The Potential Vorticity Structure and Dynamics of African Easterly Waves

JAMES O. H. RUSSELL

University of Utah, Salt Lake City, Utah

ANANTHA AIYYER

North Carolina State University, Raleigh, North Carolina

(Manuscript received 13 February 2019, in final form 26 November 2019)

ABSTRACT

The dynamics of African easterly waves (AEWs) are investigated from the perspective of potential vorticity (PV) using data from global reanalysis projects. To a leading order, AEW evolution is governed by four processes: advection of the wave-scale PV by background flow, advection of background PV by the AEW, diabatic forcing due to wave-scale moist convection, and coupling between the wave and background diabatic forcing. Moist convection contributes significantly to the growth of AEWs in the midtroposphere, and to both growth and propagation of AEWs near the surface. The former is associated with stratiform clouds while the latter with deep convection. Moist convection helps maintain a more upright AEW PV column against the background shear, which makes the wave structure conducive for tropical cyclogenesis. It is also argued that—contrary to the hypothesis in some prior studies—the canonical diabatic Rossby wave model is likely not applicable to AEWs.

1. Introduction

African easterly waves (AEWs) are the primary synoptic-scale system affecting North Africa during boreal summer (Carlson 1969; Burpee 1974). They play a role in the formation of over 60% of Atlantic tropical cyclones (TCs) (Avila and Pasch 1992; Russell et al. 2017) and are associated with variability in precipitation over West Africa and the tropical Atlantic (Duvel 1990; Fink and Reiner 2003; Mekonnen et al. 2006). In this paper, we examine the dynamics of AEWs, with a particular focus on their interaction with convection.

a. AEW dynamics and moist convection

It was recognized early on that both moist convection (Erickson 1963; Carlson 1969) and dynamical instabilities (Burpee 1972) contribute to the destabilization of AEWs. The latter was linked to the reversal in the meridional gradient of background potential vorticity (PV), which itself is a result of sustained moist convection within the intertropical convergence zone (e.g., Schubert et al. 1991). Numerous studies attempted to account for the structure and growth characteristics of AEWs using idealized models without moist convection (e.g., Rennick 1976; Thorncroft and Hoskins 1994; Paradis et al. 1995). The waves in these dry models exhibited some similarity with observations although the periods and wavelengths of the most unstable modes were often larger. Furthermore, dry simulations failed to capture the vertical structure of the waves. More recently, Hall et al. (2006) showed that when modest damping was included in an idealized model, the African easterly jet (AEJ) was rendered stable, and dry baroclinic and barotropic instability alone was insufficient to explain the growth of AEWs. A strong relationship between AEWs and convection was detailed by Payne and McGarry (1977) and Norquist et al. (1977). The latter study suggested that latent heating produced by convection can be a dominant factor for the growth and maintenance of AEWs. Inclusion of even simple parameterizations of moist convection in primitive equation models yielded more realistic AEWs (e.g., Mass 1979; Thorncroft and Hoskins 1994).

Supplemental information related to this paper is available at the Journals Online website: https://doi.org/10.1175/JAS-D-19-0019.s1.

Corresponding author: James Russell, james.russell@utah.edu

DOI: 10.1175/JAS-D-19-0019.1
© 2020 American Meteorological Society. For information regarding reuse of this content and general copyright information, consult the AMS Copyright Policy (www.ametsoc.org/PUBSReuseLicenses).
Several recent studies have examined the interaction of AEWs and moist convection (e.g., Hsieh and Cook 2008; Mekonnen et al. 2006; Mekonnen and Rossov 2011; Berry and Thorncroft 2012; Janiga and Thorncroft 2014; Tomassini et al. 2017) but there is still little consensus on how moist convection supports growth of the AEW. Some studies have shown that convection generates vorticity in the midlevels at and ahead of the AEW trough (Shapiro 1978; Schwendike and Jones 2010; Berry and Thorncroft 2012; Janiga and Thorncroft 2014; Tomassini et al. 2017). However, the issue of scale separation—how mesoscale convective systems (MCSs) and synoptic-scale AEWs interact—is unresolved, and it is unclear whether this would add to the growth or propagation of the AEW. Alternatively, Hsieh and Cook (2008) suggested that moist convection increases the environmental baroclinicity, thereby enhancing the potential for growth through dynamic interactions.

b. Baroclinic instability and diabatic Rossby waves

Some studies have suggested that the interaction between AEWs and moist convection can be interpreted in terms of moist baroclinic instability (e.g., Thorncroft and Hoskins 1994; Berry and Thorncroft 2005). Berry and Thorncroft (2005) proposed that PV generated by moist convective processes enhances the midlevel AEW-scale PV anomaly, which interacts with a low-level equivalent potential temperature anomaly in a manner characteristic of phase-locked counterpropagating edge waves (e.g., Hoskins et al. 1985). De Vries et al. (2009) and Cohen and Boos (2016) have reviewed various interpretations of dry and moist baroclinic instability from the perspective of counter propagating Rossby waves. In both flavors of baroclinic instability, upshear-tilted waves, propagating on opposite-signed background PV gradients are required. Further, mixed-baroclinic interactions between dry and moist waves are also possible.

Some studies (e.g., Moore and Montgomery 2005; Berry and Thorncroft 2005, 2012; Tomassini et al. 2017) have speculated that AEWs may be viewed as diabatic Rossby waves (DRWs; Raymond and Jiang 1990; Parker and Thorpe 1995). In such a wave, the rearrangement of PV by moist convection plays a role analogous to advection of the background PV by an adiabatic wave. In the canonical definition of a DRW as applied to extratropical cyclones, a strong low-level temperature gradient is necessary for forcing vertical motion and convection downshear of the preexisting wave trough. While the growth mechanism for DRWs is somewhat ambiguous in the literature, most attribute it to a form of mixed or moist baroclinic instability. Cohen and Boos (2016) referred to the baroclinic interaction between a low-level dry wave and a moist wave above the diabatic heating maximum as the instability driving DRWs. In this study, we investigate the relevance of moist baroclinic instability to AEW growth and whether AEWs can be described as DRWs.

c. Potential vorticity

The majority of previous studies on AEW amplification have utilized energy budgets (e.g., Hsieh and Cook 2007; Berry and Thorncroft 2012; Poan et al. 2015). However, energy budgets typically pool together moist convective and dry baroclinic energy sources into a baroclinic source term, making it difficult to separate them. PV is a useful metric to study the interactions between convection and broader dynamical features since its nonconservation can be directly attributed to diabatic processes (Hoskins et al. 1985). Further, given an appropriate balance condition (such as geostrophic or nonlinear balance), PV can be inverted to obtain the balanced wind and mass fields (e.g., Davis 1992). Thus PV also describes the balanced part of the circulation, and by understanding its sources and sinks, it is possible to describe the balanced dynamics of AEWs (Raymond et al. 2015).

