Connecting the Energy and Momentum Flux Response to Climate Change Using the Eliassen–Palm Relation

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(Manuscript received 21 November 2017, in final form 8 May 2018)

ABSTRACT

Coupled climate models project that extratropical storm tracks and eddy-driven jets generally shift poleward in response to increased CO₂ concentration. Here the connection between the storm-track and jet responses to climate change is examined using the Eliassen–Palm (EP) relation. The EP relation states that the eddy potential energy flux is equal to the eddy momentum flux times the Doppler-shifted phase speed. The EP relation can be used to connect the storm-track and eddy-driven jet responses to climate change assuming 1) the storm-track and eddy potential energy flux responses are consistent and 2) the response of the Doppler-shifted phase speed is negligible. We examine the extent to which the EP relation connects the eddy-driven jet (eddy momentum flux convergence) response to climate change with the storm-track (eddy potential energy flux) response in two idealized aquaplanet model experiments. The two experiments, which differ in their radiation schemes, both show a poleward shift of the storm track in response to climate change. However, the eddy-driven jet shifts poleward using the sophisticated radiation scheme but equatorward using the gray radiation scheme. The EP relation gives a good approximation of the momentum flux response and the eddy-driven jet shift, given the eddy potential energy flux response, because the Doppler-shifted phase speed response is negligible. According to the EP relation, the opposite shift of the eddy-driven jet for the different radiation schemes is associated with dividing the eddy potential energy flux response by the climatological Doppler-shifted phase speed, which is dominated by the zonal-mean zonal wind.

1. Introduction

Midlatitude eddies transfer momentum and energy toward the poles, modifying the atmospheric mean flow. Eddy momentum flux convergence determines the location of the jet stream, and eddy energy flux has been used to define the location of storm tracks, which influence midlatitude weather (Chang et al. 2002; Shaw et al. 2016; Barpanda and Shaw 2017). Understanding the response of jets and storm tracks to climate change has been proposed as one of the grand challenges of climate science (Bony et al. 2015).

Climate models generally predict a poleward shift of the annual-mean zonal-mean storm tracks in response to increased greenhouse gas emissions (Yin 2005; Chang et al. 2012; Shaw et al. 2016). The zonal-mean eddy-driven jet stream in CMIP5 models, as indicated by the location of the surface westerlies, also generally exhibits a poleward shift, although there are regional exceptions and a large model spread (Barnes and Polvani 2013; Simpson et al. 2014; Vallis et al. 2015). While several mechanisms have been proposed to explain the poleward jet shift, there is currently no complete theory that predicts the jet response and explains the model spread (see review in Vallis et al. 2015). Furthermore, there does not exist a theory that has been used to connect the eddy-driven jet and storm-track shifts.

Since the greenhouse gas concentration affects the energy budget directly, a natural starting point for explaining the circulation shift in response to increased CO₂ is the energy budget. When one defines the storm track energetically—for example, using dry static energy fluxes—then one can attempt to explain the poleward shift using energetic arguments. For example, arguments related to Clausius–Clapeyron scaling of water vapor and energy flux compensation have been put forward to explain the storm-track shift (Held and Soden 2006; Frierson et al. 2007; Shaw and Voigt 2016). A connection between the eddy energy and momentum...
flux would then allow for a prediction of the eddy-driven jet shift. In the literature, an alternative approach to studying the response of the midlatitude circulation to climate change has been to impose energetic perturbations in the form of diabatic heating perturbations in dry dynamical core models (e.g., Lorenz and DeWeaver 2007; Butler et al. 2010; Tandon et al. 2013; Lu et al. 2014). Representing the thermal response to increased CO$_2$ using an imposed diabatic heating is problematic because it assumes the thermal response depends only on the radiative effect of increased CO$_2$, yet the atmospheric thermal response is influenced by eddy energy and momentum fluxes. Furthermore, this approach cannot be used to connect the storm-track and eddy-driven jet responses.

Here we use a relationship derived by Eliassen and Palm (1961; herein called the EP relation) to infer the eddy momentum flux and eddy-driven jet response to increased CO$_2$ concentration given the eddy energy flux response. The EP relation states that eddy potential energy flux on pressure surfaces is equal to the eddy momentum flux times the Doppler-shifted phase speed (i.e., the eddy phase speed minus the zonal-mean zonal wind) assuming small-amplitude plane waves and neglecting nonconservative effects (see section 2 for the full derivation). According to the EP relation, the response of the eddy momentum flux to climate change is a function of the response of the eddy potential energy flux, the zonal-mean zonal wind, and the eddy phase speed.

Here, we examine the EP relation and assess the extent to which it can be employed to connect the eddy-driven jet and storm-track responses (indicated by the eddy momentum flux convergence and eddy potential energy flux, respectively) to climate change. We review the derivation of the EP relation and examine its validity for two idealized general circulation model (GCM) climate change simulations. The idealized GCMs include two different radiation schemes. The first radiation scheme is the sophisticated Rapid Radiative Transfer Model for GCMs (RRTMG; Iacono et al. 2008), and the other radiation scheme is the idealized gray radiation (GR) scheme (Frierson et al. 2006). We find that the EP relation approximately holds for the climatology and response to climate change in both models. The vertically integrated eddy potential energy flux shifts poleward in response to climate change in both models, whereas the vertically integrated eddy momentum flux convergence shifts poleward in the RRTMG model and equatorward in the GR model. Dwyer and O’Gorman (2017) and Schneider et al. (2010) previously noted the equatorward shift of the eddy-driven jet using GR. The EP relation is used here to explain the opposite shifts of the eddy momentum flux convergence in the two experiments.

In section 2, we derive the EP relation for plane waves, including all nonlinear and nonconservative terms. A derivation of the EP relation on model (sigma) surfaces is given in the appendix. The physical interpretation of the EP relation and the role of the eddy potential energy flux in the total atmospheric energy flux are also discussed in section 2. In section 3, we describe the idealized model experiments and the analysis methods. The results are described in section 4, including 1) the EP relation and its response to climate change in the model simulations; 2) the response of the eddy momentum flux and eddy-driven jet stream to climate change and the extent to which the EP relation succeeds in estimating it; and 3) the role of the Doppler-shifted phase speed in the eddy-driven jet shift in response to climate change. A summary and discussion of the results are presented in section 5.

