A cost model with several hydraulic constraints for optimizing in practice a trapezoidal cross section

Kiyoumars Roushangar, Mohammad Taghi Alami, Vahid Nourani and Aida Nouri

ABSTRACT

Open channel structures are essential to infrastructure networks and expensive to manufacture. Optimizing the design of channel structures can reduce the total cost of a channel's length, including costs of lining, earthwork, and water lost through seepage and evaporation. The present research aims to present various optimization models towards the design of trapezoidal channel cross section. First, a general resistance equation was applied as a constraint. Next, a genetic algorithm (GA) was used to determine the optimal geometry of a trapezoidal channel section based on several parameters, i.e., depth, bottom width, and side slope. Eight different models were proposed and evaluated with no other constraint besides financial cost as well as with a normal depth, flow velocity, Froude number, top width, and by ignoring the cost of seepage. Numerical outcomes obtained by the GA are compared to previous studies in order to determine the most efficient model. Results from a single application indicate that the restriction of depth, velocity, and Froude number can increase the total cost, while restriction of the top width can decrease the cost of the construction. Also, the solution for various example problems incorporating different discharge values and bed slopes caused increase and decrease in cost, respectively.

Key words | hydraulic parameters, open channels, optimization, trapezoidal section

NOTATION

$A$ flow area of channel ($m^2$)
$b$ bed width of channel ($m$)
$C$ cost per unit length of canal ($/m$)
$C_e$ cost of earthwork per unit length of channel ($/m$)
$C_L$ cost of lining per unit length of channel ($/m$)
$C_w$ capitalized cost of water lost per unit length of channel ($/m$)
$c_e$ cost per unit volume of earthwork at ground level ($/m^3$)
$c_{L}$ cost per unit surface area of lining ($/m^2$)
$c_r$ increase in unit excavation cost per unit depth ($/m^3$)
$c_w$ cost per unit volume of water ($/m^3$)
$E$ evaporation discharge per unit free surface area ($m/s$)
$Fr$ Froude number
$Fs$ seepage function (dimensionless)
$g$ gravitational acceleration ($m/s^2$)
$k$ hydraulic conductivity ($m/s$)
$m$ side slope of channel (dimensionless)
$P$ flow perimetre of channel ($m$)
$p$ penalty parameter (dimensionless)
$Q$ discharge ($m^3/s$)
$R$ hydraulic radius ($m$)
$S_0$ bed slope of channel (dimensionless)
$T_w$ top width ($m$)
$V$ average velocity ($m/s$)
$y_n$ normal depth of flow in channel ($m$)
$\epsilon$ average roughness height of canal lining ($m$)
$\lambda$ length scale ($m$)
The primary factor affecting the channel design is the channel surface forming material which determines the roughness coefficient, the minimum permissible velocity to avoid deposition of silt or debris, the constrained velocity to prevent erosion of the channel surface, and the topography of the channel route which indicates how much the section is hydraulically and/or economically efficient (Chow 1973). The choice of hydraulic parameters is a vital task in the hydraulic design of channels since they are entitled to high uncertainty. Dimitriadis et al. (2016) employed extended sensitivity analysis by simultaneously varying the input discharge, longitudinal and lateral gradients and roughness coefficient.

Monadjemi (1994) and Froehlich (1994) modeled optimized channel design using a Lagrangian undetermined multiplier method. Alternatively, for non-linear, non-convex optimization problems, the compound and implicit construction of the cost function and/or constraints makes the employment of customary gradient-based techniques very difficult, so that the optimization process cannot be applied in many locally optimized processes. This has caused the wide use of heuristic approaches, e.g., genetic algorithm (GA) (Goldberg 1989), particle swarm optimization (PSO) (Kennedy & Eberhart 1995), genetic programming (GP) (Koza 1992), gene expression programming (GEP) (Azamathulla 2012), and charged system search (Kaveh & Talatahari 2010), among many others. Different optimization algorithms are applied to open channel section design problems.

The GA has been successfully utilized for optimizing the design of open channels (Jain et al. 2004; Bhattacharjya & Satish 2007), as well as irrigation scheduling with flow of water through channel networks (Nixon et al. 2000), along with other hydraulic problems (Wu & Simpson 2002; Roushangar & Koosheh 2015). The GP has been applied in different engineering optimization problems (Sharifi et al. 2011). The GEP approach has also been used to solve engineering problems by deriving a new predictive model (Azamathulla & Ahmad 2013; Azamathulla 2015). PSO also has been successfully applied for solving water resources management problems (Janga Reddy & Nagesh Kumar 2009). Also Nourani et al. (2009) optimized composite channels using ant colony optimization.

