Comparison of different radar-raingauge rainfall merging techniques
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ABSTRACT

The improvement of precipitation estimation with the use of radar-raingauge rainfall merging techniques is influenced by several factors, such as topography, storm types, raingauge network density for adjustment, data quality and the rainfall accumulation time. However, the influence of the raingauge network configuration on the performance of radar-raingauge merging methods is often ignored. The aim of this study is to compare and evaluate the performance of different radar-raingauge merging methods on various densities of raingauge network and raingauge network configurations. The analysis of the effect of the raingauge network density shows that the performance of Kriging merging methods increases with the increase of raingauge network density. The results also showed that the influence of raingauge network configuration on the spatial distribution of precipitation of the merged products is relatively smaller for the Kriging with radar-based error correction (KRE) and Kriging with external drift (KED) methods than for the ordinary Kriging method. This indicates that the inclusion of radar data in the KRE and KED methods helps to maintain the spatial distribution of precipitation on an hourly timescale. According to the statistical performance indicators and visual inspection of the merged rainfall fields, the KED outperforms the other radar-raingauge merging techniques, regardless of raingauge network density and configuration.

Key words | geostatistical, Kriging, merging, precipitation estimation, raingauges, weather radar

INTRODUCTION

Accurate precipitation estimates in time and space are key input in hydrological studies and applications (Berndtsson & Niemczynowicz 1988; Vieux & Bedient 1998; Ogden et al. 2000). Raingauges can provide accurate rainfall measurements at individual point locations. However, often raingauge networks are too scattered to be able to measure the variability of the precipitation distribution in space and time. Tipping bucket raingauges (TBs) are the most widely used instruments to measure rainfall rates, but its accuracy suffers from several sources of errors (Habib et al. 2001; Ciach 2005). For instance, some of the typical errors in TBs are due to blockages, wetting and evaporation in the funnel, condensation errors, underestimation of high rain rates and wind effects (Upton & Rahimi 2005). Moreover, raingauge measurements suffer from spatial sampling errors because they represent point measurements often approximated (or compared) to areal estimates (Villarini et al. 2008). To obtain a distributed precipitation field based on raingauge measurements only, several spatial interpolation methods have been developed. These methods can be either geostatistical (such as Kriging) or non-geostatistical (such as inverse distance weighting and regression model). However, even a high-density raingauge network is unable to fully capture the true rainfall field at short timescales (Ciach & Krajewski 2006; Germann et al. 2006; Seo & Krajewski 2011; Peleg et al. 2015).

Weather radar, as a remote sensing instrument, estimates precipitation above the ground level by measuring the reflectivity of precipitation at a given altitude. However, the application of radar in hydrological studies has been
limited by a number of sources of errors and uncertainties in the radar rainfall estimates (Wilson & Brandes 1979; Zawadzki 1984; Joss & Waldvogel 1990; Fabry et al. 1992; Hunter 1996; Smith et al. 1996; Young et al. 1999; Borga 2002; Krajewski & Smith 2002; Villarini & Krajewski 2010). Specifically, the reflectivity measurement could be affected by the stability of radar calibration, radar signal attenuation due to heavy rain, contamination by non-meteorological echoes (e.g. ground clutter) or range effects (attenuation of the radar signal, increase of the sampling volume due to beam broadening and beam overshooting the shallow precipitation) (Kitchen & Jackson 1993). Additional errors and uncertainties could be introduced when converting the radar reflectivity $Z$ into an estimate of precipitation intensity $R$ at ground level (Kessler & Wilk 1968; Doviak 1983; Zawadzki 1984; Austin 1987; Joss & Waldvogel 1990).

Other sources of error are due to the variation of the vertical profile of reflectivity (VPR) including the radar bright band (region of enhanced reflectivity produced by melting snowflakes that can produce overestimation of precipitation) (Collier 1986; Cluckie et al. 2000) and the $Z$-$R$ relationship used to compute the rainfall rates from reflectivity measurements (Austin 1987). The relationship between radar reflectivity $Z$ and rainfall intensity $R$ considerably relies on the actual raindrop size distribution (DSD), which varies between different types of storms (Battan 1973; Austin 1987; Joss & Waldvogel 1990) and even within storms (Catanese & Stout 1968; Carbone & Nelson 1978; Smith & Krajewski 1993). Therefore, changes in the DSD introduce time-varying bias in radar rainfall estimates, which rely on the use of average $Z$-$R$ relationships. Different correction techniques have been developed in the literature to improve weather radar rainfall estimates. Despite the fact that the accuracy of radar rainfall suffers from several sources of errors and uncertainties, radars are able to provide distributed precipitation fields with high spatial and temporal resolutions over large regions. Conversely, raingauge measurements are the most reliable point ground measurements, but also have their own limitations on spatial representativeness. Thus the combination of these two precipitation measurements could provide improved rainfall estimation.

To exploit the strengths of both radar and raingauge measurement approaches, several radar-raingauge merging techniques have been proposed and developed during the last years. These merging methods range from non-statistical methods, such as Brandes spatial adjustment (Brandes 1975) and range dependent adjustment (Koistinen & Michelson 2002), to more complex statistical techniques. These statistical methods are based on univariate and multivariate geostatistical analysis, such as ordinary Kriging (Goudenhoofdt & Delobbe 2009), co-Kriging (Krajewski 1987) and Kriging with external drift (KED) (Haberlandt 2007). For instance, Krajewski (1987) formulated the merging of radar and raingauge data using a geostatistical framework. The variogram is the most important function in geostatistical analysis (Holawe & Dutter 1999; Skaien et al. 2005; Beek et al. 2011). The spatial characteristic of the rainfall field contained in the variogram is influenced by the characteristics of the storm, density of raingauge network, and rainfall accumulation period. The variogram estimation model can be either parametric or non-parametric based on raingauge or radar data, and it is often assumed that the rainfall field is isotropic. Velasco-Forero et al. (2009) developed a non-parametric methodology to compute a two-dimensional spatial correlation map in order to account for anisotropy of the rainfall field.

