Extended multi-objective firefly algorithm for hydropower energy generation
Omid Bozorg-Haddad, Irene Garousi-Nejad and Hugo A. Loáiciga

ABSTRACT
Classical methods have severe limitations (such as being trapped in local optima, and the curse of dimensionality) to solve optimization problems. Evolutionary or meta-heuristic algorithms are currently favored as the tools of choice for tackling such complex non-linear reservoir operations. This paper evaluates the performance of an extended multi-objective developed firefly algorithm (MODFA). The MODFA script code was developed using the MATLAB programming language and was applied in MATLAB to optimize hydropower generation by a three-reservoir system in Iran. The two objectives used in the present study are the maximization of the reliability of hydropower generation and the minimization of the vulnerability to generation deficits of the three-reservoir system. Optimal Paretos (OPs) obtained with the MODFA are compared with those obtained with the multi-objective genetic algorithm (MOGA) and the multi-objective firefly algorithm (MOFA) for different levels of performance thresholds (50%, 75%, and 100%). The case study results demonstrate that the MODFA is superior to the MOGA and MOFA for calculating proper OPs with distinct solutions and a wide distribution of solutions. This study’s results show that the MODFA solves multi-objective multi-reservoir operation system with the purpose of hydropower generation that are highly nonlinear that classical methods cannot solve.

Key words | firefly algorithm, hydropower generation, multi-objective optimization, multi-reservoir operation

INTRODUCTION
Most water resources management and engineering optimization problems involve conflicting objectives that must be solved by applying multi-objective approaches (Delipetrev et al. 2015; Mortazavi-Naeini et al. 2015; Aboutalebi et al. 2016a, 2016b; Bozorg-Haddad et al. 2016; Li & Qiu 2016; Sadeghi-Tabas et al. 2016; Zhang et al. 2016). Historically, multi-objective problems were solved by classical methods, such as linear programming (LP), nonlinear programming (NLP), and dynamic programming (DP), after converting the multi-objective problem into a single-objective problem. Due to the complexities and non-linearities of many multi-objective problems, classical methods often fail to attain correct Pareto fronts (PFs), that is, the set of optimal decision variables that capture the tradeoffs between objectives. Evolutionary or metaheuristic algorithms have become the methods of choice for solving complex multi-objective optimization problems.

Numerous studies have dealt with the single-objective or multi-objective operation of reservoirs system using evolutionary or meta-heuristic algorithms. Reddy & Kumar (2006) employed the multi-objective genetic algorithm (MOGA) to generate a Pareto optimal set for a multi-objective reservoir system that serves multiple purposes including irrigation, hydropower generation, and downstream water...
quality requirements in India. Kim & Heo (2006) used MOGA to solve an optimization problem involving a multi-objective multi-reservoir system. The two conflicting objectives of the latter study were the minimization of water shortage and the maximization of storage of each reservoir. Reddy & Kumar (2007) applied a multi-objective particle swarm optimization with an efficient elitist-mutation operator to generate Pareto optimal solutions for a reservoir problem in India with the objectives of minimizing irrigation deficits, maximizing hydropower generation, and maximizing water quality. Chang & Chang (2009) applied the nondominated sorting genetic algorithm (NSGA-II) to minimize shortage indices through identification of optimal joint operating strategies for a multi-reservoir system in Northern Taiwan. Zhang et al. (2013) proposed a multi-elite guide particle swarm optimization (MGPSO) and applied it to a multi-reservoir system considering the minimization of the energy deficit while subjecting to a series of hydraulic and operational constraints. Li & Qiu (2016) proposed a multi-objective reservoir optimization model incorporating ecological adaption and applied this model (using NSGA-II algorithm) to the Three Gorges Reservoir whose operation has damaged the downstream riverine ecosystem. The objectives considered by Li & Qiu (2016) were ecological adaption, flood control, and power generation. The studies cited above reached the optimal solutions to multi-objective reservoir operation problems, yet none proposed new or modified evolutionary or meta-heuristic algorithms for solving reservoir operation problems.

