Prediction of maximum scour depth around piers with debris accumulation using EPR, MT, and GEP models
Mohammad Najafzadeh, Mohammad Rezaie Balf and Esmat Rashedi

ABSTRACT
Pier scour phenomena in the presence of debris accumulation have attracted the attention of engineers to present a precise prediction of the local scour depth. Most experimental studies of pier scour depth with debris accumulation have been performed to find an accurate formula to predict the local scour depth. However, an empirical equation with appropriate capacity of validation is not available to evaluate the local scour depth. In this way, gene-expression programming (GEP), evolutionary polynomial regression (EPR), and model tree (MT) based formulations are used to develop to predict the scour depth around bridge piers with debris effects. Laboratory data sets utilized to perform models are collected from different literature. Effective parameters on the local scour depth include geometric characterizations of bridge piers and debris, physical properties of bed sediment, and approaching flow characteristics. The efficiency of the training stages for the GEP, MT, and EPR models are investigated. Performances of the testing results for these models are compared with the traditional approaches based on regression methods. The uncertainty prediction of the MT was quantified and compared with those of existing models. Also, sensitivity analysis was performed to assign effective parameters on the scour depth prediction.

Key words | bridge piers, debris accumulation, evolutionary polynomial regression, gene-expression programming, model tree, scour depth

INTRODUCTION
Debris, commonly consisting primarily of roofing materials such as tree trunks and limbs, occasionally accumulates around bridge piers during flood events. Hence, debris accumulations can obstruct, constrict, or redirect flow through bridge openings producing flooding, damaging loads, or accelerating scour phenomena at bridge piers. The occurrence of debris during a severe flood can cause irreparable economic damage and disturbance to the community. The dimensions and geometric shape of debris accumulations vary widely, ranging from a small cluster of debris around a bridge pier to a near complete blockage of a bridge waterway opening. The geometry of debris accumulation is dependent on the characteristics of flow condition around bridge piers, geometry of bridges and channels. The effects of debris accumulation on the scour process can vary from minor flow constrictions to severe flow contraction resulting in significant bridge scour (Laursen & Toch 1956; Melville & Dongol 1992; Pagliara & Carnacina 2010, 2011a, 2011b). In spite of extensive investigations of local scour at bridge piers, previous studies of the effects of debris accumulation on the scouring process have not been fully understood. Therefore, local scour around piers with debris is considered one of the most attractive issues among several research subjects.

In recent decades, a large number of experimental and field studies have been conducted to investigate the impact of debris on the sediment scour phenomena (Laursen & Toch 1956; Melville & Dongol 1992; Braudrick et al. 1997; Diehl 1997; Bradley et al. 2005; Zevenbergen et al. 2006; Pagliara & Carnacina 2010, 2011a, 2011b).

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From these studies, a general equation including governing parameter based reported experimental observations is not available to predict the scour depth around piers with debris accumulations. Occasionally, laboratory investigations have dealt with the length of time and costs related to the provision of instrumentation tools and experimental equipment in comparison with computational approaches. Artificial intelligence (AI) approaches and numerical models were extensively applied to evaluate the type of environmental issues (Wu et al. 2009; Chau & Wu 2010; Jozsa et al. 2014; Miao et al. 2014; Xu et al. 2014; Zhang et al. 2014; Zahmatkesh et al. 2014a, 2014b, 2015; Chen et al. 2015; Gholami et al. 2015; Taormina & Chau 2015). In the case of application of the AI approaches into scour depth prediction in different conditions of flow, bed sediments, and geometry of structures, artificial neural networks (ANNs), adaptive neuro-fuzzy inference systems, support vector machine, and group method of data handling (GMDH) have been employed (Kambekar & Deo 2005; Bateni & Jeng 2007a, 2007b; Etemad-Shahidi et al. 2011; Ghazanfari-Hashemi et al. 2011; Ismail et al. 2013; Najafzadeh et al. 2014; Najafzadeh 2015). The use of AI models to characterize governing parameters on scour depth provided a precise performance compared with regression models. Besides, scour depth around bridge piers with debris accumulations has not yet been predicted using AI models.

Many AI approaches based driven models, such as evolutionary polynomial regression (EPR), gene-expression programming (GEP), and model tree (MT) models can obtain a robust equation to predict scour depth around piers with perfect physical insight of problems. These models were used to find solutions for various problems in different areas of the civil engineering field, such as: prediction of rainfall-runoff (Singh et al. 2009; Shiri & Kisi 2011; Kashid & Maity 2012; Wang et al. 2015); evaluation of reference evapotranspiration (Guven et al. 2008; El-Baroudy et al. 2010; Lopez et al. 2011; Rahimikhoob 2014; Shiri et al. 2014); estimation of longitudinal dispersion coefficient in rivers (Etemad-shahidi & Taghipour 2012; Sattar & Gharabaghi 2013); prediction of friction factor in pipes (Giustolisi & Savic 2006); characterization of fluid dynamics (Giustolisi et al. 2007); analysis of soil behavior under different load conditions (Rezania et al. 2010); analysis of earthquake-induced soil liquefaction and lateral displacement (Rezania et al. 2011); prediction of permeability and compaction of soils (Ahangar-Asr et al. 2011); and evaluation of stress-strain data buried in non-homogenous structural tests (Faramarzi et al. 2014).

In the case of scour depth evaluation around piers, several investigations were found to predict local scour depth around piers using the GEP approach, such as prediction of the scour depth around piers using field data sets (Azamathulla et al. 2010) and scour depth around vertical piers under regular waves (Guven et al. 2012). The MT approach was applied to predict the local scour depth around group piers under waves and currents (Etemad-Shahidi & Ghaemi 2011; Ghaemi et al. 2013). In the case of EPR application, Laucelli & Giustolisti (2011) employed the EPR model to predict the local scour depth downstream of grade-control structures. Outperformance of the EPR approach indicated better predictions compared to the traditional models. This technique has not been utilized yet to predict the scour depth around bridge piers.

According to the above examples, it can be noted that there are no contributions regarding pier scour phenomena with debris accumulations. In this way, in the present study, to obtain generalized equation based input–output variables for pier scour depth with debris, MT, GEP, and EPR models are developed. Performance results for the proposed model based formulations are compared with the regression models.