The performance of any rainfall interpolation method highly relies on the density of the raingauge network (Chumchean et al. 2006; Schuurmans et al. 2007; Goudenhoofdt & Delobbe 2009; Jewell & Gaussiat 2013). An accurate estimation of the true rainfall field often requires a high-density raingauge network (Cherubini et al. 2002; Ballester & Moré 2007; Villarini et al. 2008). A recent study on the sensitivity analysis of the raingauge network density by Goudenhoofdt & Delobbe (2009) showed that simple adjustment methods (such as mean field bias correction) are less sensitive to the raingauge network density, while the improvement by geostatistical methods (such as KED) increases with a more dense raingauge network.

Moreover, the improvement of rainfall estimation by the different radar-raingauge merging techniques may vary between different accumulation timescales. For instance, Berndt et al. (2014) examined the effect of accumulation timescales on the performance of different merging techniques at different accumulation timescales from 10 min to 6 hour. Their results showed that the performance of the radar-raingauge merging methods improves for large
accumulation times. The improvement of precipitation estimation by the radar-raingauge merging techniques also varies between seasons (Goudenhoofdt & Delobbe 2009; Verworn & Haberlandt 2011) and between storms (Schiemann et al. 2011).

Since the radar-raingauge merging method is the final step for radar-based precipitation estimation, minimising all the sources of error in radar rainfall before the merging is applied is important to improve the precipitation estimates (Harrison et al. 2009). The radar rainfall corrections include identification of ground clutter and anomalous propagation (Rico-Ramirez & Cluckie 2008), VPR correction and attenuation correction, etc. (Borga & Tonelli 2000; Germann et al. 2006; Rico-Ramirez et al. 2007; Rico-Ramirez 2012). Several studies have looked at the correction of errors due to wind-drift effects on radar-based precipitation estimation (Collier 1999; Lack & Fox 2005; Dai et al. 2013). Furthermore, careful controlling of raingauge data quality is crucial and only good quality raingauge data should be used for adjusting radar rainfall estimates (Steiner et al. 1999).

The aim of this study is to compare and evaluate the performance of different merging methods on various densities of raingauge network for different storm types. The influence of the raingauge network configuration on the performance of radar-raingauge merging methods is often ignored and a few studies had looked at the impact of network configuration on the representativeness of the spatial distribution of rainfall (Germann & Joss 2001; Villarini et al. 2008). In addition, the performance of the different radar-raingauge merging methods is usually assessed in terms of rainfall rates using raingauge measurements, but the spatial distribution of precipitation of the merged rainfall field is often not assessed. Therefore, this paper assesses the impact of raingauge network configuration by the different radar-raingauge merging methods, not only in terms of rainfall rates, but also evaluating the resulting precipitation distribution of the radar-gauge merged products. An independent verification raingauge network is used to evaluate the performance of the merging methods in this study in terms of rainfall rates.

This paper is structured as follows. The next section introduces the study region and data sets from the raingauge network and radar measurements. Brief descriptions of selection of network density and configuration, the various radar-raingauge merging methods as well as the performance indicators used for the comparisons are given in the following section. The penultimate section is subdivided into three sections presenting the sensitivity analysis of storm types, the effect of raingauge network density on radar-raingauge merging performance, and the effect of raingauge network configuration on radar-raingauge merging performance. Finally, the summary and conclusions of this work are presented.

### RADAR AND RAINGAUGE OBSERVATIONS

The study area is located in the North of England and the area is bounded by a window of approximately $250 \times 200$ km in size. Raingauge data and composite radar rainfall fields (RAD) were accumulated and merged on the hourly timescale. On this timescale, data from 214 automatic tipping bucket raingauges were available within this area. The raingauge data were provided by the Environment Agency. The locations of the radars and raingauges are shown in Figure 1. In this study it is assumed that raingauge measurements provide the ground truth. However, as mentioned in the previous section, raingauge measurements are also prone to error and an effort has been made in this study to identify time periods with suspected invalid data.
Also, note that part of the differences between radar and raingauge measurements can be explained by the fact that the former is an areal measurement whereas the latter is a point measurement. Ciach & Krajewski (1999) developed a methodology to estimate the variance reduction factor (VRF), which is the variance between radar and raingauge measurements (point-to-area variance) with respect to the variance of the raingauge measurements (point variance). We estimated that the maximum VRF for a raingauge randomly located in a 1 km² radar pixel is less than 10% (assuming the 60-min spatial correlation given by Bringi et al. (1994)). This error is relatively small compared to other sources of error in radar rainfall estimates and is therefore ignored in this study.

