Decision support system for the optimal design of district metered areas

Enzo Galdiero, Francesco De Paola, Nicola Fontana, Maurizio Giugni and Dragan Savic

ABSTRACT

The establishment of district metered areas (DMAs) is widely recognized as one of the most effective techniques for the optimal management of water distribution networks (WDNs). However, its implementation in real cases is a very challenging task that requires decision aiding. In this work, a comprehensive methodology for the optimal design of DMAs is presented and discussed. The proposed approach consists of a two-objectives optimization problem subjected to a number of constraints related to the topology of the network, financial issues and the network hydraulics. A multi-objective evolutionary algorithm (MOEA) is combined with tools from graph theory for the solution of the problem. The validation of the model and the calibration of the parameters are performed through the application to a well-known example from the literature. Low cost as compared to the budget and a good saving in leakage are obtained.

Key words | district metered areas, multi-objective optimization, network sectorization, pressure management, water losses

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$adjD$</td>
<td>set of the supplied DMAs in adjacency to the isolated DMA</td>
</tr>
<tr>
<td>$boundD$</td>
<td>set of boundary nodes of the isolated DMA</td>
</tr>
<tr>
<td>$C_B$</td>
<td>construction cost</td>
</tr>
<tr>
<td>$C_{B,\max}$</td>
<td>budget limit for construction cost</td>
</tr>
<tr>
<td>$C_{FM}$</td>
<td>unit cost of flow-meter</td>
</tr>
<tr>
<td>$C_L$</td>
<td>cost due to water leakages</td>
</tr>
<tr>
<td>$C_{SV}$</td>
<td>unit cost of shut-off valve</td>
</tr>
<tr>
<td>$C_{WL}$</td>
<td>unit cost of water losses</td>
</tr>
<tr>
<td>$\hat{H}_i$</td>
<td>minimum required head at $i$-th node</td>
</tr>
<tr>
<td>$H_i$</td>
<td>total head at $i$-th node in the partitioned network</td>
</tr>
<tr>
<td>$H_{i,\text{nd}}$</td>
<td>total head at $i$-th node in the starting configuration</td>
</tr>
<tr>
<td>$I_{RD}$</td>
<td>Resilience Deviation Index</td>
</tr>
<tr>
<td>$I_{RD,\max}$</td>
<td>maximum value of $I_{RD}$ over time</td>
</tr>
<tr>
<td>$\text{MaxCon}$</td>
<td>maximum number of customer connections per DMA</td>
</tr>
<tr>
<td>$\text{MinCon}$</td>
<td>minimum number of customer connections per DMA</td>
</tr>
<tr>
<td>$NDMA$</td>
<td>number of DMAs</td>
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<tr>
<td>$NDMA_{\max}$</td>
<td>maximum number of DMAs</td>
</tr>
<tr>
<td>$N_{FM}$</td>
<td>number of flow-meters</td>
</tr>
<tr>
<td>$ngen$</td>
<td>number of generations</td>
</tr>
<tr>
<td>$N_{HV}$</td>
<td>normalized hyper volume</td>
</tr>
<tr>
<td>$N_{\text{leak}}$</td>
<td>number of emitter nodes in the network</td>
</tr>
<tr>
<td>$N_n$</td>
<td>number of demand nodes</td>
</tr>
<tr>
<td>$N_{SV}$</td>
<td>number of shut-off valves</td>
</tr>
<tr>
<td>$N_{tot}$</td>
<td>number of network nodes</td>
</tr>
<tr>
<td>$P_{t,j}$</td>
<td>pressure at $i$-th node at time step $t$</td>
</tr>
<tr>
<td>$P_{\min}$</td>
<td>minimum pressure target for proper customer service</td>
</tr>
<tr>
<td>$\text{popsize}$</td>
<td>size of the population</td>
</tr>
<tr>
<td>$Q_{d}$</td>
<td>water demand of $d$-th DMA</td>
</tr>
<tr>
<td>$Q_{d,\max}$</td>
<td>maximum water demand for DMA</td>
</tr>
<tr>
<td>$Q_{d,\min}$</td>
<td>minimum water demand for DMA</td>
</tr>
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**INTRODUCTION**

Among the best practices for the improvement of efficiency in water distribution network (WDN) management, the establishment of ‘supply clusters’ or district metered areas (DMAs) (Cheong 1993) is particularly suggested. This technique, also known as ‘network segmentation’ or ‘sectorization’, involves partitioning the whole system into hydraulically discrete areas through the closure of network pipes (e.g., by using shut-off valves).

