and Nigam 2000a,b; Robinson 2000; Limpasuvan and Hartmann 2000; Lorenz and Hartmann 2001; Kimoto et al. 2001).

In this three-part study, we proposed a dynamical closure for the SELF feedback that relies on separating the quadratic eddy–eddy interaction term into a climatological component and anomalous one, the slowly varying part of the anomalous component is assumed to be slaved by the anomalous low-frequency flow. By introducing the concept of a stochastic basic flow that captures both the climatological mean flow and climatological measures of the synoptic eddy flow, we focused on a linear SELF feedback closure between the anomalous synoptic activity and the anomalous large-scale flow. With the linear closure, we obtained a novel linear framework for understanding the dynamics of the low-frequency variability.

In this paper, we first raise the issue of closure for the SELF feedback in section 2. We propose a method of constructing a stochastic basic eddy flow in appendix A, where it is demonstrated that the climatological measures of the synoptic eddy flow are captured by this reconstructed basic eddy flow. In section 3, by examining the dynamical interactions between anomalies in the synoptic eddy activity and in slowly varying ensemble mean flow with respect to the basic stochastic flow, we derive an approximate closure using a barotropic model. We provide the validation of the closure using observational data in section 4. We further give brief examples in section 5 to illustrate that the SELF feedback closure is a useful tool for simulating the role of the synoptic eddy activity in regulating the atmospheric responses to remote forcing.

In Jin et al. (2006, hereafter Part II), we attempt to explore the internal dynamic modes of the linear dynamical system with SELF feedback, focusing on the role of the SELF feedback in organizing the low-frequency modes. By analyzing the structure of the least-damped low-frequency modes of the linear dynamical system, we demonstrate that with the inclusion of the SELF feedback, AO- and AAO-like modes become leading singular and eigenmodes of the dynamic system, whereas the SELF feedback also impacts the localized PNA-like mode. The results of barotropic model are compared with the observed AO, AAO, and PNA patterns. We further propose idealized model experiments and an analytical prototype model to illustrate that the scale-selective positive SELF feedback is essential for the emergence of AO- and AAO-like low-frequency modes.

In Pan et al. (2006, hereafter Part III), we extend the SELF feedback closure into a general primitive equation model. We validated the closure in a five-layer model and confirm the basic findings of the barotropic model results presented in Part II. The earlier results of this three-part study were documented in Pan (2003).

2. The concept of closure for SELF feedback

A common diagnostic methodology (cf. Lau 1988) is to separate the observed or model-simulated atmospheric flow into different parts according to their time scales. For example, the 500-hPa streamfunction \( \psi \) is often decomposed as

\[
\psi = \psi^a + \psi', \quad \psi^a = \bar{\psi}^a + \psi^a',
\]

(1)
where $\overline{\psi}$ may be obtained by applying either a low-pass filter or an appropriate time mean onto $\psi$ whereas $\psi'$ represents the transient flow. Moreover, $\overline{\psi}$ is separated into the climatological mean $\overline{\psi}$ and time–mean anomaly $\overline{\psi}'$ denoted for convenience as the low-frequency anomalies. Similarly, the time–mean variance fields of the synoptic eddy flow and associated eddy vorticity forcing from the rotational flow are also separated into two parts:

$$\overline{\psi'^2} = \overline{\psi'^2}^C + \overline{\psi'^2}^A$$

$$\overline{J(\psi', \Delta \psi')^C} = \overline{J(\psi', \Delta \psi')^C}^C + \overline{J(\psi', \Delta \psi')^C}^A.$$  \hspace{1cm} (2)

Here $\Delta$ and $J$ are Laplacian and Jacobian operators, $\overline{\psi'^2}^C$ and $\overline{J(\psi', \Delta \psi')^C}$ are defined as the climatological mean, whereas $\overline{\psi'^2}^A$ and $\overline{J(\psi', \Delta \psi')^C}^A$ are defined as time–mean anomalies. In diagnostic studies, the time–mean anomalies, denoted by $\overline{\psi'^2}^A$ and $\overline{J(\psi', \Delta \psi')^C}^A$, are usually evaluated as monthly mean anomalies (e.g., Lau 1988).

Based on the National Center for Atmospheric Research–National Centers for Environmental Prediction (NCAR–NCEP) reanalysis data, the climatological mean flow $\overline{\psi}$ and climatological variance field $\overline{\psi'^2}$ during the NH and SH cold seasons are shown in Fig. 1. In this study, the synoptic eddy flow $\psi'$ is obtained by applying a 2–8-day bandpass filter (Murakami 1979) to the daily data. The climatological means of the synoptic eddy forcing in terms of streamfunction $[-\Delta^{-1}(\overline{J(\psi', \Delta \psi')^C})$] are shown in Figs. 1b and 1d. The isolated storm tracks over the NH Pacific and Atlantic (Fig. 1b) as well as

**Fig. 1.** (a) Northern Hemisphere climatological streamfunction at 500 hPa averaged during the cold season (November to April 1949–99). Variance distributions of the bandpass-filtered streamfunction at 500 hPa (November to April 1979–95) are superimposed, as depicted by shading. (b) The climatological mean of the eddy-induced streamfunction forcing calculated using the bandpass-filtered daily 500-hPa streamfunction. (c), (d) Same as in (a), (b) except for the Southern Hemisphere cold season (May to October). The units for the mean streamfunction and the variance fields are $10^7$ m$^2$ s$^{-1}$ and $10^{13}$ m$^4$ s$^{-2}$, respectively. The unit for eddy forcing is m$^2$ s$^{-2}$; contour interval is 2 m$^2$ s$^{-2}$. 

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Lau (1988) demonstrated that there is a close asso-
ciation between monthly mean anomalies of the flow
field $\bar{\psi}$ and systematic anomalies in the monthly mean
eddy statistics including the eddy forcing $\mathcal{J}(\psi, \Delta \psi)^a$. 

Therefore, we hypothesize that at least part of the
eddy forcing $\mathcal{J}(\psi, \Delta \psi)^a$ can be directly related to $\bar{\psi}^a$
through a dynamic relationship between these two fields. Namely

$$\mathcal{J}(\psi, \Delta \psi)^a \approx L_f \bar{\psi}^a.$$  

We refer $L_f$ to as the closure operator for the SELF
feedback. Such a dynamical closure is useful for under-
standing the dynamics for the low-frequency flow $\bar{\psi}^a$
(see sections 3 and 5).

