Dynamics of Synoptic Eddy and Low-Frequency Flow Interaction. 
Part II: A Theory for Low-Frequency Modes

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ABSTRACT

Amidst stormy atmospheric circulation, there are prominent recurrent patterns of variability in the planetary circulation, such as the Antarctic Oscillation (AAO), Arctic Oscillation (AO) or North Atlantic Oscillation (NAO), and the Pacific–North America (PNA) pattern. The role of the synoptic eddy and low-frequency flow (SELF) feedback in the formation of these dominant low-frequency modes is investigated in this paper using the linear barotropic model with the SELF feedback proposed in Part I. It is found that the AO-like and AAO-like leading singular modes of the linear dynamical system emerge from the stormy background flow as the result of a positive SELF feedback. This SELF feedback also prefers a PNA-like singular vector as well among other modes under the climatological conditions of northern winters.

A model with idealized conditions of basic mean flow and activity of synoptic eddy flow and a prototype model are also used to illustrate that there is a natural scale selection for the AAO- and AO-like modes through the positive SELF feedback. The zonal scale of the localized features in the Atlantic (southern Indian Ocean) for AO (AAO) is largely related to the zonal extent of the enhanced storm track activity in the region. The meridional dipole structures of AO- and AAO-like low-frequency modes are favored because of the scale-selective positive SELF feedback, which can be heuristically understood by the tilted-trough mechanism.

1. Introduction

This paper builds on the concepts described in Part I (Jin et al. 2006) and discusses a mechanism of the low-frequency modes through the synoptic eddy and low-frequency flow (SELF) interaction. We begin by a quick review of the literature, some of which is also presented in Part I.

Decades of research have revealed a number of prominent patterns of the atmospheric low-frequency variability. One of the most significant and recurrent patterns of the Northern Hemisphere (NH) atmospheric variability is the North Atlantic Oscillation (NAO) (Walker and Bliss 1932; van Loon and Rogers 1978; Hurrell 1995; Hurrell et al. 2003). The NAO is also considered as a part of the global-scale pattern known as the Arctic Oscillation (AO) or the NH annular mode (NAM; Thompson and Wallace 1998, 2000; Wallace 2000). Another well-known recurrent pattern of the NH atmospheric variability is the so-called Pacific–North American (PNA) pattern (Wallace and Gutzler 1981). The counterpart of the AO is identified in the Southern Hemisphere (SH) as well, referred to as the Antarctic Oscillation (AAO) or the SH annular mode (SAM) (Gong and Wang 1999; Thompson and Wallace 2000).

Understanding the dynamics of these prominent patterns of the atmospheric low-frequency variability has been an active topic of research. Several mechanisms
have been proposed for the generation of the low-frequency patterns of variability in the atmospheric circulation. For instance, the slow changes in the boundary conditions to the atmosphere, such as changes in sea surface temperature (SST) associated with El Niño–Southern Oscillations (ENSO), serve as a systematic forcing to produce significant low-frequency variability in the atmospheric circulation. The linear Rossby wave propagation and dispersion theory of Hoskins and Karoly (1981) provided a simple mechanism for the remote extratropical responses to the tropical forcing. A great amount of research has been done following this pioneering work (cf. Held et al. 1989; Ting and Held 1990; Ting 1994; Jin and Hoskins 1995). The agreement of wave train patterns in theory and in the observed anomalies related to PNA pattern (Horel and Wallace 1981) supported this mechanism. Another mechanism for the generation of low-frequency variability is the instability of nonzonal flow as proposed by Simmons et al. (1983, hereafter S83). The nonzonal basic flow provides a source of internal energy that favors the development of a localized pattern having PNA-like structure as a result of local trapping of Rossby wave trains. However, these mechanisms are not adequate to explain the existence of hemispheric-scale modes such as AO and AAO.

One well-recognized source of the low-frequency variability is the so-called eddy forcing resulting from high-frequency synoptic disturbances. It was suggested that high-frequency synoptic activity through nonlinear rectification feeds back onto the low-frequency variability (Egger and Schilling 1983; Lau and Holopainen 1984; Lau 1988; Lau and Nath 1991; Branstator 1995; Limpasuvan and Hartmann 1999, 2000; Pan and Jin 2005). The synoptic eddy and low-frequency flow, that is, SELF, feedback had been recognized to play an essential role in generating the low-frequency variability (Robinson 1991; James and James 1992; Yu and Hartmann 1993; Feldstein and Lee 1996; Robinson 2000; Limpasuvan and Hartmann 2000; Pan 2003).

In Part I, we derived a dynamical closure for the SELF feedback and proposed a linear barotropic framework for the low-frequency variability. In Part II, we use this framework to investigate the role of SELF feedback in the formation of the low-frequency modes. In section 2, we analyze the leading dynamical modes of the linear barotropic model through both singular vector and eigenmode analyses. It is demonstrated that the leading singular mode for the NH (SH) winter is quite similar to the observed AO (AAO), whereas this similarity is considerably lost in the absence of the SELF feedback.