To the authors’ knowledge, despite considerable investigations into providing a protocol for minimizing cross-sectional area of channels, there are no investigations

\[ \nu \quad \text{kinematic viscosity (m}^2/\text{s)} \]
\[ \phi \quad \text{equality constraint (dimensionless)} \]
\[ \Psi \quad \text{augmented function (dimensionless)} \]
\[ $ \quad \text{monetary unit} \]
dealing with constrained hydraulic parameters as well as the role of seepage cost on optimal channel sections. Therefore, in this study, six different models of optimum design for open channels were established. The first model was evaluated with no additional constraint equation, while in the second model, the channel top width was considered as an additional constraint. The third and fourth models had additional constraints of different values of velocity and Froude numbers, respectively. The fifth model utilized additional constraint of different depth values. The sixth model was determined for no seepage cost state. Finally, the applications section shows the effect of discharge (seventh model) and longitudinal bed slopes (eighth model) on optimum design of channels. We applied all models to a real case scenario and showed that the constrained hydraulic parameters have a great influence in the design of the trapezoidal channel section.

**FORMULATION OF OPTIMAL DESIGN OF OPEN CHANNELS**

Selection of the geometric variables, e.g., side slope, bottom width, and flow depth for open channel sections varies according to the designer’s perspective. Also, the longitudinal bed slope of the channel is influenced by topography that is considered as a constant amount in seven models. One of the important objectives is minimizing the total cost of a channel section, that it has the capability of passing the channel distance safely, which must be considered. Generally, the cost per unit length of a lined open channel section is defined as the summation of three terms, namely, the depth-dependent earthwork cost, the cost of lining, and the cost of water lost as seepage and evaporation. These terms are explained in the following sections.

**Earthwork cost, lining cost, and cost of water loss**

The total cost function of the channel per unit length $C$ ($/m$) was obtained as:

$$C = C_e + C_l + C_w$$

$$= c_e A + c_r A \bar{y} + c_l P + c_{ws} y_n + c_{we} T_w$$

(1)

The earthwork cost $C_e$ (monetary unit per unit length, e.g., $$/m$$) is given as:

$$C_e = c_e A + c_r A \bar{y}$$

(2)

where $c_e =$ cost per unit volume of earthwork at ground level ($$/m^3$); $c_r =$ the additional cost per unit volume of excavation per unit depth ($$/m^4$); $A =$ flow area ($m^2$); $\bar{y} =$ depth of the centroid of the area of excavation from the ground surface ($m$).

The cost of lining $C_l$ (monetary unit per unit length, e.g., $$/m$) is expressed as:

$$C_l = c_l P$$

(3)

where $c_l =$ cost of unit lining (monetary unit per unit area of lining, e.g., $$/m^2$) and $P =$ flow perimeter ($m$). The capitalized cost of water lost $C_w$ ($$/m$$) might be expressed as:

$$C_w = c_{ws} y_n F + c_{we} T_w$$

(4)

$$c_{ws} = 3.156 \times 10^7 k_{cw} / r$$

(5)

$$c_{we} = 3.156 \times 10^7 E_{cw} / r$$

(6)

where $r =$ rate of interest ($$/$/year) and $c_w =$ cost per unit volume of water ($$/m^3$). The volumetric cost of water may differ for evaporation and seepage losses, depending upon the side effects caused by the seepage loss. $k =$ coefficient of permeability ($m/s$); $y_n =$ normal depth of flow in the channel ($m$); and $F_S =$ seepage function (dimensionless), which depends on channel geometry. $T_w =$ width of free surface ($m$); and $E =$ evaporation discharge per unit surface area ($m/s$) (Swamee *et al.* 2000). The $c_l/c_e$ and $c_w/c_r$ ratios were obtained as listed in Table 1 (Schedule of rates 1997). These ratios can be obtained for various kinds of linings, soil strata, and climatic conditions by utilizing appropriate unit rates.

The objective function of this research is taken to be the reduction of cost. The uniform flow equation is treated as a restriction and is inserted into the optimization models. A rigid boundary irrigation channel is designed by exploiting the uniform flow resistance equation. The most commonly used uniform flow resistance formula is the Manning equation.
(Chow 1973), which is suitable for rough turbulent flow and in a limited band-width of relative roughness (Christensen 1984).

Relaxing these restrictions, Swamee (1994) gave a more general resistance equation based on roughness height:

\[ V = -2.457 \sqrt{gRS_0} \ln \left( \frac{\varepsilon}{12R} + \frac{0.221 \theta}{R \sqrt{gRS_0}} \right) \]  

where \( V \) = average flow velocity (m/s); \( g \) = gravitational acceleration (m/s²); \( R \) = hydraulic radius (m); \( S_0 \) = longitudinal bed slope (dimensionless); \( \varepsilon \) = average roughness height of the channel lining (m); \( \theta \) = kinematic viscosity of water (m²/s). Utilizing the continuity equation, the discharge \( Q \) (m³/s) was obtained as:

\[ Q = AV = -2.457A \sqrt{gRS_0} \ln \left( \frac{\varepsilon}{12R} + \frac{0.221 \theta}{R \sqrt{gRS_0}} \right) \]  

Combining Equations (1) and (8) forms the general optimization algorithm for a minimum cost section of open channel. The terms of these models are all in dimensional forms. In order to facilitate the detection of the effects of variables on the models, the aforementioned equations are transformed to dimensionless forms, through defining a length scale, as follows:

\[ \lambda = \left( \frac{Q}{\sqrt{gS_0}} \right)^{0.4} \]  

The following dimensionless variables were then obtained in Table 1. The subscript * denotes the corresponding dimensionless parameters of each hydraulic parameter.