Before deriving such a closure, we give a brief ac-
count for observed relationships between $\bar{\psi}^a$ and $\mathcal{J}(\psi, \Delta \psi)^a$, which provide the bases for testing the va-
idity of the dynamic closure to be derived in section 3. We focus on the well-known prominent large-scale pat-
terns (Fig. 2) derived from monthly mean anomaly field
$\bar{\psi}$, such as the AAO, AO or NAO, and PNA, and the
associated coherent patterns of eddy forcing (Fig. 3) in

3. A SELF closure in a barotropic model

We now present an approximate dynamic closure for
the SELF feedback using a simplified form of the baro-
tropic vorticity equation:

$$\frac{\partial}{\partial t} \Delta \psi + \mathcal{J}(\psi, \Delta \psi + f) + r \Delta \psi = Q,$$

where $\psi$ and $f$ are the streamfunction and Coriolis pa-
rameter, respectively. Equation (4) includes a linear
damping term and a forcing term. It has been repeat-
eddy forcing $\overline{J(\psi', \Delta \psi')}$ is the part of eddy forcing of the climatological mean equation for $\psi'$ (not shown). In Eq. (5), we only ignored the nonlinear term $J(\psi', \Delta \psi')$ that is presumably small. The tendency term can also be neglected for monthly mean anomaly flow $\overline{\psi}'$.

Without the anomalous eddy forcing term $\overline{J(\psi', \Delta \psi')}$, Eq. (5) reduces to the conventional linear barotropic model. It was used to illustrate the basic mechanism of the atmospheric teleconnection patterns in terms of Rossby wave propagation (e.g., Hoskins and Karoly 1981; Sardeshmukh and Hoskins 1988) or in terms of the barotropic instability of a nonzonal basic flow (Simmons et al. 1983). With the inclusion of the eddy forcing term given diagnostically from data, it was also used to diagnose the impact of the eddy forcing on the atmospheric responses to tropical and other external sources of forcing (e.g., Ting and Held 1990). However, once the closure for the eddy forcing as expressed in Eq. (3) is established, the parameterized part of eddy forcing $\overline{J(\psi', \Delta \psi')}$ becomes explicitly a part of the internal dynamics of the low-frequency anomalies.

The synoptic eddy flow is highly chaotic and unpredictable beyond its typical lifetime of less than a week. Thus, on a time scale much beyond a week, it may be viewed as stochastic flow. In fact, stochastic modeling has been advanced significantly in the past decades for simulating and understanding variability of the atmospheric circulation (e.g., Farrell and Ioannou 1993; Penland and Ghil 1993; Penland and Matrosova 1994; Branstator 1995; Delsole and Farrell 1996; Newman et al. 1997; Whitaker and Sardeshmukh 1998; Zhang and Lamb 2001; Delsole 2001; Winkler et al. 2001; Majda et al. 2003).

We thus adopt a view that the evolution of the atmospheric circulation is a particular realization of a hypothetical stochastic flow ensemble. Moreover, we denote the latter as $\Psi$ for a clear distinction in notation. With this conceptual extension, we introduce another measure, the ensemble mean, denoted by the operator $(\cdot \cdot \cdot)$. We now decompose $\Psi$ into four parts:

$$\Psi = \Psi_c + \Psi_a, \quad \Psi_c = (\Psi_c) + \Psi_c', \quad \Psi_a = (\Psi_a) + \Psi_a'.$$

where $\Psi_c$ and $\Psi_a$ denotes the basic and anomalous ensembles of the flow. Each of them is further decomposed into its ensemble mean term and a deviation denoted by the prime term.

We further adopt an ergodic approximation that enables us to substitute the ensemble mean $(\cdot \cdot \cdot)$ of $\Psi$ by the time means $(\cdot \cdot \cdot)'$ of $\psi$ such that $(\Psi_c) \approx \overline{\psi}'$, $(\Psi_a) \approx \overline{\psi}'$. This is a practical and useful approximation that is valid when the interval of the time mean is much smaller than the typical lifetime of less than a week.

Fig. 3. The observed patterns of synoptic eddy-induced streamfunction forcing obtained by regressing the monthly averaged anomalies of the eddy vorticity forcing to the (a) AAO, (b) AO, and (c) PNA indices. The unit is m$^2$s$^{-2}$; contour interval is 0.5 m$^2$s$^{-2}$.
longer than the decorrelation time scale of the synoptic eddy flow. Furthermore, we consider that the ensemble eddy flow $\Psi' = \Psi'_{s} + \Psi'_{a}$ is an augmentation of the observed synoptic flow $\psi'$ that can be viewed as one particular realization of $\Psi'$. There is a significant advantage using this augmentation and the new decomposition (6). It enables us to separate $\Psi'$ into two parts, the basic and anomalous ensembles of the eddy flow and use them to characterize the climatological mean and time–mean anomalies of statistics properties expressed in Eq. (2) for the observed eddy flow $\psi'$.

Particularly, we can construct $\Psi'$ from the data (see appendix A) and show that a high-dimensional Gaussian flow $\Psi'_{s}$ captures the climatological statistics properties of the observed eddy flow $\psi'$, including the climatological variance, covariance, and eddy forcing. For instance, we show in Fig. A2 that $\langle \Psi'_{s}, \Psi'_{s} \rangle = \mathbf{J}(\psi', \psi')$. Thus, using $\Psi'_{s}$ as a given representation of the basic synoptic eddy flow, we now can make a linear approximation for the ensemble-mean eddy forcing as follows:

$$
\mathbf{J}(\Psi', \Delta \Psi') = \mathbf{J}(\Psi'_{s}, \Delta \Psi'_{s}) + \mathbf{J}(\Psi'_{a}, \Delta \Psi'_{a})
$$

We neglect the nonlinear term $\mathbf{J}(\Psi'_{a}, \Delta \Psi'_{a})$ by assuming that it is relatively small. Again using the ergodic approximation $\mathbf{J}(\Psi', \Delta \Psi') = \mathbf{J}(\psi', \Delta \psi')$, we obtain a linear expression for the anomalous eddy forcing as follows:

$$
\mathbf{J}(\psi', \Delta \psi') = \mathbf{J}(\psi', \Delta \psi') - \mathbf{J}(\psi'_{s}, \Delta \psi'_{s})
$$

$$
\approx \mathbf{J}(\psi'_{a}, \Delta \psi'_{a}) + \mathbf{J}(\psi'_{c}, \Delta \psi'_{c}).
$$

It should be noted that an anomalous flow $\tilde{\psi}'$ generates anomalous eddy flow $\Psi'_{a}$ through anomalous advection of the basic eddy flow $\Psi'_{s}$. Therefore, $\Psi'_{s}$ and $\Psi'_{a}$ are correlated to each other. As the result, the anomalous eddy forcing terms in Eq. (7) is linearly related to $\tilde{\psi}'$.

