Corrigendum

A Cohen–Lenstra phenomenon for elliptic curves


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The function $K(G)$, which is defined by equation (3), should instead be defined by

$$K(G) := \prod_{\ell \mid N} \left( 1 - \frac{(N-1)^2 \ell + 1}{(\ell - 1)2(\ell + 1)} \right) \prod_{\ell \mid N_1} \left( 1 - \frac{1}{\ell^2} \right) \prod_{\ell \mid N_2} \left( 1 - \frac{1}{\ell(\ell - 1)} \right).$$

The reason for the change is an error in the statement of Lemma 14, which was taken from [DS13, Lemma 11]. In the case that $\ell \mid (N, f)$ and $\nu_\ell(N) = 2\nu_\ell(f)$, we should have

$$c_{N,f}(\ell^\alpha) = \#C_N^{(\ell)}(1, 1, f) \begin{cases} (\ell - 1 - \frac{N_\ell}{\ell}) & \text{if } 2 \mid \alpha, \\ 1 & \text{if } 2 \nmid \alpha, \end{cases}$$

instead. This change in the statement of Lemma 14 then affects the definitions of the functions $K(N, m)$ and $K(N)$ that occur in the statement of Proposition 12. In the statement of Proposition 12, we should have

$$K(N, m) := \prod_{\ell \mid m} \left( \frac{\ell^{\nu_\ell(N) + 1} - 2\nu_\ell(m)}{\ell^{\nu_\ell(N) + 1} - \ell^{\nu_\ell(N)} - 1} \right)$$

and

$$K(N) := \prod_{\ell \mid N} \left( 1 - \frac{(N-1)^2 \ell + 1}{(\ell + 1)(\ell - 1)^2} \right) \prod_{\ell \mid N} \left( 1 - \frac{1}{\ell^{\nu_\ell(N)}(\ell - 1)} \right).$$

The following is a list of other minor changes caused by the error in the statement of Lemma 14.

1. On page 36, the definition of the function $F_2(\ell, f)$ should be changed to

$$F_2(\ell, f) := \begin{cases} \left( 1 + \frac{1}{\ell(\ell + 1)} \right) & \text{if } \nu_\ell(N) < 2\nu_\ell(f), \\ \left( 1 + \frac{1}{\ell} \right) & \text{if } \nu_\ell(N) > 2\nu_\ell(f), \\ \left( 1 - \frac{N_\ell(\ell)}{\ell(\ell^2 - 1)} + 1 \right) & \text{if } \nu_\ell(N) = 2\nu_\ell(f). \end{cases}$$

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(2) On page 37, the definitions of \( F_4(\ell) \) and \( F_5(\ell) \) do not depend on the parity of \( \nu_\ell(N) \). In either case we should have

\[
F_4(\ell) := 1 + \frac{\ell^{\nu_\ell(N)} - \ell}{F_0(\ell)^{\nu_\ell(N)2}(\ell - 1)^2};
\]
\[
F_5(\ell) := \frac{\ell^{\nu_\ell(N)+1} - \ell^{2\nu_\ell(m)}}{F_0(\ell)^{\nu_\ell(N)+2\nu_\ell(m)-1}(\ell - 1)^2}.
\]

(3) On page 38, the definition of \( K^{(\ell)}(N,m) \) does not depend on the parity of \( \nu_\ell(N) \). In either case we should have

\[
K^{(\ell)}(N,m) := \frac{\ell^{\nu_\ell(N)+1} - \ell^{2\nu_\ell(N)-1}}{\ell^{\nu_\ell(N)+1} - \ell^{\nu_\ell(N)} - 1}.
\]

(4) On page 39, the definition of \( F_6(\ell) \) should be

\[
F_6(\ell) := 1 - \frac{1}{\ell^2},
\]

instead.

Reference