Tyner and Aiyyer (2012) qualitatively described the evolution of AEWs during their transformation to tropical cyclones using isentropic PV fields. Janiga and Thorncroft (2013) showed that diabatic PV is generated in the midlevels over West Africa and Tomassini et al. (2017) showed that latent heating at and slightly ahead of the AEW trough reinforces the AEW through diabatic PV generation between 500 and 800 hPa. However, there has not yet been a comprehensive assessment of AEW PV and its sources and sinks. Given there are distinct diabatic sources of PV in the low and midlevels of AEWs, it is important to put them in context relative to their adiabatic counterparts. For example, are they as large, or larger, than the adiabatic PV sources? Finally, through scale separation, it is possible to separate PV sources such that individual sources describe different processes (Zhang and Ling 2012).

d. Objectives

The purpose of this study is to describe the structure and dynamics of AEWs from a PV perspective. The role of diabatic processes in growth and propagation of AEWs is assessed using a detailed PV budget. We also investigate the characteristics of moist baroclinic interactions in a manner similar to Cohen and Boos (2016), who examined monsoon depressions.
2. Methods

a. Data

Two reanalysis datasets are used in this study: the European Centre for Medium-Range Weather Forecasts (ECMWF) interim reanalysis (ERA-Interim, hereafter ERAI; Dee et al. 2011) and the National Centers for Environmental Prediction’s (NCEP) Climate Forecast System Reanalysis (CFSR; Saha et al. 2010). The intent is not to provide a detailed comparison between the two datasets but to note which features are independent of the various dynamics, parameterizations, and data assimilation schemes. The utility of using multiple reanalyses was highlighted by Janiga and Thorncroft (2013) as processes such as convective heating rates can be highly sensitive to model parameterizations. The grid spacing of the datasets are approximately 0.8° for ERAI and 0.5° for CFSR. Both will sufficiently resolve synoptic-scale features such as the AEW but not mesoscale features such as MCSs. In the following sections we repeat all calculations for both reanalyses, but focus on ERAI, and only show CFSR in the main text when there are key differences.

CFSR provides explicitly calculated heating rates as output from the various parameterization schemes on a 1° grid. The sum of all diabatic heating from the parameterization schemes is denoted $H$. A bilinear interpolation is used to interpolate this to the same 0.5° grid as all other CFSR variables. Since ERAI does not provide explicit diabatic heating rates, it is estimated using the thermodynamic residual (Yanai et al. 1973; Hagos et al. 2010):

$$Q_1 = \frac{D\theta}{Dt} + \frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} + \omega \frac{\partial \theta}{\partial p},$$

where $\theta$ is potential temperature, and $u$, $v$, and $\omega$ are the components of the wind velocity. Due to the various inexact centered differences involved, $Q_1$ is expected to vary from $H$. Precipitation is represented by the National Aeronautics and Space Administration (NASA) Tropical Rainfall Measuring Mission (TRMM; Huffman et al. 2007) Multisatellite Precipitation Analysis (TMPA) dataset. All results in this study are based on these datasets from the period July–September 1998–2010. This is the only period for which all the above datasets overlap.

b. Filtering

To obtain perturbations representative of AEWs, data are filtered in wavenumber–frequency space as discussed in Hayashi (1982) and implemented in Wheeler and Kiladis (1999). Figure 1 shows the spectral power calculated using ERAI meridional winds and TRMM precipitation between 5°–15°N and 60°W–60°E. Westward-propagating disturbances dominate from diurnal to synoptic time scales for both meridional winds and precipitation. This indicates a continuum from spatially large, diurnally varying MCSs to synoptic-scale AEWs. We filter the data to retain westward-propagating signals with periods between 2 and 8 days and wavelengths between 1500 and 6500 km. Filtering in this manner is very similar to the tropical depression band or “TD band” filtering typically used to identify easterly wave-like disturbances in past studies (e.g., Kiladis et al. 2006;
Russell et al. 2017). The strongest power for both meridional winds and precipitation of AEWs falls within these scales. From here on, any variable that is filtered in this manner is referred to as an AEW-scale variable.

c. Composite analysis

Representative AEWs are generated using a composite analysis technique. Base points for the composite waves are selected by examining the AEW storm tracks, depicted by the variance of AEW-scale PV and eddy kinetic energy (EKE) in Fig. 2. Two distinct tracks of AEWs can be seen: a low-level northern track (Figs. 2c,d,g,h) and a midlevel southern track, the latter of which is more commonly associated with moist convection than the former (Figs. 2a,b,e,f). The base points are selected along the southern AEW track as marked in Fig. 2a. The tracks in ERAI and CFSR are similar enough that the same base points are used for both datasets.

At each base point we average the AEW-scale meridional wind in a volume 5° latitude by 2.5° longitude over the layer 550–750 hPa. Using the resulting time series, the date and time of maxima in northerly winds are selected. A threshold of 1 m s\(^{-1}\) is used to filter out weak or noisy signals. If two maxima are within 2 days of each other then the largest is assumed to be the main passage of the AEW to avoid double counting. We focus on the northerlies since this is the typical location of strongest moist convection in AEWs over land (e.g., Janiga and Thorncroft 2016). Over the period July–September 1998–2010, this yields 200–300 AEWs depending on the base point. We focus on the base points located over the Atlantic (15°N, 30°W), West Africa (10°N, 0°), and East Africa (11°N, 30°E). For analyses using this composite method, statistical significance of the anomalies is checked using a two-sided Student’s \(t\) test with a 95% confidence interval with respect to zero. For simplicity, statistical significance is not shown but anomalies shown and discussed pass the significance tests.

d. Potential vorticity

PV, with pressure as the vertical coordinate, is written as

\[
P(x, y, p, t) = -g(\boldsymbol{\eta} \cdot \nabla \theta),
\]

where bold symbols denote vectors, \(g\) is the gravitational acceleration, \(\boldsymbol{\eta} = \zeta + f \mathbf{k}\) is the absolute vorticity vector,
\[ D_{\text{PV}} = -g\xi \cdot \nabla Q - g\nabla \theta \cdot \mathbf{V} \times \mathbf{F}, \tag{3} \]

where \( Q \) is the diabatic heating rate and \( \mathbf{F} \) is a frictional torque. For completeness, we include in the appendix, a derivation of this equation. This is the full three-dimensional form of the PV budget. The advantage of using this form is that we retain the tilting terms, which are often neglected (e.g., Zhang and Ling 2012). Isobaric surfaces are chosen for this analysis as low-level isentropic surfaces have steep slopes and intersect the ground over North Africa.