To make this point clear, we need to consider a dynamic model for $\Psi'_{a}$.

Assuming that $\Psi$ is also governed by Eq. (1) and then linearizing it with respect to the basic stochastic flow $\Psi_{s}$, we obtain the linear equation for $\Psi'_{a}$ (see detailed derivation in appendix B):

$$
\frac{\partial}{\partial t} \Delta \Psi'_{a} + J(\tilde{\psi}'^{c}, \Delta \Psi'_{a}) + J(\Psi'_{a}, \Delta \tilde{\psi}'^{c} + f) + r \Delta \Psi'_{a}
$$

$$
= Q'_{a} - J(\tilde{\psi}'^{a}, \Delta \Psi'_{a}) - J(\psi'_{s}, \Delta \tilde{\psi}'^{a}).
$$

(8)

Here, the external forcing term $Q'_{a}$ presents some independent stochastic source that is set to zero hereafter without losing generality. Without the last two terms in the right-hand side but with $Q'_{a}$, this is a simply stochastically forced model for transient eddy flow. In this paper, we set $Q'_{a}$ to zero and focus on the anomalous ensemble flow $\Psi'_{a}$ that is generated by $\tilde{\psi}'$ through the anomalous advectives as expressed by the last two terms in Eq. (8).

The set of coupled linear Eqs. (5), (7), and (8) provides a novel stochastic framework describing the dynamics of the SELF feedback. Through the basic ensemble of eddy flow $\Psi'_{s}$, the low-frequency flow anomaly $\tilde{\psi}'$ generates the anomalous stochastic eddy flow $\Psi'_{a}$. The latter facilitates energy exchange between $\tilde{\psi}'$ and $\Psi'_{c}$. Thus the SELF feedback allows the low-frequency flow $\tilde{\psi}'$ not only to draw energy from the climatological mean flow $\tilde{\psi}'$, but also to draw energy from the basic eddy flow $\Psi'_{c}$.

For a given $\tilde{\psi}'$, Eq. (8) is a linear model of anomalous storm track activities. We noted that it is somewhat equivalent to the earlier numerical ensemble approach (Branstator 1995; Zhang and Held 1999). However, our approach avoids the numerical simulation for the basic stochastic eddy–flow ensemble $\Psi'_{s}$. Moreover, we can analytically solve Eq. (8) to obtain a closure for Eq. (5).

From the expression of $\Psi'_{a}(x, y, t)$ in appendix A, the solution of linear equation (8) can be expressed as

$$
\Psi'_{a}(x, y, t) = \sum_{n=1}^{N_{c}} \sigma_{n}\psi'_{a}(x, y, t) e^{i\omega_{n}t} + \text{c.c.}
$$

(9)

Here c.c. stands for complex conjugate, and each $\Psi'_{a}(x, y, t)$ is an independent Gaussian flow related to each Gaussian process $\xi_{n}(x, y, t)$ in the right-hand side of the Eq. (8); $\sigma_{n}$ and $\omega_{n}$ are related to the variance contribution and main frequency of each independent propagating pattern of synoptic eddies, respectively. Detailed definitions for $\xi_{n}$, $\sigma_{n}$, and $\omega_{n}$ are given in appendix A.

By defining each covariance component $\tilde{\psi}'(x, y, t) = (\tilde{\psi}_{n}(t)\psi'_{n})$, where $\tilde{\psi}_{n}$ is the complex conjugate of $\xi_{n}$ and utilizing the fact that $\tilde{\psi}_{n}(t)\psi'_{n} = \tilde{\psi}_{n}(x, y, t)\delta_{n,m}(\delta_{n,m} = 0, n \neq m; \delta_{n,m} = 1)$, we obtain the following expression for the ensemble-mean anomalies of the eddy forcing:

$$
\langle \mathbf{J}(\Psi'_{a}, \Delta \Psi'_{a}) \rangle + \langle \mathbf{J}(\psi'_{c}, \Delta \psi'_{c}) \rangle = \sum_{n=1}^{N_{c}} \sigma_{n}^{2}\langle \mathbf{J}(\tilde{\psi}_{n}, \Delta \hat{E}_{n}) \rangle + \langle \mathbf{J}(\hat{E}_{n}, \Delta \hat{\psi}_{n}) \rangle + \text{c.c.}
$$

(10)

Here $\hat{E}_{n}$ is the complex conjugate of $E_{n}$, and each $E_{n}$, defined in appendix A, is an independent spatial pattern prescribed for the basic eddy flow.

It can be shown by examining the ensemble-mean...
anomalies of the variance fields that each component \( \psi_n(x, y, t) \) represents an evolving complex pattern related to the anomalous second moments of the eddy flow ensemble \( \Psi' \). Furthermore, the following steps enable us to obtain the governing equations for \( \hat{\psi}_n \); multiply Eq. (8) by \( \tilde{\varepsilon}_n \), multiply the complex conjugate of Eq. (A2) by \( \tilde{\Psi}'_n(x, y, t) \), add these two equations, and take an ensemble mean of the resulting equations. Then, the coupled stochastic model [Eqs. (5), (7), and (8)] is reduced into a set of coupled linear equations as follows:

\[
\begin{align*}
\frac{\partial}{\partial t} \Delta \tilde{\psi} + L \tilde{\psi} + \sum_{n=1}^{N_c} (\sigma_n^2 J(\hat{\psi}_n, \Delta \hat{E}_n))
+ J(\hat{E}_n, \Delta \hat{\psi}_n) + cc &= \tilde{\Omega}^n, \\
\frac{\partial}{\partial t} \Delta \hat{\psi}_n + \hat{\psi}_n + (\tau_n^{-1} + i \omega_n) \Delta \hat{\psi}_n &= -J(\hat{E}_n, \Delta \hat{\psi}_n)
- J(\tilde{\psi}^* \Delta E_n), \\
&= 1, N_c. \quad (11)
\end{align*}
\]

Here the linear operator \( \tilde{L} \) is defined as

\[
\tilde{L}X = J(\tilde{\psi}, \Delta X) + J(X, \Delta \tilde{\psi}^*) + f + rX
\]

for any field \( X \). In deriving the second half of Eq. (11), we have used \( \langle \tilde{\varepsilon}_n \varepsilon_m(t) \rangle = \delta_{n,m} \). Here \( \tau_n \) is the autodecay time scale of \( \tilde{\varepsilon}_n \). Each \( \tau_n \) is estimated from the \( e \)-folding time scale of the autocorrelation function of the time coefficient of each complex EOF derived from observed synoptic flow (see appendix A).