The main benefit related to sectorization is an increased control on the network operation that is achieved through the continuous monitoring of the water flows entering each DMA. This also results in an improved capacity to assess the current level and the evolution of the water leakage in every area of the network. Therefore, the partitioning of the WDN into DMAs is crucial for identifying the most vulnerable areas in order to schedule leak detection activities (Morrison et al. 2007).

Furthermore, in recent years the integration of sectorization with other pressure management techniques has gained increasing attention in the scientific and professional literature. In particular, combination with the use of pressure reducing valves (PRVs) has been investigated, as it can boost the efficiency of the pressure regulation provided by these devices (Creaco & Pezzinga 2014). For example, the delimitation of network areas characterized by similar hydraulic requirements is extremely useful for setting the PRVs while ensuring the minimum pressure levels. However, as highlighted in some studies, sectorization itself can be considered as a pressure management technique, because the closure of pipes can produce significant pressure drops, and therefore, mitigation of the leakages (De Paola et al. 2014). Other interesting benefits are offered in terms of protection of the water quality against incidents or malicious attacks (Di Nardo et al. 2014a).

On the other hand, there are some drawbacks that must be carefully taken into account in the establishment of DMAs, especially regarding the hydraulic performance of the partitioned network. The opening of network loops due to the closure of pipes may result in a significant reduction of system reliability. This situation can lead to the violation of the delivery of the minimum level of service when unexpected increases or changes in the spatial distribution of the water demand occur. Particular attention should also be paid to the water quality, water age in this case, which could be negatively affected by the reduced velocities and the more constrained paths that are allowed to the network flows.

Together with the intrinsic complexity of the WDNs, these issues make sectorization very difficult to implement in real cases. For this reason, many researchers have investigated this problem with different goals and optimization techniques. A number of examples have focused mainly on graph-based approaches in which the network topology was primarily taken into account (Tzatchkov et al. 2006). In other examples, one or more additional criteria were considered, such as the minimization of the ‘edge cut’ (i.e., the

\[
Q_i \quad \text{total water demand at } i\text{-th node}
\]

\[
Q_{ij} \quad \text{demand served by link } i-j
\]

\[
Q_{tot} \quad \text{total water demand of customers}
\]

\[
r \quad \text{annual discounting rate}
\]

\[
sp_{\delta} \quad \text{length of the shortest-path a network water source and the entry node of } \delta
\]

\[
SupD \quad \text{set of DMAs supplied by at least one network water source}
\]

\[
T \quad \text{number of time steps in the hydraulic simulation}
\]

\[
TotCon \quad \text{total number of customer connections}
\]

\[
tw_{ij} \quad \text{topological distance associated with link } i-j
\]

\[
unSupD \quad \text{set of DMAs not supplied by any network water source}
\]

\[
w_B \quad \text{weight for the construction cost}
\]

\[
w_L \quad \text{weight for the cost due to water leakages}
\]

\[
\alpha \quad \text{emitter coefficient}
\]

\[
\beta \quad \text{pressure exponent}
\]

\[
\delta \quad \text{generic DMA in } unSupD
\]

\[
\Delta \quad \text{duration of time step}
\]

\[
\Delta \varepsilon_{ij} \quad \text{absolute difference between ground elevations of node } i \text{ and } j
\]

\[
\eta_m \quad \text{distribution index for mutation}
\]

\[
\eta_c \quad \text{distribution index for crossover}
\]

\[
\theta \quad \text{random value between 0 and 1}
\]

\[
\lambda_d \quad \text{acceptance probability}
\]

\[
\phi \quad \text{blending factor}
\]
number of pipes to close) and the WDN hydraulics (Sempezo et al. 2009; Di Nardo & Di Natale 2011; Alvisi & Franchini 2013).

Gomes et al. (2012a, b) introduced the use of simulated annealing (SA) and user-defined criteria for setting the boundaries of DMAs. Furthermore, they introduced the minimum requirements in terms of water velocity and pressure through pipe reinforcements and replacements. However, the developed methodology was not completely automated, and a significant input by the operator was required in the optimization.

A complex methodology based on multi-agent clustering was developed by Herrera et al. (2012), which considered both the network features (node elevations and water demands) and economic issues (i.e., the edge cut). Nevertheless, it was only applicable to cases in which the number of DMAs was lower than the number of source nodes.

Diao et al. (2013) focused on the detection of ‘community structures’, namely groups of nodes characterized by a high density of links between them. The developed method was based on the evaluation of the ‘modularity index’, the use of which has also been investigated in other works (Scibetta et al. 2013; Giustolisi & Ridolfi 2014). In particular, Giustolisi & Ridolfi (2014) contemplated tailoring of the modularity index in order to overcome resolution problems in the application to water networks.