We partition the variables in Eq. (3) into a July–September (JAS) time mean (\( \mathbf{X} \) where \( \mathbf{X} \) is some arbitrary variable), an AEW scale (\( \mathbf{X}_w \)), and a residual scale (\( \mathbf{X}_r \)). Variables on the residual scale are defined by subtracting the time-mean and AEW-scale variable from the total variable (e.g., \( \mathbf{X}_r = \mathbf{X} - \mathbf{X} - \mathbf{X}_w \)). In the case of AEWs this residual encompasses any eastward-propagating anomalies (e.g., equatorial Kelvin waves, midlatitude disturbances, the Madden–Julian oscillation), any processes on a diurnal time scale (e.g., afternoon convection), and interannual variations. Following Zhang and Ling (2012), the resulting equation is

\[
\frac{\partial \mathbf{P}_w}{\partial \mathbf{A}} = -\mathbf{U} \cdot \mathbf{V}_{w} - \mathbf{U}_w \cdot \nabla \mathbf{P} - (\mathbf{U}_w \cdot \mathbf{V}_{w})_w - (\mathbf{U}_w \cdot \mathbf{F})_w - (\mathbf{U}_r \cdot \mathbf{V}_{w})_r - (\mathbf{U}_r \cdot \mathbf{F})_r - \mathbf{\eta} \cdot \mathbf{Q}_{w} - \mathbf{\xi} \cdot \mathbf{Q} - (\mathbf{\xi}_w \cdot \mathbf{Q}_{w})_w - (\mathbf{\xi}_r \cdot \mathbf{Q}_r)_r + \text{Residual}, \tag{4} \]

where \( \mathbf{U} = \mathbf{i}u + \mathbf{j}v + \mathbf{k}w \). Term A is the local tendency of wave-scale PV, terms B–F represent the advection of PV, and terms G–K represent the diabatic contributions to PV. The residual encompasses products of residual-scale processes (e.g., \( \mathbf{U}_r \cdot \mathbf{V}_{w} \)), as well as frictional processes. It should be emphasized that our focus is not on a balanced budget here—rather we wish to use the budget to diagnose the dominant terms, such that we can assess the most important processes in AEW dynamics. Nonetheless, the PV budget in these cases is typically well balanced.

In addition, we can partition all the above sources of PV into their horizontal and vertical components. For example,

\[
\mathbf{\eta} \cdot \nabla Q = \mathbf{S} \cdot \mathbf{V}_{w} Q + (\mathbf{\xi} + f) \frac{\partial Q}{\partial p}, \tag{5} \]

where \( \mathbf{S} = -(\mathbf{\partial} / \mathbf{\partial p}) \mathbf{j} + (\mathbf{\partial} / \mathbf{\partial x}) \mathbf{i} \) is the horizontal vorticity vector or shear, \( \mathbf{V}_p = \mathbf{i}(\mathbf{\partial} / \mathbf{\partial x}) + \mathbf{j}(\mathbf{\partial} / \mathbf{\partial y}) \), and \( \mathbf{\xi} \) is the vertical component of relative vorticity. Likewise, we partition our advective components into vertical and horizontal advections. Each term now represents a source of PV tendency driven by some scale interaction.

3. AEW structure

In this section we review the structure of the AEW environment and the AEW-scale PV within the context of dynamical instabilities. We also describe the relationship between PV, precipitation, and diabatic heating. While some of these topics have been examined by previous studies (e.g., Burpee 1972; Kiladis et al. 2006; Hsieh and Cook 2008; Janiga and Thorncroft 2013, 2016), this section brings all this work together in one place and is important for the interpretation of later results.

a. Structure of the mean AEW environment

Figure 3 shows the meridional PV gradient, time-mean zonal winds, and potential temperature in both ERAI and CFSR. Over Africa and the eastern Atlantic, the PV gradient in the vicinity of the AEJ reverses sign both along the horizontal at 650 hPa (Figs. 3a,b) and in the vertical, north of 10°N (Figs. 3c,d). While we show the mean PV gradient on a constant 650 hPa pressure surface here, the interpretation is similar if we choose an isentropic level close to 650 hPa in Figs. 3e and 3f as is typically done when using Ertel PV. Together with the positive meridional gradient of surface temperature, the necessary conditions for Charney–Stern instability are met by the time-mean environment (cf. Burpee 1972). At the jet level, easterly zonal flow is positively correlated with the negative PV gradient (Figs. 3c,d). Similarly, at the surface, westerly flow is positively correlated with the positive temperature gradient (Figs. 3c,d). This satisfies the Fjørtoft condition for instability (Fjørtoft 1950). Notably, despite the higher native resolution of CFSR, these gradients are still larger overall in ERAI, indicating potential differences in the dynamics of AEWs between the two reanalyses.

The instability of the mean environment can be related to interactions of waves on these PV gradients. The opposing PV gradients promote counterpropagating phase-locked Rossby waves that can mutually reinforce...
each other (Hoskins et al. 1985; Pytharoulis and Thorncroft 1999). Along the horizontal, the interaction occurs in the midtroposphere between the waves on the positive PV gradient between 5° and 10°N, and negative PV gradient between 10° and 20°N (e.g., barotropic instability). In the vertical, interactions are possible between waves on the midtropospheric negative PV gradient with those on the positive PV gradients above and below, as well as on the positive surface temperature gradient (e.g., baroclinic instability; Thorncroft and Hoskins 1994).

b. The PV structure of AEWs

Figure 4 shows the composite AEW-scale winds and PV in ERAI (shown for CFSR in the online supplemental material). The top two rows of panels show horizontal sections at 650 and 925 hPa, respectively. The bottom two rows of panels show vertical cross sections averaged between 5° and 15°N, and 12° and 22°N latitudes. Mean zonal winds are shown as vectors on the right of each panel to visualize the background horizontal and vertical shear profiles. Cyclonic and anticyclonic circulations, respectively, are associated with the AEW-scale positive and negative PV anomalies (further details in the online supplemental material).

In Figs. 4a–c, the time mean wind vectors clearly highlight the core of the AEJ between 12° and 15°N. They also illustrate the meridional shear of the jet. The PV anomalies at 650 hPa are tilted upshear in the horizontal plane on both the north and south side of the jet axis. The peak PV anomalies are located near the axis of the jet, which also corresponds to the area of negative background meridional PV gradient (cf. Fig. 3). These anomalies represent the southern-track AEWs. The associated circulations extend across the jet into the positive meridional gradients on the northern and southern flank of the jet. The PV anomalies at 925 hPa are predominantly located around 20°N. These anomalies constitute the northern AEW storm track (Figs. 4d–f). At this level, neither the meridional wind shear of the time mean flow, nor the tilt of the wave are prominent. Thus, the horizontal PV structure has an upshear configuration consistent with barotropic instability predominantly in the midtroposphere.
Figures 4g–i show the vertical structure of AEWs between 5° and 15°N, a region representative of the southern track. Over East Africa (Fig. 4i), the vertical shear is mainly easterly with the exception of a slight westerly shear around 600–500 hPa. Here, the PV anomaly is tilted downshear above 500 hPa and upshear below. Over West Africa, the PV column is tilted upshear both above and below the jet core. Over the Atlantic, the PV column is relatively upright below the jet while still tilted upshear above. Here, the AEW has a secondary low-level maximum close to 800 hPa (Fig. 4h). This will be related to the effect of moist convection in subsequent sections, as also proposed by Janiga and Thorncroft (2013). As noted in Figs. 3c and 3d the
background meridional PV gradient in this latitude range is positive throughout the troposphere. Simultaneously, the meridional temperature gradient at the surface is also positive (Figs. 3e,f). This precludes dry baroclinic instability in this latitude band. However, previous studies (Berry and Thorncroft 2005, 2012) have proposed that some form of moist baroclinic instability may occur in this region. As emphasized by Cohen and Boos (2016), an upshear tilt is a potential marker of moist baroclinic instability in the presence of diabatic sources of PV. We explore this further in subsequent sections.