Using Equations (1) and (8), and Table 1, the problem of determining the optimal channel section formed in dimensionless form is reduced to:

Minimize \( C_\ast = A_\ast + C_r A_\ast \varepsilon + C_i P_\ast + C_{\text{avE}} \alpha_\ast T_w \)  

Subject to \( \phi_1 = 1 + 2.457A_\ast \sqrt{\lambda} \ln \left( \frac{\varepsilon \lambda}{12R_\ast} + \frac{0.221 \theta}{R_\ast^{1.5}} \right) \)  

where \( \phi = \) equality restriction function.

The restriction optimization problem (i.e., Equations (10) and (11)) was solved by minimizing the augmented function \( \Psi \) distributed by:

\[ \Psi = C_\ast + \alpha \sum_{i=1}^{l} \phi_i^{\beta} \]  

where \( \beta \) = exponent of equality constraint function (0 < \( \beta \) ≤ 2); \( \alpha \) = penalty function parameter with a high positive value; \( i \) = index representing restriction; and \( l \) = total number of restrictions imposed on a particular non-linear optimization programming (NLOP). The optimum design was applied on trapezoidal channel cross sections (Figure 1). According to Figure 1, the bottom width, flow depth, and side slope are represented by \( b \) (m), \( y_n \) (m), and \( m \), respectively. Consequently, the equations are as follows:

\[ A_\ast = (b_\ast + my_n) y_n \]  

\[ P_\ast = b_\ast + 2y_n \sqrt{1 + m^2} \]  

\[ T_{w*} = b_\ast + 2my_n \]  

\[ F_r = \frac{V}{\sqrt{gD}} \]  

\[ D = \frac{A}{T_w} \]  

Table 1 | The following are dimensionless equations

<table>
<thead>
<tr>
<th>Dimensionless variables</th>
<th>( \varepsilon_\ast / \lambda )</th>
<th>( \nu_\ast = u \alpha / Q )</th>
<th>( C_\ast = C/(c_\varepsilon \lambda^2) )</th>
<th>( C_i = C_i/(c_\varepsilon \lambda) )</th>
<th>( C_r = C_r / C_\varepsilon )</th>
<th>( C_{\text{av}} = C_{av} / c_\varepsilon \lambda )</th>
<th>( C_{\text{avE}} = C_{avE} / c_\varepsilon \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_\ast = \varepsilon / \lambda )</td>
<td>( P_\ast = P / \lambda )</td>
<td>( T_{w*} = T / \lambda )</td>
<td>( A_\ast = A / \lambda^2 )</td>
<td>( R_\ast = R / \lambda )</td>
<td>( y_n = y_n / \lambda )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
PROCEDURES FOR OPTIMAL DESIGN

Validation with using empirical equations

Analysis of the optimal channel sections for a number of input variables led to the following generalized empirical equations in clear form for trapezoidal channel section (Swamee et al. 2000).

For a given set of data, a channel can be designed by minimizing Equation (10) and subjected to constraint given by Equation (11). Swamee’s method is used to verify the results.

GA-based optimization procedure

The GA is employed to solve the formulated nonlinear models. The GA is a search technique based on the genetics concept of natural selection, which combines an artificial survival of the fittest with genetic operators abstracted from nature (Holland 1975). The important difference between GA and classical optimization search techniques is that the GA generates a population of possible solutions, whereas the classical optimization techniques lead to a single solution. An individual solution in a population of solutions is analogous to a biological chromosome. While a natural chromosome specifies genetic characteristics of a human being, an artificial chromosome in GA indicates the values of varied decision variables representing a decision or a solution. For most GAs, candidate solutions are represented by chromosomes coded using a binary number system (Goldberg 1989). The GA that employs binary strings as its chromosomes is named binary-coded GA. The binary-coded GA contains three basic operators: selection, crossover or mating, and mutation. The selection function chooses parents for the next generation based on their scaled values from the fittest scaling function. In this study, the stochastic uniform selection is used in conjunction with elitism. The stochastic uniform selection lays out a line in which each parent corresponds to a section of the line of length proportional to its expectation.