We also show that SELF feedback tends to selectively destabilize the leading low-frequency modes. In section 3, we consider idealized specifications of the synoptic activity and climatological zonal mean flow as the basic conditions to further illustrate the essential role of the SELF feedback in the natural selection of AO-like and AAO-like modes. In section 4, we further propose a simple prototype model describing a highly idealized case of wave–zonal flow interaction to delineate analytically the basic mechanisms for the selective positive SELF feedback favoring the AO- and AAO-like modes. The conclusions are given in section 5.

2. Leading low-frequency modes and the SELF feedback

a. Model

In Part I, we proposed a dynamical nonlocal closure to describe the two-way SELF feedback using an anomaly modeling approach by considering a stochastic basic flow. This basic flow consists of the climatological mean basic flow, denoted as $\overline{\psi}(x, y)$, and a stochastic synoptic eddy flow, denoted as $\Psi_c^e$. The latter is expressed as

$$\Psi_c^e(x, y, t) = \sum_{n=1}^{N_c} \sigma_n \xi_n(t) E_n(x, y)e^{imt} + \text{c.c.} \quad (1)$$

Here c.c. stands for complex conjugate. Each pair of $E_n(x, y)e^{imt}$ represents an independent propagating pattern with angular frequency $\omega_n$, characterized by three other parameters: a fractional variance to the stochastic eddy flow, $\sigma_n$, and complex time coefficient $\xi_n(t)$, which follows an independent Gaussian process with an $e$-folding time $\tau_n$. The entire set $(E_n, \sigma_n, \tau_n, \omega_n; n = 1, N_c)$, which captures the observed climatological characteristics of the synoptic eddy flow, is determined from daily 500-hPa streamfunction data obtained from the National Centers for Environmental Prediction–National Center for Atmospheric Research (NCEP–NCAR) reanalysis for the cold season (April–November for NH and May–October for SH) during 1979–95. Interested readers should refer to Part I for details.

Based on the results obtained in Eq. (11) of Part I, the linear barotropic model for anomalous time–mean flow can be expressed as

$$\frac{\partial}{\partial t} \Delta \overline{\psi}^a + L \overline{\psi}^a + f \overline{\psi}^a = \mathcal{Q}^a \quad (2)$$

Here $\overline{\psi}^a$ denotes the streamfunction field of the time–mean flow anomalies, or loosely the low-frequency anomaly, whereas $\mathcal{Q}^a$ represents the external forcing. For any field, the linear operator $L$ is defined as

$$LX = J(\overline{\psi}, \Delta X) + J(X, \Delta \overline{\psi} + f) + r \Delta X \quad (3)$$
where $J$ is the Jacobian operator, $f$ is the Coriolis parameter, and $\bar{c}$ is the observed climatological mean field of the streamfunction. The linear friction coefficient $r$ is set at 1 (10 days)$^{-1}$. The only new term in Eq. (3), which makes it different from the traditional linear barotropic model such as that of S83, is the eddy feedback term $L_f \hat{\psi}$. Utilizing Eq. (10) of Part I and the so-called quasi-equilibrium approximation, we can express this extra term as follows:

$$L_f \hat{\psi}_n = \sum_{n=1}^{N_c} \sigma_n^2 (J(\hat{\psi}_n, \Delta \hat{E}_n) + J(\hat{E}_n, \Delta \hat{\psi}_n)) + c.c.$$  

$$L_f \hat{\psi}_n + (\tau_n^{-1} + i\omega_n) \Delta \hat{\psi}_n = -J(E_n, \Delta \hat{\psi}_n) - J(\hat{\psi}_n, \Delta E_n),$$  

$n = 1, N_c$.  

(4)

Here $\hat{E}_n$ is the complex conjugate of $E_n$. Each component $\hat{\psi}_n$ corresponds to a complex pattern that is related to the covariance between anomalous and basic stochastic eddy flows. Eliminating $\hat{\psi}_n$ by inverting the equations in the second line of Eq. (4), we obtain the expression for SELF feedback operator $L_f$ as shown in Eq. (13) of Part I. Equations (2)–(4) thus form a linear barotropic model for modeling low-frequency flow anomalies with the SELF interaction.

b. Singular value decomposition analysis

It has been shown that the steady response to spatially white forcing in the streamfunction field is dominated by the left singular vectors of the linear dynamical $L + L_f$ with smallest singular values (cf. Branstator 1990; Navarra 1993; Itoh and Kimoto 1999; Watanabe and Jin 2004). Thus, to investigate the origin of the low-frequency modes, we perform singular vector analysis to the linear operator of the system $L + L_f$ by means of singular value decomposition (SVD). In this section, we adopt a spectral model version of the system (2)–(4) with T42 resolution for all numerical calculations.