SCOUR AROUND PIERS WITH DEBRIS ACCUMULATION: A REVIEW

Occasionally, flow contraction due to debris accumulation can lead to an increase in bridge failure probability by accelerating the scour phenomena and extension in scour hole geometry. Several experimental and field studies have concentrated on the effects of debris accumulation on bridge pier scour (Diehl 1997; Pagliara & Carnacina 2010a). Earlier laboratory investigations in connection with debris accumulation problems and their influence on bridge scour were carried out by Laursen & Toch (1956). They performed experiments to study the effects of debris accumulation and observed that the presence of debris caused scour holes both deeper and larger in extent than those formed around a single pier.
Melville & Dongol (1992) proposed a method to determine the effective diameters to be applied in prediction of scour depth with debris accumulation. Meantime, Diehl (1997) found that one of the most important factors in bridge failures in the USA was debris accumulation. Bradley et al. (2005) studied the effect of debris impact on scour and erosion around hydraulic structures. Briaud et al. (2006) performed field studies on the accumulation with various shapes and sizes of bridge piers. Zevenbergen et al. (2006) investigated the effects of debris accumulation with inverted cone and conical shapes on bridge pier scour depth. Through their studies, comprehensive experimental data sets were reported by the National Cooperative Highway Research Program. Finally, they proposed several guidelines to evaluate local scour depth in the presence of debris for various debris sizes and flow characterizations.

Lagasse et al. (2010a, 2010b) carried out extensive experiments to discover the impact of debris on bridge pier scour. They utilized different debris with conical, square, rectangular, and triangular shapes for the tests. Their reported laboratory studies indicated that the roughness and porosity of debris do not have a significantly important effect on the pattern of scour process.

Pagliara & Carnacina (2010) conducted an experimental investigation to study the effects of roughness and porosity of debris on the scouring process around bridge piers. From their research, they found that the main variable affecting the temporal scour evolution was the flow intensity and blockage ratio. Pagliara & Carnacina (2011a) carried out experimental studies in order to understand the influences of wood debris accumulation on scour around bridge piers. Laboratory investigations were conducted with different conditions of flow and three wood debris shapes: rectangular, triangular, and cylindrical. Finally, they proposed a simple equation to evaluate the scour depth in the presence of debris accumulation, which chiefly demonstrated dependency of the scour depth with flow contraction due to debris accumulation. Pagliara & Carnacina (2011b) investigated experimentally the effects of large woody debris on the scour phenomena around piers. In their study, several pier sizes, channel widths, and bed sediment sizes were used to perform tests. They compared the results of experiments with previous research findings, and also a new explicit equation was proposed to highlight the effects of the debris accumulation on bridge pier scour in terms of relative maximum scour and pier temporal scour evolution.

Franzetti et al. (2011) designed a structure preventing the scouring process with debris accumulation on the Po River, Italy. They experimentally applied a plate to reduce the scour depth with some samples of debris. From their studies, it was found that performance of the proposed laboratory procedure cannot be considered as a practical approach at optimal status in the Po River due to the presence of several limitations.

Sok et al. (2015) performed extensive laboratory work to evaluate the effect of debris accumulated at sacrificial piles on bridge pier scour. Experiments were conducted over wide ranges of flow depths and velocities, sacrificial piles cases, debris accumulated at single pier cases, and debris accumulated at sacrificial piles cases with different geometry in terms of diameters and thickness. From their studies, it was found that the bigger diameter and thickness of debris accumulation the more deep the scour hole depth.

In the condition of single pier with debris, they concluded that local scour depth increased from 10 to 60% in comparison with isolated pier without debris. Additionally, for sacrificial piles with debris, observed data sets illustrated that local scour depth increased from 10 to 50% compared with sacrificial piles without debris.

**EFFECTIVE PARAMETERS ON THE PIER SCOUR WITH DEBRIS**

According to previous investigations’ findings, effective parameters on scour depth in the presence of debris around bridge piers are expressed as follows (e.g., Laursen & Toch 1956; Melville & Dongol 1992; Diehl 1997; Pagliara & Carnacina 2010, 2011a, 2011b):

\[
d_s = \phi(U, U_c, h, t_d, d_d, d_{50}, D, b, t)
\]

where \(d_s\) = the maximum local scour depth; \(U\) = flow velocity; \(U_c\) = critical flow velocity; \(h\) = flow depth in the unobstructed stage; \(t_d\) = submerged debris thickness; \(d_d\) = normal to flow debris diameter; \(d_{50}\) = mean grain size; \(D\) = pier diameter; \(b\) = channel width; and \(t\) = time. To perceive the effective variables on the scour depth, a
sketch of the scour process around a bridge pier in the presence of debris accumulation is shown in Figure 1. In Figure 1, \(D_e\) is equivalent bridge pier diameter (or effective diameter in the presence of debris accumulation). Melville & Dongol (1992) proposed the following equation for prediction of \(D_e\):

\[
D_e = \frac{0.52l_d d_d + (h - 0.52l_d)D}{h}
\] (2)

Based on the above equation, Lagasse et al. (2009) investigated the scour depth around piers with porous and roughened debris accumulation. They proposed the following relationship for evaluation of \(D_e\):

\[
D_e = k_{d1}t_d d_d \left( \frac{l_d}{h} \right)^{k_{d2}} + \left( h - k_{d1}t_d \right)D
\] (3)

where \(l_d\) = stream-wise debris length; \(k_{d1}\) = shape factor of debris; and \(k_{d2}\) = plunging flow intensity factor. For debris with rectangular shape, \(k_{d1} = 0.39\) and \(k_{d2} = -0.79\). Also, for triangular–conical shape, \(k_{d1} = 0.14\) and \(k_{d2} = -0.17\).

In the prediction of scour depth modeling around bridge piers, the use of dimensionless variables to develop data-driven models produced better performances than those obtained using dimensional parameters (Azamathulla et al. 2010; Etemad-Shahidi & Ghaemi 2011; Laucelli & Giustolisti 2011; Guven et al. 2012; Ghaemi et al. 2013; Najafzadeh 2015).