Crossover combines two individuals or parents to form a new individual or child for the next generation. The scattered crossover operator is employed in this study. Scatter creates a random binary vector. Mutation functions make small random changes in the individuals in the population which provide genetic diversity and enable the GA to search a broader space. In the present study, constraint-dependent default is used. Three main steps of GA generate the solutions (Goldberg 1989). Figure 2 shows these following steps: (1) select the individuals as parents considering the best objective value; (2) crossover the parents to form the next generation; (3) add random changes to the population (mutation). This method consists of eight models (Figure 3).

Model I

This model is determined by Equations (10) and (11), representing no additional constraint when consisting of three variables.

Model II

There are several cases where the top width must be constrained. Then, an additional restriction could be imposed on the NLOP represented by Equations (10) and (11). The optimization formulation remains the same, except the following additional restriction is imposed to restrict the top width to $T_{w}^{m}$:

$$
\phi_2 = T_{w}^{max} - T_{w} \geq 0
$$

where $\phi_2$ = additional equality constraint function which limits the total top width of the channel to $T_{w}^{max}$.

Model III

If the average velocity of the channel is more comparable to the permissible velocity of the channel, the average velocity can be constrained utilizing the following restriction:

$$
\phi_3 = a - v_{av} \geq 0
$$

where $a$ = maximum velocity and $v_{av}$ = average velocity of the channel.

For the designed section, the average flow velocity $V_{av}$ could be achieved by Equation (8). However, in order to safely convey the required discharge through a channel, it is necessary to ensure that the velocity of the channel does
Figure 2 | Flowchart of genetic algorithm.

Figure 3 | Solution procedures.
not exceed the corresponding maximum velocity which is related to the roughness coefficient of that segment.

**Model IV**

There are several cases where the flow of channel constrains the Froude number of the channel:

\[ \phi_4 = \frac{F_{\text{max}}}{C_0} - \frac{F_r}{C_2} \geq 0 \]  \hspace{1cm} (20)

where \( F_{\text{max}} \) = maximum permissible Froude number.

**Model V**

In the case of the existence of unfavorable strata, depth of channel should not exceed a certain limit. This may require a restriction on the maximum permissible flow depth. Thus, the limiting depth model could be addressed in the channel design problem by imposing Equation (21) as an additional constraint to the optimization formulation involving Equations (10) and (11):

\[ \phi_5 = \frac{y_{\text{max}}}{C_0} - \frac{y_r}{C_2} \geq 0 \]  \hspace{1cm} (21)

**Model VI**

The objective function has been investigated without cost of seepage to observe the influence of seepage on hydraulic parameters of a channel with constraint (Equation (11)):

Minimize \( C_r = A_r + CR_1A_r\theta + CL_1P_r + C_{ws}F_r\theta + C_{we}T \)

**RESULTS AND DISCUSSION**

Let us suppose that the channel should be designed to transport a discharge of 100 m\(^3\)/s on a longitudinal bed slope of 0.001. The channel passes through a stratum of typical soil, in which \( C_e = 7 \text{ m} \) and \( C_{we} = 10 \text{ m} \) (Table 2). Further, it is proposed to supply concrete lining with \( C_l = 12 \text{ m} \). The climatic condition of the channel area satisfies the \( C_{wE} = 2 \text{ m} \). For the purpose of design, it is assumed that \( g = 9.79 \text{ (m/s}\)\(^2\)), \( \theta = 1.1 \times 10^{-6} \text{ (m}^2\text{/s)} \) (water at 20 °C), and \( \varepsilon = 1 \text{ mm (concrete lining)} \) (Table 3).

Using Equation (9), the \( \lambda \) was determined to be 15.9 m, so the following parameters were identified using Equation (10): \( \varepsilon = 6.3 \times 10^{-5} \), \( \theta = 1.75 \times 10^{-7} \), \( C_l = 0.75 \), \( C_r = 2.27 \), \( C_{ws} = 0.63 \), and \( C_{we} = 0.125 \).

Table 4 compares the Swamee method (Swamee et al. 2000) with the proposed GA-based Model I of the present study (no additional constraint model). The sum total construction costs in both methods for Model I (no additional restriction) are 429.27 and 417.26, respectively. It can be noted from Table 4 that the proposed GA-based model shows better results compared to the Swamee method, with relatively less expensive values.

Table 5 represents the optimization results for different values of channel top width as an additional constraint (Model II). The table clearly shows that decreasing the channel top width leads to lower costs. Figure 4 displays the channel top width (\( T_{w*} \)) effect on dimensionless area (\( A* \)) and total cost (\( C* \)) values. From Figure 4 it is clear that the

<table>
<thead>
<tr>
<th>Types of data</th>
<th>( cl/ce ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Concrete tile lining</td>
</tr>
<tr>
<td>With LDPE Film</td>
<td>With LDPE Film</td>
</tr>
<tr>
<td>Ordinary soil</td>
<td>12.75 100 m (2)</td>
</tr>
<tr>
<td>Hard soil</td>
<td>10 100 m (5)</td>
</tr>
<tr>
<td>Impure lime nodules</td>
<td>8.9 100 m (8)</td>
</tr>
<tr>
<td>Dry shoal with shingle</td>
<td>6.56 100 m (11)</td>
</tr>
<tr>
<td>Slush and lahel</td>
<td>6.40 100 m (14)</td>
</tr>
</tbody>
</table>

LPDE – Low density polyethylene.