The cross sections for the northern track (Figs. 4j–l) show that the waves are more amplified at low levels compared to the southern track. Over East Africa, the surface to midtropospheric shear is weak and the PV column has a downshear tilt. Over West Africa, the shear is stronger and the wave has a clear upshear tilt. Over the Atlantic, there is a strong tilt in the PV between 700 and 900 hPa. The absence of diabatic feedback in the northern-track AEWs is evident from the lack of precipitation anomalies in this latitude band (Figs. 4d–f). The upshear-tipped wave within a region associated with reversal in PV gradients in the vertical, and no precipitation anomalies, implies that dry baroclinic instability is the likely operating mechanism (e.g., Kiladis et al. 2006).

Since our main goal is to understand the relationship between AEWs and moist convection that is mostly active south of 15°N (e.g., the green contour in Figs. 4d–f), further analysis is focused on the southern track.

c. AEW-scale diabatic heating

The structure of AEW-scale diabatic heating is important because its vertical gradient defines the magnitude and location of the nonconservative diabatic source in the PV budget. Figure 5 shows meridional–vertical cross sections of PV and diabatic heating associated with the composite AEWs. Each row of plots represents diabatic heating via a different variable (ERAI $Q_1$, CFSR $Q_1$, or CFSR $H$).

Over East Africa, the diabatic heating has a dipole structure. There is a peak at 450 hPa and a weaker opposite-signed second peak at 900 hPa. Diabatic heating at upper levels and cooling at lower levels occur to the west of the trough. The opposite pattern is found west of the ridge. The magnitude of the peak heating in the upper levels is about 2.5–3.5 K day$^{-1}$. The magnitude of the peak cooling at the lower levels ranges between 0.5 and 1.5 K day$^{-1}$. Such vertical structure of diabatic heating in the tropics is typically representative of stratiform precipitation (Schumacher et al. 2007). Composite precipitation rates (green curves) using TRMM data are also shown in Fig. 5. Anomalous precipitation peaks to the west of the heating-over-cooling dipole. This is consistent with previous studies that have shown that most precipitation occurs in conjunction with deep convection over east Africa (e.g., Janiga and Thorncroft 2014). This confirms the progression of cloud types within the wave during its early stages of development over east Africa—shallow convection in the ridge, deep convection in the northerlies, and stratiform in the trough.

Over West Africa, the heating anomalies are more vertically extended in both ERAI and CFSR $Q_1$ anomalies. The peak rates of 1.5–3 K day$^{-1}$ are centered around 400 hPa and lower values spread downward to 750 hPa. The profile for $H$ in CFSR is less top-heavy and peaks around 500 hPa in the midtroposphere. Overall, this points to a transition from a stratiform-dominated heating profile over East Africa to a mix of deep convection and stratiform cloud over West Africa.

Over the eastern Atlantic, the structure of heating is quite different. The anomalies are deep, single signed and bottom-heavy. Further, heating and precipitation are in phase with each other. These two findings indicate the dominance of deep moist convection (Schumacher et al. 2007; Janiga and Thorncroft 2016). Heating and precipitation anomalies are nearly in phase with the PV anomalies, with increased precipitation in the trough and decreased precipitation in the ridge. As noted earlier, a secondary PV maximum is in place around 700 hPa.

These results suggest that there is a transition in AEW-scale heating from that produced by dominant stratiform cloud (over east and to a lesser extent West Africa) to that produced by deep convection (over the Atlantic). This transition is most prominent around the West African coast as noted by other studies (e.g., Janiga and Thorncroft 2013, 2016). However, a deepening of the positive heating anomaly with a weak or nonexistent low-level cooling anomaly indicates that this transition begins well before the AEW reaches the coast.

Figure 5 shows that both $Q_1$ and $H$ in CFSR, produce similar spatial structures. This suggests that $Q_1$ is a reasonable estimate of the explicit diabatic heating ($H$) in AEWs. Further, explicit heating sources from the sum of all convective (shallow and deep) and microphysical processes, representing the heating due to precipitation processes only, also produce a similar structure (not shown). This indicates that $Q_1$ is also a good estimate of the heating associated with convective processes only. For this reason, the following PV budgets will use $H$ when calculating PV source terms using CFSR, but $Q_1$ when using ERAI since explicit heating sources are not available for ERAI. By using two different heating estimates from two different
4. PV sources in composite AEWs

Before we examine the individual sources of PV, it is desirable to separate the contribution of the advection of AEW-scale PV by the background horizontal flow. This basic state advection term is expected primarily to translate and deform the waves due to the presence of the AEJ. Figure 6 shows the local tendency (shaded) and advection of the AEW-scale PV by the mean horizontal wind ($-\mathbf{V} \cdot \mathbf{V}_{p}$, contours). The PV tendency below 700 hPa is weak over East Africa and progressively amplifies westward. In all three regions, horizontal advection by the mean flow dominates the PV tendency above 700 hPa but is very weak below.

To highlight the sources of AEW-scale PV relative to the basic state advection, we now define the pseudo-Lagrangian PV tendency (denoted $P_{PL}$; e.g., Hannah et al. 2016), calculated as

$$\left(\frac{D P_{w}}{Dt}\right)_{PL} = \frac{\partial P_{w}}{\partial t} + \mathbf{V} \cdot \nabla P_{w}.$$

This is referred to as pseudo-Lagrangian, since to first order the midlevel PV tendency is approximated by the advection of AEW-scale PV by the background flow reanalyses, if results are qualitatively similar, we will have better confidence in our conclusions.
(i.e., the advection by the AEJ dominates AEW-scale PV propagation). In Fig. 7, the top row of the panels show the PV anomaly (shaded) and the pseudo-Lagrangian tendency (contours) for ERAI. As expected from Fig. 6 this is largest below 700 hPa where the AEJ is weak. To understand the source of these primarily nonadvective PV tendencies, we now focus on the diabatic sources of PV.

The first diabatic source of note is the AEW-scale diabatic heating \((-\nabla \cdot Q_w/\partial p\)). Over East Africa (Fig. 7f), it has a dipole structure in the vertical. It is positive between 900 and 550 hPa at and ahead of the trough. Aloft, there is a PV sink directly above the AEW-scale PV anomaly. In and ahead of the ridge, the opposite pattern occurs. This is consistent with the vertical gradients in AEW-scale diabatic heating shown in Fig. 5. Over West Africa, this PV source is lower (Fig. 7e). In ERAI, there are two maxima—one at 600 hPa and the other at approximately 850 hPa. In contrast, using CFSR there is only one distinct maximum at around 800 hPa (Fig. S4 in the online supplemental material) consistent with the vertical diabatic heating gradient (Fig. 5h). In both datasets, the mid- to low-level positive source is located at, and to the west of the AEW-scale PV anomaly. This indicates that, over land, it contributes to both growth and propagation of the AEW. Over the eastern Atlantic (Fig. 7d), the largest PV sources are closer to the surface, and in phase with the trough. Further, the AEW PV transitions from being predominantly a midlevel anomaly over East Africa to a deep column stretching to the low levels over the Atlantic. The diabatic source associated with coupled moist convection therefore plays a role in the development of a low-level PV anomaly. This makes the wave structure conducive for tropical cyclone formation (e.g., Russell et al. 2017).