In this way, using Buckingham’s theorem, seven independent non-dimensional parameters were obtained as follows:

\[
\frac{d_s}{D} = f\left( \frac{D}{D_{50}}, \frac{h}{D}, \frac{D}{b}, \frac{U}{U_c}, \frac{d_d}{b}, T^*, \Delta A \right)
\] (4)

where \(T^*\) and \(\Delta A\) are the non-dimensional temporal factors in the presence of debris and blockage ratio due to rectangular and cylindrical debris accumulation, respectively. These parameters are computed as:

\[
T^* = \frac{U_h t}{A_b}
\] (5)

\[
A_b = D_h + \Delta A
\] (6)

\[
\Delta A = \frac{[(d_d - D) t_d]}{b h}
\] (7)

where \(A_b\) is the flow area blocked by debris accumulation and piers. For blockage ratio due to triangular debris accumulation, the \(t_d\) parameter in Equation (7) should be changed to \(0.5t_d\).

By definition of the debris contraction factor, \(K_d(T^*) = d_{s(T^*)}/d_{s-0(T^*)}\), where \(d_{s(T^*)}\) is defined to occur at \(T^*\) and \(d_{s-0(T^*)}\) is the maximum scour depth for experiment without debris contraction observed at the same \(T^*\). Equation (4) can be rewritten as follows:

\[
K_d(T^*) = q\left( \frac{D}{D_{50}}, \frac{h}{D}, \frac{D}{b}, \frac{U}{U_c}, \frac{d_d}{b}, T^*, \Delta A \right)
\] (8)

Therefore, Equation (8) is applied to develop the EPR, MT, and GEP models for evaluation of scour depth around bridge piers due to debris accumulation effects. Two hundred and forty-three scour data sets in the form of input-output parameters for the models’ development were collected from the experimental investigations of Melville & Dongol (1992), Lagasse et al. (2000), Franzetti et al. (2011), Pagliara & Carnacina (2012), and Sok et al. (2015). Table 1 presents the range of data sets. Of all data sets, about 75% (182 data sets) and 25% (61 data sets) are randomly selected to perform the training and testing stages, respectively. These data sets include four types of debris accumulations with different geometries.

Figure 1 | Schematic diagram of scour process around bridge pier with debris accumulation.
configurations of rectangular, triangular–conical, and circular are illustrated in Figure 2. For square debris accumulation (Figure 2(a)), it can be assumed that \( t_d \) is be equal to the \( d_d \) parameter. Additionally, Table 2 indicates many empirical equations based regressive models used to evaluate the scour depth around bridge pier with debris accumulation.

**FRAMEWORK OF MODELS**

In this section, descriptions of the GEP, MT, and EPR modeling approaches are presented. After that, developments of proposed techniques to extract the best relationships according to Equation (8) for prediction of the scour depth around bridge pier with debris accumulation are conducted.

**Development of GEP model**

Recently, a new technique called GEP was developed, which is an extension of the GP approach. The GEP is a search model that evolves computer programs in the form of mathematical expressions, decision trees, and logical expressions (Ferreria 2001, 2006; Shiri & Kisi 2011; Azamathulla & Haque 2012). In addition, the GEP model has attracted the attention of investigators for prediction of characterizations in hydraulic problems. This research represents GEP models for prediction of the scour depth around ridge piers with debris accumulation. The GEP approach is coded in the form of linear chromosomes, which are then expressed in expression trees (ETs).

In fact, ETs are sophisticated computer programming which have usually evolved to solve a practical problem, and are selected according to their fitness at solving that problem. The corresponding empirical expressions can be obtained from these tree structures. A population of the ETs will discover traits, and therefore will adapt to the particular problem they are employed to solve (Ferreria 2001, 2006; Shiri & Kisi 2011; Azamathulla & Ahmad 2012; Azamathulla & Haque 2012).

Development of the GEP approach includes five steps. The first step is to select the fitness function, \( f_o \), of an
individual program \((i)\). This item is evaluated as follows:

\[
f_i = \sum_{j=1}^{C_i} (M - |C_{i,j} - T_j|)
\]

in which \(M\), \(C_{i,j}\), and \(T_j\) are the selection range, value returned by the individual chromosome \(i\) for fitness case \(j\), and the largest value for fitness case \(j\).

In the second stage, the set of terminals \(T\) and the set of function \(F\) were selected to generate the chromosomes. In this study, the terminal includes three independent parameters in the form of:

\[
T(K_{d(T')}) = \left\{ \frac{D}{d_{50}}, \frac{h}{D}, U, \frac{d_d}{b}, T, \Delta A \right\}.
\]

To find the appropriate function set, it is necessary to peer review previous investigations of scour problems in this area. In this way, four basic operators (+, -, *, /) and basic mathematical functions (\(\sqrt{\cdot}\), power, sin, cos, exp) were applied to predict the local scour depth modeling. The third step is to configure the chromosomal architecture. The fourth step is selection of liking function. Finally, for the fifth stage, the sets of genetic operators casing variation and their rates are selected. The other details related to the architecture of the GEP modeling have been expressed in the literature (Azamathulla & Ahmad 2015; Azamathulla 2017).

In this study, characterizations of the local scour depth in the form of \(d_s = D\) are predicted using the GEP model. Furthermore, the functional set and the operational parameters applied in the proposed GEP models are presented in Table 3. For prediction of the scour depth around bridge piers with debris accumulation, the best individual in each generation includes 30 chromosomes and fitness values of 633.68. The best formulations of the GEP model for evaluation of the local scour depth with debris accumulation, as a function of \(D/d_{50}\), \(h/D\), \(D/b\), \(U/U_c\), \(d_d/b\), \(T\), and \(\Delta A\) are expressed as:

Table 2: List of empirical equations for prediction of pier scour depth with debris effects