2. The EP relation

In this section we derive the EP relation, which connects the eddy potential energy flux and the eddy momentum flux. We discuss the physical interpretation of the EP relation and the role of the eddy potential energy flux in the total atmospheric energy flux. We argue that the EP relation can be used to connect the eddy-driven jet stream and storm-track responses to climate change.

a. The EP relation for plane waves

The EP relation, originally developed by Eliassen and Palm (1961) for stationary waves [their Eq. (10.5)], connects the eddy potential energy flux with the eddy momentum and heat fluxes. The generalization of this relation for transient eddies and its consequence in terms of the zonal-mean momentum budget were developed by Andrews and McIntyre (1976) [see also Lindzen (1990), section 8.4, for a derivation of the EP relation]. We repeat here the derivation of the meridional component of the EP relation, starting from the eddy zonal momentum equation in pressure coordinates:

$$\frac{\partial u'}{\partial t} + \nabla \cdot \mathbf{u} = -\frac{\partial}{\partial x} \Phi' - \nu (f + \zeta) = L', \tag{1}$$

where $u'$ and $v'$ are the zonal and meridional velocity components, respectively, $\Phi$ is the geopotential, $f$ is the Coriolis parameter, and $\zeta$ is the vertical component of the vorticity. The overbar denotes zonal mean and the prime denotes deviation from the mean, that is, the eddy component. The terms on the left-hand side of (1) represent the time tendency, linear horizontal advection by the zonal-mean zonal wind, pressure gradient force, Coriolis force, and advection of the zonal-mean momentum by the eddy meridional wind. The right-hand side is a residual term $L'$, including nonlinear terms, nonconservative processes, and vertical...
advection. Next we assume a plane wave solution of the form \( A \times \exp[i(k(x - ct))] \), where \( A \) is the amplitude of the wave field. Substituting the plane wave solution into (1) gives
\[
-ik[(\pi - c)u' + \Phi'] - \nu(f + \xi) = L'.
\]
(2)

Multiplying (2) by the complex conjugate of \((\pi - c)u' + \Phi'\), applying a zonal average, taking the real part, and dividing by \(-f/(f + \xi)\), gives
\[
(\pi - c)(\overline{u'u'}) + (\overline{\Phi'\Phi'}) = -\frac{1}{f + \xi}[(\pi - c)(\overline{u'L'}) + (\overline{\Phi'L'})].
\]
(3)

The dominant balance in (3) is between the two terms on the left-hand side [the terms on the right-hand side of (3) in our simulations are relatively small, as shown in section 4]. Therefore we can approximate (3) by
\[
-\overline{(\Phi'\Phi')} = (\pi - c)(\overline{u'u'}).
\]
(4)

We will refer to (4) as "the EP relation."1 Equation (4) holds in pressure coordinates, but we perform the calculations in sigma coordinates, where an additional term is added to the equation because of variations of pressure on sigma levels (see the appendix). The EP relation connects the eddy momentum flux to the eddy potential energy flux and the Doppler-shifted phase speed. This suggests that a change in the eddy momentum flux can be attributed to a change either in the eddy potential energy flux, the zonal-mean zonal wind, the eddy phase speed, or a combination of those terms. Here we use the EP relation to evaluate the importance of each of these terms for the response of the eddy momentum flux to climate change in two idealized GCM simulations, in order to explain the eddy-driven jet shift. Alternatively, the EP relation can be used to attribute the eddy potential energy flux response to climate change to the response of the eddy momentum flux, zonal-mean zonal wind, or eddy phase speed.

b. Physical interpretation of the EP relation and its connection to the energy budget

The physical interpretation of the EP relation can be understood in terms of the eddy energy budget. Following Plumb (1983) [his Eqs. (4.12a) and (4.12b)], it can be shown that the eddy energy density \( \dot{E} \) (the sum of the eddy kinetic and available potential energy per unit mass) satisfies the following equation:
\[
\frac{\partial \dot{E}}{\partial t} = \mathbf{F} \cdot \nabla \pi - \nabla \cdot (\overline{\Phi'\mathbf{v}}),
\]
(5)

where nonconservative terms are neglected, \( \mathbf{F} \) is the EP flux whose meridional component is approximately \(-\overline{u'v'}\), and \( \mathbf{v} \) is the velocity in the meridional and vertical plane. The first term on the right-hand side of (5) represents conversion of zonal-mean flow energy to eddy energy, and the second term represents eddy potential energy flux divergence. Eddy energy density is not globally conserved because of the energy conversion between the eddies and the mean flow. It is useful to consider the pseudoenergy, which is the conserved quantity associated with eddy energy, defined as \( e = (\omega/\bar{\omega})\dot{E} \), where \( \omega \) is the wave frequency and \( \bar{\omega} \) is the intrinsic wave frequency (i.e., the Doppler-shifted wave frequency). In the absence of nonconservative processes and under the assumption of a slowly varying mean flow, pseudoenergy satisfies
\[
\frac{\partial e}{\partial t} = -\nabla \cdot (\overline{\Phi'\mathbf{v}}) - \pi \mathbf{F},
\]
(6)

(Andrews and McIntyre 1978; Bühler 2009) where \((\overline{\Phi'\mathbf{v}}) - \pi \mathbf{F}\) is the pseudoenergy flux. The conserved quantity associated with momentum is the pseudomomentum, \( \dot{p} = (k/\bar{\omega})\dot{E} \), where \( k = \omega/c \) is the wavenumber, which satisfies
\[
\frac{\partial \hat{p}}{\partial t} = \nabla \cdot \mathbf{F}
\]
(7)

(Andrews and McIntyre 1978). The pseudoenergy and pseudomomentum fluxes are in the direction of eddy propagation (i.e., the eddy group velocity). The pseudoenergy flux is the familiar EP flux. It is easy to show from the definitions of pseudoenergy and pseudomomentum and their governing equations (6) and (7) that \((\overline{\Phi'\mathbf{v}}) - \pi \mathbf{F} = -c\mathbf{F}\), which is the EP relation (4). The EP relation therefore represents the relationship between the pseudoenergy and pseudomomentum fluxes, which can be derived from Noether’s theorem (Shaw and Shepherd 2008; Bühler 2014).

Eddy potential energy flux also appears in the total atmospheric energy budget. In a steady state, the divergence of the vertically and zonally integrated total atmospheric energy flux balances the net energy input into the atmosphere at each latitude (e.g., Trenberth 1997):
\[
\frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} [\cos \phi \overline{\mathbf{F}(\overline{\mathbf{v}})}] = Q,
\]
(8)
where $a$ is Earth’s radius, $E = s + Lq + K$ is the total energy, $s = c_pT + \Phi$ is the dry static energy, $c_p$ is the specific heat at constant pressure, $T$ is the temperature, $L$ is the latent heat of vaporization, $q$ is the specific humidity, $K$ is the kinetic energy, and $Q$ is the net energy input rate into the atmosphere as a function of latitude. Angle brackets denote vertical mass-weighted integral. The energy flux can be decomposed into contributions from the zonal-mean flow and the eddies: $\mathcal{E}v = \mathcal{E} v + (\mathcal{E} \bar{v})$. The decomposition of the potential energy flux $\mathcal{E} \bar{v}$ into eddy and zonal-mean flow contributions is sensitive to the choice of vertical coordinate (see the appendix). In what follows we will use the term “eddy potential energy flux” referring to pressure surfaces, where it satisfies (4). The vertically integrated eddy potential energy flux in the atmosphere is predominantly equatorward, and it is a few percent of the total meridional energy flux in (8) (Peixoto and Oort 1992).

c. Using the EP relation to connect the eddy-driven jet and storm track

The meridional component of the EP relation (4) can be used to connect the eddy-driven jet stream (defined by the eddy momentum flux convergence) and storm-track responses to climate change, assuming 1) the eddy potential energy flux response to climate change shifts in the same direction as the storm track defined by the dry static energy flux and 2) the response of the Doppler-shifted phase speed to climate change is negligible. We find that both assumptions are satisfied in the idealized climate change simulations examined here (see section 4).