The RAD is a composite product from a network of 18 C-band weather radars provided by the UK Met Office through the British Atmospheric Data Centre. As shown in Figure 1, the study area is covered by three single-polarisation C-band weather radars (Hameldon Hill, High Moorsley and Ingham). Note that High Moorsley radar was installed in 2008 (UK Met Office, personal communication). The automatic pre-processing of weather radar data forms part of the UK Met Office Nimrod System. The description of the UK Nimrod System is included in Golding (1998). The radar reflectivity data pre-processing addresses a number of specific sources of errors, such as the removal of spurious non-meteorological echoes and corrections to account for radar sensitivity errors, variations in the VPR and signal attenuation (Harrison et al. 2000). Also the quality of the estimated radar rainfall rates is routinely assessed by comparison with ground truth raingauge measurements. The measured radar reflectivity (Z) is converted to rainfall rate (R) by using a constant Z–R relationship $Z = 200R^{1.6}$. The radar rainfall composite data applied in this study is at 5-min intervals with a spatial resolution of 1 km. More detailed information of the pre-processing of the UK radar rainfall composite product is included in Harrison et al. (2009).

Twenty storm events were selected based on the radar observations during the year of 2007 in the study area. All storms have been classified into three types of rainfall events, which are convective, stratiform and a mixture between convective and stratiform precipitation (defined as mixed precipitation in this paper). To classify the different types of storm events, Steiner et al.’s (1995) algorithm has been adapted to identify convective pixels in each radar scan. The convective pixels are also confirmed by the visual inspection of radar rainfall observations. Figure 2 shows an example of the classification result of the convective pixels at 1755 UTC 15 June 2007. Then, once the convective pixels have been identified, their size and duration have been analysed for each radar

![Figure 2](https://iwaponline.com/jh/article-pdf/17/3/422/388329/jh0170422.pdf)

scan. Based on the analyses of these parameters, a simple classification method has been adapted in this study. However, it is often difficult to judge the threshold for the discrimination of convective and stratiform events based only on the radar reflectivity. Thus, with the consideration of our study region, the following thresholds have been used for the classification of the storm types. A precipitation event can be identified as convective when the rain event has convective pixels that cover an area of at least 3% of the whole precipitation area during 3 hours or more. If the precipitation area covered by the convective structures does not exceed 3% of the total region, then that storm can be identified as mixed precipitation. An event is classified as stratiform when the convective pixels do not exceed 1% of the total precipitation area. All the selected storm events with descriptive data are summarised in Table 1.

### METHODOLOGY

#### Network density and configuration analysis

To assess the performance of radar-raingauge merging techniques, an independent validation raingauge network has been used in this study. Before selecting the raingauge network for validation, the study region has been subdivided into 20 subdivisions with the same area of 50 × 50 km² (see the grids in Figure 1). The purpose of these subdivisions is to ensure that the selected raingauges for calibration and validation are homogeneously distributed in the study region. One-hundred and sixty-one raingauges were used for calibration and 53 raingauges were used for validation as shown in Figure 1. The validation raingauges represent approximately 25% of the available raingauges in each subdivision (i.e. in each 50 × 50 km² region). Note that the

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**Table 1** Precipitation statistics of the selected storm events with hourly accumulated data from radar and raingauges, accumulated rainfall and rainfall intensity

<table>
<thead>
<tr>
<th>Case study</th>
<th>Radar on raingauge location</th>
<th>Raingauge</th>
<th>Storm types</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event No.</td>
<td>Date</td>
<td>Time-steps available (h)</td>
<td>Average accumulated rainfall (mm)</td>
</tr>
<tr>
<td>1</td>
<td>10–11 Jan 2007</td>
<td>21</td>
<td>17.8</td>
</tr>
<tr>
<td>2</td>
<td>19–20 Jan 2007</td>
<td>15</td>
<td>11.5</td>
</tr>
<tr>
<td>3</td>
<td>9–11 Feb 2007</td>
<td>46</td>
<td>21.5</td>
</tr>
<tr>
<td>4</td>
<td>13–14 Feb 2007</td>
<td>11</td>
<td>4.7</td>
</tr>
<tr>
<td>5</td>
<td>15–16 Feb 2007</td>
<td>23</td>
<td>8.7</td>
</tr>
<tr>
<td>6</td>
<td>21–22 Feb 2007</td>
<td>20</td>
<td>11.5</td>
</tr>
<tr>
<td>7</td>
<td>4–4 Mar 2007</td>
<td>13</td>
<td>11.8</td>
</tr>
<tr>
<td>8</td>
<td>5–6 Mar 2007</td>
<td>15</td>
<td>9.3</td>
</tr>
<tr>
<td>9</td>
<td>23–24 Apr 2007</td>
<td>26</td>
<td>9.8</td>
</tr>
<tr>
<td>10</td>
<td>24–25 Apr 2007</td>
<td>12</td>
<td>5.5</td>
</tr>
<tr>
<td>11</td>
<td>13–14 May 2007</td>
<td>18</td>
<td>17.0</td>
</tr>
<tr>
<td>12</td>
<td>16–17 May 2007</td>
<td>27</td>
<td>10.3</td>
</tr>
<tr>
<td>13</td>
<td>13–16 Jun 2007</td>
<td>75</td>
<td>79.7</td>
</tr>
<tr>
<td>14</td>
<td>24–26 Jun 2007</td>
<td>19</td>
<td>18.6</td>
</tr>
<tr>
<td>15</td>
<td>28–29 Jun 2007</td>
<td>17</td>
<td>12.5</td>
</tr>
<tr>
<td>16</td>
<td>30 Jun–04 Jul 2007</td>
<td>112</td>
<td>57.5</td>
</tr>
<tr>
<td>17</td>
<td>13–14 Jul 2007</td>
<td>30</td>
<td>18.5</td>
</tr>
<tr>
<td>18</td>
<td>25 Jul 2007</td>
<td>12</td>
<td>3.5</td>
</tr>
<tr>
<td>19</td>
<td>2 Sep 2007</td>
<td>20</td>
<td>3.8</td>
</tr>
<tr>
<td>20</td>
<td>24 Nov 2007</td>
<td>18</td>
<td>4.5</td>
</tr>
</tbody>
</table>
validation raingauges were not used in the radar-raingauge merging techniques and the validation raingauge network was kept fixed. Conversely, raingauge networks with three different densities were selected for calibration of the radar-raingauge merging techniques namely, entire raingauge network (161 raingauges), high-density raingauge network (125 raingauges), moderate-density raingauge network (83 raingauges) and low-density raingauge network (44 raingauges) (see Table 2). Note that the 50 × 50 km² regions shown in Figure 1 were also used for the random selection of the raingauges used for calibration (i.e. for the application of a radar-raingauge merging method). This ensures that the calibration and validation raingauges are evenly distributed in the entire region. Figure 3 illustrates the example of the distributions of high-, moderate- and low-density raingauge networks for the application of the radar-gauge merging methods.