<table>
<thead>
<tr>
<th>Reference</th>
<th>Equation</th>
<th>Equation no.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pagliara &amp; Carnacina (2011a)</td>
<td>(k_{d(T')} = 1 + 0.036\Delta A^{1.5})</td>
<td>(9)</td>
</tr>
<tr>
<td>Pagliara &amp; Carnacina (2011a)</td>
<td>(k_{d(T')} = 1 + 0.018\Delta A^{1.5})</td>
<td>(10)</td>
</tr>
<tr>
<td>Pagliara &amp; Carnacina (2011b)</td>
<td>(k_{d(T')} = \frac{1.872(h/D_e)^{0.255}D}{2.4D})</td>
<td>(11)</td>
</tr>
<tr>
<td>Pagliara &amp; Carnacina (2011b)</td>
<td>(k_{d(T')} = (D_e/D)^{0.745})</td>
<td>(12)</td>
</tr>
</tbody>
</table>
In addition, the ET of the above formulation is illustrated in Figure 3. In Figure 3, constant values illustrated in ETs are $G_1 C_3 = 1.637$, $G_1 C_4 = 12.4$, $G_2 C_5 = 0.749$, $G_2 C_1 = 9.071$, and $G_3 C_0 = 6.537$, and the actual variables are $d_0 = T^*$, $d_2 = U/U_C$, $d_3 = \Delta A$, $d_4 = D/b$, and $d_5 = D/d_{50}$.

### Development of MT model

Among the data mining techniques, methods are used to solve the problem by dividing it into several sub-problems (sub-domains) and the result is a combination of these sub-problems. Classification trees classify data records by sorting them down the tree from the root node to some leaf nodes. The difference between the better-known classification trees and the MT technique is that the latter have a numeric value rather than a class label in connection with the leaves. MT splits the entire input or parameter domain into sub-domains and a linear multivariable regression model is applied for each of them (Quinlan 1997; Rahimikhoob 2014). In this way, MT models can be applied to solve continuous class problems and obtain a structural representation of the data sets using the piecewise linear models (LM) to approximate non-linear relationships. Furthermore, this algorithm is known to be one of the most effective approaches to present meaningfully physical insight of the phenomenon. The tree-building procedure within four linear regression models and knowledge extraction from the structure for corresponding sub-domains is illustrated in Figure 4(a). Furthermore, a general tree structure of the MT approach is shown in Figure 4(b). Based on the domain-splitting criterion, various approaches such as the M5 model have been frequently utilized to generalize.

$$
 k_{d(T^*)} = \left[ \left\{ \left( \frac{12.4(D/d_{50})}{(T^*)^2(U/U_C)} \right) \times \left( \frac{-1.637(h/d)}{(D/b)} \right) + \left( \frac{(D/b)}{(D/b)} \right) \right\} \right]^{1/3}
 + \left[ \left\{ \left( \frac{-7.7996(D/d_{50})}{(D/b)^2(h/D)} \right) + \left( \frac{(h/D)}{(h/D)} \right) \right\} \right]^{1/3}
$$

(14)

### Table 3 | Parameters of the optimized GEP model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description of parameter</th>
<th>Setting of parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>Function set</td>
<td>$+, -, \times, /$, exp, power</td>
</tr>
<tr>
<td>$P_2$</td>
<td>Mutation rate</td>
<td>0.138</td>
</tr>
<tr>
<td>$P_3$</td>
<td>Inversion rate</td>
<td>0.546</td>
</tr>
<tr>
<td>$P_4$</td>
<td>One point and two point recombination rate, respectively (%)</td>
<td>0.277</td>
</tr>
<tr>
<td>$P_5$</td>
<td>Gene recombination rate</td>
<td>0.277</td>
</tr>
<tr>
<td>$P_6$</td>
<td>Gene transportation rate</td>
<td>0.277</td>
</tr>
<tr>
<td>$P_7$</td>
<td>Maximum tree depth</td>
<td>6</td>
</tr>
<tr>
<td>$P_8$</td>
<td>Number of gene</td>
<td>3</td>
</tr>
<tr>
<td>$P_9$</td>
<td>Number of chromosomes</td>
<td>30</td>
</tr>
</tbody>
</table>

In addition, the ET of the above formulation is illustrated in Figure 3. In Figure 3, constant values illustrated in ETs are $G_1 C_3 = -1.637$, $G_1 C_4 = 12.4$, $G_2 C_5 = 0.749$, $G_2 C_1 = 9.071$, and $G_3 C_0 = -6.537$, and the actual variables are $d_0 = T^*$, $d_2 = U/U_C$, $d_3 = \Delta A$, $d_4 = D/b$, and $d_5 = D/d_{50}$.

**Figure 3** | Optimal ET structures for the GEP model for prediction of the scour depth around bridge pier with debris accumulation.
the MT technique (Quinlan 1992; Wang & Witten 1997). Through the MT approach, the basic tree is first generated using the splitting criterion of the standard deviation reduction (SDR) factor:

$$SDR = sd(E) - \sum_{i} \frac{|E_i|}{|E|} sd(E_i)$$

in which $E$, $sd$, and $E_i$ are the set of examples (data records) that reach the node, the set that results from splitting the node according to the chosen attribute (parameter), and standard deviation, respectively. The M5 utilizes the $sd$ parameter as an error measure of the class values that reach a node. Testing all parameters at a node, it calculates the expected reduction in error and then selects the parameter that maximizes SDR. This process stops when the SDR becomes less than a certain percent of the standard deviation of the original data set or when only a few data records remain (Quinlan 1992; Wang & Witten 1997). Then, a linear regression model is developed for each sub-domain. Only the data in connection with the variables tested in that sub-domain are used in the regression. Other descriptions of the MT model have been presented in the literature (Rahimikhoob 2014).

In this study, mathematical formulations in the form of linear equations and corresponding rules for prediction of scour depth around bridge piers with debris accumulation are presented in Table 4. The proposed MT technique includes six input and one output parameters. MT technique was developed using five rules in the form of linear equations. A schematic diagram of tree-building of the MT approach in the form of rules for prediction of the scour depth around bridge piers with debris accumulation is illustrated in Figure 5. From the diagram, the splitting variable for providing Equations (16) and (17) (first and second LM) is $\Delta A$ and the corresponding value obtained is 6.93. In addition, $D/b$ with a value of 0.008 was considered as splitting variable for production of the third and fourth LM. In Equations (16) and (17), all input parameters except $T/C_3$ were taken into account in the prediction of the local scour depth with debris accumulation and also are considerably significant in developing the proposed LM. As well, $h/D$, $D/b$, $\Delta A$, $d_d/b$ as input variables contributed in producing Equations (18) and (19) and this indicated these parameters have more important effects in comparison with other ones. In the fifth model (Equation (20)) given by the MT approach, $h/D$, $U/U_c$, and $\Delta A$ parameters play key roles in prediction of scour depth.