3. Data and methods

a. Idealized model simulations

We use an idealized aquaplanet GCM [described in Frierson et al. (2006) and O’Gorman and Schneider (2008)]. All the simulations use a spectral dynamical core with T42 resolution and 30 vertical levels, but with different radiative schemes. The solar insolation is in the same equinox configuration as in Frierson et al. (2006). The ocean is represented by a slab ocean with a mixed layer depth of 30 m and no heat transport. The model is run for 18 model years, and the results are averaged over the last 12 years and both hemispheres. The analysis is based on 6-hourly model output.

We made use of two different radiation schemes. The first radiation scheme called GR is a two-stream gray radiation scheme with a specified longwave optical thickness and no water vapor radiative effects (Frierson et al. 2006). The GR experiment uses the model setup and parameters as in O’Gorman and Schneider (2008), except for the ocean mixed layer depth. Climate change is simulated by increasing the optical depth parameter $\alpha$, which controls the absorption of longwave radiation in the atmosphere; $\alpha$ is set to 1 in the control simulation and 2.5 in the climate change simulation. The mean surface temperature is 289.1 K in the control simulation and 303.9 K in the climate change simulation. Dwyer and O’Gorman (2017) previously noted the eddy-driven jet shifts equatorward in response to increased longwave optical depth in the GR model, which is opposite to the shift of the eddy heat flux.

The second radiation scheme called RRTMG is a sophisticated radiation scheme based on a correlated k-distribution band model (Iacono et al. 2000, 2008). Climate change is simulated by increasing the CO2 mixing ratio from 355 ppmv in the control simulation to 5680 ppmv (16 times the control value) in the climate change simulation. The mixing ratios of other gases are uniform and prescribed as follows: O3 = 30 ppbv, CH4 = 1700 ppbv, and N2O = 320 ppbv. There are no chlorofluorocarbons (CFCs) and no stratospheric ozone layer in the RRTMG simulations. Water vapor is radiatively active and its concentration is determined by the hydrological cycle. The simulations use a clear sky radiation scheme with no clouds. The surface albedo is set to 0.25. The mean surface temperature is 291.1 K in the control simulation and 305.1 K in the climate change simulation. A more detailed discussion of the GR and RRTMG radiative schemes and a comprehensive investigation of the climate change response, including heating rates, will be presented in a separate study (Z. Tan et al. 2018, unpublished manuscript).

Here we use the GR and RRTMG simulations to examine the EP relation and its ability to connect the shift of the eddy-driven jet and eddy potential energy flux. The GR and RRTMG simulations represent qualitatively different climate change experiments, as evident from the different climatologies and responses of the zonal-mean flow to climate change. In the GR experiment there is amplified warming aloft in the tropics and at the surface in high latitudes (Fig. 1a), consistent with coupled climate models (Vallis et al. 2015). However, the temperature response in
The stratospheric jet is connected to a tropospheric jet in the upper troposphere. In response to climate change, the stratospheric jet strengthens and the tropospheric (eddy-driven) jet shifts poleward.

The poleward jet shift in response to increased CO₂ concentration using the RRTMG scheme and the eddy-driven jet shift in response to increased optical thickness using the GR scheme are robust across a wide range of CO₂ and optical thickness values (Z. Tan et al. 2018, unpublished manuscript). The relation between the eddy-driven jet shift and the stratospheric jet response in the GR and RRTMG experiments is consistent with previous studies, which showed that an enhanced (weakened) stratospheric jet leads to a poleward (equatorward) shift of the tropospheric eddy-driven jet (see Kidston et al. 2015 and references therein). In section 4, we analyze the EP relation in each experiment and examine its ability to connect the eddy momentum and energy flux responses to climate change for the different responses to increased greenhouse gases simulated by the different radiation schemes.

### b. Cospectra

We use the phase speed cospectrum of the eddy momentum flux as in Randel and Held (1991). In particular, we multiply the eddy momentum flux cospectrum by \((\overline{\pi} - c)\overline{u'}\) for each phase speed and sum over all phase speeds to calculate \((\overline{\pi} - c)(\overline{u'}^2)\). The phase speed cospectra are calculated for time segments of 120 days and then averaged over all segments in the averaging period. The phase speed range is from \(-35\) to \(35\, \text{m s}^{-1}\) with an interval of \(0.5\, \text{m s}^{-1}\). We use a similar method when calculating \(((\overline{\Phi u'})/(\overline{\pi} - c))[\text{see section 4, (10)}]; however, we exclude from the sum regions where \((\overline{\pi} - c) < 7\, \text{m s}^{-1}\) or where the absolute value of \((\overline{\Phi u'})\) is smaller than \(5\%\) of its maximum value to avoid unphysical values near the critical layers.

### c. Calculating shifts of the eddy-driven jet and storm track

We use the latitude of maximum mass-weighted vertically integrated eddy momentum flux convergence for the RRTMG experiment has a more realistic zonal-mean zonal wind profile (Fig. 1d), with a single jet concentrated around \(30°\) latitude in the upper troposphere. The vertically integrated eddy momentum flux convergence by \(-\partial\overline{\psi}/\partial y(\overline{u'v'})\) for brevity. The full expression in spherical coordinates is \(-1/(a \cos\phi) \partial\overline{\psi}/\partial\phi\cos^2\phi \int_0^\phi (1/g)(\overline{u'v'})\, dp\). According to our definition of the vertical integral operator (section 2), this expression is equal to \(-1/(2\pi a^2 \cos^2\phi) \partial\overline{\psi}/\partial\phi\cos\phi(\overline{u'v'}))\). The vertically integrated eddy momentum flux convergence thus defined has units of pascals and is approximately equal to the surface wind stress in steady state. We use a similar definition for \(-\partial\overline{\psi}/\partial y(\overline{\Phi u'})/(\overline{\pi} - c)\) below.
and eddy potential energy flux as indicators of the latitude of the eddy-driven jet and storm track, respectively. The eddy-driven jet latitude is measured by the vertically integrated eddy momentum flux convergence because the zonal-mean surface zonal wind (u_, defined as the zonal-mean zonal wind at the lowest model level) is proportional to the surface wind stress (τ), which is approximately equal to the vertically integrated eddy momentum flux convergence (Vallis 2006). We follow Hoskins and Valdes (1990) and define the storm track energetically, for example, the latitude of maximum vertically integrated dry static energy flux. The shift of the vertically integrated eddy potential energy flux follows the vertically integrated eddy dry static energy flux in the simulations presented here (as shown in section 4), thus it is connected to the storm-track shift. The eddy momentum flux and eddy momentum flux convergence response to climate change can be connected to changes in the eddy potential energy flux or changes in the Doppler-shifted phase speed. The latitude of maximum is calculated by 1) interpolation to a latitudinal grid with a 0.1° interval and 2) finding the latitude where the derivative of the interpolated field crosses zero from positive to negative.