Moreover, to analyse the effect of the raingauge network configuration in the final radar-raingauge merged product,

<table>
<thead>
<tr>
<th>Network density</th>
<th>Percentage (%)</th>
<th>Gauges for merging</th>
<th>Gauges per 10,000 km²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entire</td>
<td>100</td>
<td>161</td>
<td>32.2</td>
</tr>
<tr>
<td>High</td>
<td>75</td>
<td>125</td>
<td>25.0</td>
</tr>
<tr>
<td>Moderate</td>
<td>50</td>
<td>83</td>
<td>16.6</td>
</tr>
<tr>
<td>Low</td>
<td>25</td>
<td>44</td>
<td>8.8</td>
</tr>
</tbody>
</table>

Figure 3: Distributions of three different densities of raingauge networks (high, moderate and low) for radar-raingauge merging and key geometric statistics.
10 different raingauge network configurations were randomly selected for each raingauge network density, except when using the entire calibration raingauge network. To get a first impression of the difference of raingauge network configurations for each network density, the clustering factor (CF) (Garcia et al. 2008) for the raingauge network distribution has been calculated for each independent raingauge network, and it is given by

\[
CF = \frac{N}{(N-1)(N-2)} \sum_{n=1}^{N} \left( \frac{d_n - \bar{d}}{\sigma} \right)^3
\]

where \(N\) is the number of raingauges, \(d_n\) is the distance to the nearest neighbour of the raingauge \(n\), and \(\bar{d}\) and \(\sigma\) are the mean and standard deviation for the distribution of nearest-neighbour distances, respectively. A network with a smaller value of CF indicates the raingauges are mainly distributed within larger nearest-neighbour distances. In comparison, the larger CF value represents the network distribution dominated by smaller nearest-neighbour distances and this produces gaps in coverage in the study region. Figure 3 also demonstrates the examples of the geometric statistics of raingauge network configurations for different network densities. As shown in Figure 3, the mean and standard deviation of nearest-neighbour distances significantly increases with the decrease of network density. This is indicative that a dense raingauge network may be able to represent the distribution of precipitation better than a low-density gauge network over the study region. In comparison, although the mean and standard deviation of a moderate-density network is higher than a high-density network, its CF with the value of 0.77 is much lower than that of a high-density network with the value of 0.96. This is in part due to the fact that when randomly selecting the raingauges for calibration, the 50×50 km grid shown in Figure 1 was taken into account for this selection. This indicates that the sparse raingauge network may still be able to represent the distribution of the rainfall field. However, the merged rainfall field obtained by a moderate-density raingauge network will be much smoother than the one generated by a high-density raingauge network. In comparison, the low-density raingauge network may not be able to represent the spatial distribution of precipitation (Figure 3). The CF values of different network configurations for high-, moderate- and low-raingauge network densities are included in Table 3.

### Radar-raingauge merging methods

In this section, several radar-raingauge merging methods have been implemented to assess the improvement in the precipitation estimation. It is worth noting that the topography has not been taking into account when merging radar and raingauge measurements, since the study region is relatively flat. The following radar-raingauge merging methods were implemented in the study region shown in Figure 1.

#### Mean field bias correction

The mean field bias (MFB) correction is a simple and effective method, which was developed by Smith & Krajewski (1991) for adjusting radar-based quantitative precipitation estimates and it is widely used in several studies (Fulton et al. 1998; Harrison et al. 2000; Chumchean et al. 2006). The assumption is that the radar estimates are affected by a single uniform multiplicative error. This error may be due to a poor electronic calibration or an erroneous coefficient in the Z–R relationship (Borga 2002). The MFB method applied in this study is an additional correction that is applied using data from the specific region of

<table>
<thead>
<tr>
<th>Network configuration no.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>0.96</td>
<td>0.72</td>
<td>0.80</td>
<td>1.40</td>
<td>0.76</td>
<td>0.62</td>
<td>0.86</td>
<td>0.92</td>
<td>1.02</td>
<td>0.70</td>
</tr>
<tr>
<td>Moderate</td>
<td>0.77</td>
<td>0.22</td>
<td>1.80</td>
<td>0.69</td>
<td>0.62</td>
<td>0.93</td>
<td>1.04</td>
<td>0.64</td>
<td>1.96</td>
<td>0.41</td>
</tr>
<tr>
<td>Low</td>
<td>1.51</td>
<td>1.65</td>
<td>0.46</td>
<td>0.13</td>
<td>0.38</td>
<td>0.14</td>
<td>0.88</td>
<td>0.38</td>
<td>−0.09</td>
<td>1.00</td>
</tr>
</tbody>
</table>
interest. The adjustment factor is estimated as a mean field bias at the 60-min timescale

\[
MFB = \frac{\sum_{i=1}^{N} G_i}{\sum_{i=1}^{N} R_i}
\]  \hspace{1cm} (2)

where \(N\) is the number of valid radar-raingauge pairs, \(G_i\) and \(R_i\) are the raingauge and radar values, respectively, associated with raingauge \(i\) at a particular time step. The bias-adjusted RAD (\(\bar{R}\)) is given by \(\bar{R} = MFB \times R\), where \(R\) represents the original RAD. Note that this correction adjusts the radar field so that on average the bias between radar and raingauge measurements is close to 1.