The leading singular modes of the dynamical system (2) are shown in Figs. 1 and 2, respectively. The spectra
of the singular values suggest that there is one distinct leading singular mode in each hemisphere. Although the magnitudes of the leading singular value depend on the linear damping rates, the patterns of the singular modes are robust to the choice of this relatively arbitrary parameter of the model.

The leading singular mode in the SH has a dipole component of the zonal flow and it resembles the observed pattern of the AAO as seen in Fig. 3a of Part I. In addition to the dipole component of the zonal flow, the observed streamfunction pattern associated with the AAO has some pronounced stationary wave features dominated by wavenumbers 2 and 3, as marked by two negative centers in the polar region and three positive centers in the midlatitude belt (Fig. 3a of Part I). Remarkably, the simulated AAO-like leading singular mode presented in Fig. 1a also has similar features, with arguably two negative centers in the polar cap and three positive centers in the midlatitude belt, although the centers in the solution here are somewhat shifted and weaker when compared to observations.

Similarly, the leading singular mode (Fig. 2a) in the NH resembles the AO/NAO, which also accompanies a zonal mean flow anomaly (Fig. 2b) in the leading mode. The dipole structure in the streamfunction becomes locally pronounced over the Atlantic sector, akin to the NAO. When it is compared with the observation as shown in Fig. 3b of Part I, the simulated center in the Atlantic is relatively weak and shifted toward the eastern Atlantic. Nevertheless, the dipole in the zonal flow and local enhancement over the Atlantic sector of the observed AO/NAO are qualitatively captured by this leading mode.

To isolate the role of the SELF feedback, we examine the leading modes of the dynamical system without $L_f$. In this case, the linear model described in Eq. (2) returns to its traditional form (e.g., S83 model), except for a different choice of the basic state. Without $L_f$, the leading singular modes of the barotropic model linearized with respect to the climatological mean flow of NH or SH winter circulation, as shown in Figs. 3 and 4, bear little resemblance to the AO or AAO. The second mode in the NH has a dipole structure in the zonal mean wind, but the circulation pattern is still drastically
different from the observed AO (not shown). The above results suggest that in the barotropic framework (1) the SELF feedback is responsible for generating the AO- and AAO-like leading singular vectors as the leading low-frequency modes of the system. Although the leading singular vector of baroclinic models without SELF feedback may exhibit some AO likeness as well (Kimoto et al. 2001; Watanabe and Jin 2004), we found in Pan et al. (2006, hereafter Part III) that the SELF feedback play an important role in the generation AAO- and AO-like modes in the baroclinic framework (Part III).

In the system (2), the AAO- and AO-like modes have two major energy sources in the basic flow, namely, the climatological mean flow and the basic synoptic eddy flow. Both of them are also the sources for zonal asymmetries seen in the AAO- and AO-like modes. The climatological mean flow is not zonally uniform owing to the presence of stationary waves. The storm tracks are not zonally uniform, even in the SH, whereas there are so-called Pacific and Atlantic storm tracks in the NH. These two zonal asymmetries in the basic stochastic flow are ultimately due to topography and land–sea contrasts underlying the atmosphere; they are dynamically related, as can be inferred from the collocations of the jet streams and storm tracks in the basic state as seen in Fig. 1 of Part I, for example. In our linearized framework, these two parts of basic flow and the asymmetries in this flow are prescribed. Thus, we can examine separately the impacts of these asymmetries on the leading low-frequency modes.

Because we have already examined the case without $L_f$, that is, without the climatological storm track activity in the basic state thus without the energy source from the basic synoptic eddy flow, we now study the case without the effect of stationary waves of the basic flow in the linear operator $L_f$. We do so by considering only the zonal mean part of the climatological flow $\overline{\psi}_c$ for the linear advection terms $J(\overline{\psi}_c, \overline{\psi}_a^u) + J(\overline{\psi}_c^u, \overline{\psi}_c + f)$ in the linear operator $L$ of Eq. (2). In this way, we have suppressed the S83 mechanism of nonzonal flow instability for low-frequency modes. All asymmetric features of the leading modes in this case can be completely attributed to the zonal asymmetry associated with $L_f$. For the SH leading mode, as shown in Fig. 5, there is a pronounced local enhancement between the Australia and Antarctic sector, downstream of the location of the maximum storm track intensity. This localized dipole feature is present in the AAO-like mode and can now be attributed to the zonal variation in the SELF feedback. In the NH case, we again find that the leading mode (Fig. 6) is an AO/NAO-like mode and the solution is very similar to that shown in Fig. 2, indicating that the local features in the northern Atlantic thus are also attributable to the SELF feedback. The modest differences in the patterns of the singular modes shown in Figs. 1 and 2 and Figs. 5 and 6 indicate that the SELF interaction controls the main features of the leading modes of Eq. (1).