However, in these approaches, the system topology was mainly considered while less emphasis was given to the hydraulic performance of the partitioned network and to the other advantages offered by the sectorization. More technical principles were adopted in the studies by Di Nardo et al. (2014a) and Ferrari et al. (2014). Finally, the protection of water quality as design criteria was particularly addressed in Grayman et al. (2009) and Di Nardo et al. (2014b). In fact, the establishment of DMAs also allows the water utility to isolate the areas eventually affected by contamination due to incidents or malicious attacks. In addition, it facilitates the collection of samples for the necessary physical-chemical analysis and it reduces the complexity of the mixing of different water sources (Herrera Fernandez 2011).

Despite the number of contributions, there are still some authors who identify shortcomings in approaches developed so far, as highlighted by Hajebi et al. (2014), who criticized them for one or more of the following: (i) the applicability to small networks only; (ii) the neglected consideration of the ground elevations among the design criteria; (iii) the lack of objective functions related to the hydraulics of the network. The same authors introduced a multi-objective approach based on the use of NSGA-II (Deb et al. 2002). Nevertheless, the large number of considered objectives significantly complicated the selection of the best solution.

To this aim, De Paola et al. (2014) introduced a comprehensive approach for the automatic and multi-objective sectorization of WDNs. An innovative perspective was proposed, namely, the use of sectorization as a pressure management technique for mitigating water leakages.

In this paper, a more detailed description of the optimization algorithm is provided and discussed through an application to a simple case from the literature. A calibration procedure that was followed for setting the parameters of the meta-heuristic optimizer is presented as well. Finally, an in-depth analysis of the results is carried out through the technical evaluation of the obtained solutions.

**METHODOLOGY**

A summary of the proposed approach is presented here, as well as some novel developments and details of the optimization algorithm previously introduced by De Paola et al. (2014). Three categories of design criteria are considered in this methodology, which takes into account the topology, the operational cost and the hydraulics of the WDN. However, the mathematical formulation of the optimization problem is focused on the two main aspects involved in the design of the network segmentation. In fact, a total cost function (TCF) is compared to the change in the hydraulic behaviour of the WDN in terms of reduction of system resilience.

Together with the required investment ($C_B$), which is the cost for setting the boundaries of the DMAs, the daily cost due to the water leakages ($C_L$) is explicitly taken into account in the definition of TCF, that should be minimized:

\[
\text{minimize: } TCF = \omega_B (r \cdot C_B) + \omega_L (365 \cdot C_L)
\]

(1)

Except for the presence of the two weights $\omega_B$ and $\omega_L$, Equation (1) expresses the total cost of the WDN in a
single year of operation ($r$ is the annual discounting rate). The provided formulation of the TCF is intended to give to the decision-maker the possibility of influencing the optimization. In fact, it is possible to put the desired emphasis on the two different cost items considered in this approach by changing the values of $w_B$ and $w_L$, which should be selected according to the following:

$$w_B, w_L \in [0; 1] ; \quad w_B + w_L = 1 \quad (2)$$

For example, if there is more interest in achieving long-term paybacks (i.e., through the reduction of the water leakages), higher values of $w_B$ can be adopted, while the opposite can be assumed in case of limited budget for the implementation of the detected solutions. Furthermore, the formulation of a single cost function improves the understanding of the solutions because it allows a more straightforward comparison between different designs of DMAs.

The evaluation of $C_B$ involves the assessment of the numbers ($N_{SV}$, $N_{FM}$) and of the unit costs ($C_{SV}$, $C_{FM}$) of respectively: (i) the pipes to close with shut-off valves; and (ii) the flow-meters to install on the accesses to the DMAs.

$$C_B = \sum_{j=1}^{N_{SV}} C_{SV,j} + \sum_{k=1}^{N_{FM}} C_{FM,k} \quad (3)$$

The value of $C_L$ is estimated according to the monomial formulation of the leakage law (Lambert 2003). A proper characterization of the parameters of this relationship for every node is therefore required. Assuming that the hydraulic simulation is performed with reference to a daily time pattern for the water demand (consisting of $T$ time steps), the value of which is calculated through the following:

$$C_L = C_{WL} \cdot \sum_{i=1}^{N_{leak}} \sum_{t=1}^{T} (\alpha_i P_{i,t}^\beta_i) \Delta t \quad (4)$$

in which $N_{leak}$ is the number of emitter nodes, $\alpha$ and $\beta$ are the parameters of the ‘leakage law’ (i.e., emitter coefficient and pressure exponent, respectively), $C_{WL}$ is the unit cost of lost water, $\Delta t$ is the generic time step and $P_{i,t}$ is the pressure at the $i$-th leaking node (assumed constant during the whole duration of the time step $\Delta t$). It is useful to highlight that the leakage flow (as well as the water demand) may present significant variations between workdays and weekends, and between different seasons of the year. However, this aspect is not addressed in this work, and its characterization is postponed to the future development of the research.