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channel area ($A^*$) presents a direct linear relation with $T_{w^*}$: the higher the $T_{w^*}$, the greater the $A^*$ value. In the case of $C^*$ variations, however, there is no direct linear relationship, where $C^*$ tends to yield a constant value for $T_{w^*}$ values of approximately 0.45. Compared with no additional constraint model (Model I) in Table 5, it is clear that decreasing the channel top width would decrease the total cost by 30%.

In order to safely convey the required discharge through the channel, it is necessary to make sure that the actual average velocity in the channel will not exceed the maximum permissible velocity. To analyze this effect, Model III was evaluated (in which different values of velocity are introduced as additional constraints) and the results are presented in Table 6 and Figure 5. According to this model, decreasing flow velocity leads to higher construction cost values. Again, comparing the results of Tables 4–6, it is seen that for higher velocity values, the influence of introducing velocity as an additional constraint diminishes due to the higher cost needed for constructing a specific channel for delivering high-velocity flows. Nevertheless, for velocity values smaller than 2.1 m/s, the cost increases up to 30%.

Table 3 | Modeling parameters of the trapezoidal channel

<table>
<thead>
<tr>
<th>Flow factors</th>
<th>Average roughness of height</th>
<th>Cost of terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$ (m$^3$/s)</td>
<td>$S_0$</td>
<td>$g$ (m/s$^2$)</td>
</tr>
<tr>
<td>100</td>
<td>0.001</td>
<td>9.79</td>
</tr>
</tbody>
</table>

Note: For water at 20°C.

Table 4 | Optimum results for trapezoidal channels design (Method I and II) with no additional restriction (Model I)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Method I (Swamee et al. (2000), Model I)</th>
<th>Method II (GA, Model I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$ (m)</td>
<td>5.829</td>
<td>5.159</td>
</tr>
<tr>
<td>$y$ (m)</td>
<td>3.878</td>
<td>3.784</td>
</tr>
<tr>
<td>$m$</td>
<td>0.512</td>
<td>0.540</td>
</tr>
<tr>
<td>$A$ (m$^2$)</td>
<td>30.304</td>
<td>27.253</td>
</tr>
<tr>
<td>$Fr$</td>
<td>0.6</td>
<td>0.68</td>
</tr>
<tr>
<td>$V$ (m/s)</td>
<td>3.3</td>
<td>3.66</td>
</tr>
<tr>
<td>Cost</td>
<td>429.27</td>
<td>417.26</td>
</tr>
</tbody>
</table>

Note: Cost equivalent $k = C/\lambda^2$; $C = C_k k$.

Table 5 | Optimization result for different values of top width as additional restriction (Model II)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$T_{w^*} \leq 0.45$</th>
<th>$T_{w^*} \leq 0.40$</th>
<th>$T_{w^*} \leq 0.35$</th>
<th>$T_{w^*} \leq 0.30$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$ (m)</td>
<td>4.388</td>
<td>3.752</td>
<td>3.100</td>
<td>2.575</td>
</tr>
<tr>
<td>$y$ (m)</td>
<td>4.086</td>
<td>4.324</td>
<td>4.070</td>
<td>3.625</td>
</tr>
<tr>
<td>$m$</td>
<td>0.337</td>
<td>0.300</td>
<td>0.300</td>
<td>0.300</td>
</tr>
<tr>
<td>$A$ (m$^2$)</td>
<td>23.655</td>
<td>21.895</td>
<td>17.718</td>
<td>13.374</td>
</tr>
<tr>
<td>$V$ (m/s)</td>
<td>4.222</td>
<td>4.58</td>
<td>5.636</td>
<td>7.466</td>
</tr>
<tr>
<td>Cost</td>
<td>391.855</td>
<td>381.743</td>
<td>345.338</td>
<td>298.821</td>
</tr>
</tbody>
</table>

Note: Cost equivalent $C = C_k k^2; k = C_1 \cdot \lambda^2$.