In addition to the leading AAO- and AO-like modes, other singular modes also reveal some interesting features. For instance, the third mode of system (1) under the same background conditions as the case shown in Fig. 6 has a significant projection onto the observed PNA pattern (Fig. 7). The third singular mode under the background conditions as in the case shown in Fig. 2 also contains the PNA-like structure in the Pacific–North America sector. Thus, the SELF feedback plays an important role in the generation of PNA-like teleconnection pattern, as also indicated in an example in Part I.

c. Eigenanalysis

The SVD analysis to the linear dynamical system implies that spatially random forcing will preferentially

![Fig. 5. As in Fig. 1 except that the nonzonal part of the time-mean flow in the linear advection terms of Eq. (1) is eliminated.](https://example.com/fig5.png)
excite the most singular, or near-neutral, modes of the system. The leading singular modes are often similar to the least damped eigenmodes of the linear dynamical operator \( L + L_f \), particularly when we focus on the low-frequency modes of zero or near-zero frequencies. In other words, the least-damped eigenmodes of \( L + L_f \) also contain AO-, PNA-, and AAO-like features. For brevity, we will not show the patterns of leading eigenmodes of the linear dynamical system (1). Instead, we examine how the leading eigenvalues change between the cases without and with the SELF feedback. This is done by considering an artificial nondimensional parameter \( \mu \), varying from zero to one and then tracing the numerically changes in eigenvalues of \( L + \mu L_f \) as the function of \( \mu \).

The decay rates of the leading low-frequency modes systematically reduce as \( \mu \) increases (Fig. 8a) from zero to one. For the least damped modes in both NH and SH (Fig. 8b), the decay rate is reduced from about 8–9 days\(^{-1}\) to 11–12 days\(^{-1}\), indicating that the SELF feedback serves as a significant positive feedback corre-

**3. Results of an idealized model**

To gain better understanding of the origin of AO- and AAO-like modes, we further simplify the background conditions for the linear dynamical system \( L + L_f \) as follows. First, we only consider the zonal mean of the climatological mean flow as the basic flow in both \( L \) and \( L_f \). Second, the set \( (E_n, \sigma_n, \tau_n, \omega_n; n = 1, N_c) \), which gives the measures for the climatological synoptic eddy activity, is greatly simplified by considering only one idealized wave stochastic wave packet with the following specifications:

\[
E(\lambda, \varphi) = F(\lambda)G(\varphi)e^{i\sigma_1(\varphi - \omega_1 t)}, \quad \sigma = 3\sigma_1, \quad \tau = \tau_1, \quad \omega = \omega_1, \quad N_c = 1.
\]

Here \( F(\lambda) \) is an envelop function prescribed for describing the zonal extend of this single idealized packet of
synoptic wave activity, \( G(\varphi) = |\mathbf{E}_1(\lambda, \varphi)|^2 \), which is the zonal-averaged meridional structure of the first complex EOF (COEF) mode for synoptic eddies; \( m \) is its zonal wavenumber, which is set to be six; \( \sigma \) is the level of the variance for the single pattern and it is set to be a factor of 3 of that for the first observed CEOF so that the total level of eddy variance in this idealized storm track is on the same order as what is observed. The phase propagation speed and autodecay time scale of this stochastic wave packet are set the same as the first observed CEOF. The details for \( (E_n, \sigma_n, \omega_n; n = 1, N_e) \) are given in appendix A of Part I.

For all cases of the idealized eddy packet with different choices of \( F(\lambda) \), as shown in Fig. 9a, there are always dipole structures in the meridional distributions of the zonal-mean flow components of the leading modes (Fig. 9b). We also found that the leading mode changes from a zonal symmetric mode [when \( F(\lambda) = 1 \)] to a locally intensified mode (Figs. 9c–e) over the storm track region, as the idealized eddy packet becomes gradually localized following the choices of \( F(\lambda) \). The zonal scale of the locally intensified pattern varies proportionally with the specified zonal extent of the background synoptic eddy activity. When the storm track activity becomes localized, the leading mode (Figs. 9c–e) acquires a localized dipole pattern reminiscent of the AO-like mode in section 3.

Moreover, the second leading modes of all of these cases are similar to the first leading modes except that dipole structures in the meridional distributions in zonal-mean flow components and in their zonal asymmetric components are all replaced by tripole structures (not shown). This selection of the dipole and tripole meridional scales is related to the zonal extent of the storm track. This fundamental dipole feature of AO-like modes is due to the scale-selective positive SELF feedback, as will be demonstrated in section 4.