As noted in appendix B, the first equation in (11) is the same as the equation for the anomalous first moment of \( \Psi \). The eddy forcing term of this equation is now expressed as a linear function of the prognostic variables \( \tilde{\psi}_n \) that is related to anomalies of the second moments of \( \Psi \). Therefore, the linear system (11) may be viewed as an anomaly model linearized with respect to the first two moments of the stochastic basic flow \( \Psi' \).

Further inspection of the system (11) shows that the eddy \( e \)-folding time scales appear in the left-hand side of the equations for the second moment anomalies \( \tilde{\psi}_n \).

As shown in appendix A, these time scales are on the order of 3–4 days. Thus, we may consider that \( \tilde{\psi}_n \) is approximately in quasi equilibrium with \( \tilde{\Psi}^n \) and drop the time derivative of \( \tilde{\psi}_n \) in the system (11). Under this quasi-equilibrium approximation, all \( \psi_n \) can be expressed as a linear function of \( \tilde{\psi}^n \). We therefore obtain a linear equation for the low-frequency anomaly \( \tilde{\psi}^n \) with a closure for the SELF feedback:

\[
\frac{\partial}{\partial t} \Delta \tilde{\psi} + L \tilde{\psi} + L \tilde{\psi} = \tilde{\Omega}^n, \quad (12)
\]

where the SELF feedback operator \( L_i \) is defined as

\[
L_i \tilde{\psi} = \sum_{n=1}^{N_c} \sigma_n^2 \tilde{L}_n[(\tau_n^{-1} + i \omega_n) \Delta + L]^{-1} L_n \tilde{\psi}^n. \quad (13)
\]

Here \( \{(\tau_n^{-1} + i \omega_n) \Delta + L\}^{-1} \) denotes the inverse operator of \( \{(\tau_n^{-1} + i \omega_n) \Delta + L\} \) and the linear operators \( L_n \) are defined as

\[
L_n X = J(E_n, \Delta X) + J(X, \Delta E_n)
\]

\[
\tilde{L}_n X = J(\tilde{E}_n, \Delta X) + J(X, \Delta \tilde{E}_n)
\]

for any field \( X \). Although the closure \( L_i \) is a rather complicated operator, it can be evaluated directly for a given climatological basic state \( \tilde{\psi}^n \) and a prescribed set of \( (E_n, \sigma_n, \omega_n, \tau_n; n = 1, N_c) \) obtained from climatological properties of the observed eddy flow (appendix A). Therefore, the linear dynamic system (12) for low-frequency variability depends on both climatological mean flow \( \tilde{\psi}^n \) and climatological measure \( (E_n, \sigma_n, \omega_n, \tau_n; n = 1, N_c) \) of the synoptic eddy flow, as illustrated schematically in Fig. 4.

Figure 4 also illustrates the main difference of the traditional and the extended linear frameworks for studying dynamics of the low-frequency variability. The traditional linear framework corresponds to the processes depicted by the two boxes of the left-hand side of Fig. 4. In such a framework, the linear dynamics of the low-frequency variability (denoted by the lower left box) is dictated by the climatological mean flow, and other sources for low-frequency variability may be attributed to external forcing denoted by \( F \). Equation (5) is an example of such a system in which the basic mean flow effectively controls the forced Rossby wave propagations (e.g., Hoskins and Karoly 1981; Jin and Hoskins 1995) and the nature of normal modes (e.g., Simmons et al. 1983), whereas eddy forcing is determined diagnostically and attributed as a part of the forcing.

The extended linear framework has two additional boxes as depicted by the right half of Fig. 4. One is for the basic stochastic eddy flow that captures the climatological measures of the synoptic eddy flow (denoted by upper right box) and another is for anomalous synoptic eddy activity (denoted by a lower right box) that is modulated by low-frequency variability. In this new framework, the climatological measures of synoptic eddy flow become a part of the essential factors controlling the dynamics of the low-frequency variability.
For instance, through anomalous interactions with the basic stochastic synoptic eddy flow, low-frequency flow anomalies, excited by some external forcing for example, can generate anomalous eddy activities that can feed back onto the low-frequency flow anomalies, as denoted by the two opposite arrows between the two lower boxes. As much as the traditional linear dynamics depends on the climatological flow, the dynamics of the SELF feedback and thus dynamics of extended framework, such as system (12), also depends on the climatological measures of the synoptic eddy flow.

It should be pointed out that our closure is different from empirical closures, which relate the observed $\mathcal{J}(\psi', \Delta \psi')$ to the observed $\overline{\psi}'$ through linear regressions (e.g., Peng et al. 2005). The so-called linear inverse modeling is to derive empirically the entire linear operators $L_f \pm L_s$ based on the covariance and lagged covariance of the observed low-frequency flow variability $\overline{\psi}'$ (e.g., Winkler et al. 2001). Our SELF feedback operator $L_f$, derived through a series of assumptions and approximations, depends only on the climatological mean flow and climatological measures of the synoptic eddy flow. It needs neither any information about observed anomalous eddy forcing $\mathcal{J}(\psi', \Delta \psi')$ nor information about observed low-frequency variability of $\overline{\psi}'$, so the feedback operator $L_s$ has no direct built-in answer about the observed anomalous eddy activity and low-frequency flow anomalies. Therefore, the validity of this analytically derived SELF feedback operator can be independently tested, by applying $L_f$ onto observed $\overline{\psi}'$ to calculate a parameterized eddy forcing as $L_f \overline{\psi}'$ and then comparing it with the observed eddy forcing anomalies $\mathcal{J}(\psi', \Delta \psi')$. The details of the validation are presented in the next section.