Table 6 | Optimization result for different values of velocity as additional restriction (Model III)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$V \leq 3.5$</th>
<th>$V \leq 3$</th>
<th>$V \leq 2.9$</th>
<th>$V \leq 2.5$</th>
<th>$V \leq 2.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$ (m)</td>
<td>5.399</td>
<td>5.135</td>
<td>3.434</td>
<td>8.792</td>
<td>10.478</td>
</tr>
<tr>
<td>$y$ (m)</td>
<td>3.699</td>
<td>3.084</td>
<td>3.545</td>
<td>3.434</td>
<td>3.657</td>
</tr>
<tr>
<td>$m$</td>
<td>0.632</td>
<td>1.804</td>
<td>1.766</td>
<td>0.823</td>
<td>0.690</td>
</tr>
<tr>
<td>$A$ (m$^2$)</td>
<td>28.625</td>
<td>33.291</td>
<td>34.439</td>
<td>39.949</td>
<td>47.559</td>
</tr>
<tr>
<td>$Fr$</td>
<td>0.66</td>
<td>0.67</td>
<td>0.63</td>
<td>0.48</td>
<td>0.38</td>
</tr>
<tr>
<td>Cost</td>
<td>429.194</td>
<td>505.620</td>
<td>511.181</td>
<td>537.979</td>
<td>570.086</td>
</tr>
</tbody>
</table>

Note: Cost equivalent $C = C_k k^2; k = C_1 \cdot \lambda^2$. 
because it requires greater cost and effort to deliver low-velocity (and high-normal depth, consequently) flows through the channel. Figure 5 illustrates that increasing flow velocity has a direct effect on \( \frac{V}{C_*} \), while there is an inverse relation between the depth variations vs. \( \frac{C_*}{V} \).

Table 7 sums up the optimization results of channel design for different Froude number (\( Fr \)) values. As a predecessor, it is clear that for better stability of the designed channel, the flow regime must be sub-critical, for which \( Fr < 1 \). From Figure 6, it can be seen that total area and the cost of channel construction increase with the reduction of the maximum \( Fr \) number value, due to the reduction in velocity of the channel and subsequent increase in the cross-sectional area. It is clear from the table that the cost reduction with increasing the smaller \( Fr \) number is more significant than those observed for larger \( Fr \) values increasing. The total cost of construction obtained by Model IV for \( Fr < 0.15 \) is approximately 1.93 times more than that obtained from Model I (\( Fr = 0.68 \)). In Table 7, it is clear that for \( Fr \leq 0.3 \) side slopes (\( m \)) tend to zero (\( m \to 0 \)), for which the trapezoidal cross section would transform to a rectangular cross section. Comparison between Model IV for \( Fr = 0.9 \) (Table 6) and Model I with \( Fr = 0.68 \) (no additional restriction) shows that total cost of construction decreases by approximately 21%.

Figure 7 plots the variation of \( b_* \) versus \( \frac{V}{C_*} \), with different Froude numbers around the critical flow regime (\( m = 0.5 \)). Results indicate that for near critical state condition, as \( b_* \) increases \( \frac{V}{C_*} \) decreases.

**Table 7** | Optimization result for different Froude numbers for subcritical-supercritical flow condition as additional restriction (Model IV)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( Fr \leq 0.15 )</th>
<th>( Fr \leq 0.2 )</th>
<th>( Fr \leq 0.3 )</th>
<th>( Fr \leq 0.4 )</th>
<th>( Fr \leq 0.5 )</th>
<th>( Fr \leq 0.6 )</th>
<th>( Fr \leq 0.7 )</th>
<th>( Fr \leq 0.8 )</th>
<th>( Fr \leq 0.9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b ) (m)</td>
<td>2.368</td>
<td>2.573</td>
<td>3.554</td>
<td>3.175</td>
<td>4.532</td>
<td>4.554</td>
<td>4.404</td>
<td>4.927</td>
<td>4.675</td>
</tr>
<tr>
<td>( y ) (m)</td>
<td>20.070</td>
<td>15.678</td>
<td>9.648</td>
<td>7.743</td>
<td>5.430</td>
<td>4.496</td>
<td>4.078</td>
<td>3.717</td>
<td>3.553</td>
</tr>
<tr>
<td>( m )</td>
<td>( 1.9 \times 10^{-5} )</td>
<td>( 4.44 \times 10^{-5} )</td>
<td>( 9.09 \times 10^{-5} )</td>
<td>0.121</td>
<td>0.172</td>
<td>0.408</td>
<td>0.471</td>
<td>0.309</td>
<td>0.313</td>
</tr>
<tr>
<td>( V ) (m/s)</td>
<td>2.102</td>
<td>2.477</td>
<td>2.914</td>
<td>3.138</td>
<td>3.368</td>
<td>3.479</td>
<td>3.873</td>
<td>4.425</td>
<td>4.860</td>
</tr>
<tr>
<td>( A ) (m²)</td>
<td>47.503</td>
<td>40.314</td>
<td>34.263</td>
<td>31.823</td>
<td>29.650</td>
<td>28.707</td>
<td>25.784</td>
<td>22.568</td>
<td>20.546</td>
</tr>
<tr>
<td>( P ) (m)</td>
<td>42.48</td>
<td>33.909</td>
<td>22.837</td>
<td>18.765</td>
<td>15.543</td>
<td>14.260</td>
<td>13.416</td>
<td>12.702</td>
<td>12.115</td>
</tr>
<tr>
<td>Cost</td>
<td>1224.326</td>
<td>976.2052</td>
<td>669.768</td>
<td>558.526</td>
<td>469.243</td>
<td>435.686</td>
<td>406.570</td>
<td>381.325</td>
<td>362.526</td>
</tr>
</tbody>
</table>