4. Results

We start by examining the extent to which the EP relation (4) holds for the simulations described in section 3, using the GR and RRTMG radiation schemes. Next we use the EP relation to estimate the eddy momentum flux response to climate change, given the eddy potential energy flux and the Doppler-shifted phase speed responses. We show that the EP relation can be used to estimate the eddy-driven jet shift in response to climate change, given the response of the eddy potential energy flux and the climatological zonal-mean zonal wind and eddy phase speed, because the response of the Doppler-shifted phase speed is negligible. Finally, we examine the relative roles of the climatological zonal-mean zonal wind and the eddy phase speed in determining the eddy momentum flux and eddy-driven jet responses.

a. The EP relation and its response to climate change

The climatological (Φw) and (u - c)(u'w') are concentrated in the subtropical upper troposphere close to the latitude of maximum zonal-mean zonal wind for both radiation schemes (Fig. 2). In both experiments, (Φw) and (u - c)(u'w') exhibit differences near the surface, arising from boundary layer drag, which contributes to L’ [(3)]. In the upper troposphere the residual is larger in the RRTMG experiment than in the GR experiment. It arises mostly from nonlinear advection and advection of the zonal momentum by the eddy vertical velocity. The contribution of the different residual terms to the vertical integral of (4) is discussed below. The responses of −(Φw) and (u - c)(u'w') to climate change consist of an upward and poleward shift in both experiments (Fig. 2). The poleward shift is seen in the vertically dependent response for the RRTMG experiment (Figs. 2c,d). For the GR experiment it is seen more clearly after vertically integrating.

Next we examine the mass-weighted vertical integral of the terms in the EP relation (see section 2 for the definition of the vertical integral operator). While the magnitudes of minus the vertically integrated eddy potential energy flux and eddy momentum flux multiplied by the Doppler-shifted wind, that is, −(Φw) and (u - c)(u'w'), are not the same, with (u - c)(u'w') being generally larger especially in the RRTMG experiment, the two terms have a similar shape and exhibit a poleward shift in response to climate change (Fig. 3). The qualitative similarity demonstrates the usefulness of the EP relation in connecting the energy and momentum flux responses to climate change.

The residual term in the EP relation [right-hand side of (3)] explains the discrepancy between −(Φw) and (u - c)(u'w'), which is especially large for the RRTMG radiation scheme (Figs. 3c,d). To determine the dominant contributions to the residual term, we calculate each contribution separately, by replacing L’ in (3) with one of its components, where L’ is the sum of the terms

![Fig. 2. The terms in the EP relation (4) for the (top) GR and (bottom) RRTMG experiments: (a),(c) −(Φw) and (b),(d) (u - c)(u'w'). Contours show the control simulation and color shading shows the response to climate change in m³ s⁻¹. Note the different scales of the shading colors for the GR and RRTMG experiments. Contour interval is 250 m³ s⁻¹. Solid (dashed) contours denote positive (negative) values and the zero contour is omitted.](http://journals.ametsoc.org/doi/abs/10.1175/JCLI-D-17-0792.1)
in the eddy zonal momentum equation that are not explicitly written in (1):

\[
L' = -\frac{\tau}{a \cos \phi} \frac{\partial (u' \cos \phi)}{\partial \phi} - \frac{\partial u'}{\partial p} - \omega \frac{\partial \omega}{\partial p} - \left[ u' \frac{\partial u'}{\partial x} + \frac{\partial}{\partial x} \left( \frac{u' \cos \phi}{a \cos \phi} \frac{\partial (u' \cos \phi)}{\partial \phi} \right) + \omega \frac{\partial u'}{\partial p} \right] + F',
\]

where \( \omega \) is the vertical velocity in pressure coordinates and \( F \) is the boundary layer drag, which is represented in the model as vertical diffusion. We will refer to the term in the parentheses on the right-hand side of (9) as the nonlinear term.

In all the simulations the sum of the residual terms explains the discrepancy between \( -\langle \overline{\Phi' v'} \rangle \) and \( \langle (\overline{\tau - c})(\overline{u' v'}) \rangle \) (cf. solid and dashed black lines in Fig. 4). The total residual is much larger in the RRTMG experiment than in the GR experiment (note the different scaling of the y axis in Fig. 4). In the GR climatology simulation, boundary layer drag dominates (Fig. 4a). However, in the GR climate change simulation (Fig. 4b) and in both RRTMG simulations (Figs. 4c,d), the nonlinear term and the advection of the zonal-mean zonal wind by the eddies. This term is small compared with \( \langle (\overline{u - c})(\overline{u' v'}) \rangle \) and \( -\langle \overline{\Phi' v'} \rangle \) (Fig. 3), which justifies its neglect in (4).

The above results show that the EP relation (4) is approximately valid in our simulations. Next we examine the response of the EP relation to climate change. In both experiments the EP flux terms shift poleward in response to climate change (Table 1). In the GR experiment there is also a reduction of both terms on their poleward flanks around 50° latitude (Figs. 3a,b). In the RRTMG experiment \( -\langle \overline{\Phi' v'} \rangle \) and \( \langle (\overline{u - c})(\overline{u' v'}) \rangle \) increase in response to climate change (Figs. 3c,d).

We examine the magnitude and direction of the latitudinal shift of the vertically integrated fluxes in order to assess the relation between them (see section 3 for the definition of the latitudinal shift). The shifts of the EP relation terms and the storm track (eddy dry static energy flux) are summarized in Table 1. In both experiments the EP relation terms shift poleward; however, the shift of \( \langle (\overline{u - c})(\overline{u' v'}) \rangle \) (2.6° and 5.7° in the GR and RRTMG experiments, respectively) is larger than the shift of \( -\langle \overline{\Phi' v'} \rangle \) (1.2° and 4.8° in the GR and RRTMG experiments, respectively). The shift of the EP relation terms is consistent with the shift of the eddy dry static energy flux \( \langle (\overline{u' \overline{\delta \Phi}}) \rangle \) in terms of its direction; however, the shift of the eddy dry static energy flux is much larger (7.8° and 7.2° in the GR and RRTMG experiments, respectively). This shows the EP relation terms only approximate the magnitude of the storm-track shift for the simulations presented here.

### b. Using the EP relation to connect the eddy momentum and energy flux response

The results of the previous subsection demonstrate that the EP relation (4), which connects the energy and momentum fluxes, approximately holds for the climatology and climate change simulations in the idealized model. In both GR and RRTMG simulations the climate change response of the eddy potential energy flux is similar to that of the Doppler-shifted phase speed times the eddy momentum flux, that is, a poleward shift. The poleward shift of the eddy potential energy flux is consistent with the poleward shift of the storm track, as indicated by the eddy dry static energy flux.

In the GR experiment the eddy-driven jet shifts equatorward in response to climate change, as indicated by the surface wind stress (Fig. 5a) and the vertically integrated eddy momentum flux convergence (Fig. 5b). The eddy momentum flux convergence and surface wind stress become negative across a larger range of latitudes poleward of the eddy-driven jet in the climate change simulation, which is typical for a transition from a double to a single upper-tropospheric jet state (Son and Lee...
as indeed occurs in this case. This is consistent with the eddy momentum flux becoming negative at high latitudes in response to climate change (Fig. 5c). Note that the maximum vertically integrated eddy momentum flux does not shift significantly in latitude (Fig. 5c). In the RRTMG experiment there is a consistent poleward shift of the surface wind stress, eddy momentum flux convergence, and eddy momentum flux, without a significant change in structure (Figs. 5d–f).