Ordinary Kriging

Ordinary Kriging (ORK) is the most widely used geostatistical method which deals with the spatial interpolation of a random field from observations at several locations. Note that the ORK method is not a radar-raingauge merging method, but can be used as a benchmark to compare against other radar-raingauge merging methods. A general description of ORK is presented in Goovaerts (1997). In this method, the spatial variability of the precipitation field is characterised by a predefined model semivariogram obtained from raingauge observations. The description of the semivariogram is presented in Schabenberger & Gotway (2005), and the semivariogram is defined as

\[
\gamma(x_i-x_j) = \frac{1}{2|N(x_i-x_j)|} \sum_{i}^{N(x_i-x_j)} (Z_{x_i} - Z_{x_j})^2
\]  \hspace{1cm} (3)

where \(\gamma\) is the semivariogram, \(N(x_i-x_j)\) is the number of pairs of raingauge observations for a given lag distance of \(x_i-x_j\), \(Z_{x_i}\) and \(Z_{x_j}\) are the values of raingauge data at locations \(x_i\) and \(x_j\), respectively. Figure 4 shows the examples of semivariograms using different raingauge network densities. The quality of the interpolated product depends on the number of raingauges within the region and to some extent on the selection of the semivariogram model used to represent the spatial variability of the rainfall field (Schiemann et al. 2011). In this method, the spatial variability of the precipitation field is characterised by a predefined model semivariogram obtained from raingauge observations.

The spherical model semivariogram (dashed line in Figure 4), assumed to be isotropic, was applied in this study.

The interpolated rainfall value \(\hat{Z}_{ORK}(x_0)\) at a specific location \(x_0\) is a linear combination of the raingauge measurements \(Z_G(x_i)\) and weights \(\lambda_i^{ORK}\) at the corresponding locations \(x_i\)

\[
\hat{Z}_{ORK}(x_0) = \sum_{i=1}^{n} \lambda_i^{ORK} Z_G(x_i)
\]  \hspace{1cm} (4)

where \(n\) is the number of available raingauges. The optimal weights for the best linear unbiased estimation are obtained by following the ORK system of \((n + 1)\) linear equations:

\[
\sum_{j=1}^{n} \lambda_j^{ORK} \cdot \gamma(x_i, x_j) + \mu = \gamma(x_i, x_0), \hspace{0.5cm} i = 1, 2, \ldots, n
\]  \hspace{1cm} (5)

\[
\sum_{j=1}^{n} \lambda_j^{ORK} = 1
\]  \hspace{1cm} (6)
where Equation (5) is required for the unbiased constraint of the estimation, \( \gamma(x_i, x_j) \) is the semivariogram value between points \( x_i \) and \( x_j \), \( n \) is the number of available raingauges and \( \mu \) is the Lagrange multiplier. The weights \( \lambda_i^{\text{ORK}} \) will change if the estimation location \( x_0 \) changes. It can be seen that ORK is not a merging technique, as it only uses one set of data (e.g. raingauge data) as input. However, the ORK method provides a more reliable reference field or benchmark to evaluate the radar-raingauge merging methods.

**Kriging with radar-based error correction**

The Kriging with radar-based error (KRE) method is included as part of this study owing to its simplicity and computational efficiency. This method produces a rainfall field that follows a mean field of raingauge interpolation based on Kriging (Ehret et al. 2008). At the same time, it also takes advantage of the spatial variability of radar data, which is generally representative of the true spatial pattern of rainfall. A general description of this merging method was originally presented in Sinclair & Pegram (2005) and later refined in Ehret et al. (2008).

In this study, the raingauge data and radar measurements at the corresponding raingauge locations are interpolated separately by the ORK method at the first stage of the KRE process. The deviation of the original radar field (\( R_{\text{ORI}} \)) and the interpolated radar field using Kriging (\( R_{\text{ORK}} \)) is obtained by the following equation (Ehret et al. 2008):

\[
C = \exp \left( \arctan \left( \ln \frac{R_{\text{ORI}}}{R_{\text{ORK}}} \right) \right)
\]

where \( R_{\text{ORI}} \) is the original RAD and \( R_{\text{ORK}} \) is the interpolated RAD using ORK and radar data at raingauge locations. Finally, the KRE corrected rainfall field (\( Z_{\text{KRE}} \)) is given by \( Z_{\text{KRE}} = C \times Z_{\text{ORK}} \), where \( Z_{\text{ORK}} \) represents the interpolated rainfall field using ORK and raingauge measurements. The merged rainfall field not only follows the spatial structure of the original radar field but also preserves the mean field of the raingauge data (Ehret et al. 2008).