Note: Cost equivalent \( C = C_e k^2 \); \( k = \frac{C}{C_e} \times \lambda^2 \).
Table 8 sums up the optimization results for Model V, where \( y_{\text{max}} \) is considered a constraint in the event that unfavorable strata depth of channel should not go beyond a certain limit, because the excavation may not be economical or due to some other problem, such as the presence of shallow ground water table. This may require restriction on the maximum permissible flow depth. Therefore, the bottom width of the channel should be significantly increased to provide the necessary cross-sectional area for conveying the required flow discharge, which leads to increasing section area as well as the total cost and evaporation loss.

![Table 8](https://iwaponline.com/jh/article-pdf/19/3/456/391946/jh0190456.pdf)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>When depth is:</th>
<th>( y \leq 1.5 )</th>
<th>( y \leq 2 )</th>
<th>( y \leq 2.5 )</th>
<th>( y \leq 3 )</th>
<th>( y \leq 3.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b ) (m)</td>
<td>18.60</td>
<td>13.22</td>
<td>9.77</td>
<td>7.55</td>
<td>5.91</td>
<td></td>
</tr>
<tr>
<td>( y ) (m)</td>
<td>1.49</td>
<td>1.98</td>
<td>2.49</td>
<td>2.98</td>
<td>3.49</td>
<td></td>
</tr>
<tr>
<td>( m )</td>
<td>0.55</td>
<td>0.54</td>
<td>0.53</td>
<td>0.5</td>
<td>0.53</td>
<td></td>
</tr>
<tr>
<td>( A ) (m(^2))</td>
<td>29.05</td>
<td>28.41</td>
<td>27.72</td>
<td>25.27</td>
<td>27.25</td>
<td></td>
</tr>
<tr>
<td>Cost of evaporation ((C_{\text{we}}))</td>
<td>40.29</td>
<td>30.55</td>
<td>24.72</td>
<td>21.55</td>
<td>19.21</td>
<td></td>
</tr>
<tr>
<td>Cost</td>
<td>597.87</td>
<td>417.59</td>
<td>449.01</td>
<td>425.91</td>
<td>417.59</td>
<td></td>
</tr>
</tbody>
</table>

Note: Cost equivalent \( k = C_x \times \lambda^2 \); \( C = C_x \times \lambda^2 \times x^2 \).

Table 9 | Optimization result without cost of seepage

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model VI</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b ) (m)</td>
<td>4.770</td>
</tr>
<tr>
<td>( y ) (m)</td>
<td>4.070</td>
</tr>
<tr>
<td>( m )</td>
<td>0.505</td>
</tr>
<tr>
<td>( A ) (m(^2))</td>
<td>27.779</td>
</tr>
<tr>
<td>Cost</td>
<td>218.68</td>
</tr>
</tbody>
</table>

Applications

In this section, several examples with different discharge values and longitudinal bed slopes (Models VII and VIII), which are considered as a constant problem in the previous models, are analyzed and the optimal dimensions calculated based on the new discharges and the longitudinal bed slopes. As can be observed from Figure 8, increasing of discharge values leads to growth trend in optimal cost, cross section area, and the top width of channel; while a decreasing trend is observed in the Froude number. For example, for a discharge of 300 (m\(^3\)/s) compared to the discharge of 80 (m\(^3\)/s), the cost has been increased by approximately 79%. The depth and width of the channel for the discharge of 200 (m\(^3\)/s) are equal to 6.92 m and 4.98 m and for the
discharge of 100 (m$^3$/s) are equal to 5.159 m and 3.784 m, which represents an increment in channel dimensions. However, the amounts of side slopes are close to each other and equal to about 0.5. For $Q = 250$ (m$^3$/s), the Froude number is 0.27, which has been decreased by 60% compared to the discharge of 100 (m$^3$/s). In the next model, different longitudinal bed slopes ranging from 0 to 0.0016 are examined for discharges of 100 m$^3$/s and 250 (m$^3$/s) and the optimum size obtained. It is also seen that there is a reduction in optimal cross section area and cost of channel construction and an increased Froude number and velocity by increasing longitudinal bed slope (Tables 11 and 12). For example, for $S_0 = 0.0013$, depth, bottom width, and slope are 4.89 m, 3.60 m and 0.53, respectively. For the discharge of 100 (m$^3$/s) and $S_0 = 0.0016$ compared to $S_0 = 0.0001$, it can be seen that there is a significant decline in the cost by 45%. Comparison between the discharge of 100 (m$^3$/s) with bed slope of 0.001 (Table 2) and discharge of 250 (m$^3$/s) with bed slope of 0.0016 shows that the cost and area cross section are raised, respectively, by approximately 36% and 41%, however the Froude number has been decreased by 48%. Here, the final results are shown

![Figure 8](https://iwaponline.com/jh/article-pdf/19/3/456/391946/jh0190456.pdf)

**Figure 8** Changes in the top width (a), the Froude number (b), the cross section area (c) and the total cost of construction (d) with different discharges ($S_0 = 0.001$).

