A multi-criteria decision-making model dealing with correlation among criteria for reservoir flood control operation

Feilin Zhu, Ping-an Zhong, Bin Xu, Ye-nan Wu and Yu Zhang

ABSTRACT

Flood control operation in a multi-reservoir system is a multi-criteria decision-making (MCDM) problem, in which the considered criteria are often correlated with each other. In this paper, we propose an MCDM model for reservoir flood control operation to deal with correlation among criteria. Considering the flood control safety of reservoirs and downstream protected regions, we establish the hierarchical structure of the criterion system. We use the principal component analysis method to eliminate the correlation, and transform the original criterion system into an independent comprehensive criterion system. The comprehensive decision matrix coupled with the weight vector obtained by the improved entropy weight method serves as the input to TOPSIS method, fuzzy optimum method, and fuzzy matter-element method, by which we determine the ranking order of the alternatives. We apply the proposed model to a cascade system of reservoirs at the Daduhe River basin in China. The results show that the dimensionality of the criterion system is reduced and the correlation among criteria is eliminated simultaneously, and the ranking order of the alternatives is reasonable. The proposed model provides an effective way to deal with correlation among criteria, and can be extended to wider applications in many other MCDM problems.

Key words | criteria correlation, flood control operation, multi-criteria decision-making, multi-reservoir system, principal component analysis (PCA) method, TOPSIS

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ABBREVIATIONS AND NOTATION

MCDM	Multi-criteria decision-making
PCA	Principal component analysis
TOPSIS	Technique for order performance by similarity to ideal solution
DPR	Downstream protected region
N	The number of reservoirs in a multi-reservoir system
K	The number of downstream protected regions in a multi-reservoir system
Z_{max}	The highest water level
Z_e	The terminal water level
R_v	Ratio of the used flood control capacity
Q_{max}	Peak discharge in downstream protected regions
T	Duration of stream flow exceeding the safety discharge in downstream protected regions
W	Spillover volume exceeding the safety discharge in downstream protected regions

INTRODUCTION

Flood control operation in a multi-reservoir system is an important non-engineering measure to mitigate flood damage through the complementarity of each single reservoir. For a multi-objective optimization problem, it is impossible to determine a single optimal alternative that

optimizes all objectives, such as flood control, hydropower generation, water supply, irrigation, shipping, etc., because some of them are incommensurable and conflicting with each other (Malekmohammadi *et al.* 2011). Instead, it is more appropriate to select the most satisfying alternative from a set of feasible alternatives (Chen & Hou 2004; Yu *et al.* 2004; Fu 2008; Malekmohammadi *et al.* 2011; Wang *et al.* 2011), and thus, reservoir flood control operation can be actually defined as a multi-criteria decision-making (MCDM) problem. Unlike most of MCDM problems, reservoir flood control operation is generally a complex, multi-objective and multi-stage process in nature due to the influences of social, economic, environmental, technical, and political factors (Fu 2008). In addition, reservoir flood control operation is also a real-time dynamic adjustment process. Therefore, it is of great theoretical and practical significance to establish an MCDM model to help reservoir operators make informed decisions.

Plenty of methods have been developed to solve MCDM problems since the 1960s, and these methods can be classified into six categories (Hajkowicz & Collins 2007). Moreover, some MCDM models that consider fuzzy uncertainty related to human judgments (Shafiqul Islam *et al.* 2014; Xu & Qin 2014) and stochastic uncertainty related to data (Zarghami & Szidarovszky 2009; Akbari *et al.* 2011; Madani *et al.* 2014; Yager 2014) have also been studied. In an MCDM problem, the criteria are usually employed to measure the performance of each alternative in different aspects, and they should be independent, comprehensive, and non-redundant (Keeney & Raiffa 1993). However, it is difficult or impossible to ensure that each criterion is strictly independent of each other. In a complex system, all criteria are correlated, either directly or indirectly (Chen *et al.* 2010). Correlation among criteria implies that repeated and inter-ferential information exists in the criterion system, which will affect the assessing results when decision-makers use MCDM methods to evaluate the alternatives. Obviously, the rationality and accuracy will be influenced by the correlated criteria in an MCDM process (Larichev & Moshkovich 1995; Raju *et al.* 2000). Consequently, dealing with correlation among criteria is an important issue for MCDM problems and has received deserved attention. For example, Brans & Mareschal (1994) pointed out that it is particularly important to analyze the conflicting aspects of

the criteria that express similar, independent, or opposite preferences, and they used the GAIA (geometrical analysis for interactive assistance) visual modeling method to provide decision-makers with a powerful tool for analysis. Chen *et al.* (2010) indicated that previous works to evaluate environment plans have been done under the assumption that the criteria are independent. They proposed a novel hybrid MCDM model combining the decision-making trial and evaluation laboratory (DEMATEL) and analytical network process methods to address the dependency among the criteria for environment watershed plans.