According to the provided definition, the minimization of the TCF is likely to produce lower water pressures. Therefore, this objective is clearly in conflict with the preservation of the hydraulic reliability of the WDN. The latter is usually evaluated in terms of ‘network resilience’ (Todini 2000; Prasad et al. 2003; Prasad & Park 2004; Farmani et al. 2005).

In this methodology, a particular measure of this quantity is adopted as the second objective, namely the Resilience Deviation Index ($I_{RD}$) introduced by Di Nardo & Di Natale (2011). Instead of its original structure, a manipulated expression of this performance indicator is adopted here:

$$I_{RD} = \frac{\sum_{i=1}^{N_i} Q_i (H_{i,\text{ind}} - H_i)}{\sum_{i=1}^{N_i} Q_i (H_{i,\text{nd}} - H_i)} \quad (5)$$

where $i$ is the generic demand node, $Q_i$ is the total water demand, including the pressure-dependent leakage flow, $H_{i,\text{ind}}$ is the minimum head required for proper supply, while $H_{i,\text{nd}}$ and $H_i$ are the total heads detected at the same node in the starting configuration of the WDN and in the partitioned one, respectively. As stated before, the water pressures in the network are expected to decrease with the establishment of the DMAs, and therefore an increase in the Resilience Deviation Index indicates a worse hydraulic performance. Consequently, the system reliability levels can be maintained through the minimization of the following objective, namely, the maximum of $I_{RD}$ detected over time ($I_{RD,\text{max}}$):

minimize: $I_{RD,\text{max}} = \max_I \{I_{RD}\} \quad (6)$

The set of problem constraints is defined as well. First, the water pressures and flows must satisfy the network hydraulic laws, and therefore the corresponding equations are considered (i.e., mass balance and head-loss formulas, pump curves, etc.). However, in this case the solution of
the system of equations is determined through the hydraulic simulator EPANETpdd (Morley & Tricarico 2008). This modelling approach employs an improved version of the well-known software EPANET 2.0 (Rossmann 2000). In particular, EPANETpdd handles pressure-dependent leakages without causing negative demands (i.e., flows entering the emitter nodes) when negative pressures occur.

A further constraint is introduced in order to satisfy the service requirement for the level of service requirement, which is to achieve the minimum pressure target \( (P_{\text{min}}) \) at every demand node (in number equal to \( N_n \)):

\[
P_{i,t} \geq P_{\text{min}} \quad \forall i \in \{1, \ldots, N_n\}, \forall t \in \{1, \ldots, T\}
\]  

(7)

A financial constraint can also be defined in case of limited budget for the required investment \( (C_B, \max) \). The corresponding function is easily formulated as follows:

\[
C_B \leq C_B, \max
\]  

(8)

Finally, another design criterion commonly adopted in practice is considered. A constraint can be introduced which takes into account the number of customer connections per DMA (Morrison et al. 2007), which should range between 500 (MinCon) and 5,000 (MaxCon). According to Ferrari et al. (2014), when the total number of customer connections (TotCon) and the total demand \( (Q_{\text{tot}}) \) are known this condition can be expressed in terms of water demand per DMA \( (Q_d) \), which should range between a minimum \( (Q_{d,\text{min}}) \) and a maximum \( (Q_{d,\text{max}}) \) value:

\[
Q_{d,\text{min}} = \frac{\text{MinCon}}{\text{TotCon}} \cdot Q_{\text{tot}}
\]

\[
Q_{d,\text{max}} = \frac{\text{MaxCon}}{\text{TotCon}} \cdot Q_{\text{tot}}
\]

\[
Q_{d,\text{min}} \leq Q_d \leq Q_{d,\text{max}}
\]  

(9)