The firefly algorithm (FA) is a meta-heuristic algorithm introduced by Yang (2008). The efficiency of the FA was first assessed with single-objective problems in which Yang (2009) applied the FA to solve ten multi-optimization test problems and reported a better performance of the FA than those of the GA and particle swarm optimization (PSO) algorithm in obtaining the optimal solution of various problems. Yan et al. (2012) developed an adaptive FA (AFA) to cope with large-dimensionality optimization problems. They concluded the greater accuracy of the AFA compared with those of FA, differential evolution (DE) algorithm, and PSO algorithm. Yang (2013) applied multi-objective FA (MOFA) to solve design optimization benchmarks and compared its results with those from the NSGA-II, vector evaluated GA (VEGA), multi-objective differential evolution (MODE), and the strength Pareto evolutionary algorithm (SPEA). Yang (2013) showed a better convergence rate of the MOFA compared with those of NSGA-II, VEGA, MODE, and SPEA. Silva et al. (2013) determined the optimal geometric dimensions of hydro cyclones using MOFA. Garousi-Nejad et al. (2016a) evaluated the performance of the FA to the optimal operation of reservoirs with the purpose of irrigation supply (Aydoghmoush reservoir) and hydropower production (Karoun 4 reservoir) in Iran. The results demonstrated the superior performance of the FA in terms of the convergence rate to global optima and of the variance of the results regarding global optima when compared with the results of GA. Garousi-Nejad et al. (2016b) proposed a modified firefly algorithm (MFA) for reservoir operation problems that was successfully tested with three benchmark multi-reservoir operation problems. The results of Garousi-Nejad et al. (2016a) were compared to other classic and evolutionary or meta-heuristics algorithms (such as GA), and the better performance of MFA was reported. However, the MFA cannot solve multi-objective reservoir operation problems. The present study extends the MFA to solve multi-objective reservoir operation problems.

Multi-objective optimal operation of multi-reservoir systems is complex and multi-dimensional. It is, therefore, essential to develop, extend, and re-formulate optimization algorithms to meet the demands posed by such operational problems. This paper proposes and applies an extended multi-objective developed firefly algorithm (MODFA) to the three-reservoir system composed of the Karoun 4, Kherasan 1, and Karoun 3 reservoirs in Iran. The MODFA script code is applied (using MATLAB R2012a software) to optimize the operation of these reservoirs for hydropower generation while considering two conflicting objectives: (1) maximization of the reliability and (2) minimization of the vulnerability of hydropower production.

**METHODOLOGY**

This section contains four sub-sections outlining a multi-reservoir operation model for hydropower generation, MOFA, MODFA, and the Pareto performance criteria employed for evaluating algorithmic performance.
Multi-reservoir operation model for hydropower production

The two objective functions of the multi-reservoir operation problem are maximizing the numeric reliability and minimizing the vulnerability of hydropower production expressed by Equations (1) and (2), respectively:

\[
\text{Maximize } \text{OFR}_{\text{HP}} = \sum_{r=1}^{n_{\text{Res}}} \left[ \sum_{t=1}^{T} \frac{P_r(t) \geq \text{Rate} \times \text{PPC}_r}{n_{\text{Res}} \times T} \right] \\
\text{Minimize } \text{OFV}_{\text{HP}} = \sum_{r=1}^{n_{\text{Res}}} \left[ \frac{(\text{Rate} \times \text{PPC}_r - P_r(t))}{\text{Rate} \times \text{PPC}_1 + \text{Rate} \times \text{PPC}_2 + \ldots + \text{Rate} \times \text{PPC}_{n_{\text{Res}}}} \right]
\]

in which \( \text{OFR}_{\text{HP}} \) = numeric reliability objective function of the multi-reservoir operation problem for hydropower production; \( \text{OFV}_{\text{HP}} \) = vulnerability objective function of the multi-reservoir operation problem for hydropower production; \( r \) = the reservoir index \((r = 1, 2, \ldots, n_{\text{Res}})\); \( n_{\text{Res}} \) = the total number of reservoirs; \( t \) = the period index \((t = 1, 2, \ldots, T)\); \( T \) = the total number of operational periods; \( P_r(t) \) = power generated by power plant of reservoir \( r \) during period \( t \); \( \text{PPC}_r \) = rated installed capacity of the power plant of reservoir \( r \); and \( \text{Rate} \) = performance threshold.

It is worth mentioning that in the type of reservoir operation problems herein considered it is practically impossible to achieve 100% of the hydropower generation goals. Therefore, it is imperative to define performance thresholds in order to determine approximate optimal Paretos (OPs) and to assess algorithmic performance.

Equation (1) expresses the maximization of the numeric reliability of the multi-reservoir operation problem for hydropower production. The numeric reliability is defined as the fraction of the number of the periods in which the value of the generated hydropower is equal or greater than the rated capacity of the power plant over the total number of operational periods. The reliability of the multi-reservoir system is calculated for the entire operational system. In other words, for each reservoir, the number of periods at which the value of the generated hydropower is equal or greater than the rated capacity of the power plant is calculated and the summation of all of these periods for all of the reservoirs is computed as shown in the numerator of Equation (1). The latter value is divided by the total number of reservoirs multiplied by the total number of operational period shown in the denominator of Equation (1). According to Equation (1), a system is 100% reliable when it meets the rated capacity of the power plant in every period for all of its reservoirs. Additionally, a system has 0% reliability when none of its reservoir meets the rated capacity of power plant in any period.