In reservoir flood control operation, many MCDM methods (Chen & Hou 2004; Yu *et al.* 2004; Fu 2008; Wang *et al.* 2011) have been developed and applied in recent years. Chen & Hou (2004) used a subjective preference and iterative weights method for assessing the weights, and proposed a fuzzy recognition model. Yu *et al.* (2004) established a multi-person MCDM model, which incorporates the influence of multi-objectives and knowledge of decision-makers. Fu (2008) presented a fuzzy optimization method based on the concept of ideal and anti-ideal points under fuzzy environments. Wang *et al.* (2011) proposed an MCDM model based on the theory of variable fuzzy sets. These studies provide more available choices for MCDM of reservoir flood control operation. However, correlation among criteria is ignored in these models, and all of the test cases in these researches are conducted in a single reservoir. In addition, the flood control operation is no longer limited to a single reservoir, cascaded reservoirs are normally optimized jointly so that the overall benefit can be maximized in the literature (e.g., Wang *et al.* 2014; Li & Ouyang 2015; Peng *et al.* 2015). Compared with flood control operation of a single reservoir, the MCDM problem of a multi-reservoir system is more complex and challenging due to the fact that more criteria should be selected for assessing the performances of alternatives, and the complexity and challenge enhances with increasing number of reservoirs. In addition, due to the hydraulic connection between reservoirs, the criteria are more likely to correlate with each other in a multi-reservoir system. Sometimes, the used criteria are even highly correlated. Consequently, for complex large-scale multi-reservoir systems, dealing with correlation among criteria and reducing dimensionality are both essential in the MCDM process.

In this paper, we propose an MCDM model dealing with correlation among criteria for flood control operation in a multi-reservoir system. First, we establish the criterion system for ranking alternatives. Second, based on the correlation analysis of criteria, principal component analysis (PCA) method is utilized to transform the original criterion system into an independent comprehensive criterion system for eliminating the correlation among criteria. Third, we use an improved entropy weight method to determine weights for the criteria. Thereafter, we obtain the ranking order of all the feasible alternatives by using the classical TOPSIS method, fuzzy optimum method, and fuzzy matter-element method simultaneously. Finally, we apply the proposed methodology to a case study.

ESTABLISHMENT OF THE CRITERION SYSTEM

For establishing a criterion system, the structure of criterion system, criteria selection, and explanation of each criterion should be taken into account. A flood control system can be generalized as a multi-reservoir system that includes N reservoirs and K downstream protected regions. During a flood event, reservoir operators are mainly concerned about the flood control safety of reservoirs and downstream protected regions. Therefore, we select three criteria for

assessing the status of each reservoir in the multi-reservoir system within a given flood control alternative, including the highest water level (denoted by Z_{max}), the terminal water level (denoted by Z_e), and ratio of the used flood control capacity (denoted by R_v). In addition, we choose another three criteria for assessing the status of each downstream protected region within a given flood control alternative, including peak discharge in downstream protected regions (denoted by Q_{max}), duration of stream flow exceeding the safety discharge in downstream protected regions (denoted by T), and spillover volume exceeding the safety discharge in downstream protected regions (denoted by W). All of the above criteria are cost criteria which can be quantified during the reservoir flood routing process. The hierarchical structure of the criterion system, which consists of the objective layer, principle layer, and criterion layer, is shown in Table 1.

As shown in Figure 1, for a multi-reservoir system, each alternative uses three criteria to assess the status of a reservoir, including Z_{max}, Z_e, and R_v. Similarly, three other criteria are applied to evaluate the status of each downstream protected region, including Q_{max}, T, and W. Suppose that the criterion system includes n criteria, the criterion set can be denoted as $C = \{c_1, c_2, K, c_n\}$, and we assume that there are m feasible alternatives to be ranked, which can be expressed as $A = \{a_1, a_2, K, a_m\}$. We can get

Table 1 | Hierarchical structure of the criterion system

| Objective layer	Principle layer	Criterion layer	Meaning of criteria	Type of criteria
A_1 The overall objective of flood control operation in multi-reservoir system	B_1 Flood control safety of reservoirs	C_1 The highest water level (Z_{max}, m)	Reservoirs' own safety during the operation	Cost type, quantitative
		C_2 The terminal water level (Z_e, m)	Capacity of storing the subsequent floods	Cost type, quantitative
		C_3 Ratio of the used flood control capacity (R_v, %)	Ratio of the occupied flood control capacity	Cost type, quantitative
	B_2 Flood control safety of downstream protected regions	C_4 Peak discharge in downstream protected regions (Q_{max}, m^3/s)	Determining whether the downstream protected regions are damaged by floods	Cost type, quantitative
		C_5 Duration of stream flow exceeding the safety discharge in downstream protected regions (T, h)	Duration of the downstream protected regions being damaged by floods	Cost type, quantitative
		C_6 Spillover volume exceeding the safety discharge in downstream protected regions (W, million m^3)	Degree of the downstream protected regions being damaged by floods	Cost type, quantitative
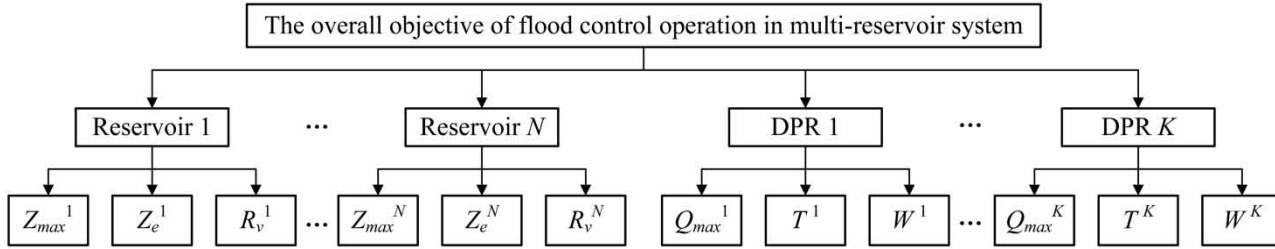
Figure 1 │ Criterion system for a multi-reservoir system (DPR denotes the downstream protected region).

the following decision matrix:

$$X = (x_{ij})_{m \times n} \tag{1}$$

where x_{ij} represents the value of alternative a_i with respect to criterion c_j, $i = 1, 2, …, m$ and $j = 1, 2, …, n$.