**Kriging with external drift**

KED is a more sophisticated geostatistical method and a variant of the ORK method that allows the incorporation of one or more additional variables. In this paper, only radar data are included as additional variable. The KED estimator \( \lambda_i^{\text{KED}} \) for the unknown point \( x_0 \) is a weighted sum of the raingauge observations \( Z_G(x_i) \) from the \( n \) surrounding points \( x_i \), using the same equation as in the ORK method

\[
\hat{Z}_{\text{KED}}(x_0) = \sum_{i=1}^{n} \lambda_i^{\text{KED}} Z_G(x_i)
\]

where \( \hat{Z}_{\text{KED}}(x_0) \) is the estimated rainfall value at location \( x_0 \), \( \lambda_i^{\text{KED}} \) for \( i = 1, \ldots, n \) are the weights. However, the KED weights, \( \lambda_i^{\text{KED}} \), are unlike those used in the ORK method, since the estimator in the KED equation system has two constraints to satisfy the external drift hypothesis. Firstly, the sum of the weights \( \lambda_i^{\text{KED}} \) must add up to 1

\[
\sum_{i=1}^{n} \lambda_i^{\text{KED}} = 1
\]

The additional and exclusive constraint to the KED system is that the radar value \( Z_R(x_0) \) at the unknown point \( x_0 \) must equal to the sum of the weights multiplied by the radar value \( Z_R(x_i) \) at raingauge point \( x_i \), that is (Haberlandt 2007)

\[
\sum_{i=1}^{n} \lambda_i^{\text{KED}} Z_R(x_i) = Z_R(x_0)
\]

The weights \( \lambda_i^{\text{KED}} \) are the solution of the KED system of equations (Haberlandt 2007)

\[
\sum_{i=1}^{n} \lambda_i^{\text{KED}} \gamma(x_i, x_j) + \mu_0 + \mu_1 Z_R(x_i) = \gamma(x_i, x_0), \quad i = 1, 2, \ldots, n
\]

where \( \gamma(x_i, x_j) \) is the semivariogram value between points \( x_i \) and \( x_j \), \( \mu_0 \) and \( \mu_1 \) are Lagrange multipliers, and \( n \) is the number of available raingauges. The semivariogram \( \gamma \) was fitted to a spherical model, assuming that the rainfall field is isotropic. Therefore, in the KED technique, the radar data provides the external drift term and it is important that the radar rainfall measurements are highly linearly correlated to the raingauge measurements.

Performance assessment

The performance of the radar-raingauge merging techniques described in the previous section has been evaluated by comparing the merged rainfall estimates with the rainfall measurements from the validation raingauge network. The performance assessment has been conducted for all storms and for the different raingauge network densities. The performance indicators described in the following paragraphs were calculated using time series of merged rainfall estimates and gauge measurements (from the validation raingauge network) at point locations. That is, merged rainfall products at 1 km spatial resolution were compared against point raingauge measurements. Also, different rainfall thresholds (0, 0.1 and 1.0 mm) were selected and only rainfall measurements above a given threshold were used to compute the performance indicators. The following paragraphs describe the performance indicators used in the analysis.

The mean bias (MBIAS) assesses the overall systematic error of a merging method and it is given by

$$\text{MBIAS} = \frac{\sum_{i=1}^{N} Z_i}{\sum_{i=1}^{N} G_i}$$

(12)

where $N$ is the number of data points, and $Z_i$ and $G_i$ represent the merged rainfall product and raingauge measurements, respectively, at locations $i$. The root mean square error (RMSE) is the most common performance indicator used as verification method

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{N} (Z_i - G_i)^2}{N}}$$

(13)

The mean absolute error (MAE) was also used here as main performance indicator, due to the fact that it is less sensitive to large outliers

$$\text{MAE} = \frac{\sum_{i=1}^{N} |Z_i - G_i|}{N}$$

(14)

The Nash–Sutcliffe’s efficiency (NSE) coefficient proposed by Nash & Sutcliffe (1970) is defined as

$$\text{NSE} = 1 - \frac{\sum_{i=1}^{N} [G_i - Z_i]^2}{\sum_{i=1}^{N} [G_i - G]^2}$$

(15)

where $G$ is the mean rainfall value measured by raingauges. The NSE coefficient can range from $-\infty$ to 1. A perfect estimator will have a NSE of 1. An efficiency of 0 represents that the estimator is as accurate as the mean of the raingauge observations.

RESULTS AND DISCUSSION

In this section, the results of the comparison of different radar-raingauge merging methods are presented and analysed including the effect of raingauge network density and configuration. Several merging methods combining rainfall measurements from a given raingauge network and radar rainfall composite data have been implemented. The evaluation of different merging methods has been conducted on all selected storms events. A number of statistical parameters are independently calculated for each case study, for each time step and for each merging method.

Radar-raingauge merging performance

The analysis of the performance of different merging methods on different storm types is included in this section. The MAE, RMSE, MBIAS and NSE scores of all radar-raingauge merging methods including the original radar during convective, mixed precipitation, stratiform events and all storm events for the entire, high-, moderate- and low-density of calibration raingauge networks are plotted in Figures 5–8, respectively. Note that these results are averaged over 10 different calibration raingauge networks per storm and over all the storm types. The results show that the MAE and RMSE scores of all the radar-raingauge merging methods (i.e. KRE and KED) are lower than the MAE and RMSE scores of the original radar rainfall estimates for all the different types of storms. The MAE and RMSE scores for stratiform events are much lower than those for
convective storms. This is in part due to the fact that convective events show a large spatial variability in comparison to stratiform events. However, the MAE and RMSE scores of all radar-raingauge merging methods for mixed precipitation are somewhere between the scores obtained for convective and stratiform events. Moreover, the MAE and RMSE values of all radar-raingauge merging methods increase with the increase of the rainfall threshold amount. All Kriging interpolation methods (ORK, KRE and KED) perform better than the simple MFB correction method for each storm type, especially for high rainfall threshold amount. The Kriging interpolation methods provide additional benefit at higher rainfall thresholds in all storm types. Specifically, the KED method performs better than the other methods when using all different raingauge network densities, in terms of the MAE scores. However, as shown in Figures 5–8, there are some cases where the ORK method performs slightly better than KED in terms of

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**Figure 5** | Statistical scores of key parameters of the radar-raingauge merging techniques and original radar during different types of storms, when using the entire calibration raingauge network.
RMSE in particular during stratiform precipitation and for low rainfall thresholds.