The other diabatic source of note is associated with background heating but coupled to the AEW by the AEW-scale vorticity \((\nabla \cdot Q_w/\partial p\)). This represents the imprint of the AEW kinematics on the background heating within the intertropical convergence zone (ITCZ). This source has its largest magnitude over East Africa (cf. Fig. 7i with Figs. 7g,h). Though not as large as the diabatic source associated with AEW-scale heating, it is for the most part, directly in phase with the AEW-scale PV anomaly. This indicates that it contributes mostly to growth of the AEW. Over West Africa, the two reanalyses diverge on the magnitude of this source. ERAI has only a weak midlevel source, while CFSR (Fig. S4) diagnoses a much stronger midlevel source of PV, the effect on the AEW of which will be shown in Figs. 9 and 10 here. Over the Atlantic, both reanalyses indicate a significant low-level source of PV in phase with the AEW-scale PV anomaly. However, this source is tilted relative to the AEW-scale PV anomaly such that at 750 hPa it is out of phase with the AEW-scale PV anomaly. These results suggest the importance of background diabatic heating to the growth of the AEW in midlevels, especially over East Africa.

The final PV source of interest in this section is the advection of background PV by the AEW-scale winds \((-\nabla \cdot \mathbf{V}_w\)). This term can contribute to both growth and propagation of the wave. It propagates a wave westward (eastward) when a wave is in a region of positive (negative) meridional PV gradient (Fig. 3). From the perspective of wave growth, this source represents the combined effect of barotropic and baroclinic instabilities. This is because, the induced flow from one member of the counterpropagating wave can advect background PV into the other. Given the right phasing of the PV anomalies in the horizontal or vertical, this will yield amplification of the interacting waves (Hoskins et al. 1985).
Figure 8 shows horizontal plots of this source term averaged over 800–550 hPa, the layer where there is a reversal in the background PV gradient (cf. Fig. 3). At all locations and in both reanalyses there is a checkerboard pattern of PV sources. Over East Africa and the Atlantic there are three rows since the background PV gradient is positive equatorward of 12°N, negative between 12° and 20°N, and positive north of 20°N (Fig. 3). Over West Africa there are only two rows since the northernmost positive background PV gradient is weak over the Sahara (Fig. 3). These PV tendencies indicate the presence of counterpropagating waves in the horizontal that, given the shear associated with the AEJ (Fig. 3), can be phase locked. This gives rise to barotropic instability through interactions of the waves in each “row.”

5. Contributions of PV sources to propagation and growth

Following Andersen and Kuang (2012) and Arnold et al. (2013), we diagnose the contribution of the source terms in the PV budget to AEW growth and propagation. For any individual source term $Y$ in Eq. (4), its
contribution to growth must be in phase (positively correlated) with the AEW-scale PV anomaly and can be represented by

$$Y_g = \langle Y \cdot P_w \rangle / \langle P_w \cdot P_w \rangle,$$  
(7)

where \(\langle \rangle\) represents an average in latitude. Since each term denoted by \(Y\) represents a process that yields a local AEW-scale PV tendency, the growth contribution \(Y_g\) can be interpreted as the fractional growth rate:

$$Y_g = \frac{1}{P_w} \frac{\partial P_w}{\partial t}.$$  
(8)

Similarly, the contribution to the propagation must be in phase with the AEW-scale PV tendency and can be represented as

$$Y_p = \left\langle Y \cdot \frac{\partial P_w}{\partial t} \right\rangle / \left\langle \frac{\partial P_w}{\partial t} \right\rangle.$$  
(9)

For the intent here, we chose latitudes of 5°–15°N as this encompasses the southern track of AEWs. Table 1 shows the contribution of each term in Eq. (4) averaged over 400–700 hPa, 30°W–40°E, and in time. Additionally, specific terms are averaged in time and examined further in the cross sections shown in Figs. 9 and 10.

a. Propagation

From Table 1, horizontal advection is the primary driver of AEW propagation in a volume average sense. Of the terms that compose this advection [cf. Eq. (4)], the dominant source is \(-\nabla \cdot V_P\), which represents advection of the AEW-scale PV by the time-mean basic state. The next largest contribution arises from the advection of the time-mean PV by the AEW-scale circulation \((-V_w \cdot V_P\)). With the exception of \(-\nabla \omega Q_w / \partial \), most diabatic terms are negligible. We now focus on the spatial structure of individual sources highlighted as important to propagation of the AEW.

Figures 9a and 9b show the contribution from the advection of the AEW-scale PV anomalies by the horizontal time-mean flow \((-\nabla \cdot V_P\)). Here, positive shading indicates westward motion. As expected, this term is dominant above 700 hPa and almost entirely

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Source & Contribution to & Contribution to & \\
 & propagation & growth (day\(^{-1}\)) & \\
\hline
\(\nabla \cdot V_P\) & 0.71 & 0.79 & — & — \\
\(\nabla \cdot V_P\) & 0.12 & 0.06 & 0.09 & 0.06 \\
\(\nabla \cdot V_P\) & — & — & — & — \\
\(\nabla \cdot V_P\) & 0.04 & 0.03 & — & — \\
\(\nabla \cdot V_P\) & 0.03 & 0.03 & — & 0.02 \\
\(\nabla \cdot V_P\) & 0.02 & 0.02 & 0.02 & 0.02 \\
\(\nabla \cdot V_P\) & — & — & — & — \\
\(\nabla \cdot V_P\) & 0.02 & — & 0.02 & — \\
\(\nabla \cdot V_P\) & 0.02 & 0.03 & — & — \\
\(\nabla \cdot V_P\) & — & — & — & — \\
\(\nabla \cdot V_P\) & — & — & — & — \\
\hline
\end{tabular}
\caption{Source terms from Eq. (4) contributing to propagation and growth of AEW-scale PV [calculated using Eq. (9)] averaged in the volume encompassed by 400–700 hPa, 4°–20°N, and 30°W–40°E. Anything with a fractional contribution of 0.01 (i.e., 1%) or less was deemed negligible and for presentation is represented by an em dash (—).}
\end{table}
accounts for the westward phase speed of AEWs. Averaging the zonal wind over this region gives a mean wind speed of 7 m s\(^{-1}\) in ERAI and 8 m s\(^{-1}\) in CFSR. This is consistent with past literature. For example, Laing et al. (2008) reported that the phase speed of AEWs ranged between 5 and 11 m s\(^{-1}\) and the median value was about 7 m s\(^{-1}\). Near the surface, however, as a result of the westerly monsoonal flow, this term contributes to a weak eastward motion. Thus this term can be expected to shear the AEW-scale PV column. To maintain a deep PV column as seen in Figs. 4g–i, there must be another process contributing positively to low-level AEW-scale PV tendency.

The largest contributor to AEW propagation below 750 hPa is the AEW-scale diabatic heating \((- \nabla \cdot \mathbf{V}_w / \partial p\)) shown in Figs. 9c and 9d. This term contributes to westward propagation east of 30°W since at these longitudes diabatic heating leads the trough. Therefore, while the mean flow acts to tilt the PV anomaly downshear, convection coupled to the AEW opposes it and likely leads to the observed upshear tilt below 700 hPa (Figs. 4g–i). Furthermore, a more upright PV column with deep convection will more likely present the opportunity for TC genesis.