SIMPLIFICATION OF THE CRITERION SYSTEM

Since the criterion system is complex, the correlation between criteria involved in the MCDM problem of flood control operation in a multi-reservoir system may widely exist. The approach of deleting the correlated criteria directly cannot ensure that the remaining criteria are strictly independent of each other, and usually they are still correlated. Besides, it is hard to delete these cross-correlated criteria since they measure the performance of each alternative from different aspects. Thus, we suggest using the PCA method to handle this issue.

Correlation analysis

The linear correlation level between two criteria can be measured by Pearson's coefficient. A large absolute value of the Pearson's coefficient between two criteria indicates that the two criteria considered are highly correlated. The Pearson's coefficient can be calculated by the following expression:

$$r_{ij} = \frac{\sum\limits_{k=1}^{m} (x_{ki} - \bar{x}_i)(x_{kj} - \bar{x}_j)}{\sqrt{\sum\limits_{k=1}^{m} (x_{ki} - \bar{x}_i)^2 (x_{kj} - \bar{x}_j)^2}} \tag{2}$$

where x_{ki} and x_{kj} denote the value of alternative a_k regarding

criterion c_i and criterion c_j, respectively; $\bar{x}_i$ and $\bar{x}_j$ represent the average value of criterion c_i and criterion c_j, respectively.

PCA method

As a type of multivariate statistical method, PCA is often utilized to reduce the dimensionality of a criterion system (Singh *et al.* 2009; Wan *et al.* 2010), and numerous correlated criteria can be transformed into a few independent criteria while retaining the information contained in the original criterion system as much as possible. These simplified criteria are named the principal components. PCA includes the following procedures:

(1) Normalize the decision matrix $X = (x_{ij})_{m \times n}$ using the following equation:

$$x_{ij}^* = \frac{x_{ij} - \bar{x}_j}{\sigma_j} \tag{3}$$

where x_{ij}^* denotes the value of alternative a_i with regard to criterion c_j, $i = 1, 2, …, m$ and $j = 1, 2, …, n$; σ_j is the standard deviation of criterion c_j.

(2) Calculate the correlation matrix $R = (r_{ij})_{n \times n}$ using Equation (2).

(3) Obtain the eigenvalues (denoted by $\lambda_1, \lambda_2, …, \lambda_n$) of correlation matrix R via solving the characteristic equation $|\lambda I - R| = 0$. Suppose that $\lambda_1 \geq \lambda_2 \geq K \geq \lambda_n \geq 0$, the corresponding eigenvectors (denoted by $e_1, e_2, …, e_n$) are obtained.

(4) If the contribution rate of accumulative variance ($\sum\limits_{j=1}^{p} \lambda_j / \sum\limits_{i=1}^{n} \lambda_i$) is greater than 90%, we can obtain the first p principal components Y_s ($s = 1, 2, …, p$):

$$Y_s = e_s'X^* = e_{1s}X_1^* + e_{2s}X_2^* + K + e_{ns}X_n^* \tag{4}$$

where $X_1^*, X_2^*, …, X_n^*$ are column vectors of the

normalized matrix X^*; $e_s = (e_{1s}, e_{2s}, K, e_{ns})'$ is the sth eigen vector of the correlation matrix R.

(5) The principal component loadings, which measure the correlation level between the original criterion c_j and Y_s, are determined by:

$$\rho(c_j, Y_s) = \sqrt{\lambda_s} e_{sj} \tag{5}$$

(6) Define y_{is} as the value of alternative a_i with regard to principal component Y_s, $i = 1, 2, \ldots, m$ and $s = 1, 2, \ldots, p$, which can be calculated by the following formula:

$$y_{is} = \sum_{t=1}^{n} y_{it} e_{ts} \tag{6}$$

Through the above steps, we replace n criteria by p comprehensive criteria (i.e., the principal components) for evaluating alternatives, and transform the original decision matrix $X = (x_{ij})_{m \times n}$ into the new decision matrix $Y = (y_{is})_{m \times p}$. Therefore, we reduce the dimensionality of the criterion system, and eliminate the correlation among criteria simultaneously.

EVALUATION OF THE ALTERNATIVES

Unlike with those forecasting models that can be tested by comparing the forecasted results with the benchmark results (such as hydrological forecasts, forecasted results are usually compared with the measured results to assess the performance of hydrological forecasting models), it is difficult to select a benchmark alternative from the MCDM problem of reservoir flood control operation and use it to examine the effectiveness of the proposed methodology. Furthermore, the majority of studies usually apply more than one MCDM method to test the sensitivity and rationality of results (Hajkowicz & Collins 2007). Therefore, we use the TOPSIS method, fuzzy optimum method, and fuzzy matter-element method to evaluate reservoir flood control operation alternatives simultaneously, and compare the evaluation results of the three methods that use PCA procedures with the results of these methods without applying PCA

procedures to show the advantages of the proposed method.

Entropy weight method

Entropy is a measure of the system chaos in information theory, and it reflects the amount of useful information provided by the criterion system. For the p comprehensive criteria, we can calculate their weight vectors according to the variation degree of criteria values. The main steps are as follows:

(1) The decision matrix $Y = (y_{is})_{m \times p}$ is normalized to a new matrix $Y^* = (y_{is}^*)_{m \times p}$. For benefit criteria, we utilize the following formula to normalize:

$$y_{is}^* = (y_{is} - y_{s\,min})/(y_{s\,max} - y_{s\,min}) \tag{7}$$

and we use the following equation to normalize the cost criteria:

$$y_{is}^* = (y_{s\,max} - y_{is})/(y_{s\,max} - y_{s\,min}) \tag{8}$$

where $y_{s\,max}$ and $y_{s\,min}$ denote the maximum and minimum value, respectively, of the sth comprehensive criterion.