However, in the proposed methodology, this constraint is relaxed because of the purely informative nature of the best practice limits. Therefore, a ‘soft’ constraint is defined, which is based on the generation of a random number \( (\theta) \) and on the calculation of an acceptance probability \( (\lambda_d) \). In particular, this probability is equal to one when the water demand for each DMA is in the above mentioned range, while it linearly decreases to zero when \( Q_d \) is out of the bounds of the interval but within the 50% of their values:

\[
\lambda_d = \begin{cases} 
0 & \text{if } Q_d \leq 0.5Q_{d,\text{min}} \\
\frac{(Q_d - 0.5Q_{d,\text{min}})}{0.5Q_{d,\text{min}}} & \text{if } 0.5Q_{d,\text{min}} < Q_d < Q_{d,\text{min}} \\
1 & \text{if } Q_{d,\text{min}} \leq Q_d \leq Q_{d,\text{max}} \\
\frac{(1.5Q_{d,\text{max}} - Q_d)}{0.5Q_{d,\text{max}}} & \text{if } Q_{d,\text{max}} < Q_d < 1.5Q_{d,\text{max}} \\
0 & \text{if } Q_d \geq 1.5Q_{d,\text{max}}
\end{cases}
\]  

(10)

The design of the sectorized network is therefore accepted when the following condition is verified for every DMA:

\[
\lambda_d \geq \theta
\]  

(11)

The optimization problem described so far is solved through the algorithm schematized in Figure 1. At first, the procedure is initialized through the assignment of the input data, some of which are used for performing preliminary calculations. These data consist of the general information about the network and its hydraulic model, the analysis options (i.e., the maximum number of DMAs per solution, the weights in the TCF and the values of \( C_{B,\max}, r, \text{MinCon}, \text{MaxCon} \) and \( P_{\text{min}} \)), the unit costs and the parameters of the NSGA-II (Deb et al. 2002), which is the multi-objective optimization algorithm adopted in this methodology. In particular, the size of the population \( (\text{pop-size}) \), the number of generations \( (\text{ngen}) \), the distribution indices for crossover \( (\eta_c) \) and mutation \( (\eta_m) \) are required. The candidate solutions for generating the offspring are chosen through a binary tournament selection (BTS) (Miller & Goldberg 1995). The genetic operators adopted in this case are the simulated binary crossover (SBX) (Deb & Agrawal 1995) and the polynomial mutation (PM) (Rahman & Kakde 2004), and they are applied with respective probabilities of 0.9 and 0.1.

The NSGA-II is initialized with a randomly generated population of solutions, each one consisting of the following decision variables:

- the number of DMAs \( (\text{NDMA}) \) in the current solution (integer, ranged between 1 and \( \text{NDMA}_{\text{max}} \));
the node indices for a number of DMA centroids equal to \( NDMA \) (integer, ranged between 1 and the total number of network nodes, \( N_{\text{tot}} \));

- a blending factor (\( \phi \)) for the evaluation of a ‘topological distance’ between nodes (real, ranged between 0 and 1).

Starting from these variables, the corresponding designs of the DMAs are created according to the procedure described later in the text. Before, it is important to note some advantages provided by this feature of the proposed approach. First of all, the number of DMAs is often considered as an input parameter, while here it is obtained as a result of the optimization. Therefore, less knowledge about the problem is required and the analysis is not biased by preliminary assumptions. In addition, the characterization of every solution requires a limited number of decision variables, that is also independent of the size of the network. In fact, the number of decision variables is always equal to \( NDMA + 2 \). This is a finite and relatively low quantity, and it applies both to small and large networks. Consequently, the methodology can be successfully applied to problems of any size without significant differences in the computational effort, except for the share required by the hydraulic simulations.

The ‘centroids’ could be intended as the ‘barycentres’ of the DMAs. In order to take into account the topology of the network, the other nodes (including the water sources) are assigned one by one to the DMAs in consideration of the minimum ‘topological distance’ from the centroids. This distance is identified as the shortest-path between the node and the centroid, which is here calculated through the Floyd–Warshall algorithm (FWA) (Floyd 1962; Fontana et al. 2015). The weights (\( tw_{ij} \)) on the edges of the network graph that correspond to the pipes are assigned according to both the demand and the elevation of the end nodes (see Equation (12)). The initial status of the pipes in the original configuration of the network is considered as well. In particular, an infinite weight is assigned to closed pipes,
which are therefore preferred for setting the boundaries of the DMAs. A null weight is assigned to valves and pumps, so that the end nodes are always included in the same DMA.

\[
tw_{ij} = \begin{cases} 
0 & (i, j) \text{ is pump/valve or } i = j \\
\varphi \frac{Q_{ij}}{\max \{Q_{ij}\}} + (1 - \varphi) \frac{\Delta z_{ij}}{\max \{\Delta z_{ij}\}} & (i, j) \text{ is open pipe and } i \neq j \\
\infty & \text{otherwise} 
\end{cases}
\] (12)