Equation (2) calculates the reservoir system’s power production deficit and divides it by the reservoir system’s production capacity. This is the term within parentheses on the right-hand side of Equation (2). The maximum normalized power-production deficit over the entire operational period is defined as the system’s vulnerability. The value of vulnerability ranges between 0 and 1. A system that generates the rated capacity in all periods at all reservoirs would equal 100%.

The storage in the reservoirs, the evaporative loss of water from reservoirs, the water spilled from reservoirs, and the power generated by the power plants in each reservoir are computed using Equations (3)-(6), respectively.

\[
S_r(t + 1) = S_r(t) + Q_r(t) - \text{Loss}_r(t) + \text{M}_{n_{\text{Res}}n_{\text{Res}}} \text{Re}_r(t) \\
- \text{M}_{n_{\text{Res}}n_{\text{Res}}} \text{Sp}_r(t)
\]

\[
\text{Loss}_r(t) = A_r(t) \times \text{Ev}_r(t)
\]

\[
\text{Sp}_r(t) = \left\{ \begin{array}{ll}
S_r(t + 1) - \text{Smax}_r(t) & \text{if } S_r(t + 1) > \text{Smax}_r(t) \\
0 & \text{if } S_r(t + 1) \leq \text{Smax}_r(t)
\end{array} \right.
\]

\[
P_r(t) = \frac{\gamma \times \eta_r \times \Delta H_r(t) \times \text{DisRe}_r(t)}{10^6 \times \eta_r(t)}
\]

in which:

\[
\Delta H_r(t) = \left( \frac{H_r(t + 1) + H_r(t)}{2} \right) - TR_r(t)
\]

\[
\text{DisRe}_r(t) = \frac{\text{Re}_r(t)}{\text{CF}_r(t)}
\]

\[
\text{CF}_r(t) = \frac{24 \times 3,600}{1,000,000} \text{ day}(t)
\]
$S_r(t + 1) = $ the storage of reservoir $r$ at the beginning of period $t + 1$; $S_r(t) = $ the storage of reservoir $r$ at the beginning of period $t$; $Q_r(t) = $ the reservoir inflow to reservoir $r$ during period $t$; $\text{Loss}_r(t) = $ the loss volume of water from reservoir $r$ during period $t$; $M_{nRes \times Res} = nRes$-order matrix of indexes of reservoir hydraulic connections with $-1$ along the main diagonal, $+1$ if a reservoir releases into a downstream reservoir, and zeros elsewhere (more detail about the representation of the relationships between reservoirs in a multi-reservoir systems in terms of a matrix is provided in the study of Garousi-Nejad et al. 2016b); $\text{Dis}_r(t) = $ the release of reservoir $r$ during period $t$; $\text{Sp}_r(t) = $ volume of spilled water during period $t$; $A_r(t) = $ the area of reservoir $r$ at the beginning of period $t$ which is a function of $S_r(t)$; $\text{Ev}_r(t) = $ the depth of evaporation from reservoir $r$ during the period $t$; $\text{Smax}_r = $ the maximum allowable storage of reservoir $r$ during period $t$; $\gamma = $ the specific weight water; $\eta_r = $ the efficiency of the power plant of reservoir $r$; $\Delta H_r = $ the difference between the average level of water surface of reservoir $r$ at the beginning and at the end of period $t$ and the tailwater level of reservoir $r$, which is a function of $\text{Dis}_r(t)$; $\text{Dis}_r(t) = $ the discharge of the water from the power plant of reservoir $r$ during period $t$; $n_r = $ the performance coefficient of the power plant of reservoir $r$ during period $t$; $H_r(t + 1) = $ the level of the water surface in the $r$-th reservoir at the end of period $t$; $H_r(t) = $ the level of the water surface in the $r$-th reservoir at the beginning of period $t$; $\text{Tr}_r(t) = $ the tailwater level of reservoir $r$ during period $t$; $\text{CF}_r(t) = $ the conversion factor from million cubic meters per day to cubic meters per second in reservoir $r$ during period $t$; and $\text{day}(t) = $ the number of days in the operation period $t$.