(2) Calculate the entropy H_s of the comprehensive criterion Y_s:

$$H_s = -\frac{1}{\ln m} \sum_{i=1}^{m} f_{is} \ln f_{is} \tag{9}$$

$$f_{is} = y_{is}^* / \sum_{i=1}^{m} y_{is}^* \tag{10}$$

Specifically, we assume $\ln f_{is} = 0$ when $f_{is} = 0$ in order to ensure that $\ln f_{is}$ is valid.

(3) The entropy weight w_s is typically obtained via a conversion formula according to the entropy H_s. However, when H_s is close to 1, the entropy weight obtained by the conventional conversion formula will fluctuate violently with the tiny change of entropy. Zhou *et al.* (2007) proposed an improved conversion formula to solve this problem. This paper uses the improved

conversion formula to determine the weight vector $E = (w_1, w_2, L, w_p)$.

$$\begin{cases} w_s = \dfrac{1-H_s}{n-\sum\limits_{t=1}^{n}H_t} & \text{Conventional conversion formula} \\[4mm] w_s = \dfrac{\sum\limits_{t=1}^{n}H_t + 1 - 2H_s}{\sum\limits_{l=1}^{n}\left(\sum\limits_{t=1}^{n}H_t + 1 - 2H_l\right)} & \text{Improved conversion formula} \end{cases}$$

(11)

TOPSIS method

Based on the concept of ideal and anti-ideal point, the most satisfying alternative obtained by the TOPSIS method (Hwang & Yoon 1981) is defined as the alternative that is simultaneously closest to the ideal alternative and farthest from the anti-ideal alternative. United Nation Environmental Program recommends the use of the TOPSIS method to evaluate water resource development projects, and it has been widely applied to solve MCDM problems (e.g., Chen 2000; Abo-Sinna *et al.* 2008; Afshar *et al.* 2011). The main steps are as follows:

(1) Normalize the decision matrix $Y = (y_{is})_{m \times p}$ to $Z = (z_{is})_{m \times p}$ using the following formula:

$$z_{is} = \frac{y_{is}}{\sqrt{\sum\limits_{i=1}^{m} y_{is}^2}}$$

(12)

(2) Multiply the normalized decision matrix $Z = (z_{is})_{m \times p}$ by the weight vector $E = (w_1, w_2, L, w_p)$ to determine the weighted decision matrix $B = (b_{is})_{m \times p}$, where $b_{is} = z_{is} w_s$.

(3) Obtain the ideal alternative $G^+ = \left[g_1^+, g_2^+, L, g_p^+\right]$ and anti-ideal alternative $G^- = \left[g_1^-, g_2^-, L, g_p^-\right]$:

$$g_s^+ = \begin{cases} \max\limits_{1 \leq i \leq m}\{b_{is}\} & \text{For benefit criteria} \\ \min\limits_{1 \leq i \leq m}\{b_{is}\} & \text{For cost criteria} \end{cases}$$

(13)

$$g_s^- = \begin{cases} \min\limits_{1 \leq i \leq m}\{b_{is}\} & \text{For benefit criteria} \\ \min\limits_{1 \leq i \leq m}\{b_{is}\} & \text{For cost criteria} \end{cases}$$

(14)

(4) Calculate the Euclidean distances from alternative a_i to the ideal and anti-ideal alternatives, as given by:

$$\begin{cases} d_i^+ = \sqrt{\sum\limits_{s=1}^{p}(g_s^+ - b_{is})^2} \\[4mm] d_i^- = \sqrt{\sum\limits_{s=1}^{p}(g_s^- - b_{is})^2} \end{cases}$$

(15)

(5) The closeness coefficient c_i provides the global evaluation for alternative a_i with regard to all criteria, by which the ranking order of all feasible alternatives can be identified. We can calculate c_i by:

$$c_i = \frac{d_i^-}{d_i^+ + d_i^-}$$

(16)

Fuzzy optimum method

Chen (1994) proposed fuzzy optimum method based on the fuzzy sets theory to evaluate the alternatives of water conservancy and hydropower system. This method is performed by the following procedures:

(1) Normalize the decision matrix $Y = (y_{is})_{m \times p}$ to $Y^* = (y_{is}^*)_{m \times p}$ according to Equations (7) and (8).

(2) Similar to the concept of ideal and anti-ideal points in TOPSIS method, fuzzy optimum method also determines the ideal alternative $G = [g_1, g_2, L, g_p]$ and anti-ideal alternative $B = [b_1, b_2, L, b_p]$, where $g_s = \max\limits_{1 \leq i \leq m}\{y_{is}^*\}$, $b_s = \min\limits_{1 \leq i \leq m}\{y_{is}^*\}$.

(3) Calculate the weighted Euclidean distances from alternative a_i to the ideal and anti-ideal alternatives, as follows:

$$dg_i = \sqrt{\sum\limits_{s=1}^{p}\left[w_s(g_s - y_{is}^*)\right]^2}$$

(17)

$$db_i = \sqrt{\sum\limits_{s=1}^{p}\left[w_s(y_{is}^* - b_s)\right]^2}$$

(18)

where w_s is the entropy weight of the sth criterion.