The response of the eddy momentum flux to climate change can be estimated using the EP relation, given the response of the eddy potential energy flux and the Doppler-shifted phase speed. According to the EP relation, the eddy momentum flux is

\[
(u'v') = - \frac{(\Phi v')}{\bar{u} - c}. \tag{10}
\]

The latitudinal shift of the eddy-driven jet, which is equal to the shift of the maximum eddy momentum flux convergence, can be estimated by taking the meridional divergence of (10). Note that (10) is not valid near the critical latitudes, where \(\bar{u} = c\), but since these latitudes

![Figure 4](image)

**FIG. 4.** The vertically integrated residual terms in the EP relation [right hand side of (3)] for (a),(c) the control simulations and (b),(d) the climate change simulations of the (top) GR and (bottom) RRTMG experiments. The different terms are denoted in the legend by the components of \(L\) substituted into (3) [see (9) for the different components of \(L\), the residual term in the wave zonal momentum equation]. The residual terms were set to zero equatorward of \(\Phi\) latitude to avoid nonphysical values due to the term \(\frac{f}{z}\) in the denominator on the right-hand side of (3) approaching zero at the equator. Note the different scales of the \(y\) axes for the GR and RRTMG experiments. The similarity between the sum of all the residual terms (solid black curve) and the left-hand side of (3) (dashed black curve) demonstrates the accuracy of the calculation.

<table>
<thead>
<tr>
<th>Term</th>
<th>GR control</th>
<th>GR climate change</th>
<th>GR shift</th>
<th>RRTMG control</th>
<th>RRTMG climate change</th>
<th>RRTMG shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\langle\Phi v'\rangle)</td>
<td>28.7</td>
<td>29.9</td>
<td>1.2 (\uparrow)</td>
<td>29.8</td>
<td>34.6</td>
<td>4.8 (\uparrow)</td>
</tr>
<tr>
<td>(&lt;(\bar{u} - c)(u'v')&gt;)</td>
<td>28.2</td>
<td>30.8</td>
<td>2.6 (\uparrow)</td>
<td>29.1</td>
<td>34.8</td>
<td>5.7 (\uparrow)</td>
</tr>
<tr>
<td>(\Pi(u'v'))</td>
<td>28.5</td>
<td>30.8</td>
<td>2.3 (\uparrow)</td>
<td>29.1</td>
<td>34.7</td>
<td>5.5 (\uparrow)</td>
</tr>
<tr>
<td>(\Pi_{diss}(u'v'))</td>
<td>28.5</td>
<td>30.8</td>
<td>2.3 (\uparrow)</td>
<td>29.1</td>
<td>32.7</td>
<td>3.6 (\uparrow)</td>
</tr>
<tr>
<td>(\langle u'v' \rangle)</td>
<td>42.9</td>
<td>50.8</td>
<td>7.8 (\uparrow)</td>
<td>39.9</td>
<td>47.2</td>
<td>7.2 (\uparrow)</td>
</tr>
</tbody>
</table>
are located at the flanks of the jet, (10) is still useful for estimating the jet shift.

The vertical integral of the right-hand side of (10) 
\[ \int_{h} \left( \frac{F_{y0}}{u_{c}^{2}} \right) \frac{u_{0} y_{0}}{i} \] and its meridional convergence \[ \frac{\partial}{\partial y} \int_{h} \left( \frac{F_{y0}}{u_{c}^{2}} \right) \frac{u_{0} y_{0}}{i} \] in the GR and RRTMG experiments are shown in Fig. 6. The actual vertically integrated eddy momentum flux and its convergence are shown in dashed lines for comparison, multiplied by a factor of 0.5 for the GR simulations (Figs. 6b,c) and 0.3 for the RRTMG simulations (Figs. 6e,f). The magnitudes of \[ -\langle \frac{\Phi_{y0}}{\pi - c} \rangle \] and \[ \frac{\partial}{\partial y} \langle \frac{\Phi_{y0}}{\pi - c} \rangle \] are smaller than the magnitudes of \[ \langle \frac{u_{0} y_{0}}{i} \rangle \] and \[ -\frac{\partial}{\partial y} \langle \frac{u_{0} y_{0}}{i} \rangle \], respectively. However, the structure of the eddy momentum flux and eddy momentum flux convergence is qualitatively captured by the EP relation estimate far from the critical layers. The eddy momentum flux estimate from the EP relation underestimates the actual eddy momentum flux because of 1) the exclusion of the residual term and 2) not including the unphysical values of \( \langle \frac{F_{y0}}{\pi - c} \rangle \) in regions where the Doppler-shifted phase speed is small (see section 3).

Since the estimate of the eddy momentum flux based on the EP relation captures the structure of the eddy momentum flux well, we can use it to estimate the shift of the eddy-driven jet in response to climate change. Table 2 shows the latitudinal shift of the eddy-driven jet (i.e., the shift of the latitude of maximum vertically integrated eddy momentum flux convergence), surface westerlies, and surface wind stress in response to climate change and compares them with the estimates based on the EP relation. In the GR simulation the surface westerlies, the surface wind stress, and the vertically integrated eddy momentum flux convergence shift equatorward (by 8.4°, 8.0°, and 7.0°, respectively). The estimate of the eddy-driven jet shift based on the EP relation (the shift of \( \frac{\partial}{\partial y} \langle \frac{\Phi_{y0}}{\pi - c} \rangle \)), which is 5.4° equatorward, captures the direction of the shift and 77% of its magnitude. In the RRTMG simulation the surface westerlies, surface wind stress, and eddy momentum flux

---

Fig. 5. Eddy momentum flux and its convergence in the (top) GR and (bottom) RRTMG experiments: (a),(d) zonal-mean zonal surface wind stress, (b),(e) vertically integrated eddy momentum flux convergence, and (c),(f) vertically integrated eddy momentum flux. Blue and red curves are for the control and climate change simulations, respectively.

4 Unphysical values of \( -\langle \frac{\Phi_{y0}}{\pi - c} \rangle \) and \( \frac{\partial}{\partial y} \langle \frac{\Phi_{y0}}{\pi - c} \rangle \) near the critical latitudes were excluded from the calculation (see section 3). The variables in Fig. 6 were vertically integrated between 635 hPa and the top of the atmosphere in order to avoid large contributions of boundary layer drag and other residual terms in (3) in regions far from the jet core. Note that dividing by \( \langle u_{0} \rangle \) gives more weight to regions far from the jet core where the EP relation is less accurate because of the relatively large contribution of nonlinear and nonconservative terms. Subsequently, a three-point moving average filter was applied.
The estimate of the eddy-driven jet shift based on the EP relation (5.0° poleward) captures the direction of the shift and 76% of its magnitude. In summary, the response to climate change in the RRTMG experiment shows a consistent poleward shift of the storm track, eddy potential energy flux, eddy momentum flux convergence, and eddy-driven jet. In contrast, the storm-track and eddy potential energy flux shift in the opposite direction to the eddy-driven jet and eddy momentum flux convergence in the GR experiment. In both experiments the EP relation captures approximately 77% of the magnitude of the eddy-driven jet shift. Since the eddy potential energy flux shift follows the storm track in the simulations presented here, an analysis of the EP relation can reveal the conditions.