4. A prototype SELF interaction model for AAO and AO

In sections 2 and 3, we used the linearized barotropic model with SELF interaction to demonstrate that the SELF feedback is responsible for the generation of AAO- and AO-like modes that dominate the low-frequency variability in the NH and SH. Motivated by these results, we undertake a simpler analysis of the SELF feedback under a highly idealized situation to provide an analytical theory to explain the meridional scale selections of AO- and AAO-like modes.

a. Analytical model

To extract essence, we make the following simplifications. We consider a \( \beta \)-plane channel version of the
barotropic vorticity equation in Cartesian coordinates. The climatological flow $\vec{\psi}$ is now given as

$$\vec{\psi} = -y\vec{\pi}^\epsilon,$$

and $\vec{\pi}^\epsilon$ is a constant zonal flow. The stochastic basic eddy flow for the climatological storm track activity is also drastically simplified by considering only a single propagating wave:

$$\Psi'_e = \xi(t)G(y)e^{ikx} + \text{c.c.},$$

where $\Psi'_e$ is an idealized representation of the storm track with zonally uniform variance, $G(y)$ is the meridional structure of the transient wave, $k$ is the zonal wavenumber, and $c$ is the propagation speed. As in Eq. (1), we assume that $\xi(t)$ represents a complex normalized red noise process. Therefore, in this simple case, the set $(E_n, \sigma_n, \tau_n, \omega_n; n = 1, N_e)$ for characterizing the background synoptic eddy activity is reduced to

$$E_1(x,y) = G(y)e^{ikx}, \quad \sigma_1 = 1, \quad \tau_1 = \tau, \quad \omega_1 = kc;$$

$$N_e = 1,$$

where $\tau$ is the $e$-folding time of the autocorrelation function from $\xi(t)$.

Substituting Eqs. (5) and (7) into Eqs. (2)–(4), we can obtain a linear equation for the low-frequency flow anomaly with respect to this simple stochastic basic flow $\vec{\psi} + \Psi'_e$. For further simplicity, we focus on the zonal mean part of the low-frequency flow anomaly $\vec{\psi}'$. Thus, we only consider the zonal mean part of Eq. (1), which can be rewritten as

$$\partial\vec{\pi}'/\partial t = -r\vec{\pi}' - 2k\omega \Re(i\vec{G}\partial\vec{\psi}'/\partial y + i\vec{\psi}'\partial G/\partial y)/\partial y,$$

Fig. 9. (a) Different zonal distributions $[F(\lambda)]$ of the synoptic wave packet in dash, dot, and solid curves, representing zonally uniform, wide, and narrow storm tracks, respectively. (b) Meridional distributions of zonal-mean zonal flow of leading SVD modes in dash, dot, and solid curves, corresponding to the choices of $F(\lambda)$ in (a). Streamfunction patterns of the leading SVD modes under the basic flows with the (c) zonally uniform, (d) wide, and (e) narrow storm tracks, according to the three choices of $F(\lambda)$ in (a) in dash, dot, and solid curves, respectively.
where \( \Pi^a = -\partial \tilde{\Omega}^a / \partial y \), denoting the low-frequency zonal-mean flow anomaly. In the above equation for \( \Pi^a \), the last term, obtained by integrating with respect to \( y \) for the zonal mean of Eq. (4), represents the anomalous convergence of the ensemble mean of eddy momentum flux. In Eq. (8), \( \hat{\psi} \) is the complex conjugate of \( \psi(y, t) \) and \( \tilde{\psi}(y, t) \), which is governed by the following equation:

\[
\left( \Pi_c - c \hat{\psi} \frac{\partial^2}{\partial y^2} - k^2 \right) + \beta - \left[ i(r + r_s)/k \left( \hat{\psi} \frac{\partial^2}{\partial y^2} - k^2 \right) \right] \hat{\psi} = -\left[ \Pi_c \left( d^2/\partial y^2 - k^2 \right) G - G \tilde{\eta} \right]/\eta.
\]

This is a simplified version of Eq. (4) by setting \( \hat{\psi}_1 = \hat{\psi}(y, t) e^{i \omega t} \) and \( r_c = 1/\tau \). Equations (8)–(9) describe the zonal wave interaction with respect to the stochastic basic flow \( \tilde{\psi} + \Psi' \). The zonal flow anomaly \( \Pi^a \) induces an anomalous stochastic Rossby wave through anomalous advectations of the basic stochastic eddy flow \( \Psi' \). Such an anomalous Rossby wave is related to \( \xi(t) \) and their covariance field \( \hat{\psi}_1 \) thus follows the variations in \( \Pi^a \). The amplitude of \( \hat{\psi}_1, \hat{\psi}(y, t) \), is described by Eq. (9). This anomalous Rossby wave leads to an anomaly in the ensemble mean of eddy momentum flux, which feeds back to the zonal flow anomaly \( \Pi^a \), as described by the last term in Eq. (8).

b. Tilted trough mechanism and SELF feedback

Eliminating \( \hat{\psi}_1 \) in the Eq. (8), one obtains the SELF closure for \( \Pi^a \). The closure operator is still rather complicated, and we will assess the effect of the SELF feedback by directly solving Eqs. (8)–(9). We consider the following approximations:

\[
\partial^3 \Pi^a / \partial y^2 = -l^2 \Pi^a, \quad d^2 G / \partial y^2 = -l_e^2 G.
\]

Here, \( l^2 \) and \( l_e^2 \) represents typical meridional scales for anomalous zonal mean flow and the stochastic synoptic wave, respectively.