(4) The membership degree u_i is defined as a variable that provides the global evaluation for alternative a_i with

respect to all criteria, by which the ranking order of all feasible alternatives can be obtained. From the angle of fuzzy sets theory, u_i can be explained as the membership degree of alternative a_i to the fuzzy concept of 'optimum for all criteria', i.e., the ideal alternative, and $1 - u_i$ denotes the membership degree to the anti-ideal alternative (Fu 2008). In order to solve the optimal evaluation of u_i, the objective function is established as follows:

$$Minimize \quad F(u_i) = u_i^2 dg_i^2 + (1 - u_i^2)db_i^2 \tag{19}$$

and its differentiating function satisfies the following equation:

$$\frac{dF(u_i)}{du_i} = 0 \tag{20}$$

The membership degree u_i can be determined:

$$u_i = \left\{ 1 + \frac{\sum\limits_{s=1}^{p} \left[w_s (g_s - y_{is}^*) \right]^2}{\sum\limits_{s=1}^{p} \left[w_s (y_{is}^* - b_s) \right]^2} \right\}^{-1} \tag{21}$$

Fuzzy matter-element method

Cai (1994) first proposed the matter-element analysis. In the matter-element analysis, three key elements are events, characteristics, and values, which are utilized to describe these factors as the order basic element, and it is called the matter-element. Fuzzy matter-element method (Cai 1994) is an MCDM technique based on the theory of matter element analysis, and has been widely used.

For a given MCDM problem with M alternatives to be evaluated and the characteristic C is x, we can constitute the sequential matter-element $R = (M, C, x)$. Suppose that x is fuzzy, each of the M alternatives has n criteria C_1, C_2, ..., C_n and the n values are x_1, x_2, ..., x_n, R is named a fuzzy matter-element of dimension n. M alternatives and n-dimensional fuzzy matter-elements constitute the compound fuzzy matter-element R_{nm}, which can be expressed as:

$$R_{nm} = \begin{bmatrix} & M_1 & M_2 & \cdots & M_m \\ C_1 & \mu(x_{11}) & \mu(x_{12}) & \cdots & \mu(x_{1m}) \\ C_2 & \mu(x_{21}) & \mu(x_{22}) & \cdots & \mu(x_{2m}) \\ \vdots & \vdots & \vdots & & \vdots \\ C_n & \mu(x_{n1}) & \mu(x_{n2}) & \cdots & \mu(x_{nm}) \end{bmatrix} \tag{22}$$

where M_i is the ith alternative ($i = 1, 2, ..., m$); C_j is the jth criterion ($j = 1, 2, ..., n$); x_{ij} is the corresponding value of alternative M_i with respect to criterion C_j; $\mu(\cdot)$ is a preferable fuzzy membership grade function:

$$\mu(x_{ij}) = \begin{cases} \dfrac{x_{ij}}{\max x_{ij}} & \text{For benefit criteria} \\ \dfrac{\min x_{ij}}{x_{ij}} & \text{For cost criteria} \end{cases} \tag{23}$$

where $\max x_{ij}$ and $\min x_{ij}$ refer to the maximum and minimum value of the jth criterion, respectively.

Standard fuzzy matter element R_{0n} is defined as the maximum or minimum value of preferable fuzzy membership grade. We stipulate the maximum value as the optimal one, which means the preferable membership grade for each criterion equals to 1. Δ_{ij} ($i = 1, 2, ..., n; j = 1, 2, ..., m$) is the square sum of element difference between standard fuzzy matter element R_{0n} and compound fuzzy matter element R_{nm}, which can be calculated by the following formula:

$$\Delta_{ij} = [1 - \mu(x_{ij})]^2 \tag{24}$$

In the fuzzy matter-element method, the Euclid closeness degree ρH_i is defined as the degree of proximity between the evaluated alternatives and a standard alternative. The larger the Euclid closeness degree is, the better the alternative. Therefore, the Euclid closeness degree decides the ranking order of all feasible alternatives, and can be calculated as follows:

$$\rho H_i = 1 - \sqrt{\sum_{j=1}^{n} w_j \Delta_{ij}} \tag{25}$$

where w_j is the weight of the jth criterion.

CASE STUDY

Alternatives generation

We applied the proposed methodology to the case study of a cascade reservoir system, which includes the Shuangjiangkou reservoir, Houziyan reservoir, and Pubugou reservoir, and is located at the Daduhe River basin in China. The total storage capacities of the three reservoirs are 3.115 billion m^3, 1.164 billion m^3, and 5.522 billion m^3, respectively. Leshan city is located at the downstream of the Pubugou reservoir, which is an important flood control protected region with a population size of 3,544,000 and GDP of 113.479 billion CNY. A generalized diagram of the considered system is shown in Figure 2.

An actual flood event in the upstream of Shuangjiangkou reservoir and the corresponding interval flood events are used as the input to the cascade system. We generate the alternatives according to the following principle. According to the importance weights between the safety of reservoirs and downstream protected regions, we first determine the operation model of the upstream reservoir,
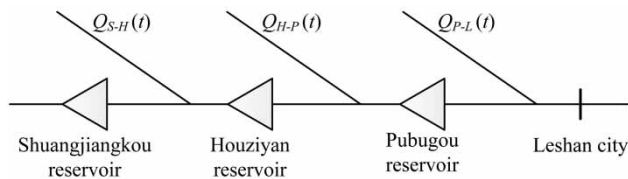
Figure 2 │ Generalized diagram of the multi-reservoir system.

then determine the operation model of the downstream reservoirs in sequence. In real-world reservoir flood control operation, the discharge capacity operation model, the regular operation model, and the maximum flood peak reduction operation model are the three most widely used models. Because the flood event is large (the return period is close to a hundred years), in order to avoid the risk of dam failure, the Houziyan reservoir only uses the discharge capacity operation model due to its small flood control capacity. The Shuangjiangkou reservoir uses the discharge capacity operation model and the maximum flood peak reduction operation model. The Pubugou reservoir uses all of the three operation models above for reservoir flood routing. As shown in Table 2, we consider six combinations of the three operation models in the cascade system, and generate six corresponding flood control operation alternatives.