The results also show that the NSE score improves with the application of radar-raingauge merging techniques for all storm types. The values of NSE for all Kriging interpolation methods (ORK, KRE and KED) are higher than the simple MFB correction method for all raingauge network densities. The KED outperforms the other radar-raingauge merging methods, which is followed by the ORK method. The MBIAS score is also plotted in Figures 5–8. The best estimator will have an MBIAS of 1. The results indicate that the KRE performs slightly better than the other methods during different storm events when using different raingauge network densities. The MFB correction method consistently underestimates the rainfall amounts when comparing to the raingauge observations. Overall, the results showed that the KED method consistently performs better than any other radar-gauge merging methods for the different storm types.
at different rainfall thresholds when using different density of raingauge networks.

**Effect of raingauge network density on radar-raingauge merging performance**

The results shown in the previous section of the effect of the raingauge network density on the different radar-raingauge merging methods were averaged over 10 different calibration raingauge networks per storm and over all the storm types. The sensitivity of key statistical parameters of radar-raingauge merging methods on different raingauge network densities is illustrated in Figures 9 and 10 for convective and stratiform events, respectively. The relative MAE and RMSE scores are normalised with respect to the errors shown by the original radar rainfall data.

As shown in Figure 9, all radar-raingauge merging methods are effective to reduce the errors in the original

![Figure 7](https://iwaponline.com/jh/article-pdf/17/3/422/388329/jh0170422.pdf)
radar rainfall, which is reflected by the lower relative scores of MAE and RMSE relative to the original radar errors. The results show that a simple MFB correction is less sensitive to the raingauge network density, showing in some cases relative errors even lower than the ORK method when using a low-density raingauge network during convective events. However, as expected, the raingauge network density has a dominant role in the performance of the Kriging merging methods. The relative errors for ORK, KRE and KED methods increase with the decrease of raingauge network density during convective storms. This indicates that the performance of the ORK, KRE and KED methods improves with the increase of the number of raingauges, in terms of the MAE and RMSE errors (Figure 9). Specifically, the KRE and KED methods show a relatively small increase of error if the raingauge density is reduced, whereas the ORK method shows the worst performance with the decrease of the raingauge density.
Figure 9 | Relative errors normalised with respect to the radar errors for the different radar-raingauge merging techniques during convective events and for different raingauge densities.

Figure 10 | The same as Figure 9, but for stratiform events.
Similar results have been obtained for stratiform events. According to Figure 10, the results show that the quality of the merged products, except the MFB correction method, shows an increase in error (MAE and RMSE) when decreasing the density of raingauges for stratiform conditions. All Kriging interpolation methods (ORK, KRE and KED) are effective to reduce the errors of original radar, particularly at highest rainfall threshold. The results also indicate that the effectiveness of error reduction by the Kriging methods increase with the increase of rainfall threshold amount. However, the radar observation outperforms the MFB method for high rainfall thresholds when using all raingauge network densities during stratiform events. Overall, Figures 9 and 10 indicate that the KED method performs better than the other methods in terms of the relative MAE and RMSE scores.

**Effect of raingauge network configuration on radar-raingauge merging performance**

The effect of raingauge network configuration on the different merging methods is analysed in this section. Ten different raingauge network configurations were selected with a high, moderate and low density of raingauges. The results are shown in Figures 11 and 12 for convective and stratiform events, respectively, and using the MAE score normalised by the MAE of the original radar data. As shown in Figure 11, it is evident that the normalised MAE values of

![Graphs showing relative MAE errors for different configurations and rainfall thresholds.](image-url)
different radar-raingauge merging methods depend less on the geometry of the raingauge network distribution when using a high-density raingauge network for convective events. However, for a low-density raingauge network, the fluctuations of all merging methods become larger than for a high- or moderate-density raingauge network for convective events. It is obvious that the normalised MAE values of all Kriging interpolation methods and simple adjustment methods show more fluctuations at highest rainfall threshold when using low-density networks. This is due to the fact that there are more potential raingauge network configurations that can be generated when using a low-density raingauge network, and thus the results show more variations of rainfall spatial distributions. As shown in Figure 12, the fluctuations of the normalised MAE values during stratiform events are obvious for all radar-raingauge merging methods. It is evident that the quality of the merged products depends on the geometry of the raingauge network distribution, in particular at highest rainfall threshold. The results indicate that the quality of the merged rainfall fields highly depend on the geometry of the raingauge network distribution for the estimation of high rainfall amounts. Results for the other normalised statistical parameters are not shown here, but they follow a similar trend as the normalised MAE scores.