4. The validation of the SELF closure

In principle, the closure Eq. (13) can be tested directly using the observed low-frequency flow $\overline{\psi}'$ and eddy forcing $\mathcal{J}(\psi', \Delta \psi')$. In reality, however, the observed eddy forcing $\mathcal{J}(\psi', \Delta \psi')$ is neither completely determined by the low-frequency flow nor entirely controlled by the barotropic processes. Moreover, both $\mathcal{J}(\psi', \Delta \psi')$ and $\overline{\psi}'$, normally defined as the monthly mean anomalies, also contain low-frequency noises. In other words, as in all parameterization closures, there are some parts of the observed anomalies in the eddy forcing $\mathcal{J}(\psi', \Delta \psi')$ that cannot be expressed by the low-frequency flow $\overline{\psi}'$.

As shown in section 2, there are observed correlations between the low-frequency flow $\overline{\psi}'$ and eddy-induced streamfunction forcing $-\Delta^{-1}\mathcal{J}(\psi', \Delta \psi')$ for the leading modes of the low-frequency variability. To illustrate that the SELF closure does capture these observed relationships associated with the most energetic modes of low-frequency variability, we test the validity of the closure (13) by examining if it can reproduce the observed eddy-forcing patterns shown in Fig. 3.

Using the patterns in Fig. 2 as the known $\psi'$ fields, we apply the feedback operator onto them to calculate the parameterized eddy forcing $-\Delta^{-1}L_f \overline{\psi}'$ (Fig. 5). The pattern correlations between the parameterized eddy forcing (Fig. 5) and the observed $-\Delta^{-1}(\mathcal{J}(\psi', \Delta \psi'))$ (Fig. 3) are 0.81 for the AAO, 0.85 for AO, and 0.71 for

Fig. 4. Schematic diagram of the linear frameworks for understanding the dynamics of the extratropical atmospheric low-frequency variability. The left half of the diagram depicts the two components in traditional linear framework. The extended linear framework has two additional components illustrated by the left half of the diagram. The two opposite arrows between the two lower components indicate the SELF feedback captured in the extended framework.
PNA, indicating a reasonable success of the closure. In the parameterized eddy forcing associated with the AAO, positive tendencies in the polar region and low latitudes and negative tendencies in midlatitudes are all captured. In particular, the magnitudes of these eddy-induced streamfunction tendencies are also well simulated in the midlatitude and polar regions. Only in the subtropics, the parameterized positive forcing is somewhat too strong. For the case of the AO, as shown in Fig. 5b, the streamfunction forcing derived from the SELF feedback operator simulates the observed three-latitude belts with minus–plus–minus signs over the Atlantic sector. The parameterized and observed results for the PNA are also in good agreement (Figs. 4c and 5c). The locations of the major action centers in the eddy-induced forcing over the vicinities of the PNA region are all reproduced. The details in the areas away from the PNA region have discrepancies. Nevertheless, considering the great simplifications in the framework, these agreements are still remarkable.

To further demonstrate that the dynamical closure for the SELF feedback is generally valid for large-scale features of the low-frequency variability, we extend the above validation by examining systematically the parameterized eddy forcing patterns of \(-\Delta^{-1}L_f\psi^a\) associated with each EOF pattern of \(\psi^a\) for the period 1979 to 1995. They are then compared with the observed synoptic eddy-forcing patterns obtained by regressing \(-\Delta^{-1}\mathcal{F}(\psi^a, \Delta \psi^a)\) onto the time coefficients associated with each EOF. The spatial correlations of the parameterized and observed eddy forcing patterns for the first 10 EOFs of \(\psi^a\) are shown in Fig. 6. For the first 10 EOFs, which explain about 75% of the total variance of the low-frequency variability of \(\psi^a\) in both hemispheres, the correlations are all above 0.5.

We also used the data to obtain leading patterns of \(\psi^a\) and associated eddy forcing patterns for the period of 1995–2004, and then used the same \(L_f\) to calculate the corresponding parameterized eddy forcing patterns. The results are similar (not shown) therefore the SELF feedback operator \(L_f\) is shown to be robust. Thus, the closure operator is reasonably successful in capturing the SELF feedback involved in the low-frequency variability.

The closure operator \(L_f\) (13) is insensitive to the two free parameters: the truncation number \(N_c\), and the linear damping rate \(r\). In all calculations for testing \(L_f\), \(N_c\) was set at 10. The operator \(L_f\) converges rather quickly with respect to the truncation number \(N_c\), and it is not very sensitive to further increase in \(N_c\). The closure operator is also insensitive to the linear damping rate \(r\). This is because the time scale of the linear friction (10 days in the present computation) is overwhelmed by \(\tau_{\psi^a}\), the intrinsic e-folding decay time scale for the transient eddies.
In addition to the closure for anomalous eddy forcing, anomalies of the eddy variance, covariance, and lagged covariance fields can also be expressed as the functions of $\tilde{\psi}^{a}$ as follows:

$$
\tilde{\psi}^{2, a} \approx 2\langle \Psi_{a}^{i} \Psi_{a}^{j} \rangle = \sum_{n=1}^{N_{c}} \hat{E}_{a}(x, y) \hat{E}_{a}(x, y, t) + c.c. = L_{f, 2} \tilde{\psi}^{a}
$$

$$
\tilde{\psi}(x, y, t) \tilde{\psi}(x, y, t - \tau) \approx \langle \Psi_{a}(x, y, t) \Psi_{a}(x, y, t - \tau) \rangle + \langle \Psi_{a}(x, y, t) \Psi_{a}(x, y, t - \tau) \rangle
$$

$$
= \sum_{n=1}^{N_{c}} [\hat{E}_{a}(x, y) \hat{E}_{a}(x, y, t) e^{-i\omega_{a} t + i\phi_{a} \tau} + \hat{E}_{a}(x, y) \hat{E}_{a}(x, y, t) e^{-i\omega_{a} t + i\phi_{a} \tau}] + c.c. = L_{f, 2} \tilde{\psi}^{a}.
$$

(14)

Here $(x, y)$ can be any chosen base point for calculating the covariance fields and $\tau$ is the time lag. The closure operators $L_{f, 1}$ and $L_{f, 2}$ can be readily obtained by inverting the equations of $\Psi_{a}$ under the quasi-equilibrium approximation. We take the same validation approach employed to test these auxiliary closures. Namely, we first calculate the observed patterns of anomalous storm activity represented by $\tilde{\nu}^{2, a}$ and anomalous eddy covariance $\tilde{\psi}(x, y, t) \tilde{\psi}(x, y, t - \tau)$ of any given lag. Both are obtained by regressing monthly mean anomalies of these fields onto the time series of the leading low-frequency modes. Then we compare them with the parameterized patterns that are obtained by applying $L_{f, 1}$ and $L_{f, 2}$ onto the streamfunction patterns of these low-frequency modes.