The discharge capacity operation model

The discharge capacity operation model is typically adopted under emergencies (e.g., extreme large flood events) to ensure the safety of reservoirs. This model assumes that all the flood discharge facilities including floodgates, top openings, bottom outlets, and diversion bottom outlets are all opened to their maximum capacity. This operation model is conducive to ensure the flood control safety of the reservoir itself during a large flood event, however, the safety of the downstream regions is considered to be a lower priority. In order to avoid excessive decline of the reservoir water

Table 2 │ Combination of reservoir operation models for different alternatives

Operation models

Shuangjiangkou reservoir	Houziyan reservoir	Pubugou reservoir	Alternative no.
The discharge capacity operation model	The discharge capacity operation model	The discharge capacity operation model	1
		The regular operation model	2
		The maximum flood peak reduction operation model	3
The maximum flood peak reduction operation model		The discharge capacity operation model	4
		The regular operation model	5
		The maximum flood peak reduction operation model	6

level, the outflow is expressed as follows:

$$Q_{out}(t) = \min\left[Q_{\max}(t), Q_{in}(t)\right] \tag{26}$$

where $Q_{\max}(t)$ denotes the discharge capacity at time t, $Q_{in}(t)$ is the reservoir inflow at time t.

The regular operation model

In the regular operation model, the operation rule is usually applied to guide reservoir operation according to the forecasted inflow and the current storage state of reservoirs. This model is widely applied to real-world cases due to its flexibility. Compared with the discharge capacity operation model, the regular operation model can ensure the safety of downstream protected regions in the early stage of reservoir operation as much as possible. When the subsequent flood is large, this model is unfavorable to the safety of reservoirs and downstream protected regions because of the small flood control capacity in the later stage, as this model uses limited forecasted flood information to guide operations.

The maximum flood peak reduction operation model

The maximum flood peak reduction operation model is a widely applied reservoir flood control optimal operation model. The objective of this model is to minimize the peak discharge of reservoir outflow as well as mitigating the flood damage of the downstream protected regions. Additionally, the highest water level constraint (Equation (29)) is employed to ensure the safety of reservoirs. Therefore, the maximum flood peak reduction operation model considers the overall flood control benefit of reservoirs and downstream protected regions simultaneously. We use the stepwise trial-and-error method (Zhong *et al.* 2003) to solve the solution. The objective function of this optimization model can be formulated as follows:

$$Minimize \quad F = \sum_{t=1}^{T}\left[Q_{out}(t) + Q_e(t)\right]^2 \tag{27}$$

where t and T denote the index of the time, number of time periods, respectively; $Q_{out}(t)$ is the reservoir outflow at time t; $Q_e(t)$ is the interval inflow at time t.

The constraints of the optimization model include water balance constraint, the highest water level constraint, terminal water level constraint, discharge limit constraint, and outflow variation limit constraint. These constraints can be formulated as follows:

$$V(t) = V(t-1) + \left[\frac{Q_{in}(t) + Q_{in}(t-1)}{2} - \frac{Q_{out}(t) + Q_{out}(t-1)}{2}\right]\Delta t \tag{28}$$

$$Z(t) \le Z_m(t) \tag{29}$$

$$Z_{end} = Z_e \tag{30}$$

$$Q_{out}(t) \le Q_{\max}(t) \tag{31}$$

$$|Q_{out}(t) - Q_{out}(t-1)| \le \nabla Q_m \tag{32}$$

where $V(t)$ denotes the reservoir storage at time t; Δt is time interval; $Z(t)$ and $Z_m(t)$ are the water level and the upper limit of water level at time t, respectively; Z_{end} and Z_e represent the terminal water level and the target terminal water level, respectively; ∇Q_m is the allowed outflow variation limit.

RESULTS AND DISCUSSION

Considering the characteristics of the cascade system, we select 12 criteria to measure the safety of the three reservoirs and Leshan city during the flood event. We can obtain the criteria values of the six alternatives after reservoir flood routing, as shown in Table 3.

Pearson's coefficients between each two criteria are shown in Table 4. It is obvious that the correlations between criteria are universal, and some of the criteria are highly correlated. The results of correlation analysis indicate that repeated and interferential information exists in the criterion system, which will affect the evaluation result. Correlation between criteria not only occurs in this case study, but also exists in previous studies (e.g., Yu *et al.* 2004; Fu 2008; Wang *et al.* 2011). These studies mainly discussed how to improve and apply MCDM methods for

Table 3 │ Criteria values of the six alternatives

Alternative no.	Z_{max}^1	Z_e^1	R_v^1	Z_{max}^2	Z_e^2	R_v^2	Z_{max}^3	Z_e^3	R_v^3	Q_{max}	T	W
1	2,490.5	2,481.6	0.7	1,835.6	1,835.2	1.3	845.5	841.7	6.2	6,161.8	130	286
2	2,490.5	2,481.6	0.7	1,835.6	1,835.2	1.3	850.1	850.1	13.4	7,732.4	122	331
3	2,490.5	2,481.6	0.7	1,835.6	1,835.2	1.3	846.2	843.0	7.55	5,632.0	103	159
4	2,493.0	2,485.8	3.4	1,835.0	1,835.0	0	844.7	842.0	5.35	5,819.5	103	155
5	2,493.0	2,485.8	3.4	1,835.0	1,835.0	0	850.0	850.0	13.2	6,552.1	96	197
6	2,493.0	2,485.8	13.4	1,835.0	1,835.0	0	845.8	843.0	7	5,252.9	86	22

Z_{max}, m; Z_e, m; R_v, %; Q_{max}, m³/s; T, h; W, million m³.