In the second formula of Equation (12), \(Q_{ij}\) is the demand served by the link connecting the nodes \(i\) and \(j\), while \(\Delta z_{ij}\) is the absolute difference between the ground elevations of the same two nodes. Both quantities are normalized by dividing by their maximum values and then combined with the blending factor \(\varphi\). The function of this parameter is then clarified: through the minimization of the shortest-paths, for low values of \(\varphi\) the DMAs are identified with groups of nodes having similar elevation. On the contrary, the end nodes of very long pipes (which supply large demands) are likely to be separated in different DMAs, thus involving an increased probability of not violating the constraint in Equation (11). The blending factor is also used to increase diversity in the solutions. In fact, different combinations of centroids could be obtained by slightly adjusting the position of some of them (which means that the closest neighbouring nodes are selected as centroids). This situation might easily bring about the same design of DMAs if fixed and constant topological distances were adopted. Instead, the blending factor introduces further variability in these quantities, and it makes it possible to obtain different DMAs starting from the same centroids.

Once all the nodes are assigned to the DMAs, a possible sectorization of the network is designed. This is introduced in the hydraulic model by closing the pipes connecting different DMAs. However, in some cases, this operation may lead to the isolation of the areas of the network which may be disconnected from any of the water sources. For example, this occurs when the number of network water sources is lower than the desired number of DMAs. In such cases, the supply must be provided by re-opening the most appropriate connection between different DMAs (i.e., cascade supply).

The domain of possible solutions to this sub-problem has a very large dimension that rapidly increases with the size of the network and the number of DMAs. Furthermore, in order to satisfy the minimum service requirements, a sufficient hydraulic head must be preserved at the entry point of the indirectly supplied DMAs. For both the mentioned issues, the procedure represented in Figure 2 is applied here.

First, two groups of DMAs are identified, namely those that are supplied (SupD) and those that are not (unSupD). Then, the following steps are applied to all the \(\delta\) in unSupD.

(i) The presence of adjacent supplied DMAs is checked. If this is verified, these DMAs are included in the set adjD, while their boundary nodes having connections with those of \(\delta\) are assigned to the set boundD. Otherwise, \(\delta\) is skipped and queued. (ii) The FWA is used again for evaluating the shortest-paths from the water sources in the DMAs of adjD to the nodes in boundD. The weights on the graph edges are assumed equal to zero for pumps and valves, while for the initially open pipes the hydraulic resistance is adopted. The presence of check-valves is also taken into account, and the corresponding weight is set to infinity (like that of closed pipes) in the direction opposite to that allowed by the valve. (iii) The connection belonging to the shortest-path with minimum length is opened. (iv) The entry node of \(\delta\) (the boundary node of \(\delta\) belonging to the shortest-path) is labeled as ‘source’ node. (v) The length of the shortest-path \(sp_{\delta}\) is stored. (vi) \(\delta\) is moved from unSupD to SupD.

The described sequence is repeated until unSupD is emptied. Furthermore, when calculating the shortest-paths from the source nodes (i.e., the entry nodes) of the indirectly supplied DMAs, the length of the shortest-paths must be increased by the quantities \(sp_{\delta}\).

This process makes it possible to run the hydraulic simulation on the sectorized network, which is required to evaluate the objective functions and the constraints. The NSGA-II operates the selection of the best solutions in order to improve the objectives iteration by iteration until the desired number of generations is reached. In the end, an optimal set is obtained (Pareto front, PF) that is an approximation of the true Pareto front (true-PF) for the problem. Together with the evaluation of technical indicators about the solutions (e.g., number and diameters of closed pipes, reduction of water leakages, etc.), the PF provides the wanted decision support.
RESULTS AND DISCUSSION

The application of the methodology to a well-known case study from the literature is presented here. The 25-node network (Bargiela 1984) represented in Figure 3 is analysed, whose features are retrieved from the work by Jowitt & Xu (1990). The network consists of 22 demand nodes, three water sources (reservoirs) and 37 pipes of different lengths and diameters (six classes are identified, namely 152, 229, 305, 381, 457, 475 mm). The original configuration also includes three flow-control valves, but their presence is neglected in the present application. The Hazen–Williams formula is adopted for the head-loss computation.

The water demands assigned to the nodes and the levels of the reservoirs vary according to a daily time pattern. Pressure-dependent leakages are introduced as well through the assignment of leakage coefficients to the pipes. Nevertheless, in order to run the hydraulic simulation with EPANETpdd, an equivalence is established for modelling the water leakages through emitter nodes. The same emitter exponent equal to 1.18 is assumed, while different coefficients are assigned depending on the length and the

Figure 2 | Procedure for opening the best connections between different DMAs.