As an example, the observed and parameterized patterns of $\tilde{\psi}^{2, a}$ associated with the AAO is shown in Fig. 7. The observed pattern of $\tilde{\psi}^{2, a}$ in Fig. 7a reflects AAO-related shifts in the storm track location and change in its intensity. It agrees with the corresponding parameterized pattern of $L_{f, 2} \tilde{\psi}$ to a reasonable degree, although significant discrepancy remains. Similarly, a typical example is shown in Fig. 8 to illustrate a reasonable agreement among the patterns of the observed anomalous covariance $(\tau = 0)$ and lagged covariance field $\tilde{\psi}(x, y, t) \tilde{\psi}(x, y, t - \tau)$ with those obtained through $L_{f, 2} \tilde{\psi}^{a}$. These anomalies provide examples for AAO-related changes in the synoptic eddy patterns and propagations, whereas the corresponding fields of the basic synoptic flow are shown in the appendix (Fig. A4). The base point for the covariance field is marked by a cross in Fig. 8a. The structural changes of synoptic eddy flow associated with the AAO, as measured by the covariance maps (Figs. 8a and 8b) and lagged covariance maps (Figs. 8c and 8d), are well captured. The parameterized patterns of anomalies in the eddy variance, covariance, and lagged covariance for AO and PNA also resemble those observed, but to a lesser degree (not shown).

5. Teleconnections and SELF feedback

Localized external forcings produce remote responses owing to Rossby wave dispersion in the climatological background flow (Hoskins and Karoly 1981). However, it is well documented that remote responses in the middle latitudes are partly attributable to the so-called eddy forcing (e.g., Held et al. 1989). With the newly proposed SELF closure, we here briefly revisit this issue to illustrate the importance of SELF feedback as an internal dynamical process in regulating the atmospheric teleconnections.

We conducted a set of forced experiments with Eq. (13). Without the eddy feedback, a Rossby wave train is generated by a localized divergent forcing located at the central equatorial Pacific, which is specified in a similar manner to Branstator (1985). It gives rise to a
modest remote response in the midlatitudes (Fig. 9a). With the inclusion of the SELF feedback, the midlatitude response is significantly enhanced (Fig. 9b). The difference between the responses with and without the SELF feedback, as shown in Fig. 9c, has a significant projection onto the PNA-like pattern. Therefore, the forced Rossby wave train tends to perturb the synoptic eddy activity in the stormy basic flow in an organized manner to intensify the remote response. In other words, the SELF feedback is an important part of the relay for the atmospheric teleconnections.

The signature of PNA-like pattern in Fig. 9c also implies that leading dynamic modes of the system (13) may be of relevance to the observed low-frequency mode such as the PNA pattern. This detailed analysis of the leading dynamical modes of the system (13) will be reported in Part II.

6. Conclusions

In this paper, we propose a dynamic SELF closure that relates the time-mean anomalies in the synoptic eddy forcing to the time-mean anomalies of the flow. The central concept proposed in this paper is to consider a basic stochastic flow that captures climatological mean flow through its ensemble-mean flow and climatological measures of the synoptic eddy statistics through its ensemble-mean covariance fields. We viewed the evolution of the atmospheric circulation as a particular realization of a hypothetic stochastic flow ensemble. The ensemble-mean flow and ensemble-mean variance/covariance fields of the stochastic flow are assumed to be approximately equivalent to the time means of the particular realization of the atmospheric circulation over a time interval much longer than the typical synoptic eddy lifetime scale. Using observed daily streamfunction at 500 hPa as an example, we reconstructed the stochastic basic flow (cf. appendix A).

By considering the stochastic basic flow, we took an anomaly modeling approach for the SELF feedback. Using a barotropic vorticity equation framework, we demonstrated that the SELF feedback is approximately governed by a set of coupled linear equations for the low-frequency anomalies related to the first and second moments of the stochastic atmospheric flow. Under a quasi-equilibrium approximation, a linear closure for the SELF feedback is derived. With the dynamically derived SELF feedback closure, a new linear dynamical framework is established for studying the dynamics of the low-frequency variability in a stormy background flow that realistically captures the climatological mean flow and climatological measures of the synoptic eddy flow.

In deriving this dynamic closure, we use the ergodic
assumption to relate the time mean to the ensemble mean, which relies on the time-scale separation between the synoptic eddy flow and low-frequency flow, the latter being represented by time-mean flow anomalies in this paper. This is one of the main sources of inaccuracy for the linear dynamic closure, particularly when the interval for the time mean is not sufficiently long. The reasonable success of the closure in capturing the observed relationship between the leading low-frequency modes and their associated synoptic eddy-forcing patterns in the monthly mean anomaly fields provides supports for our approach. It should be pointed out that the successful validation of our dynamic closure can be extended to other independent data periods. Moreover, our definition of the climatological mean flow can also be extended to the seasonally varying basic flow, namely, including seasonal cycles in both climatological mean flow and climatological measures of the synoptic eddy flow. This will allow a more refined validation of the closure and the results will be reported elsewhere (Pan and Jin 2005).

By contrasting the steady responses to a remote tropical forcing with and without the SELF feedback, we illustrated that the SELF feedback plays a profound role in the atmospheric teleconnections that involve the excitation of the well-known prominent low-frequency modes such as the PNA pattern. As will be demonstrated more explicitly in Part II, our results also suggest that low-frequency modes may emerge amidst a turbulent atmospheric flow as a result of the dynamic self-organization in the sense that they get self-reinforcement by perturbing the turbulent synoptic activity in a systematic manner to draw energy from the stochastic basic eddy flow through the SELF feedback. With the inclusion of this internal dynamic mechanism that enables the upscale transfer of energy from the basic eddy flow to the low-frequency anomalous flow, our new linear dynamic framework with the SELF closure may become a useful tool for understanding the basic dynamics of the low-frequency variability.