Furthermore, a visual inspection of all the merging products on different raingauge network configurations shows some differences in the precipitation distribution. Figure 13 illustrates some examples of the distribution of precipitation...
Figure 13 | Examples of the estimated rainfall fields at 1700 UTC 1 July 2007 (Event 16). The estimated rainfall fields were computed using ORK, KRE and KED methods on different densities of raingauge networks. The original RAD is shown in the top row.
estimated by the different radar-raingauge merging methods including radar observations for different raingauge network configurations when using a high-, moderate- and low-raingauge network densities on 1700 UTC 1 July 2007. As shown in the figure, the KRE and KED methods are able to preserve the spatial rainfall distribution as captured by radar. There are, however, some differences in the precipitation spatial patterns estimated on the different raingauge densities and it is evident that a high-density gauge network is able to better capture the spatial variability of precipitation when compared to the original RAD. Interestingly, the rainfall distribution estimated by KED is smoother compared to the estimated rainfall distribution by the KRE method. However, the rainfall fields estimated by the ORK do not follow the spatial distributions of the original RAD and there are some clear differences between different raingauge densities. As expected, the rainfall distributions estimated by the ORK method are very smooth compared to the KRE or KED methods and they are not able to represent the true rainfall spatial patterns, even if a high-density raingauge network is used. However, this problem is less evident for the KRE and KED methods. For instance, the spatial distributions of precipitation for the ORK field vary between raingauge networks and the estimated rainfall spatial patterns by the ORK method highly rely on the distribution and raingauge network density. As described earlier, the ORK method is an interpolation method that relies only on raingauge observations and therefore the raingauge network density plays a crucial role. The KRE and KED methods, on the other hand, rely not only on raingauge measurements, but also on radar observations and this tends to improve the precipitation estimation in regions where there is a lack of raingauge data.

To investigate the spatial variability of the merged products, the corresponding spectral density maps of the rainfall fields estimated by radar-raingauge merging methods are computed in the frequency domain by using fast Fourier transform (FFT) and compared to the original RADs. Figure 14 illustrates the computed frequency spectrum of the corresponding rainfall fields estimated by the ORK, KRE, KED and radar fields shown in Figure 13. The results show that the FFT frequency spectrums of the KRE and KED merging methods closely follow the original frequency spectrum computed from the RAD, whereas the ORK method has a completely different frequency spectrum. This is clearly due to the fact that the KRE and KED methods use gauge and radar data, whereas the ORK method only uses gauge measurements. Note that the frequency spectrum of the MFB method (not shown in Figure 14) is exactly the same as the frequency spectrum of the original radar data. Furthermore, the correlations between the frequency spectrums of the MFB, ORK, KRE and KED methods and the frequency spectrum of the original RADs for the different storm types on different raingauge network density are shown in Figure 15. The MFB adjustment method outperforms the other Kriging interpolation methods and completely preserved the frequency spectrum of the original RAD for all raingauge network densities and storm types. This is due to the fact the RADs are simply adjusted by a single uniform multiplicative factor. As shown in Figure 15, the KRE and KED methods get higher correlations compared to the ORK method for all storm types. Specifically, the correlation scores of KRE and KED are above 0.75 and close to 0.8, whereas the correlations scores of ORK are less than 0.6 for all storm events on all raingauge network densities. Moreover, the correlation scores of the ORK method decrease with the decrease of raingauge network density for all storm events. Once again, the visual inspection of the merged products and the correlation scores show the great benefits of the KRE and KED methods, not only in terms of quantitative rainfall estimates, but also preserving the spatial distribution of precipitation as observed by radar.

**SUMMARY AND CONCLUSION**

Several rainfall merging methods that combine rainfall measurements from a raingauge network and radar rainfall data have been implemented in this study. Comparisons of different radar-raingauge merging methods have been conducted on different densities of raingauge networks with different configurations.

The effect of the density and configuration of the raingauge network on the performance of merging methods has been analysed. The results showed that the merging methods KRE and KED are effective to reduce the errors
Figure 14 | Frequency spectra of the rainfall fields shown in Figure 13.
from the original radar rainfall for both convective and stratiform events at different rainfall thresholds, which is reflected by the lower relative scores of MAE and RMSE compared to the original radar. However, their performances highly rely on the density of raingauges. For instance, the quality of these merged products increases if the number of raingauges increases. In contrast, the simple MFB correction method is less sensitive to the raingauge network density. However, the MFB is the worst method for the reduction of errors from original radars, particularly at the highest rainfall threshold. The KED method was found to be the best method for the reduction of error (in terms of both MAE and RMSE) in radar rainfall estimation when compared to an independent validation raingauge network. The KED method also performs better than the other methods as shown by the NSE score. Moreover, the KRE and KED methods perform better than the ORK and MFB methods for the correction of MBIAS from the original radar data. The MFB correction method consistently underestimates the true rainfall amounts during all storm types at all rainfall thresholds.

Furthermore, the analysis of the effect of raingauge network configurations shows that the performance of the merging methods varies between different raingauge network configurations. According to the visual inspection of estimated rainfall fields and the correlations of the frequency spectra of the rainfall fields of the merged products, the results showed that the KRE and KED methods obtain higher correlations and are therefore able to preserve the rainfall distribution patterns of the original radar fields regardless of raingauge network density. The ORK is the worst method in representing the true rainfall spatial patterns as shown by the frequency spectra, and the correlation performance decreases with the decrease of raingauge network density. This indicates that the use of both radar and raingauge measurements such as in the KRE and KED methods improves the rainfall distribution estimation, in particular when the raingauge network density is poor. Moreover, the overall results indicate that the KED merging method is the best method for merging radar and raingauge measurements when comparing to an independent raingauge network.
Further analysis on the sensitivity of the radar-raingauge merging techniques is needed in order to explore shorter accumulation timescales (e.g. 15 min), which are commonly used for hydrological applications. There is also a scope to look at the performance of the different merging methods in hydrological modelling.

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REFERENCES


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