As seen in Figs. 5a,g and 7d, and Fig. S4d, enhanced convection transitions into the trough over the Atlantic; therefore, there is little contribution to propagation from diabatic processes there. This is corroborated by Figs. 9c and 9d. Over the Atlantic, the low-level background PV gradient gets stronger and wave propagation via advection of background PV is the dominant process (relative to the background flow). This is evident in Figs. 9e and 9f where advection of mean PV by AEW-scale flow \((- \mathbf{V}_w \cdot \nabla \Phi\)) contributes a much larger proportion of the PV tendency west of 15°W, especially in the low levels.

b. Growth

Table 1 shows that, in the volume average sense, AEW growth can be attributed to three main sources: advection of background PV by the AEW-scale winds \((- \mathbf{V}_w \cdot \nabla \Phi\), wave-scale diabatic heating \((- \nabla \cdot \mathbf{V}_w / \partial p\),
and background heating \((-\zeta_w \, \nabla \cdot (Q + \nabla) / \partial p)\). We now focus on the spatial structure and interpretation of these three terms.

Figures 10a and 10b show the contribution due to advection of time-mean PV by the AEW \((-\nabla_w \cdot \nabla P\)). In both reanalyses, this term yields a growth rate of 0.1–0.2 (i.e., 10%–20%) day\(^{-1}\) (i.e., 10%–20%) between 600 and 800 hPa. There is a similar contribution to growth in the low levels over the Atlantic. As noted in section 4, this term includes barotropic and baroclinic instabilities. Assuming a fractional growth rate of 0.15 per day, and beginning with a PV anomaly over East Africa of magnitude 0.015 PVU (e.g., Fig. 4c), we can expect a PV anomaly of 0.04 PVU around the West African coast after approximately 7 days (5000 km at approximately 8 m s\(^{-1}\)). This is only half the magnitude of PV in our composite AEWs near the West African coast (Fig. 4a). Further, this does not explain the maximum in PV that occurs above 600 hPa. Thus, we must look to other sources to explain the total growth of AEW-scale PV.

The contribution to growth associated with vertical gradients in AEW-scale diabatic heating \((-\nabla_w \cdot (Q + \nabla) / \partial p)\) is shown in Figs. 10c and 10d. This term contributes to growth between 500 and 700 hPa, coincident with the maxima in AEW-scale PV (e.g., Fig. 4c). Further, in ERAI, growth rates of 0.5–0.7 day\(^{-1}\) are centered at 850 hPa across a broad region from 30\(^\circ\)W to 30\(^\circ\)E, although in CFSR, this is only 0.1–0.2 day\(^{-1}\) over east Africa. Small variations in the heating profile (Figs. 5c,i) can therefore significantly impact the structure of the PV source (Fig. 7f), and thus the magnitude of growth. Overall, this term amplifies the midlevel AEW PV anomaly and plays the dominant role in generating the low-level PV anomaly. Thus, the wave-scale diabatic heating is instrumental in creating a deep column of PV associated with the AEW.

The vertical gradients in time-mean diabatic heating \((-\zeta_w \, \nabla \cdot (Q + \nabla)\) lead to a growth of PV between 500 and 800 hPa at the rate of 0.2–0.35 day\(^{-1}\) (Figs. 10e,f). Over East Africa, large MCSs occur nearly every day during July–September. The persistent heating in this region is dominated by large areas of stratiform clouds within MCSs. The heating is maximized between 400 and 500 hPa and accounts for the positive PV anomalies below 500 hPa.
This source represents stretching of wave-scale relative vorticity and both anticyclonic and cyclonic vorticity will be amplified by such time-mean heating gradients. While this source and the AEW-scale diabatic source diminish over West Africa, they account for the growth of PV observed between East and West Africa (e.g., cf. Figs. 5c.f with Figs. 5b,e, respectively).

Vertical gradients in the time-mean heating generate large contributions over the ocean below 800 hPa. This is associated with the large amount of deep moist convection in the ITCZ over ocean. Thus, both time-mean and AEW-scale diabatic heating support the growth of a low-level circulation over ocean. Over land, there is a negative contribution below 800 hPa by this source. This is likely associated with the low-level gradient in evaporative cooling below the time-mean stratiform cloud.

6. Discussion

a. A conceptual model of an AEW

From the perspective of PV, we find that AEW dynamics can be represented to a leading order as follows:

\[
\frac{dP_w}{dt} = -\nabla \cdot \nabla P_w - \nabla \cdot \nabla P + \frac{\partial Q_w}{\partial p} - \frac{\partial Q}{\partial p}.
\]  

(10)

Here the AEW-scale PV tendency is approximated by four terms: advection of the AEW PV by the background flow (source A), AEW-scale advection of the background PV (source B), vertical gradients in AEW-scale diabatic heating (source C), and a stretching of the AEW-scale vorticity by the background heating (source D).

The propagation of southern-track AEWs is primarily a result of two physical processes. In the mid- and upper levels, advection (terms A and B) is dominant. Source A represents the action of the mean flow (i.e., the AEJ) while source B represents the canonical adiabatic Rossby wave propagation. At low levels, the PV source associated with AEW-scale diabatic heating (term C) is dominant. Generation of PV at the low levels is characteristic of deep moist convection (as opposed to stratiform cloud). Since deep moist convection is associated with the northerlies of the midlevel AEW (e.g., Fig. 5 and Janiga and Thorncroft 2016), this suggests that there is a coupling between a midlevel dry wave and a low-level moist wave.

Figure 11 depicts a conceptual diagram representing possible growth mechanisms for the AEW. We will describe each mechanism in turn. Both barotropic and baroclinic instabilities are encompassed in source term B in Eq. (10). The upshear tilts in the horizontal indicate growth by barotropic instability. This can be interpreted as the interaction between counterpropagating waves on the midlevel positive and negative PV gradients. This process is described in the first row of Table 2 and is shown as an interaction between the two midlevel dry PV anomalies in Fig. 11b. Any dry baroclinic instability will occur due to an interaction between waves on the midlevel negative PV gradient and those on the northern low-level positive PV gradient since there are only conducive upshear tilts and PV gradients in this latitude band. This is highlighted in the second row of Table 2 and is represented by the interaction between the two northernmost dry PV anomalies in Fig. 11b. Previous studies have suggested that a form of mixed or moist baroclinic instability may be applicable to AEW growth (e.g., Berry and Thornicroft 2005). In our analysis, we see evidence for interactions between a midlevel dry wave and a lower-level moist wave in the southern track, with favorable upshear tilts for instability. The third row in Table 2 describes the potential interaction between these two. The midlevel dry wave generates moist convection to the west of its PV anomaly (e.g., Fig. 5 and Janiga and Thorncroft 2016). This moist convection will enhance the low-level moist wave (e.g., Figs. 10c,d) through diabatic generation of PV. There then exists the potential for feedback of the low-level moist wave on the midlevel dry wave through induced circulations given the upshear tilt as shown in Fig. 11. However, as shown in the previous section, the contribution to growth by AEW-scale advection of the time-mean PV [source B in Eq. (10); encompassing all the instabilities we have just described] only accounts for half of the total growth of the midlevel AEW and does not explain the growth of PV above 600 hPa. Other terms such as the AEW-scale and background heating contribute more strongly to growth of the midlevel AEW, especially over East Africa.