Development of EPR model

EPR can be defined as a non-linear global stepwise regression that provides symbolic formulas of models. Differently from the original stepwise regression of Draper & Smith (1998), EPR is non-linear because the relationships between variables may result in non-linear functions although they are linear with respect to regression parameters. It is global since the search for optimal model structure is based on the exploration of the entire space of models by leveraging a flexible coding of the candidate mathematical expressions.

The expressions achievable by EPR are made of a number of additive terms multiplied by as many coefficients
Table 4 | General features of the proposed MT approach

<table>
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<tbody>
<tr>
<td>D/d50 &lt;= 96.429 : h/D &lt;= 5.892 :</td>
<td>ΔA &lt;= 6.93 :</td>
<td>LM1</td>
<td>6.93 :</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ΔA &gt; 6.93 :</td>
<td>LM2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>h/D &gt; 5.892 :</td>
<td>D/b &lt;= 0.008 :</td>
</tr>
<tr>
<td></td>
<td></td>
<td>D/b &gt; 0.008 :</td>
<td>LM4</td>
</tr>
<tr>
<td>D/d50 &gt; 96.429 :</td>
<td></td>
<td>LM5</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5 | Proposed MT structure for prediction of the scour depth around bridge pier with debris accumulation.

(i.e., as for polynomials) as reported in the following general expression:

\[
\hat{Y} = a_0 + \sum_{i=1}^{m} a_i \cdot (X_1)^{ES_{(j,1)}} \cdot \ldots \cdot (X_k)^{ES_{(j,k)}} \cdot f(\cdot (X_1)^{ES_{(j,k+1)}} \cdot \ldots \cdot (X_k)^{ES_{(j,2k)}})
\]

(21)

where \(m\) is the maximum number of additive terms, \(X_i\) and \(\hat{Y}\) are model input and output variables, function \(f\) is chosen by

the user and exponents of variables (i.e., \(ES_{(j,i)}\)) are selected from a set \(EX\) of candidates defined by the user (see Giustolisi & Savic 2006) for details.

The genetic algorithm is used to select the exponents \(ES_{(j,i)}\) from among the values in set \(EX\). This means that an integer coding of possible alternative exponents \(ES_{(j,i)}\) is adopted to achieve non-linear relationships. It is worth noting that, if the set of exponents contains zero and \(ES_{(j,i)} = 0\), the relevant input disappears from the final expression. Thus, although simple, structures like Equation (21) are quite versatile and flexible at reproducing patterns in data.

A key point of the EPR model development strategy is that final expressions are linear with respect to coefficient \(a_i\) so that they are estimated using classical numerical regression (e.g., least squares). The parameter estimation is solved as a linear inverse problem in order to guarantee a two-ways (i.e., unique) relationship between each model structure and its parameters (Giustolisi & Savic 2006). In terms of numerical regression strategy, EPR may produce
a non-linear mapping of data (like that achievable by ANNs (Haykin 1999)) although with few constants to estimate and using linear regression for parameters’ estimation. These features, in turn, help in avoiding over-fitting to training data especially when the data set is not large. Furthermore, prior assumptions on mathematical structures, functions (i.e., $f(\cdot)$) and number of parameters can be the user’s initial hypotheses for the automatic model construction. More details on the EPR working sequence are reported in Giustolisi & Savic (2006) and Laucelli & Giustolisi (2011).

The most significant upgrade of the initial EPR paradigm encompasses the multi-objective optimization strategy (i.e., EPR-MOGA), where accuracy of data reproduction and parsimony of model structures are simultaneously maximized (Giustolisi & Savic 2009). Maximizing the parsimony of resulting formulas is aimed at facilitating the physical meaning of final expressions and, in turn, achieving a general description of the underlying phenomenon.

The search space in EPR-MOGA is defined by the user in terms of the base structure of mathematical expressions (e.g., as in Equation (21) and type of function $f$), the maximum number of additive terms $m$, the cardinality of set $	ext{EX}$ of candidate exponents, and number of candidate explanatory variables (i.e., $k$).

The model search is performed by using the optimized multi-objective genetic algorithm (OPTIMOGA – Laucelli & Giustolisi 2011) that is based on the Pareto dominance criterion (Pareto 1896; Van Veldhuizen & Lamont 2000) to accomplish multi-objective optimization.

EPR-MOGA explores the space of $m$-term formulas using two or three from the following objectives: (a) the maximization of model accuracy, (b) the minimization of the number of model coefficients (i.e., number of additive terms), and (c) the minimization of the number of actually used model inputs (i.e., whose exponent is not 0 in the resulting model structure). Note that the last two objectives represent several measures of model parsimony. The EPR-MOGA finally obtains a set of optimal solutions (i.e., the Pareto front) that can be considered as trade-off between structural complexity and accuracy (Reed et al. 2007). Advantages of the EPR-MOGA strategy can be found in Giustolisi & Savic (2006) and Savic et al. (2006) and are documented by a number of applications in different research areas.

The EPR application starts from the functional relationship in Equation (21) for predicting the local scour depth, thus $D/d_0$, $h/D$, $D/b$, $U/U_c$, $d_d/b$, $T^*$, and $\Delta A$ parameters were considered as candidate inputs.

The range of exponents $\text{EX}$ is $[-2; -1.5; -1; -0.5; 0; 0.5; 1; 1.5; 2]$; the maximum number of polynomial terms is set to $m = 3$, without assuming a bias $a_0$, and admitting only positive coefficients, i.e., $a_i > 0$. The optimization strategy made use of the following objective functions: (i) the maximization of model accuracy and (ii) the minimization of the number of actually used model inputs (i.e., whose exponent is not 0 in the resulting model structure). Different runs were performed using different options for the definition of $f(\cdot)$ in Equation (21).