The redistribution of energy through interactions among motions of different scales is one of the key difficulties and challenge issues of long historic interests not only in understanding the dynamics of interactions between motions of planetary scales and synoptic-scales in the atmosphere, but also in understanding the dynamics of interactions between motions of gyre scales and mesoscales in the ocean. It is demonstrated in Part III that our approach can be extended to a general primitive equation framework. Similarly, our approach is potentially applicable for formulating linear frameworks that may become useful for understanding the low-frequency variability in the ocean gyre circulations.

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Reconstruction of a Gaussian Basic Eddy Flow

To define and reconstruct the so-called basic ensemble of the eddy flow $\Psi'$, we hypothesize that we may use a multivariate Gaussian flow to represent the basic nature of the observed synoptic eddy flow. We propose the following expression:

$$\Psi'(x, y, t) = \sum_{n=1}^{N} \sigma_n \xi_n(t) E_n(x, y)e^{i\omega_nt} + \text{c.c.}, \quad (A1)$$

where c.c. stands for complex conjugate. The above expression consists of a number of typical, independent, propagating patterns $[E_n(x, y)e^{i\omega_nt}, n = 1, N]$ associated with a set of independent time coefficients $[\xi_n(t), n = 1, N]$. The contribution to the total variance by each of these independent propagating patterns is measured by $\sigma_n^2$. The complex time coefficients $\xi_n(t)$ is an independent Gaussian red-noise process:

$$\frac{d\xi_n}{dt} = -\frac{\xi_n}{\tau_n} + w_n(t), \quad (A2)$$

where $w_n(t)$ represents complex white noise normalized such that each $\xi_n(t)$ is of unit variance and $\langle \xi_n(t)\xi_{m}(t) \rangle = \delta_{n,m} (\delta_{n,m} = 0, n \neq m; \delta_{n,n} = 1)$. Here $\langle \cdots \rangle$ denotes the ensemble mean while $\tilde{\xi}_n(t)$ is the complex conjugate of $\xi_n(t)$.

To ensure that $\Psi'$ captures the climatological covariance fields of the observed synoptic flow $\psi'$, we determine the entire set $(E_n, \sigma_n, \omega_n, \tau_n; n = 1, N)$ by performing a complex empirical orthogonal function (CEOF) analysis (Barnett 1983) to daily observations of $\psi'$. The 500-hPa streamfunction data, derived from National Centers for Environmental Prediction–National Center for Atmospheric Research (NCEP–NCAR) reanalysis (Kalnay et al. 1996), are interpolated onto a T21 Gaussian grid. After subtracting the monthly mean from the daily data, a bandpass filter by Murakami (1979) is applied, which retains high-frequency variability with periods of 2–8 days. In this paper, the daily streamfunction field for the period from January 1979 to December 1995 is used. There is no specific reason for using this relatively short 17-yr duration of the daily dataset; our basic results in characterizing climatological features of the eddy flow will not have substantial changes with a longer dataset.

With the CEOF expansion, $\psi'$ can be expressed as

$$\psi'(x, y, t) = \sum_{n=1}^{N} A_n(t) E_n(x, y) + \text{c.c.} \quad (A3)$$

Here $N$ is the total number of CEOFs, $E_n(x, y)$ is the $n$th CEOF pattern, and $A_n(t)$ is the corresponding com-
plex time coefficient, represented as $A_n(t) = |A_n(t)|e^{i\phi_n(t)}$, where $|A_n(t)|$ and $\phi_n(t)$ are the amplitude and phase, respectively. From each time series of $A_n(t)$, a linear fit for the constant rate of phase change, $\omega_n$, is calculated so that $A_n(t) = |A_n(t)|e^{i\omega_n t + i\phi_0}$ with $\phi_0(t)$ describing the residual phase fluctuations. The climatological variance of each $A_n(t)$ is $|A_n(t)|^2$, which is equated to $\sigma_n^2$. Furthermore, each $\tau_n$ is estimated from the $e$-folding time in the autocorrelation of the time series $|A_n(t)|$. Some typical examples of these autocorrelations are shown in Fig. A1. Thus, the entire set $(E_n, \sigma_n, \omega_n, \tau_n, n = 1, N_c)$, serving as the basis functions and parameters for the representation in (A1)–(A2), are completely determined from the data.

A major and conceptual difference between (A1) and (A3) is that each observed time series $|A_n(t)|e^{i\phi_n(t)}$ in (A3) is substituted by an independent and idealized red noise process $\sigma_n \xi_n(t)$ in (A1). With this substitution, the stochastic basic flow ensemble $\Psi'$ in (A1) becomes a multivariate Gaussian flow with $2N_c$ degrees of freedom. Another difference is that we only consider a rather small number $N_c$, $N_c \ll N$ because the majority of the variance is normally explained only by a small set of leading EOFs. From the variance spectra for both hemispheres (Figs. A1a,b), if $N$ is truncated at 20 it still explains more than 70% and 80% of the total variance for NH and SH, respectively.

In the above representations of the transient eddies in the storm track, the main characteristics of the synoptic eddy flow include the spatial structures $E_n(x, y)$ and the associated variances $\sigma_n^2$, the primary frequencies $\omega_n$, and the $e$-folding time scales $\tau_n$. An example of the leading CEOF patterns is presented in Fig. A2. The first CEOF of the NH synoptic eddy field explains about 10% of the total variance, whereas the second EOF accounts for about 9% of the total variance. The real and imaginary parts of these complex EOFs have a nearly 90° phase shift in the zonal direction, indicating that these wave packetlike patterns propagate eastward at a primary angular phase speed $\omega_n$ about 60° day$^{-1}$. Since the dominant wavenumber is 5–7 in the patterns shown in Fig. A2 this value of $\omega_n$ corresponds to a phase speed of roughly 10 m s$^{-1}$. As shown in Table A1, the $e$-folding decay time $\tau_n$ is 3.5 days and the primary frequency $\omega_n$ is also nearly a constant for the first 10 EOFs. The short autodecay time scale of the tran-
sient eddies provides an essential demarcation that separates the short-lived weather activity from the low-frequency variability.