### Table 11 | Optimization result for different values of side slope as ($Q = 100$ m$^3$/s), Model VII

<table>
<thead>
<tr>
<th>$S_0$</th>
<th>Cost ($$/m)</th>
<th>m</th>
<th>Fr</th>
<th>V (m/s)</th>
<th>A (m$^2$)</th>
<th>b (m)</th>
<th>y (m)</th>
<th>T (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>702.90</td>
<td>0.55</td>
<td>0.22</td>
<td>1.48</td>
<td>67.55</td>
<td>8.27</td>
<td>5.86</td>
<td>14.77</td>
</tr>
<tr>
<td>0.0003</td>
<td>551.94</td>
<td>0.54</td>
<td>0.37</td>
<td>2.25</td>
<td>44.33</td>
<td>6.49</td>
<td>4.85</td>
<td>11.78</td>
</tr>
<tr>
<td>0.0005</td>
<td>492.10</td>
<td>0.53</td>
<td>0.47</td>
<td>2.76</td>
<td>36.13</td>
<td>5.92</td>
<td>4.36</td>
<td>10.63</td>
</tr>
<tr>
<td>0.0007</td>
<td>456.81</td>
<td>0.53</td>
<td>0.56</td>
<td>3.16</td>
<td>31.58</td>
<td>5.55</td>
<td>4.07</td>
<td>9.93</td>
</tr>
<tr>
<td>0.0009</td>
<td>432.16</td>
<td>0.53</td>
<td>0.64</td>
<td>3.50</td>
<td>28.57</td>
<td>5.29</td>
<td>3.87</td>
<td>9.45</td>
</tr>
<tr>
<td>0.0011</td>
<td>413.45</td>
<td>0.53</td>
<td>0.71</td>
<td>3.79</td>
<td>26.35</td>
<td>5.06</td>
<td>3.72</td>
<td>9.08</td>
</tr>
<tr>
<td>0.0013</td>
<td>398.67</td>
<td>0.53</td>
<td>0.77</td>
<td>4.05</td>
<td>24.65</td>
<td>4.89</td>
<td>3.60</td>
<td>8.78</td>
</tr>
<tr>
<td>0.0014</td>
<td>392.15</td>
<td>0.53</td>
<td>0.80</td>
<td>4.17</td>
<td>23.93</td>
<td>4.82</td>
<td>3.55</td>
<td>8.65</td>
</tr>
<tr>
<td>0.0016</td>
<td>381.06</td>
<td>0.54</td>
<td>0.85</td>
<td>4.40</td>
<td>22.69</td>
<td>4.69</td>
<td>3.45</td>
<td>8.43</td>
</tr>
</tbody>
</table>
CONCLUSIONS

In the present research, the effect of parameter restrictions on optimal design of trapezoidal channels was investigated. The proposed non-linear optimization formulation consists of minimizing the cost of lining, the depth-dependent unit volume earthwork, water lost by seepage, and evaporative losses of the open channel that are constrained by uniform flow conditions and the resistance equation. For the aforementioned optimization issue, six different models were proposed. These non-linear models were evaluated as including no restriction, constrained normal depth, constrained velocity of flow, constrained Froude number, and constrained top width. The optimization formulations corresponding to all of the models are investigated in the present research and solved using GA. The results indicate that for Models III and V (with constrained velocity and normal depth), the total cost of construction is high and for Models II and IV (with constrained top width and Froude number \((Fr > 0.7)\), the cost of construction in the open channels is smaller. Also, the results indicate that a model which disregards seepage in the objective function causes a considerable decrease in the cost. The obtained result suggests that seepage cost plays an important role on optimal design of open channels. Also, the different applications in the last two models for different discharge with constant bed slope (\(S_0 = 0.001\)) and varying longitudinal bed slopes with constant discharge \((Q = 100 \text{ m}^3/\text{s}, Q = 250 \text{ m}^3/\text{s})\) was evaluated. It was concluded that with increasing discharge and bed slope, cost of channel construction increases and decreases, respectively.

It is to be noted that the present study is conducted for a specified set of input values and it can be easily extended to any other combination of input design parameters. Also the proposed models for design of open channels are simpler to implement and effective for practical applications, thus it can be used for reliable design of irrigation channels.

REFERENCES


Schedule of rates 1997 Irrigation Department. Government of Uttar Pradesh, Lucknow, India.

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