When \( l^2 \gg l_e^2 + k^2 \), the meridional scale of the zonal flow anomaly is much smaller than the scale of the synoptic eddies in the basic flow. In this case, Eq. (9) can be approximated as

\[
[\Pi_c - c - i(r + r_s)/k] \hat{\psi} = G \Pi^a.
\]

The anomalous eddy flow is mainly due to the vorticity advection of the anomalous zonal flow by the basic synoptic eddy flow.

If we only keep the dominating term of the anomalous eddy momentum flux in Eq. (8), we get

\[
\partial \Pi^a / \partial t = -\kappa \Pi^a + \partial \Pi^a / \partial y,
\]

where the eddy viscosity coefficient is

\[
\kappa = 2[kG(r + r_s) + [1 + k(r - c)/r_s])].
\]

If the eddies are nearly isotropic so that \( l_e \sim k \), then \( 2[kG(r + r_s) \) is a measure of eddy kinetic energy and the eddy diffusion coefficient is thus proportional to the total eddy kinetic energy in the basic state. Here \( \kappa \) depends also on \( k(r - c)/(r + r_s) \), the ratio of the advection (propagation) and decay time scales of the synoptic eddies. Normally this ratio is larger than unity. If \( c = \Pi_c - \beta(l_e^2 + k^2) \) and \( k(r - c)/(r + r_s) \gg 1 \), then \( \kappa = 8|kG[l_e^2(r + r_s)/\beta^2] \). Setting the linear friction time scale to be about 10 days, the e-folding time of eddy lifetime to be about 3–4 days, \( 2[kG]^2 \) to be \( O(100 \text{ m}^2 \text{s}^{-2}) \), and \( l_e \) to be \( O(10^6 \text{ m}) \), we find that \( \kappa \approx 10^6 \text{ m}^2 \text{s}^{-1} \). This is a significant eddy viscous damping for zonal flow anomalies with relatively small spatial scales \( l^2 \gg l_e^2 + k^2 \).

The schematic diagram in Fig. 10 illustrates how the SELF feedback tends to damp the zonal flow anomalies of relatively small meridional scales. Let us start with an initial zonal flow anomaly with an anticyclonic shear or a negative vorticity. Since the meridional scale of the zonal flow is smaller than that of the stochastic wave in the basic flow, the anomalous eddy flow is mainly owing to the meridional advection of vorticity in the anomalous zonal flow by the stochastic wave in the basic flow. The anomalous eddy flow thus has a similar meridional scale to that of the zonal flow anomalies. Superimposed on the stochastic wave in the basic flow, it results in systematic NW–SE tilted troughs and ridges, as shown.
in the third column of Fig. 10. The ensemble and zonal mean of these tilted transient waves induce a net ensemble-mean and zonal mean northward eddy-momentum transport in the perturbed region, which act to reduce the initial vorticity in the zonal flow anomaly. Thus amidst the idealized basic synoptic eddy field representing the climatological storm track, the zonal flow modes with the meridional scales much smaller than the meridional scale of synoptic eddies are damped by the eddy viscosity due to this negative SELF feedback.

Now consider another case in a different flow regime where the zonal-mean flow anomalies have relatively large meridional scales; that is, \( l^2 \ll l_z^2 + k^2 \). In this case, we have the following approximation from Eq. (9),

\[
\{[\pi^i - c - \beta/(l_z^2 + k^2)] - i(r + r_s)/k\} \tilde{\psi} = -\pi^i G.
\]

Now the eddy flow is mainly the result of the zonal advection of the vorticity in the stochastic basic wave by the anomalous zonal-mean flow. The two terms for the anomalous eddy momentum fluxes in the zonal mean flow Eq. (8) are of the same order. We now get the following approximate equation:

\[
\frac{\partial \tilde{\Pi}}{\partial t} = -\sigma \tilde{\Pi} + \partial_x \partial_y \tilde{\Pi} - \kappa^{\gamma} - 2[kG]^2(r + r_s)^{-1}.
\]

Here we have assumed \( c = \pi^i - \beta/(l_z^2 + k^2) \). If we again consider that \( 2[kG]^2 \) is \( O(10^7 \text{m}^2 \text{s}^{-2}) \) and the eddy e-folding time is \( O(3-4 \text{ days}) \), we obtain \( \kappa^{\gamma} \sim -10^7 \text{m}^2 \text{s}^{-1} \). This is a significant negative viscosity for relatively large-scale zonal flow anomalies. Though \( l^2 \) small for zonal-mean flow anomalies in this flow regime, the destabilization for the zonal-mean flow anomalies due to this positive SELF feedback still can be significant.