**TABLE 2.** Latitude of maximum surface westerlies, surface wind stress, vertically integrated eddy momentum flux convergence, and estimates of the vertically integrated eddy momentum flux convergence based on the EP relation (in degrees) for the control and climate change GR and RRTMG simulations, and the latitudinal shifts between the control and climate change simulations. Equatorward and poleward shifts are marked by downward and upward arrows, respectively.

<table>
<thead>
<tr>
<th></th>
<th>GR control</th>
<th>GR climate change</th>
<th>GR shift</th>
<th>RRTMG control</th>
<th>RRTMG climate change</th>
<th>RRTMG shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi)</td>
<td>45.3</td>
<td>36.9</td>
<td>-8.4</td>
<td>35.8</td>
<td>42.6</td>
<td>6.8</td>
</tr>
<tr>
<td>(\tau)</td>
<td>45.5</td>
<td>37.5</td>
<td>-8.0</td>
<td>36.0</td>
<td>42.8</td>
<td>6.8</td>
</tr>
<tr>
<td>(-\partial\pi/\partial y)</td>
<td>45.6</td>
<td>38.6</td>
<td>-7.0</td>
<td>37.2</td>
<td>43.8</td>
<td>6.6</td>
</tr>
<tr>
<td>(\partial\pi/\partial y)</td>
<td>43.2</td>
<td>37.8</td>
<td>-5.4</td>
<td>37.7</td>
<td>42.7</td>
<td>5.0</td>
</tr>
<tr>
<td>(-\partial\pi/\partial y)</td>
<td>43.2</td>
<td>37.0</td>
<td>-6.2</td>
<td>37.7</td>
<td>43.4</td>
<td>5.7</td>
</tr>
</tbody>
</table>
that allow for the opposite shifts of the storm track and eddy-driven jet in the GR experiment and the role of the Doppler-shifted phase speed, as shown in the next subsection.

c. The role of the Doppler-shifted phase speed for the eddy-driven jet response

According to the EP relation (10) the opposite shift of the storm track and eddy-driven jet in response to climate change using the GR radiation scheme could be due to 1) dividing the eddy potential energy flux response by the climatological Doppler-shifted phase speed or 2) the response of the Doppler-shifted phase speed. In this subsection we examine whether the climatological Doppler-shifted phase speed is sufficient for reproducing the eddy-driven jet shift in response to climate change given the eddy potential energy flux. In addition, we examine the contributions from the zonal-mean zonal wind and the eddy phase speed to the EP relation response. We focus mostly on the GR experiment in order to explain the opposite shifts of the storm track and eddy-driven jet.

Using the eddy momentum flux convergence estimate based on the EP relation, \( \frac{\partial}{\partial y} \left( \langle \Phi' \rangle \right) / (\langle \tau - c \rangle) \), the relative roles of the eddy potential energy flux and the Doppler-shifted phase speed responses to climate change for the eddy-driven jet shift can be examined by perturbing each of the factors separately. Ideally, one would calculate the contributions of the eddy potential energy flux, zonal-mean zonal wind, and eddy phase speed responses to the eddy momentum flux convergence estimate. This would require calculating the eddy phase speed, which can be defined as an eddy momentum flux–weighted average of the phase speed \( \langle c(u'v') \rangle / (\langle u' \rangle) \). However, this leads to unreasonable values where the eddy momentum flux is small. Therefore we examine the eddy momentum flux convergence estimate based on the EP relation when the zonal-mean zonal wind and the phase speed cospectrum of \( \langle \Phi' \rangle \) are changed separately. When the phase speed cospectrum is taken from the climate change simulation and \( \tilde{u} \) is taken from the control simulation, the eddy momentum flux convergence estimate based on the EP relation \( \left[ \frac{\partial}{\partial y} \langle \Phi' \rangle \right] \text{change}/(\langle \tau \text{clim} - c \rangle) \), where “clim” refers to climatology and “change” refers to climate change. Yellow curves in Figs. 6a,d] is similar to the eddy momentum flux convergence estimated from the control simulation (blue curves in Figs. 6a,d). This demonstrates that the climatological zonal-mean zonal wind and the response of the eddy potential energy flux phase speed cospectrum to climate change are sufficient to estimate the eddy-driven jet response using the EP relation. The eddy momentum flux convergence estimate using the climatological zonal-mean zonal wind \( \langle \Phi' \rangle \text{clim}/(\langle \tau \text{clim} - c \rangle) \) captures the eddy-driven jet shift in both the GR and RRTMG experiments (6.2° equatorward, which is 89% of the jet shift in the GR experiment, and 5.7° poleward, which is 86% of the jet shift in the RRTMG experiment: Table 2).

The relative roles of the zonal-mean zonal wind and the eddy phase speed in the EP relation response to climate change can be demonstrated by looking at \( \langle \tau - c \rangle \text{clim}/(\langle u' \rangle) \) and \( -\langle c(u'v') \rangle \) in the control and climate change simulations (Fig. 7). In both the GR and RRTMG experiments the poleward shift of \( \langle \tau - c \rangle \text{clim}/(\langle u' \rangle) \) is mostly due to \( -\langle c(u'v') \rangle \). This can be seen by comparing \( \langle \tau - c \rangle \text{clim}/(\langle u' \rangle) \) (Figs. 7a,c) and \( -\langle c(u'v') \rangle \) (Figs. 7b,d) with \( \langle \tau - c \rangle \text{clim}/(\langle u' \rangle) \) (Figs. 3b,d) and \( \langle u' \rangle \) (Figs. 5c,f). We find that 1) the meridional shift and the change in structure of \( \langle \tau - c \rangle \text{clim}/(\langle u' \rangle) \) in response to climate change are well captured by the response of \( \langle \tau - c \rangle \text{clim}/(\langle u' \rangle) \) in both experiments and 2) the term \( -\langle c(u'v') \rangle \) is much smaller than \( \langle \tau - c \rangle \text{clim}/(\langle u' \rangle) \) and its structure resembles that of minus \( \langle u' \rangle \) for the control and climate change simulations in both models. Consistently, the eddy momentum
flux–weighted phase speed \((c(u'v'))/((u'v'))\), measured at latitude 30°, stays around 8 m s\(^{-1}\) in the control and climate change simulations in the GR experiment, and reduces from around 9 to 8 m s\(^{-1}\) in the RRTMG experiment. Both the climatological values and the response of the phase speed are small compared with those of the zonal-mean zonal wind (Figs. 1c,d).