The reconstructed basic eddy flow \( \Psi'(x, y, t) \) must capture all of the major climatological properties of the synoptic eddy flow \( \psi' \). All statistical properties of \( \Psi'(x, y, t) \) can be analytically derived from (A1), once the set \( \{ E_n, \sigma_n, \omega_n, \tau_n; n = 1, N_c \} \) is given. For instance, using (A1) the ensemble mean variance field, eddy-induced vorticity forcing field, and covariance field of the eddy flow \( \Psi'(x, y, t) \) can be expressed as

\[
\langle \Psi'^2 \rangle = 2 \sum_{n=1}^{N_c} \sigma_n^2 |E_n(x, y)|^2
\]

\[
\langle J(\Psi', \Delta \Psi') \rangle = \sum_{n=1}^{N_c} \sigma_n^2 J(E_n, \Delta E_n) + \text{c.c.}
\]

\[
\langle \Psi'(x, y, t)\Psi'(x, y, t) \rangle = \sum_{n=1}^{N_c} \sigma_n^2 E_n(x, y) \hat{E}_n(x, y, \omega_n, \tau_n) + \text{c.c.}
\]  

(A4)

Here \( \hat{E}_n \) is the complex conjugate of \( E_n \). It is straightforward to prove that the quantities in (A4) are good approximations for the corresponding climatological statistical properties, such as \( \psi'^2 \) and \( J(\psi', \Delta \psi') \), that can be calculated from observed \( \psi' \) directly following the definition of the climatological average as long as \( N_c \) is sufficiently large. The results in Fig.A3 are eddy-induced streamfunction forcing \( -\Delta^* \mathcal{J}(\Psi', \Delta \Psi') \), which is in excellent agreement with that in Fig. 1, as expected. Similarly, as a typical example shown in Fig.A4a,b, the one-point covariance maps obtained from the basic eddy flow \( \Psi' \) using the third equation in (A4) are indeed almost the same as that from the observed eddy flow \( \psi' \) using the definition of \( \hat{\psi}'(x, y, t) \hat{\psi}'(x, y, t) \).

The quantities calculated in (A4) only depend on the spatial patterns \( E_n \) and the associated variances \( \sigma_n^2 \); they do not depend on the characteristic time scales \( (\omega_n, \tau_n) \) of the eddy flow. However, these time scales are an important part of the lagged covariance field of the eddy flow \( \Psi'(x, y, t) \), which can be expressed as

\[
\langle \Psi'(x, y, t)\Psi'(x, y, t, \tau) \rangle = \sum_{n=1}^{N_c} \sigma_n^2 E_n(x, y) \hat{E}_n(x, y, \omega_n, \tau_n) \times \exp(-\tau/\tau_n + i\omega_n\tau) + \text{c.c.}
\]  

(A5)

Here \( \tau \) is the lag. As shown in Figs. A4c,d, the climatological lagged covariance maps \( \hat{\psi}'(x, y, t)\hat{\psi}'(x, y, t, \tau) \)

\[\text{FIG. A3. As in Figs. 1b and 1d except for the results based on the basic stochastic representation of the synoptic eddy flow following Eq. (A4).}\]
calculated based on the observed eddy flow is also well captured by that calculated from the synthetic eddy flow $\Psi'(x, y, t)$ using (A5). These results support the concept that the Gaussian basic eddy flow $\Psi'_b$, as expressed in (A1), is an adequate surrogate to represent the climatological statistics of the observed eddy flow $\Psi'$.

**APPENDIX B**

**A Linear Equation for the Anomalous Eddy Flow**

We assumed that $\Psi$ also follows the system similar to Eq. (4) as follows:

$$\frac{\partial}{\partial t} \Delta \Psi + J(\Psi, \Delta \Psi + f) + r \Delta \Psi = Q.$$  \hspace{1cm} \text{(B1)}

Here $Q$ denotes the external forcing. Linearizing this equation with respect to the basic flow $\Psi'_b$, we obtain a linear model for the anomalous ensemble of the stochastic flow:

$$\frac{\partial}{\partial t} \Delta \Psi'_a + J(\Psi'_a, \Delta \Psi'_a + f) + J(\Psi'_b, \Delta \Psi'_a) + r \Delta \Psi'_a = Q'_a.$$  \hspace{1cm} \text{(B2)}

Here $Q'_a$ is an anomalous forcing with respect to a basic forcing $Q_c$, which maintains the basic flow $\Psi'_b$ as the solution of the model. By taking the ensemble average of the above equation and applying the ergodic approximations $(\overline{\Psi'}) \approx \overline{\psi'}$, $(\overline{\Psi'_b}) \approx \overline{\psi'_b}$, and $(\overline{Q'_a}) \approx \overline{Q'}$, we obtain the following equation for the $\overline{\psi'}(x, y, t)$:

$$\frac{\partial}{\partial t} \Delta \overline{\psi'} + J(\overline{\psi'}, \Delta \overline{\psi'}) + J(\overline{\psi'_b}, \Delta \overline{\psi'}) + f + r \Delta \overline{\psi'} + \langle J(\Psi'_a, \Delta \Psi'_a) \rangle + \langle J(\Psi'_b, \Delta \Psi'_a) \rangle = \overline{Q'}.$$  \hspace{1cm} \text{(B3)}

This equation becomes the same as Eq. (5) in section 3. By subtracting this ensemble mean equation from the Eq. (B2), we obtain the following equation for anomalous ensemble of the eddy flow $\Psi'_a$:

$$\frac{\partial}{\partial t} \Delta \Psi'_a + J(\overline{\psi'_b}, \Delta \Psi'_a) + J(\Psi'_a, \Delta \Psi'_a + f) + r \Delta \Psi'_a = Q'_a - J(\overline{\psi'}, \Delta \Psi'_a) - J(\overline{\psi'_b}, \Delta \overline{\psi'}) - \langle J(\Psi'_a, \Delta \Psi'_a) \rangle - \langle J(\Psi'_a, \Delta \Psi'_a) \rangle.$$  \hspace{1cm} \text{(B4)}
For simplicity, we will ignore the high-order terms in the third line, then we obtain Eq. (8) used in section 3. Under the intensity of the observed storm track activity, we found that the impact of these high-order temporal terms is indeed small (Jin and Lin 2006).

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