Table 4 │ Correlation coefficient matrix

	Z_{max}^1	Z_e^1	R_v^1	Z_{max}^2	Z_e^2	R_v^2	Z_{max}^3	Z_e^3	R_v^3	Q_{max}	T	W
Z_{max}^1	1*	1*	1*	−1*	−1*	−1*	−0.10	0.09	−0.10	−0.40	−0.78	−0.67
Z_e^1	1*	1*	1*	−1*	−1*	−1*	−0.10	0.09	−0.10	−0.40	−0.78	−0.67
R_v^1	1*	1*	1*	−1*	−1*	−1*	−0.10	0.09	−0.10	−0.40	−0.78	−0.67
Z_{max}^2	−1*	−1*	−1*	1*	1*	1*	0.10	−0.09	0.10	0.40	0.78	0.67
Z_e^2	−1*	−1*	−1*	1*	1*	1*	0.10	−0.09	0.10	0.40	0.78	0.67
R_v^2	−1*	−1*	−1*	1*	1*	1*	0.10	−0.09	0.10	0.40	0.78	0.67
Z_{max}^3	−0.10	−0.10	−0.10	0.10	0.10	0.10	1*	0.99*	1*	0.80	0.09	0.48
Z_e^3	0.01	0.01	0.01	−0.01	−0.01	−0.01	0.99*	1*	0.99*	0.80	0.02	0.43
R_v^3	−0.10	−0.10	−0.10	0.10	0.10	0.10	1*	0.99*	1*	0.80	0.08	0.48
Q_{max}	−0.40	−0.40	−0.40	0.40	0.40	0.40	0.80	0.80	0.80	1*	0.58	0.85*
T	−0.78	−0.78	−0.78	0.78	0.78	0.78	0.09	0.02	0.08	0.58	1*	0.90*
W	−0.67	−0.67	−0.67	0.67	0.67	0.67	0.48	0.43	0.48	0.85*	0.90*	1*

*Represents the correlation coefficient at 95% confidence level.

reservoir flood control operation. However, they ignored correlation between criteria.

In order to correct the bias produced by the evaluation result under the correlated criteria, we use the PCA method to eliminate the correlation that exists in the original criterion system. According to the principle that the contribution ratio of accumulative variance should be greater than 90%, we obtain two principal components, the first principal component Y_1 and the second principal component Y_2. The contribution ratio of accumulative variance reaches 96.7%, indicating that the two principal components preserve most of the information included in the original criterion system. As shown in Table 5, we obtain the loading matrix by varimax orthogonal rotation, and identify the meaning of the two principal components. Table 5 shows that the first principal component is highly correlated to the criteria that are used to assess the status of Shuangjiangkou reservoir (i.e., Z_{max}^1, Z_e^1, R_v^1), the criteria that are used to assess the status of Houziyan reservoir (i.e., Z_{max}^2, Z_e^2, R_v^2), and the criteria that are

Table 5 │ Loading matrix of the principal components

Principal component no.	Z_{max}^1	Z_e^1	R_v^1	Z_{max}^2	Z_e^2	R_v^2	Z_{max}^3	Z_e^3	R_v^3	Q_{max}	T	W
Y_1	−0.99	−0.99	−0.99	0.99	0.99	0.99	0.02	−0.08	0.02	0.39	0.85	0.71
Y_2	−0.05	−0.05	−0.05	0.05	0.05	0.05	0.98	0.99	0.98	0.87	0.16	0.55

used to assess the status of Leshan city (i.e., T, W); the second principal component is highly correlated to the criteria that are used to assess the status of Pubugou reservoir (i.e., Z_{max}^3, Z_e^3, R_v^3) and Q_{max}.

Table 6 lists the values of the six alternatives regarding Y_1 and Y_2. The Pearson's coefficient between Y_1 and Y_2 equals to 0, which indicates that Y_1 and Y_2 are linearly independent. Therefore, the PCA method provides an effective way to deal with correlation among criteria in an MCDM process, which transforms the original criterion system into an independent comprehensive criterion system, and uses the loading matrix to explain the meaning of each comprehensive criterion.

According to Equation (11), we calculate the weight vector of the two principal components, $E = (0.528, 0.472)$. We use the TOPSIS method, fuzzy optimum method, and fuzzy matter-element method to evaluate reservoir flood control alternatives simultaneously, and compare the evaluation results of the three methods that do not apply the PCA procedures. The results shown in Table 7 indicate that: (1) when PCA procedure is not conducted, i.e., the original decision matrix with criteria correlation (shown in Table 3) serves as the input of the three

methods, the optimal alternative, the suboptimal alternative, and the worst alternative obtained by the three methods are consistent with each other, but these methods show a difference in the ranking of other alternatives (alternative one, three, and five); (2) when PCA procedure is conducted, i.e., the new decision matrix without criteria correlation (shown in Table 6) serves as the input of the three methods, the evaluation results of the three methods are consistent. This is because the repeated and interferential information exists in the original criterion system. The PCA eliminates the correlation and improves the consistency of the evaluation results.