Figure 3 | Example network (Jowitt & Xu 1990).
number of the pipes connected to the nodes. As a result, a 20% leakage is estimated under uncontrolled conditions.

Starting from the total demand and reasonable values of the per-capita consumption (200 litres/inhabitant/day) and number of inhabitants per customer connection, an estimate of \( \text{TotCon} \) is provided (approximately 1,900 connections). Other relevant quantities adopted in this analysis and the unit costs are reported in Tables 1 and 2. In particular, equal emphasis is given to the required investment and the unit costs are reported in Tables 1 and 2. In particular, other relevant quantities adopted in this analysis and the unit costs were retrieved through a market survey among Italian manufacturers, and they were estimated on the basis of the closest commercial values. However, it must be highlighted that the reported values are only intended to provide a reasonable combination of parameters for the evaluation of the objectives, and that they do not involve any loss of generality.

In the present work, the parameters of the NSGA-II are not assigned a priori, but they are estimated through a calibration analysis. To this aim, 90 different test cases (see Table 3) are considered, which consisted of different combinations of the parameters at varying levels. For each of them, several iterations were carried out in order to improve the optimization. In particular, ‘short’ tests (30, 50 or 100 generations) were repeated five times, while longer tests were iterated only three times. Then, the total number of tests was 432, with a total of 3,429,000 evaluations of the objective functions and of the constraints.

The calibration is carried out with the aim of finding the combination of parameters that provides the best approximation of the true-PF. The procedure described in Wang et al. (2014) is adopted for this purpose. This methodology is based on the ranking of the algorithms according to the normalized hyper volume \( \text{NHV} \) (Deb 2001), that is a robust measure of the performance of the multi-objective optimization algorithm. The results show that the best combinations of parameters \( (\text{NHV} = 0.9641) \) is that consisting of a population size of 50 and 100 generations, with the distribution indices for crossover and mutation being, respectively, 1 and 20. The performance of this algorithm is significantly better than that of the second best \( (\text{NHV} = 0.7799) \), in which the largest values of \( \text{popsize} \) and \( \text{ngen} \) are assumed (respectively, 100 and 1,000). This outcome is apparently in contrast to the general assumption according to which the performance of the optimization algorithms improves with the number of evaluations. However, in this case, the detected anomaly can be ascribed to the reduced size of the considered network. In fact, in this small example, it is difficult to maintain diversity in the provided solutions, and this increases the probability of converging to local minima. However, this aspect is properly taken into account by running the optimization model several times.

The best solutions for the optimal design of the DMAs are identified through the extrapolation of the quasi-Pareto front (quasi-PF), that is obtained by applying a non-dominated sorting (Deb et al. 2002) on the solutions of the PF obtained in all the tests. The quasi-PF is completely described in Figure 4, where some technical features of the obtained solutions are presented as well.

The obtained values of the TCF range between 72,611 and 73,838, while \( I_{RD,max} \) goes from 0 (no sectorization) to 0.107. This means that the reduction of the system resilience induced by the partitioning is 10.7% in the worst case, in which a minimum water pressure of 30.1 m is determined at the most critical node.

Most solutions consist of just two DMAs (Figure 4(a)), and in many cases the required investment (Figure 4(b)) is somewhat lower than the assumed budget limit. The technical feasibility of the sectorization designs is also emphasized in Figure 4(d), which shows that the maximum number of pipes to close is seven. Moreover, these pipes are predominantly those characterized by the smallest diameters, as specified in best practices.

Figure 4(c) displays the effect of the partitioning of the network on the reduction of the water leakages. Potential

### Table 1 | Analysis options

<table>
<thead>
<tr>
<th>Min. number of connections per DMA (MinCon)</th>
<th>Max. number of connections per DMA (MaxCon)</th>
<th>Max. number of DMAs (NDMA(_{\text{max}}))</th>
<th>Budget limit for the required investment (CB(_{\text{max}}))</th>
<th>Weight for the required investment ( (w_{\text{R}}) )</th>
<th>Weight for the cost of water leakages ( (w_{\text{L}}) )</th>
<th>Min. pressure required at demand nodes (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>5,000</td>
<td>3</td>
<td>10,000 €</td>
<td>0.50</td>
<td>0.50</td>
<td>30</td>
</tr>
</tbody>
</table>
reduction levels are estimated to be between 0.30 and 1.81%, and they are found to be highly correlated to the values of the TCF. However, the obtained correlation might depend on several features (e.g., the weights in the TCF, the unit costs, the network topology), whose effects should be evaluated case by case.