The role of the climatological zonal-mean zonal wind in the EP relation response can be demonstrated by looking at the mixed product \((\overline{\tau_{\text{dim}}(u'v')}_\text{change})\) (yellow curves in Figs. 7a,c), which is similar in its latitudinal structure to \((\overline{u(u'v')}\)) from the climate change simulation in both the GR and RRTMG experiments (red curves in Figs. 7a, c) and to the EP relation terms \(\left[\left(-\overline{(\Phi'v')}\right)\right]\) and \(\left(\overline{(u-c)}(u'v')\right)\) from the climate change simulation (red curves in Fig. 3). The poleward shift of \((\overline{\tau_{\text{dim}}(u'v')})\) (Table 1) captures well the shift of \((\overline{(u-c)}(u'v'))\) (2.3° poleward, which is 88% of the shift of \((\overline{(u-c)}(u'v'))\) in the GR experiment, and 3.6° poleward, which is 63% of the shift of \((\overline{(u-c)}(u'v'))\) in the RRTMG experiment). This shows that weighting the eddy momentum flux by the climatological zonal-mean zonal wind in the EP relation is a dominant factor in the connection between the eddy potential energy flux and eddy momentum flux responses to climate change for both radiation schemes.

The above results demonstrate that the eddy momentum flux convergence response to climate change, and thus the eddy-driven jet response, can be estimated in both the GR and RRTMG experiments using the EP relation given the eddy potential energy flux response because the response of the Doppler-shifted phase speed is negligible. Thus, according to the EP relation, the response of the Doppler-shifted phase speed is not important for the eddy momentum flux response. According to the EP relation, the eddy-driven jet and storm-track shifts in response to climate change have opposite signs in the GR simulation because the storm-track response (eddy potential energy flux response) must be divided by the climatological Doppler-shifted phase speed to infer the eddy-driven jet response.

5. Summary and conclusions

Eddy energy and momentum fluxes dominate the extratropical circulation and can be used to define the location of the storm track and the eddy-driven jet stream. Eddy energy and momentum fluxes are connected according to the EP relation (4), which states that the eddy potential energy flux is equal to the eddy momentum flux times the Doppler-shifted eddy phase speed. The EP relation assumes linear plane waves in the absence of nonconservative processes. According to the EP relation, the response of the eddy-driven jet to climate change can be connected to the storm-track response assuming 1) the storm-track shift is consistent with the eddy potential energy flux and 2) the response of the Doppler-shifted phase speed is negligible. Here we examine the validity of the EP relation and the extent to which it can be used to estimate the eddy-driven jet response to climate change in idealized GCM simulations of the response to increased greenhouse gases.

We use two idealized GCM simulations, which differ in their radiation schemes. The RRTMG simulation uses a sophisticated radiation scheme and the GR simulation uses a gray radiation scheme. The eddy-driven jet shifts poleward in response to climate change in the RRTMG experiment and equatorward in the GR experiment. In contrast, the storm track, as measured by the eddy dry static energy flux, shifts poleward using both radiation schemes. Eddy potential energy flux also shifts poleward using both radiation schemes, consistent with the storm-track shift.

The main conclusions are as follows:

1) The EP relation (4) approximately holds in the idealized GCM for the climatology and response to climate change. The residual term in the EP relation is dominated by boundary layer drag, nonlinear terms, and vertical advection of momentum. Despite the residual term, the response of the eddy potential energy flux to climate change is well captured by the response of the eddy momentum flux times the Doppler-shifted phase speed, both in terms of its shape and poleward shift.

2) The eddy momentum flux estimated from the EP relation (10) qualitatively captures the eddy momentum flux response to climate change in the GR and RRTMG experiments. The latitude of maximum vertically integrated eddy momentum flux convergence estimated from the EP relation also captures the shift of the eddy-driven jet in response to climate change. The EP relation captures 77% of the equatorward eddy-driven jet shift in the GR experiment and 76% of the poleward jet shift in the RRTMG experiment.

3) The EP relation can be used to connect the eddy-driven jet and storm-track shifts because the response of the Doppler-shifted phase speed is negligible. The EP relation estimate of the eddy-driven jet shift using the eddy potential energy flux response to climate change and the climatological zonal-mean zonal wind captures 89% of the shift in the GR experiment and 86% of the shift in the RRTMG experiment.

4) According to the EP relation, the opposite shift of the storm track and eddy-driven jet in response to
climate change in the GR experiment is associated with dividing the storm-track response by the climatological Doppler-shifted phase speed, dominated by the zonal-mean zonal wind, which consists of a strong stratospheric jet with zonal winds increasing toward the top of the atmosphere.

The results presented here show that the eddy-driven jet shift in response to climate change can be connected to the storm-track response using the EP relation because 1) the storm track follows the eddy potential energy flux response and 2) the response of the Doppler-shifted phase speed is negligible.

Explaining the storm track and eddy-driven jet shift in response to climate change is one of the grand challenges of climate science. The general approach in the literature has been to focus on either the eddy-driven jet (e.g., Chen et al. 2008; Rivière 2011; Lorenz 2014; Lu et al. 2014) or storm-track (e.g., Lorenz and DeWeaver 2007; Lu et al. 2008; Butler et al. 2010; Mbengue and Schneider 2013) response to imposed diabatic perturbations. For the eddy-driven jet response, the starting point is typically a zonal-mean zonal wind perturbation, which is then connected to changes in mean potential vorticity gradient, index of refraction, Doppler-shifted phase speed, or eddy length scale. For the storm-track response, the focus is typically on how the imposed diabatic perturbation impacts Eady growth rate or available potential energy. In both cases imposing a diabatic perturbation is problematic because it assumes a connection between the radiative effect of increased CO2 and the thermal response, which is influenced by eddies. Furthermore, both approaches exclusively focus on either the eddy-driven jet or the storm track but not on their connection.

The EP relation provides, to our knowledge, the only connection relating the storm-track and eddy-driven jet responses to climate perturbations. It can be used to either infer the eddy-driven jet response from the storm-track response or vice versa. Here we take an energetic approach and consider the storm-track (energetic) response as a starting point, because increased CO2 directly perturbs the energy budget, and use it to infer the eddy momentum flux and eddy-driven jet shift. The response of the energy budget to increased greenhouse gas concentration has been used to explain the storm-track shift. For example, Held and Soden (2006) argued that the dry static energy flux response to climate change can be predicted via its compensation for the increased latent energy flux following the Clausius–Clapeyron relation. Dry static energy flux, which peaks poleward of the latent heat flux, is expected to compensate for the increase in latent heat flux by decreasing on the equatorward side of its maximum, leading to a poleward shift of the dry static energy flux. If the eddy potential energy flux shifts in the same direction as the eddy dry static energy flux, this argument could be used to predict the poleward shift of the eddy potential energy flux in response to climate change. The results presented here support the assumed connection between the response of the eddy dry static energy flux and the eddy potential energy flux, but this assumption requires further investigation.

Conversely, if the climate change involves a mechanical forcing (changes in the surface torque), as occurs during the Last Glacial Maximum, then the EP relation could be used to connect the eddy-driven jet response to the storm-track response. Overall, these ideas highlight the usefulness of the EP relation in connecting the mid-latitude energy and momentum flux response to climate change.