Conceptually, this positive feedback can be also understood by the tilted trough mechanism. As illustrated in Fig. 11, an initial sheared zonal flow anomaly systematically alters the synoptic waves in the climatological storm track through the advective process. This leads to a systematic NE–SW tilting in eddies. The systematic tilted synoptic eddies in this case further results in an enhancement of the net ensemble- and zonal-mean momentum fluxes. As the result, the altered storm track has a net effect to reinforce the initial zonal-mean flow anomaly, constituting a positive SELF feedback. This kind of mechanism was also found to be operating in the stationary wave and zonal flow feedback (Kimoto et al. 2001).

c. Scale selection of SELF feedback

The above qualitative analysis clearly indicates that the SELF feedback is scale-dependent. When \( l^2 \gg l_z^2 + k^2 \), the negative SELF feedback suppresses the formation of small-scale zonal flow anomalies through a viscous-type damping. When \( l^2 \ll l_z^2 + k^2 \), the SELF feedback results in a positive feedback for zonal flow modes. Somewhere in the regime near \( l^2 \approx l_z^2 + k^2 \), the SELF feedback changes sign. Thus the zonal flow mode has a positive feedback in the regime \( 0 \ll l^2 \ll l_z^2 + k^2 \).

This scale dependence of the SELF feedback suggests that there is a remarkable internal dynamical selection that favors a particular meridional scale of the zonal-mean flow modes that emerge from the stochastic basic flow.

We can derive an analytical expression for this scale-dependence of the growth rate for the zonal-mean flow modes by using the following approximation method. Assuming \( G \sim \sigma \sin l_y \), with \( \sigma \) a constant measuring the intensity of the variance in the synoptic eddy flow, and denoting the imaginary part of \( \tilde{\psi} \) as \( Y \), and further letting

\[
D_r = (\pi^i - C)(\partial^2_y \tilde{\Pi}) - k^2 + \beta,
\]

\[
D_l = (r + r_s)/(k)(\partial^2_y \tilde{\Pi}) - k^2 - l_z^2
\]

we can rewrite Eqs. (8)–(9) as

\[
\partial_t + r \tilde{\Pi} = 2kG(\partial^2_y \tilde{\Pi})Y + l_y \tilde{\Pi} Y
\]

\[
(D_r^2 + D_l^2) = D_r(l_z^2 + k^2) + \partial^2_y \tilde{\Pi} = -l_y \tilde{\Pi} Y.
\]

We consider an approximate form of the solution for the above equations:

\[
Y \approx Y_l \sin l_y \sin l_y, \quad \tilde{\Pi} = U_l \sin l_y
\]
and take a truncation $\partial^2 Y/\partial y^2 \sim -(l^2 + l_c^2) Y$, which ignores some terms having little projection on the structures of $Y$ or $\Pi$. Then, we obtain an approximate equation for zonal modes as follows:

$$\alpha \approx \frac{(r + r_z)}{r} \frac{\sigma^2(k^2 + l_c^2 - l^2)y^2(l_c^2 + l^2 + k^2)}{[(r - (l_c^2 - c))(l_c^2 + l^2 + k^2)]^2 + [(r + r_z)(l_c^2 + l^2 + k^2)]^2},$$

(17)

where we have set $2|G|^2 \approx \sigma^2$, which is the variance measure of the zonally uniform storm track in terms of a streamfunction. It becomes apparent that the sign of the SELF feedback depends on the sign of $k^2 + l_c^2 - l^2$. It is also clear that the SELF feedback is proportional to the strength of the storm track variance $\sigma^2$; that is, the stronger the storm track activity, the stronger the SELF feedback. For the zonal modes with meridional scales such that $l^2 < k^2 + l_c^2$, the positive SELF feedback tends to reduce the linear damping by a significant fraction, which is also seen from the eigenanalysis in section 2c. When $\sigma^2$ is very large, the positive feedback may overcome the linear frictional damping; hence the zonal model becomes linearly unstable. In the nonlinear case, this instability leads to multiple stochastic equilibria (Farrell and Ioannou 2003; Kravtsov et al. 2003, 2005). However, numerical results shown in section 2c suggest that the positive SELF feedback based on the observed levels of the storm track activity is modest. The linear system (2) is stable unless the linear friction is set to be unrealistically weak.

From Eq. (17), the least-damped mode of the zonal flow has a meridional scale about $l_m \approx \sqrt{(k^2 + l_c^2)/2}$. With a reasonable choice of parameters, the most favored zonal flow mode is around $l = 2$, which corresponds to a dipole structure in the zonal flow. The dipole structure of the zonal flow, $\pi_w = U_i \sin 2y$, qualitatively resembles the observed structure in the zonal components of the AO and AAO patterns in Figs. 1 and 2. A similar equation to (16) for the AAO index or annular mode index of the SH was empirically derived by Lorenz and Hartmann (2001). However, our analytical model provides a mechanism for the observed meridional structure of the zonal mean flow anomalies associated with AAO and AO modes.

Moreover, the associated anomalous variance field of synoptic eddies also has a dipole structure which is proportional to the meridional structure of $GY$ or $|G|^2 \pi$. The dipole-structured low-frequency zonal flow anomalies associated with the AO and AAO shifts the position of jet streams. At the same time, the associated anomalous transient eddy activity shifts the position of maximum variance in the storm track as well. Our analysis suggests that the atmospheric internal dynamics allows anomalous global-scale circulations of AO- and AAO-like structures to organize the synoptic eddies in the turbulent background atmospheric circulation in such a way as to gain selective reinforcement. This dynamic self-organization of the AO and AAO mode is likely essential for them being the leading modes of variability in the extratropical atmospheric circulation.