The source of PV driven by AEW-scale diabatic heating [source C in Eq. (10)] represents the effect of coupled moist convection, predominantly stratiform cloud over eastern and central Africa (e.g., Fig. 5), generating PV anomalies that add to the already present AEW PV anomaly [similar to that proposed in Berry and Thornicroft (2005)]. In Fig. 11 this is represented by the midlevel moist PV anomaly below the predominant stratiform cloud in the AEW trough. The direct amplification of the preexisting dry midlevel PV anomalies by AEW-scale diabatic heating does not fit the classical models of moist baroclinic instability. A somewhat related mechanism was proposed by Adames and Ming (2018), who in effect showed that a neutral, lower-tropospheric Rossby wave could be destabilized in the presence of moist convection in an idealized model for monsoon depressions. The mechanism for instability
relies on a prognostic specification for moisture that is coupled to surface vorticity. This in turn ensures that deep convection and vortex stretching reinforce the adiabatic tendency, thereby leading to propagation and amplification of the wave. Finally, the PV source driven by the background heating [source D in Eq. (10)] represents the effect of ever-present stratiform clouds in the ITCZ that can stretch the AEW vorticity anomalies as they pass through the region, thereby enhancing the AEW PV anomalies. In Fig. 11 this is represented by the colored vertically pointing arrows. It is these two diabatic mechanisms—driven by the AEW-scale and background heating—that allow AEWs to grow to their typical observed magnitudes.

b. AEWs and DRWs

Previous studies have suggested the applicability of the DRW conceptual model to AEWs (e.g., Moore and Montgomery 2005; Berry and Thorncroft 2005; Tomassini et al. 2017). We argue that the AEW system does not resemble a canonical DRW for two reasons.

Table 2. Possible interactions between waves in the AEW. PV anomalies are denoted by $P$ with $d$ for dry waves and $m$ for moist waves, and low for low-level waves and mid for midlevel waves. $Q$ represents induced diabatic processes enhancing a moist wave and $v$ represents an induced cross-advection enhancing a dry wave. PV gradients referred to are those shown in Fig. 3.

<table>
<thead>
<tr>
<th>Instability</th>
<th>Waves involved</th>
<th>Interactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barotropic instability</td>
<td>Two midlevel dry waves (positive and negative PV gradients)</td>
<td>$P_{\text{mid}}^d \rightarrow v \rightarrow P_{\text{mid}}^d \rightarrow v \rightarrow P_{\text{mid}}^d$</td>
</tr>
<tr>
<td>Dry baroclinic instability</td>
<td>Midlevel dry wave (negative PV gradient) and surface dry wave (northern track)</td>
<td>$P_{\text{mid}}^d \rightarrow v \rightarrow P_{\text{low}}^d \rightarrow v \rightarrow P_{\text{mid}}^d$</td>
</tr>
<tr>
<td>Moist instability</td>
<td>Midlevel dry wave (negative PV gradient) and low-level moist wave (both southern track)</td>
<td>$P_{\text{mid}}^d \rightarrow Q \rightarrow P_{\text{low}}^m \rightarrow v \rightarrow P_{\text{mid}}^d$</td>
</tr>
</tbody>
</table>
First, there is no coupling between a surface dry wave and the low-level moist wave in the southern track. The baroclinic instability associated with a coupling of waves in this manner is the growth mechanism for DRWs described by Cohen and Boos (2016). Thus the growth mechanism for DRWs described by Cohen and Boos (2016) is likely not present in AEWs. While other studies suggest that a baroclinic interaction between two upshear-tilted moist waves is the growth mechanism for DRWs (Raymond and Jiang 1990; Parker and Thorpe 1995), we find little evidence that such a mechanism could be the dominant growth mechanism in AEWs. Rather, the direct addition of PV anomalies by AEW-scale heating and the stretching of existing AEW vorticity anomalies by the background heating are dominant.

Second, in a DRW, convection is coupled to isentropic ascent in a region of strong meridional temperature gradient. Although some studies have shown that convection is modulated by quasigeostrophic synoptic-scale ascent, other factors such as moisture convergence, thermodynamics, and organization of convection by wind shear are important (e.g., Kiladis et al. 2006; Hall et al. 2006; Janiga and Thorncroft 2016; Tomassini et al. 2017). Tomassini et al. (2017) and Tomassini (2018) argued that moisture is the primary factor coupling moist convection with the AEW. In fact, studies such as Fink and Reiner (2003) have questioned whether the magnitude of adiabatic ascent is sufficient to overcome the convective inhibition. Further, the low-level moist wave in the southern track is not associated with strongly sloping isentropes (e.g., Figs. 3e,f). As shown by Hannah and Aiyyer (2017), the dynamical scaling for the southern-track AEWs is more appropriately described by the weak temperature gradient (WTG) approximation. Thus, it is unlikely that adiabatic ascent is the primary driver of coupled moist convection.

7. Conclusions

The propagation and growth of AEWs is mediated by both adiabatic and diabatic processes. In the southern track of AEWs, the midlevel wave propagates primarily via adiabatic PV advections while the low-level wave propagates through diabatic PV generation. This is consistent with past literature such as Mass (1979), who argued that propagation of the midlevel wave is not strongly influenced by moist convection. These waves are coupled to each other via the generation of moist convection in the northerlies of the midlevel AEW. While the necessary environmental conditions and PV structure for barotropic and baroclinic instability exist, even when considering forms of moist instability, they do not completely explain the growth of AEWs. Rather, vertical gradients in AEW-scale and background diabatic heating, likely driven by stratiform cloud regions, contribute most strongly to the direct amplification of the midlevel PV, especially over East Africa.

The discussion in the previous section describes the AEW over land. Over ocean, there is a clear transition toward a more upright PV column where deep moist convection plays a much larger role than stratiform clouds. In fact this transition begins long before the AEW reaches the Atlantic and appears to be driven, at least partly, by the AEW-relative distribution of latent heating. AEW-scale diabatic heating transitions from a distribution representative of stratiform cloud over East Africa, to a distribution representative of deep convection over the Atlantic, with a vertical structure indicating a mix of the two over West Africa. This leads to diabatic PV tendencies that transition from the midlevels over East Africa to the low levels over the Atlantic.

Finally, this study reveals that moist convective coupling with the AEW is essential for the enhancement of low-level circulation in the southern track. Without coupled moist convection, the background flow would tilt the PV column downshear due to the environmental shear between low-level monsoon westerlies and the AEJ. The continual generation of new PV by deep moist convection in the northerlies maintains a column with an upshear tilt. There are significant implications here for the generation of TCs since a low-level circulation is positively correlated with TC genesis (Russell et al. 2017).

A caveat of this analysis is that we rely on non-convection-permitting reanalyses. These data use convective parameterization to diagnose the distribution of latent heating, which in our study area is mostly driven by MCSs (e.g., Laing et al. 2008; Schwendike and Jones 2010). A companion study (Russell et al. 2020) examines the evolution of AEWs in convection-permitting simulations. That study corroborates many of the results presented here indicating that conclusions based on the latent heating distributions presented in Fig. 5, are appropriate when the structure of MCSs is taken into account. Further, it highlights the role of stratiform cloud in enhancing the midlevel AEW and presents a conceptual model for AEW intensification and maintenance by diabatic processes based, in part, on the results presented in this study.