Among several models returned by EPR-MOGA-XL, the following models have been selected, as trade-off between accuracy (on the training set) and parsimony:

$$k_{d(T^*)} = 0.32808 \left( \frac{(\Delta A)/(D/b)^{0.5}}{(d_d/b)^{0.5}} \right) + 0.00091662 \left( \frac{(U/U_c)^{0.5}}{(D/b)^{0.5}} \right) \times \ln \left( \frac{(T^*)^{0.5}(U/U_c)}{(\Delta A)^{0.5}(D/d_0)^{0.5}(h/D)} \right) + 0.11332 \left( \frac{(U/U_c)^{0.5}}{(h/D)^{0.5}} \right) \ln \left( \frac{(T^*)^{1.5}(U/U_c)^{0.5}(d_d/b)^{1.5}}{(D/d_0)^{0.5}} \right)$$

(22)

The selected model contains inputs that are recursively present in all the returned models by EPR. This allows a possible identification of the most meaningful input variables among those available (Giustolisi & Savic 2009). All calculations were performed using the software package EPR-MOGA-XL, working in the MS-Excel environment (Laucelli et al. 2012).

### MEASUREMENT OF COMPUTATIONAL ERRORS FOR MODELS’ EVALUATION

To compare the efficiency of the proposed models and empirical equations for training and testing stages, several statistical error indicators listing correlation coefficient (R), the root mean squared error (RMSE), the mean absolute
error (MAE), the relative absolute error (RAE), and relative squared error (RSE), are applied as follows:

\[
R = \frac{\sum_{i=1}^{M} (O_i - \bar{O})(P_i - \bar{P})}{\sqrt{\sum_{i=1}^{M} (O_i - \bar{O})^2 \sum_{i=1}^{M} (P_i - \bar{P})^2}} 
\]

\[
RMSE = \left( \frac{\sum_{i=1}^{M} (P_i - O_i)^2}{M} \right)^{0.5} 
\]

\[
MAE = \frac{\sum_{i=1}^{M} |P_i - O_i|}{M} 
\]

\[
RAE = \frac{\sum_{i=1}^{M} |P_i - O_i|}{\sum_{i=1}^{M} |O_i - \bar{O}|} 
\]

\[
RSE = \frac{\sum_{i=1}^{M} (P_i - O_i)^2}{\sum_{i=1}^{M} (O_i - \bar{O})^2} 
\]

where \( \bar{O} \) is the mean of \( O \) (observed target), \( \bar{P} \) is the mean of \( P \) (predicted target), and \( M \) is the number of data sets sample.

**PERFORMANCES OF THE PROPOSED MODELS AND EMPIRICAL EQUATIONS**

Results of statistical error functions for both training and testing of the EPR, MT, GEP, and empirical equations are listed in Table 4. Table 4 indicates that Equation (22) given by the EPR model produced the scour depth under debris flow for the training stage with higher accuracy (\( R = 0.959 \)) and lower prediction (RMSE = 0.318 and RSE = 0.0789) compared with MT (\( R = 0.898 \), RMSE = 0.533, and RSE = 0.477) and GEP (\( R = 0.86 \), RMSE = 0.578, and RSE = 0.295) techniques. In addition, the EPR model predicted local scour depth with lower statistical error criteria in terms of MAE (0.253) and RAE (0.405) than those using MT (MAE = 0.295 and RAE = 0.477) and GEP (MAE = 0.336 and RAE = 0.512) models. Equations (16)–(20) extracted by MT model based linear equations gave a precise prediction in comparison with the GEP (Equation (22)) technique.

Figure 6 illustrates the scatter plots between the predicted and observed scour depths for the training of the EPR, MT, and GEP models. From Figure 6, for \( d_s/D < 3 \), all proposed models have good performances in prediction of the scour depth with debris accumulation, whereas the GEP model over-predicts for \( 3 < d_s/D < 6 \) in comparison with the other approaches.

In the testing stage, it can be seen that coefficient correlation obtained by Equations (16)–(20) (\( R = 0.944 \)) demonstrated relatively better prediction than those yielded by EPR (\( R = 0.909 \)) and GEP (\( R = 0.925 \)). Also, the MT approach provided more accurate results with RMSE = 0.241, MAE = 0.178, and RAE = 0.089 compared with EPR (RMSE = 0.310, MAE = 0.243, and RAE = 0.167) and GEP (RMSE = 0.286, MAE = 0.232, and RAE = 0.105) techniques. In accordance with quantitative comparisons of performances in Table 5, it can be generally said that the proposed equations given by the MT approach are the most efficient model to provide the scour depth around bridge piers with debris accumulations. The scatter plots between the predicted and observed scour depths for the testing of the proposed models are presented in Figure 7.

In this section, Equations (9) and (10) (Pagliara & Carnacina 2011a) and Equations (11) and (12) (Pagliara & Carnacina 2011b) were utilized to predict the local scour depth with debris accumulation. From Table 5, performance of empirical equations demonstrated that Equation (9) provided lower error of scour depth prediction in term of RMSE (0.513),
RSE (0.516), and MAE (0.446) in comparison with Equation (10) (RMSE = 0.554, RSE = 0.602, and MAE = 0.489), Equation (11) (RMSE = 0.773, RSE = 0.995, and MAE = 0.605), and Equation (12) (RMSE = 0.669, RSE = 0.878, and MAE = 0.579). As seen in Table 5, it should be said that the R parameters given by empirical equations have not as considerable meaningful accuracy for the scour depth prediction as the proposed techniques based formulation. The scatter plots between the predicted and observed scour depths for the empirical equations are shown in Figure 8. As seen in Figure 8, for non-dimensional scour depth lower than 1, all empirical equations indicated remarkable over-predictions. Also, Equation (10) illustrated under-prediction of the scour depth for \( 1 < \frac{d_s}{D} < 3 \).

Availability of effective parameters on the scour depth with debris accumulation plays a key role in obtaining an accurate performance. For instance, Equations (9) and (10) only have the function of \( \Delta A \) parameter and Equations (11) and (12) include those of \( h/D_e \) and \( D \).