Reservoir flood control operation involves two conflicting objectives, i.e., the safety of reservoirs and the safety of downstream protected regions. To accomplish the first objective, the reservoirs should release more water and reduce the highest water level as much as possible. In contrast, to accomplish the second objective, reservoirs should store more water and reduce peak discharge of reservoir outflow as much as possible. Consequently, reservoir operators are required to schedule outflows reasonably so that the two objectives can be balanced. As shown in Table 7, the ranking order of the six alternatives obtained by the three methods applying PCA procedures is (6, 4, 5, 3, 1, 2), and alternative six is the optimal alternative. In alternative six, the maximum flood peak reduction operation model is selected by the Shuangjiangkou reservoir and Pubugou reservoir for reservoir flood routing. To ensure the flood control safety of the reservoirs, the two reservoirs use their flood control capacity to store upstream flood according to the target terminal water level constraints. Furthermore, in order to alleviate the flood control burden of the downstream reservoirs and

Table 6 | Principal components' values of the six alternatives

Alternative no.	Y_1	Y_2
1	1.095	−0.599
2	0.855	1.374
3	0.759	−0.633
4	−0.720	−0.683
5	−0.990	1.203
6	−0.998	−0.662

Table 7 | Ranking order of the six alternatives obtained by different methods

Alternative no.	TOPSIS method				Fuzzy optimum method				Fuzzy matter-element method			
	Without PCA		With PCA		Without PCA		With PCA		Without PCA		With PCA	
	c_i	Rank	c_i	Rank	u_i	Rank	u_i	Rank	ρH_i	Rank	ρH_i	Rank
1	0.299	5	0.457	5	0.047	5	0.423	5	0.361	4	0.273	5
2	0.097	6	0.084	6	0.022	6	0.008	6	0.154	6	0.059	6
3	0.406	4	0.509	4	0.234	3	0.528	4	0.453	3	0.370	4
4	0.740	2	0.903	2	0.271	2	0.989	2	0.465	2	0.903	2
5	0.576	3	0.554	3	0.140	4	0.597	3	0.262	5	0.390	3
6	0.884	1	0.993	1	0.400	1	1.000	1	0.486	1	0.995	1

protected regions, the maximum outflows of these two reservoirs are reduced as much as possible. The criteria values listed in Table 3 indicate that the values of alternative six regarding Q_{max}, T, and W are obviously less than other alternatives. Therefore, alternative six considers the importance weights between the safety of reservoirs and downstream protected regions, and it is a relatively satisfying alternative. In addition, the discharge capacity operation model only considers the safety of current reservoir during the flood event. Although the value of current reservoir's criteria (i.e., Z_{max}, Z_e, R_v) are less than the other two flood control operation models, it is easy to cause great danger in the downstream reservoirs and protected regions. Thus, the discharge capacity operation model ignores the overall flood control benefit of the cascade reservoir system. When the regular operation model is used for reservoir flood routing in the Pubugou reservoir, the values of criteria Z_{max}^3, Z_e^3, R_v^3, Q_{max}, and W are obviously greater than other alternatives due to the large subsequent volume of storm water, which is unfavorable to the safety of reservoirs and downstream protected regions during the flood event. Consequently, the PCA-based MCDM method obtains a reasonable ranking order of the six alternatives.

It should be noted that many uncertainties (e.g., reservoir inflow forecasting uncertainty, reservoir operation delay time uncertainty, outflow uncertainty, etc.) exist in the processes of reservoir flood control operation, which may lead to the uncertainties of criteria values and then reverse the ranking order of the alternatives. In this paper, we do not consider these uncertainties, and just develop the MCDM model under a deterministic environment. Despite this lack, the study offers an efficient way to deal with the correlation among criteria for flood control operation in a multi-reservoir system. In further studies we may try to extend the MCDM model dealing with correlation among criteria to fuzzy and stochastic environments.

CONCLUSIONS

Evaluating alternatives for flood control operation in a multi-reservoir system by MCDM approaches is a significant way to help reservoir operators make a fast and desirable decision. However, the widespread criteria correlation will affect the rationality and accuracy in the MCDM process. In this paper,

we proposed an MCDM model for dealing with correlation among criteria. In consideration of the flood control safety of reservoirs and downstream protected regions, we established the hierarchical structure of the criterion system consisting of the objective layer, principle layer, and criterion layer. To eliminate the correlation among criteria and reduce dimensionality, we used the PCA method to transform the original criterion system into the independent comprehensive criterion system after conducting the correlation analysis. The comprehensive decision matrix coupled with the weight vector obtained by the improved entropy weight method served as the input to three MCDM methods (TOPSIS method, fuzzy optimum method, and fuzzy matter-element method), by which we determined the ranking order of all the feasible alternatives, and then we compared the evaluation results of the three methods that do not apply the PCA procedures. We applied the proposed model to a cascade system of reservoirs at the Daduhe River basin in China, in which six combinations of three flood control operation models are designed to generate six feasible alternatives. The results show that 12 original criteria are transformed into two independent comprehensive criteria, the dimensionality of the criterion system is reduced and the correlation among criteria is also eliminated simultaneously. The PCA-based MCDM method improves the consistency of the evaluation results, and obtains a reasonable ranking order of the feasible alternatives.

The novel aspects and major contributions of this work are summarized as follows:

1. The PCA-based MCDM model solves the correlation issue of criteria effectively by transforming the correlated criteria into independent criteria. The new independent criteria are explained by the loading matrix, which keeps the calculation process intelligible and simple.
2. This paper extends the study object from a single reservoir to a multi-reservoir system. The PCA-based MCDM model shows clear advantages in dealing with correlation among criteria and dimensionality reduction, and provides a powerful tool for MCDM of reservoir flood control operation under correlation.
3. The proposed model can be easily integrated with decision support system for reservoir flood control operation to evaluate alternatives automatically, providing decision support for decision-makers.

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