Acknowledgments. This research was sponsored by NSF through Award 1433763 and NASA through Awards NNX16AD76G and NNX13AH47G. We thank the staff at the ECMWF, NCEP, and NASA for free access to the ERAI, CFSR, and TRMM datasets. We also thank Joshua Dylan White, Gary Lackmann, Matthew Parker, William Boos, and Carl Schreck for constructive discussion.
We are grateful to George Kiladis, Chris Thorncroft, and an anonymous reviewer for their suggestions to improve the article.

APPENDIX

Wave-Relative Isobaric PV Tendency Equation

For completeness, we begin from the primitive equations in isobaric coordinates [Vallis 2006, p. 79, Eq. (2.153)]:

\[ \frac{D\mathbf{V}}{Dt} = -f\mathbf{k} \times \mathbf{V} - \mathbf{V}_p \Phi + \mathbf{F}, \quad (A1) \]

\[ \frac{D\theta}{Dt} = Q, \quad (A2) \]

\[ \mathbf{V} \cdot \mathbf{U} = 0, \quad (A3) \]

\[ \frac{\partial \Phi}{\partial p} = -\alpha, \quad (A4) \]

where \( \mathbf{V} = \mathbf{u} + \mathbf{v} \), \( \mathbf{U} = \mathbf{u} + \mathbf{v} + \omega_0 \mathbf{k}, \ f = 2\Omega \sin \phi, \ \mathbf{v}_p = (\partial/\partial x)\mathbf{i} + (\partial/\partial y)\mathbf{j}, \ \mathbf{V} = (\partial/\partial x)\mathbf{i} + (\partial/\partial y)\mathbf{j} + (\partial/\partial p)\mathbf{k}, \ \mathbf{F} \) is a frictional torque, \( Q \) is the diabatic heating, and all other variables have their typical meteorological meanings. We form a three-dimensional vorticity equation by taking \( \mathbf{V} \times \) Eq. (A1) to give

\[ \frac{D\eta}{Dt} = (\eta \cdot \mathbf{V})\mathbf{U} + \mathbf{V} \times \mathbf{V_p} \Phi + \mathbf{V} \times \mathbf{F}, \quad (A5) \]

where \( \eta \) is the hydrostatic absolute vorticity given as

\[ \eta = \mathbf{V} \times \mathbf{V} + f\mathbf{k}. \quad (A6) \]

Operating \( \mathbf{V} \theta \) on Eq. (A5) gives

\[ \mathbf{V} \theta \cdot \frac{D\eta}{Dt} = (\eta \cdot \mathbf{V})\mathbf{U} \cdot (\mathbf{V} \theta) + \mathbf{V} \theta \cdot \mathbf{V} \times \mathbf{F}. \quad (A7) \]

The term involving \( \mathbf{V} \theta \cdot (\mathbf{V} \times \mathbf{V_p} \Phi) \) is not present in Eq. (A7) because it vanishes due to the vector identities involved. Now, operating \( \eta \cdot \mathbf{V} \) on Eq. (A2) gives

\[ \eta \cdot \nabla \frac{D\theta}{Dt} = \eta \cdot \mathbf{V} Q, \quad (A8) \]

which can be manipulated using the vector product rule to get

\[ \eta \cdot \frac{D\mathbf{V} \theta}{Dt} = -(\eta \cdot \mathbf{V})\mathbf{U} \cdot (\mathbf{V} \theta) + \eta \cdot \mathbf{V} Q. \quad (A9) \]

Adding Eqs. (A9) and (A7) yields

\[ \frac{D}{Dt} (\eta \cdot \mathbf{V} \theta) = \eta \cdot \mathbf{V} Q + \mathbf{V} \theta \cdot \mathbf{V} \times \mathbf{F}. \quad (A10) \]

Defining \( P \), the isobaric PV as

\[ P = -g(\eta \cdot \nabla \theta). \quad (A11) \]

Equation (A10) then becomes the isobaric PV budget [Eq. (3)]:

\[ \frac{DP}{Dt} = -g \eta \cdot \mathbf{V} Q - g \mathbf{V} \theta \cdot \mathbf{V} \times \mathbf{F}. \quad (A12) \]

We now wish to separate the terms in the above isobaric PV budget such that we can isolate the contributions to PV on the wave scale. If we neglect the effects of frictional torque and apply a filter (denoted by the subscript \( w \)) for the waves, we can write Eq. (A12) as

\[ \frac{\partial P}{\partial t} = \frac{\partial P}{\partial p} - \frac{\omega}{\partial q} \left[ g \left( S \cdot \mathbf{V} Q \right)_w - g \left( \xi + f \right) \frac{\partial q}{\partial p} \right]_w, \quad (A13) \]

where \( S \) is the quasi-horizontal relative vorticity vector and \( \xi \) is the vertical relative vorticity. Beginning with term \( B \) as an example, we can expand this by stating \( \omega = \omega_0 + \omega' \) and \( P = P_0 + P' \):

\[ \frac{\partial P}{\partial t} = \frac{\partial P}{\partial p} + \omega' \frac{\partial P'}{\partial p} + \omega \frac{\partial P'}{\partial p} + \omega_0 \frac{\partial P_0}{\partial p} + \omega_0 \frac{\partial P'}{\partial p} + \omega_0 \frac{\partial P_0}{\partial p} \quad (A14) \]

and then

\[ \left( \frac{\omega}{\partial p} \right) = 0 + \omega_0 \left[ \frac{\partial P}{\partial p} \right]_w + \frac{\partial P}{\partial p} + \omega_0 \frac{\partial P'}{\partial p} \quad (A15) \]

If we now define \( \omega' = \omega_0 + \omega \) and \( P' = P_0 + P_r \), where \( r \) is a residual after the wave-filtered and time-mean variables have been subtracted from the full variable, the fourth term on the rhs of Eq. (A15) becomes

\[ \left( \omega_0 \frac{\partial P}{\partial p} \right)_w = \left( \omega_0 \frac{\partial P}{\partial p} \right)_w + \left( \omega_0 \frac{\partial P'}{\partial p} \right)_w \]

and term \( B \) in Eq. (A13) becomes

\[ \left( \omega_0 \frac{\partial P}{\partial p} \right)_w = \left( \omega_0 \frac{\partial P}{\partial p} \right)_w + \left( \omega_0 \frac{\partial P}{\partial p} \right)_w + \left( \omega_0 \frac{\partial P}{\partial p} \right)_w \]

\[ + \left( \omega_0 \frac{\partial P}{\partial p} \right)_w + \left( \omega_0 \frac{\partial P}{\partial p} \right)_w + \left( \omega_0 \frac{\partial P}{\partial p} \right)_w. \quad (A17) \]
Following the same process with the other terms in Eq. (A13) we can define the wave-scale PV tendency equation [Eq. (4)]:

$$\frac{dP_w}{dt} = -\nabla_x \cdot \nabla \cdot \nabla - (\nabla_x \cdot \nabla_w^2 - \nabla_y \cdot \nabla_w^2 - \nabla_z \cdot \nabla_w^2) - \nabla_x \cdot \nabla_w^2 - \nabla_y \cdot \nabla_w^2 - \nabla_z \cdot \nabla_w^2.$$  
(A18)

REFERENCES