An example of sectorization is provided in Figure 5, in which the solution that minimizes the TCF is presented. Two DMAs are designed, one (DMA1) supplied by a single reservoir and the other (DMA2) fed by two sources. The separation between the DMAs involves the closure of seven pipes (represented as dotted lines) and an expenditure of 7,400 € compared to water savings estimated at 63,785 m³ per year (1.81% of the initial amount of water leakage).

Finally, the ‘closure frequency’ is presented in Figure 6. In this plot, the thickness of the pipes is increased according to the number of solutions in which they are assumed as closed. It is clearly shown that pipes 20 and 21 (see Figure 3) have the highest closure frequency, and they are followed by pipes 19 and 35.

**CONCLUSIONS**

An innovative model for the optimal sectorization of water distribution networks is discussed in this work. The presented methodology follows a multi-objective approach, and it takes into account several design criteria related to the topology, the economics and the hydraulics of the network. In particular, the possible use of sectorization as a pressure management technique for the mitigation of water leakages is addressed.

However, in order to improve the understanding of the optimization process, only two objectives having a clear meaning are considered in the optimization problem. A set of constraints related to the minimum service requirements, financial issues and best practices is introduced as well.

The combination of an evolutionary algorithm with graph analysis tools allows a simple implementation of the model via software. Furthermore, the particular selection of the decision variables, as well as the intermediate procedures included in the algorithm, are intended to provide the applicability to a broad range of cases, regardless of the size of the analysed networks.

The intervention of the analyst is only required in the assignment of the input parameters, most of which are related to essential features of the network operation. Furthermore, the application of the model provides decision aiding in the optimal design of DMAs, by providing feasible solutions representing the best trade-offs between the two adopted objectives: the total operational cost against the reduction of the hydraulic reliability of the network.

The effectiveness of the methodology is demonstrated on an example from the literature. The developed application is focused on both the calibration of the optimization algorithm and the technical description of the outcomes. The obtained results demonstrate the potential of the proposed model in providing feasible solutions to the problems, which are also characterized by the independence on the number of water sources in the network, reduced impact on the network resilience and low cost compared to the assumed budget. The possibility of achieving a

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**Table 2 | Unit costs and annual discounting rate**

<table>
<thead>
<tr>
<th>Pipe diameter (mm)</th>
<th>Cost of shut-off valve (€)</th>
<th>Cost of flow-meter (€)</th>
</tr>
</thead>
<tbody>
<tr>
<td>152</td>
<td>245.00</td>
<td>2,020.00</td>
</tr>
<tr>
<td>229</td>
<td>865.00</td>
<td>2,580.00</td>
</tr>
<tr>
<td>305</td>
<td>1,930.00</td>
<td>3,345.00</td>
</tr>
<tr>
<td>381</td>
<td>3,450.00</td>
<td>4,315.00</td>
</tr>
<tr>
<td>457</td>
<td>5,420.00</td>
<td>5,490.00</td>
</tr>
<tr>
<td>475</td>
<td>5,955.00</td>
<td>5,800.00</td>
</tr>
</tbody>
</table>

| Cost of water leakages (€/m³) | 0.15 |
| Annual discounting rate | 0.05 |

**Table 3 | Calibration of NSGA-II. Summary of the tested parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
<th>Test cases</th>
<th>No. of runs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size</td>
<td>10; 30; 50</td>
<td>1 to 81</td>
<td>5</td>
</tr>
<tr>
<td>Number of generations</td>
<td>30; 50; 100</td>
<td>82 to 90</td>
<td>3</td>
</tr>
<tr>
<td>Distr. index for crossover</td>
<td>1; 10; 20</td>
<td>100</td>
<td>1; 10; 20</td>
</tr>
<tr>
<td>Distr. index for mutation</td>
<td>1; 10; 20</td>
<td>1,000</td>
<td>1; 10; 20</td>
</tr>
</tbody>
</table>

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Figure 4 | Quasi-PF and technical features of the obtained solutions.

Figure 5 | Design of the DMAs for the minimization of the total cost function.

Figure 6 | Closure frequency of pipes.
reduction in the water leakages through the simple closure of network pipes is emphasized as well.

REFERENCES


Gomes, R., Sa Marques, A. & Sousa, J. 2012 Decision support system to divide a large network into suitable district metered areas. Water Sci. Technol. 65 (9), 1667–1675.


Scibetta, M., Boano, F., Revelli, R. & Ridolfi, L. 2013 *Community detection as a tool for complex pipe network clustering*. *Procedia Eng.* 70, 1518–1523.


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