The analytical result in Eq. (17) may not be very accurate when $l$ is close to $l_i$. Thus, we solve the eigenvalue numerically using Eqs. (8)–(9). The numerical and analytical results agree well in terms of feedback factors as shown in Fig. 12, suggesting that the approximate analytical solution is qualitatively correct. The results suggest that modes with meridional wavenumber 2, namely the modes of dipole structure, is favored, which is also consistent with numerical result in section 3 for the case under the basic flow with the zonally uniform storm track in section 3.

It is difficult to obtain analytical results for background conditions with longitudinally confined synoptic activity as in the idealized cases section 3. Numerical results in section 3 suggest that the zonal extent of the enhanced synoptic activity in the background conditions play a crucial role in controlling the local features of the AO and AAO. The analytical results in this section show clearly that the key feature of the AO and AAO, namely their dipole structures, is the consequence of natural selection by the SELF feedback.

5. Conclusions

In this Part II, we illustrate that, once we use the observations to prescribe the cold season climatological mean flow and the climatological measures of the storm track eddy flow for NH and SH, the leading singular and/or eigenmodes of the linear barotropic model with closure for the SELF feedback are similar to the observed AO and AAO, respectively. A mode of certain features of the observed PNA pattern is also obtained as the third leading mode of the linear model under northern winter background conditions. In particular, without the SELF feedback the leading modes of the

$$\frac{d}{dt} U_i = -r(1 - \alpha) U_i.$$  

Here the feedback factor is
linear barotropic model bear little resemblance to the observed AAO and AO. We propose that the observed AO, AAO, and PNA patterns may be related to the leading dynamical modes generated by the internal self-organization through interaction between the planetary circulation and storm tracks, or the SELF feedback.

What is critical for the emergence of these low-frequency modes is that they can organize the transient eddies and feed on their energy so as to gain reinforcement preferentially. We have used the term dynamic self-organization to describe this positive SELF feedback and a natural selection of the leading dynamical modes of the atmospheric circulation such as the AAO and AO. The usage of this term of self-organization is often referred to describe the emergence of orderly structures of otherwise disorderly behavior of nonlinear dynamic systems through a structurally discriminative or selective growth mechanism. Unlike the relatively simple biological or chemical systems with clear transitions of orders, the atmospheric low-frequency flow variability and its leading modes are not highly energetic and they coexist with a more dominating chaotic synoptic flow. In other words, the positive SELF feedback that facilitates the dynamic self-organization of low-frequency modes of the atmospheric circulation is normally modest and inadequate to overcome the frictional damping. The low-frequency modes are thus normally weakly damped. Nevertheless, because they are least damped modes or are the most singular modes of the dynamic system, they tend to dominate the low-frequency variability in the presence of abundant external excitation.

Further, using an idealized example described in section 3 and analytical model in section 4, we provide a simple conceptual model for understanding the dynamics of SELF feedback and a natural scale selection of zonal flow modes through the positive SELF feedback. Our results suggest that the selection of the meridional scale of AAO- and AO-like low-frequency modes is controlled by the scale of the dominant synoptic eddies in the storm track. In particular, we find that, if the meridional scale of the most unstable or least damped modes of the zonal flow mode is measured in terms of wavenumber $k_m$, it satisfies $l_m \approx \sqrt{(k^2 + l^2)}$, whereas $k$ and $l$ are the zonal and meridional wavenumbers of the typical synoptic eddies in the storm track. Therefore, the meridional scale of the AAO- and AO-like modes is largely controlled by the background activity of synoptic eddies in the storm track. Without the SELF feedback, this fundamental scale selection cannot be captured. The physical mechanism of this scale selection can be heuristically understood by the tilted-trough mechanism.

Moreover, we showed through an idealized numerical experiment that the main zonal scales and locations of action centers in the AO (AAO) are largely controlled by the extent and location of the locally enhanced storm track intensity in the North Atlantic (southern Indian Ocean). Thus, the observed AO and AAO may be viewed as fundamentally near-annular modes governed by similar dynamics, whereas the dominating local features, such as the NAO feature of the AO, are largely a result of the enhanced SELF feedback owing to local storm track activity, such as the distinct North Atlantic storm track.

Our analyses also demonstrated that the SELF feedback contributes to the formation of a mode resembling the PNA pattern. However, the relative roles of Pacific and Atlantic storm track activity in forming this PNA-pattern-like mode in the model of section 2 remains to be further investigated. Nevertheless, the linear dy-
namical framework with closure for SELF feedback provides a useful approach for understanding the internal dynamical origin of the observed low-frequency modes of variability. The development and analysis of a general primitive equation framework with a similar SELF feedback closure will be reported in Part III.

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