In the present study, proposed empirical equations do not include properties of bed sediments. In fact, the mathematical

| Table 5 | Results of performances for proposed models and empirical equations |
|---------|-------------------------|-------------------|-----------------|-----------------|
| Model   | Training stage         |           |           |           |
|         | \( R \) | \( RMSE \) | \( RSE \) | \( MAE \) |
| EPR     | 0.959 | 0.318 | 0.0789 | 0.253 |
| MT      | 0.898 | 0.533 | 0.477 | 0.295 |
| GEP     | 0.86  | 0.578 | 0.295 | 0.336 |
|         | Testing stage          |           |           |           |
|         | \( R \) | \( RMSE \) | \( RSE \) | \( MAE \) |
| EPR     | 0.909 | 0.310 | 0.289 | 0.243 |
| MT      | 0.944 | 0.241 | 0.137 | 0.178 |
| GEP     | 0.925 | 0.286 | 0.136 | 0.232 |
| Equation (9) | 0.832 | 0.513 | 0.516 | 0.446 |
| Equation (10) | 0.832 | 0.554 | 0.602 | 0.489 |
| Equation (11) | 0.738 | 0.713 | 0.995 | 0.605 |
| Equation (12) | 0.650 | 0.669 | 0.878 | 0.579 |

**Figure 7** | Scatter plot of observed and predicted scour depth in live-bed condition for testing of the proposed models.

**Figure 8** | Scatter plot of observed and predicted scour depth in live-bed condition for testing of the empirical equations.
shapes of the proposed equations are more appropriate to give accurate scour depth predictions with debris effects than those empirical equations proposed by Pagliara & Carnacina (2011a, 2011b). In the case of practical engineering, there is no disguising the fact that lack of generalized capacity for the empirical methods corresponds to the restrictions of the governing parameters tested in a laboratory set-up and therefore not all the physical behaviors of the scour process were met precisely. Accurate performance of empirical equations depends on ranges of input and output parameters.

EXTERNAL VALIDATION OF THE PROPOSED MODELS

Tropsha et al. (2005) recommended new external validation criteria for checking models based on their performance with testing data subsets. At least one of the gradients of the regression line through the origin for the predicted versus observed values, or for the observed versus predicted values, should be close to 1 (Sattar 2014):

\[ K = \frac{\sum_{i=1}^{n} (T_i \times P_i)}{P_i^2} \]  

\[ K' = \frac{\sum_{i=1}^{n} (T_i \times P_i)}{T_i^2} \]  

Additionally, the coefficient of determination for the regression line through the origin should be less than 0.1:

\[ m = \frac{(R^2 - R_0^2)}{R^2} \]  

\[ n = \frac{(R^2 - R_{00}^2)}{R^2} \]

Moreover, the condition of cross validation should satisfy:

\[ R_m = R^2 \times \left( 1 - \sqrt{R^2 - R_0^2} \right) > 0.5 \]

where the squared correlation coefficients through the origin between the predicted and observed values \( R_0^2 \) and between the observed and predicted values \( R_0'^2 \) are given as:

\[ R_0^2 = 1 - \frac{\sum_{i=1}^{n} P_i^2 (1 - K)^2}{\sum_{i=1}^{n} (P_i - \bar{P})^2} \]  

\[ R_0'^2 = 1 - \frac{\sum_{i=1}^{n} T_i^2 (1 - K')^2}{\sum_{i=1}^{n} (T_i - \bar{T})^2} \]

The validation criteria and relevant performance measures of the developed models are presented in Table 6. Models will be considered valid for prediction should they satisfy some or all of the required conditions. As observed, the proposed models satisfy all of the pertained validation criteria; thus, they benefit from strong prediction power and are not random correlations.

UNCERTAINTY ANALYSIS FOR THE PROPOSED MODELS PREDICTION

In this section, a quantitative assessment of the uncertainties in the prediction of the scour depth around piers with debris accumulation is presented using the EPR, MT, and GEP models. The uncertainty analysis is employed to the data set of 432 experimental measurements used in this study, which was used to derive the proposed models. The uncertainty analysis defines the individual prediction error as \( e_j = P_j - T_j \). The calculated prediction errors for the entire data set are used to calculate the mean and standard deviation of the prediction errors as \( \bar{e} = \frac{1}{n} \sum_{i=1}^{n} e_j \) and

\[ S_e = \sqrt{\frac{\sum_{i=1}^{n} (e_j - \bar{e})^2}{n - 1}} \]

respectively. A negative mean
value demonstrates that the prediction model underestimated the observed values, and a positive value shows that the equation overestimated the observed values. Using the values of \( \hat{e} \) and \( S_e \), a confidence band can be defined around the predicted values of an error using the Wilson score method without continuity correction (Newcombe 1998; Sattar 2014); the use of \( \pm 1.96S_e \) yields an approximately 95% confidence band. The results of the uncertainty analysis, the mean prediction errors of the various models, the width of the uncertainty band, and the 95% prediction interval error are given in Table 7. The three proposed approaches have produced absolute mean prediction errors much less than those of the empirical equations. The proposed models showed approximately similar behavior, whereas empirical equations showed the opposite behavior. In the proposed models, the MT model demonstrates better behavior than the GEP and EPR models. The uncertainty band for the MT model ranged from \(-3.05\) to \(+3.03\). This range is smaller than that of the EPR and GEP models, which were \(\pm 3.14\) and \(\pm 3.08\), respectively. Similarly, the lowest 95% confidence prediction error interval was observed for the MT model. The MT model had the lowest mean prediction error and the smallest uncertainty bands in comparison with other ones.