Acknowledgments. OL and TAS acknowledge support from NSF (AGS-1538944), and TAS is also supported by the David and Lucile Packard Foundation and the Alfred P. Sloan Foundation. The model simulations in this paper were completed with resources provided by the University of Chicago Research Computing Center. The authors thank Zhihong Tan for setting up the RRTMG radiation scheme in the idealized GCM and for helpful discussions, and two anonymous reviewers for useful comments that helped improve the manuscript.

APPENDIX

The EP Relation in Sigma Coordinates

The EP relation includes an eddy potential energy flux term, which depends on the choice of vertical coordinate. We are interested in the EP relation on pressure surfaces \( \sigma \); however, the idealized model solves the equations in sigma coordinates, defined as \( \sigma = \rho / \rho_s \).

There are two options for calculating the EP relation from the model output: 1) interpolating all the data to pressure surfaces, which introduces a source of inaccuracy and an additional computational cost, and 2) using the EP relation in sigma coordinates and adding a correction term to the eddy potential energy flux, so that (4) is satisfied, as explained below. In this study we used the second method. We will show below that the two methods give similar results.

To derive the EP relation on \( \sigma \) levels, we start from the wave zonal momentum equation on \( \sigma \) levels. The pressure gradient force in the momentum equation on \( \sigma \) levels can be derived by applying the chain rule of
partial derivatives on the geopotential (Holton 2004, section 10.3.1):

\[
\left( \frac{\partial \Phi}{\partial x} \right)_p = \left( \frac{\partial \Phi}{\partial x} \right)_\sigma - \left( \frac{\partial \Phi}{\partial p} \right)_\sigma \left( \frac{\partial p}{\partial x} \right)_\sigma,
\]

(A1)

where the subscripts denote the variable that is kept constant in the derivative. Replacing the hydrostatic equation \{(\partial \Phi/\partial p)_s = -RT/p\}, where R is the gas constant and the definition of \(\sigma\) into (A1) gives

\[
\left( \frac{\partial \Phi}{\partial x} \right)_p = \left( \frac{\partial \Phi}{\partial x} \right)_\sigma + RT \frac{\partial \ln p}{\partial x}.
\]

(A2)

Substituting (A2) into the momentum equation in pressure coordinates gives the momentum equation in sigma coordinates [Holton 2004, Eq. (10.30)]. Taking the eddy component and using similar assumptions as used in section 2 for deriving (4), we get the wave zonal momentum equation in sigma coordinates:

\[
\frac{\partial u'}{\partial t} + \pi \frac{\partial u'}{\partial x} + \rho \frac{\partial v'}{\partial x} + \frac{\partial}{\partial x} \left[ \rho \frac{\partial \ln p}{\partial x} (\Phi')' \right] - v' (f + \zeta) = 0.
\]

(A3)

Assuming a plane wave and following the same derivation as in section 2 leads to the EP relation in sigma coordinates:

\[
-(\langle \Phi' v' \rangle - RT' \langle \ln p \rangle \langle \Lambda' \rangle') = (\pi' - c) \langle \Lambda' \rangle' - \zeta',
\]

(A4)

where \(\langle \cdot \rangle'\) denotes zonal averaging over \(\sigma\) levels and primes are the deviations from the average. Vertically integrating (A4) and (4), and assuming that the vertically integrated momentum flux is not sensitive to the choice of vertical coordinates, we get

\[
\langle \Phi' v' \rangle + RT' \langle \ln p \rangle \langle \Lambda' \rangle' = \langle \Phi' v' \rangle,
\]

(A5)

where \(\langle \cdot \rangle'\) denotes zonal averaging over pressure levels. In this study, we used the corrected eddy potential energy flux on sigma surfaces

\[
\langle \Phi' v' \rangle_{\text{corr}} = \langle \Phi' v' \rangle + RT' \langle \ln p \rangle \langle \Lambda' \rangle' - \zeta'.
\]

when referring to the eddy potential energy flux. Note that the corrected eddy potential energy flux on sigma surfaces satisfies (4).

To demonstrate the importance of the correction to the eddy potential energy flux on \(\sigma\) surfaces we examine the different terms in (A5) for year 18 of the control RRTMG simulation. The eddy potential energy flux on pressure levels was calculated by interpolating \(\Phi\) and \(v\) from the full model levels to pressure levels. Terms \(\langle \Phi' v' \rangle\) and \(\langle \Phi' v' \rangle_{\text{corr}}\) are almost identical, consistent with (A5), though the interpolation to pressure levels introduces a small error (Fig. A1a). The \(\langle \Phi' v' \rangle\) and the correction term, \(RT' \langle \ln p \rangle \langle \Lambda' \rangle'\), have opposite signs, with a magnitude approximately 5 times higher than \(\langle \Phi' v' \rangle_{\text{corr}}\) (Fig. A1b). The \(\langle \Phi' v' \rangle\) is actually poleward, opposite to the direction of \(\langle \Phi' v' \rangle_{\text{corr}}\). This

---

5 The interpolation from full model levels to pressure levels was done for each longitude, latitude, level, and time grid point, using the NCL code "int2p" with log interpolation and no extrapolation.
demonstrates the importance of considering the vertical coordinate when calculating the eddy potential energy flux.

It might seem counterintuitive that a vertically integrated quantity would depend on the choice of vertical coordinate. The solution for this paradox is that it is the partitioning between eddy and mean flow fluxes that depends on the coordinate system, and not the full vertically integrated potential energy flux \( \langle F \rangle \). Using the identity \( \overline{Fv} = \overline{F}v + \langle \overline{F}v' \rangle \) for both coordinate systems leads to the result that the mean flow fluxes change between the two systems:

\[
\langle \overline{F}v' \rangle - \langle \overline{F}v'' \rangle = \left\langle \left( \frac{1}{2} \langle \overline{F}v' \rangle \right) \right\rangle - \left( \frac{1}{2} \left\langle \langle \overline{F}v' \rangle \right\rangle \right).
\]

Substituting (A5) into (A6) gives

\[
\langle \overline{F}v' \rangle - \langle \overline{F}v'' \rangle = -\langle \overline{R} \overline{\tau} \rangle \left( \frac{1}{2} \left\langle \langle \overline{F}v' \rangle \right\rangle \right).
\]

The red and yellow curves in Fig. A1b show that this is approximately satisfied for the RRTMG climatology simulation. It should be noted that the mean flow fluxes are on the order of 20 PW, so that inaccuracies in \( \langle \overline{F}v' \rangle - \langle \overline{F}v'' \rangle \) on the order of 0.1 PW as shown in Fig. A1b are no more than 0.5% of the original values and can be attributed to the numerical error arising from the interpolation. This demonstrates that eddy potential energy flux changes between different coordinate systems because of the different partitioning between eddy and mean flow fluxes.

The sensitivity of the eddy-mean flow decomposition to the choice of vertical coordinates was also noted by Trenberth et al. (1993) in the context of time averaging on model surfaces. Another example of this sensitivity is the fact that all the heat flux is included in the mean flow component in isentropic coordinates (Vallis 2006).

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