**SENSITIVITY ANALYSIS**

To assign the importance of each input variable on the scour depth, the MT model was selected to perform a sensitivity analysis. The analysis was conducted such that, one parameter of Equation (4) was eliminated each time to evaluate the effect of that input on output. Results of the analysis demonstrated that \( D/d_{50} \) (\( R = 0.48 \), \( RMSE = 5.52 \), \( RSE = 22.86 \), \( MAE = 1.94 \), and \( RAE = 26.42 \)) is the most effective parameter on the maximum scour depth whereas \( d_d/b \) (\( R = 0.95 \), \( RMSE = 0.21 \), \( RSE = 0.11 \), \( MAE = 0.16 \), and \( RAE = 0.07 \)) has the least influence on the \( K_d(T^*) \) for the MT model, respectively. The other effective parameters on the \( K_d(T^*) \) parameter include \( h/D, T^*, \Delta A, d_d/b, \) and \( U/U_c \) which were ranked from higher to lower values, respectively. The statistical error parameters yielded from the sensitivity analysis are given in Table 8. Also, the results of sensitivity analysis indicated that \( D/d_{50} \) is the most important parameter in modeling of the maximum scour depth by the MT network. This study has proved that the MT model as an adaptive learning network can be used as a powerful soft computing tool for predicting the prediction

| Table 7 | Uncertainty estimate for \( K_d(T^*) \) in models |
|-----------------------------------|------------------|------------------|------------------|------------------|------------------|
| Models               | Mean prediction error | Width of uncertainty band | 95% prediction error interval |
| EPR                  | +0.0306             | \( \pm 3.1472 \)       | \(-0.7185 \) to \( 0.9253 \) |
| GEP                  | +0.0305             | \( \pm 3.0812 \)       | \(-4.9626 \) to \( 1.7305 \) |
| MT                   | +0.0189             | \( \pm 3.0282 \)       | \(-4.8749 \) to \( 1.3591 \) |

| Table 8 | Sensitivity analysis for independent parameters |
|-------------------|------------------|------------------|------------------|------------------|------------------|
| Input parameters                      | R         | RMSE   | RSE   | MAE   | RAE   |
| \( K_d(T^*) \) = \( f\left(\frac{D}{d_{50}}, \frac{h}{D}, \frac{D}{b}, \frac{U}{U_c}, \frac{d_d}{b}, T^*\right) \) | 0.89     | 0.36   | 0.27   | 0.25   | 0.18   |
| \( K_d(T^*) = \( f\left(\frac{D}{d_{50}}, \frac{h}{D}, \frac{D}{b}, \frac{U}{U_c}, \frac{d_d}{b}, \Delta A\right) \) | 0.78     | 0.52   | 19.23  | 0.44   | 2.32   |
| \( K_d(T^*) = \( f\left(\frac{D}{d_{50}}, \frac{h}{D}, \frac{D}{b}, \frac{U}{U_c}, T^*\right) \) | 0.95     | 0.21   | 0.11   | 0.16   | 0.07   |
| \( K_d(T^*) = \( f\left(\frac{D}{d_{50}}, \frac{h}{D}, \frac{D}{b}, \frac{U}{U_c}, \Delta A\right) \) | 0.94     | 0.26   | 0.43   | 0.21   | 0.17   |
| \( K_d(T^*) = \( f\left(\frac{D}{d_{50}}, \frac{h}{D}, \frac{U}{U_c}, \frac{d_d}{b}, T^*\right) \) | 0.93     | 0.25   | 0.24   | 0.20   | 0.12   |
| \( K_d(T^*) = \( f\left(\frac{D}{d_{50}}, \frac{D}{b}, \frac{U}{U_c}, \frac{d_d}{b}, T^*\right) \) | 0.71     | 0.52   | 0.92   | 0.35   | 0.5    |
| \( K_d(T^*) = \( f\left(\frac{h}{D}, \frac{D}{b}, \frac{U}{U_c}, \frac{d_d}{b}, T^*\right) \) | 0.48     | 5.52   | 22.86  | 1.94   | 26.42  |
of maximum scour depth around piers with debris accumulation as well as the other AI methods.

CONCLUSIONS

In this study, the EPR, MT, and GEP approaches were developed to evaluate the scour depth around bridge piers with debris accumulation. Performances of the proposed techniques for training and testing stages were carried out using experimental data sets collected from the literature. As well, empirical equations, as proposed by Pagliara & Carnasina (2010, 2011a, 2011b), were utilized to compare results with the proposed models. Regarding the EPR, MT, and GEP, to obtain the optimum functions on the basis of the best formulations, a dimensional analysis was used to extract parameters affecting the scour process around piers under debris flow conditions.

From the statistical error parameters presented in the training stage, it can be concluded that the EPR method predicted the local scour depth with more efficient performance compared with MT and GEP approaches. In addition, performance of the testing stages demonstrated that the MT method in the forms of linear equations and five rules captured the scour depth under debris flow conditions with more suitable accuracy (RMSE = 0.241 and MAE = 0.178) than EPR (RMSE = 0.3105 and MAE = 0.243) and GEP (RMSE = 0.286 and MAE = 0.232) techniques. In this study, one of the most interesting issues is that MT approaches provided better predictions of the scour depth with the simplest mathematical shape (Equations (16)–(20)) in comparison with EPR (Equation (22)) and GEP (Equation (14)) techniques.

Linear formulations given by the MT approach demonstrated that $T^*$ variable has no meaningful effects on the scour depth.

Performances of empirical equations (Equations (9)–(12)) indicated that Equation (9) provided lower error of scour depth predictions in term of RMSE, RSE, and MAE than Equations (10)–(12). Meanwhile, empirical equations exhibited considerable over-prediction of the scour depth with debris accumulation.

The use of the EPR, MT, and GEP models, as explicit equations for evaluation of the scour depth, have been proven to be more practically efficient with somewhat higher accuracy in comparison with empirical equations.

Selection criteria based on various statistical measures and on external validation measures and the output of uncertainty analyses were used to select the best MT with the highest prediction accuracy and the least uncertainty. The prediction errors and uncertainties associated with the developed MT were smaller than those associated with all of the available proposed models. $D/d_{50}$ was found to have considerable weight in all of the selected predictive models. Finally, the robustness of the developed MT predictive model was verified by sensitivity analysis. The results of MT agreed with experimental data from previous work, producing similar variation of $K_d(T^*)$ with $R$ over local scour depth without the correct impact of influencing parameters, such as $D/b$ and $U/U_c$.

As seen in the present study, controlled experimental data sets used to develop the proposed approaches are an indication of the research limitation. In terms of any improvement, proposed equations can be developed by field data sets to get results with more insight and reality. Furthermore, the GMDH model would be applied so as to predict the local scour around bridge piers with debris accumulations. Also, performance of the GMDH models can be compared with the current investigation. The proposed equations based model can be of interest for use in engineering applications as contemporary formulations based on experimental data sets with a wide range of input and output variables for scour prediction around bridge piers with debris accumulation.

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