Figure 1 shows that the volume ($\text{Re}_r(t)$) of water entering the power plant of reservoir $r$ passing through the penstock has two components: (1) $\text{RPH}_r(t)$ and (2) $\text{SpPH}_r(t)$. The former is the water volume used to generate electricity and the latter is the bypass, or the water volume that plays no role in the generation of electricity. When dealing with multi-reservoir operation systems, it is required to treat both components as decision variables. This is so because during the periods when a reservoir is full (of water), some part of the water is released to enhance hydropower generation in downstream reservoirs. In fact, when both water components are treated as decision variables, the system can reach higher reliability and less vulnerability since the downstream reservoirs benefit from the water released from upstream reservoirs (even if the release water is not used to generate power in the upstream reservoir). In such systems, the volume $\text{SpPH}_r(t)$ is calculated as follows:

\[
\begin{align*}
&\text{if } P_r(t) > \text{Rate} \times \text{PPC}_r, \quad \text{DisSpPH}_r(t) = \frac{[\text{Rate} \times \text{PPC}_r - P_r(t)] \times 10^6 \times n_r(t)}{\gamma \times \eta_r \times \Delta H_r(t)} \\
&\text{Otherwise} \quad \text{SpPH}_r(t) = \text{DisSpPH}_r(t) \times \text{CF}_r(t) \\
\end{align*}
\]

in which $\text{DisSpPH}_r(t)$ and $\text{SpPH}_r(t)$ are the discharge and the volume of the water that has no role in the generation of electricity in reservoir $r$ during period $t$, respectively.

The constraints on reservoir storages, reservoir releases, and generated hydropower are expressed as follows, respectively:

\[
\begin{align*}
&S_{\text{min}}(t) \leq S_r(t) \leq S_{\text{max}}(t) \\
&\text{Remax}_r(t) \leq \text{Re}_r(t) \leq \text{Remax}_r(t) \\
&0 \leq P_r(t) \leq \text{PPC}_r
\end{align*}
\]

in which $S_{\text{min}}(t)$ = the minimum allowable reservoir storage of reservoir $r$ during period $t$; $\text{Remin}_r(t)$ = the minimum allowable reservoir release of reservoir $r$ during period $t$; and $\text{Remax}_r(t)$ = the maximum allowable reservoir release of reservoir $r$ during period $t$.

This study deals with the deterministic operation reservoir systems. Thus, stochastic processes (say, river inflow) fall outside the scope of the present work, although it is a subject worthy of research in future studies by these authors.

**The multi-objective firefly algorithm**

Classical multi-objective methods, such as the weighted sum, goal programming, goal attainment, and $\epsilon$-constraint
combine all objectives into a single objective, this being one of their key limitations. Evolutionary or meta-heuristic algorithms, on the other hand, do not reduce multi-objective problems to single-objective ones. There are different approaches employed by researchers to extend evolutionary or meta-heuristic algorithms to solve multi-objective problems, among which the elitist non-dominated sorting genetic algorithm (NSGA-II) approach was introduced by Deb (2001). The NSGA-II is applied in this study because it has been successfully applied in solving multi-objective problems (Srinivas & Deb 1994; Li 2003; Tsai et al. 2014; Zhang et al. 2015). Multi-objective problems do not calculate single optima as single-objective optimization does. Instead, a set of non-dominated solutions, known as the PF, is obtained. The solutions in a PF have a superior ranking and crowding distance compared to other solutions that do not form part of the PF. The multi-objective version of the FA, or MOFA, is summarized next.

The FA and the MOFA are meta-heuristic methods inspired by the behavior of fireflies. The unisex fireflies with lower light intensity (attractiveness) move toward those with higher light intensity. The attractiveness of a firefly is determined by its brightness, which in turn, is associated with the encoded objective function. Equation (14) expresses the attractiveness of a firefly:

$$\beta(p_{ij}) = \beta_0 e^{-r p^2_{ij}}$$

in which $\beta(p_{ij}) =$ firefly's attractiveness; $\beta_0 =$ the attractiveness at a distance of $p_{ij} = 0; r =$ the coefficient of light absorption; and $p_{ij} =$ the distance between a pair of fireflies $i$ and $j$.

Equation (15) expresses the distance $p_{ij}$ for a pair of fireflies $i$ and $j$ at positions $x_i$ and $x_j$, respectively:

$$p_{ij} = \|x_i - x_j\| = \sqrt{\sum_{d=1}^{TD} (x_{i,d} - x_{j,d})^2}$$

in which $\| \|$ = distance vector between a pair of fireflies $i$ and $j$ at the space; $d =$ the index of the dimensions (decision variables); $TD =$ the total number of the dimensions (decision variables); $x_{i,d} =$ $d$-th dimension of the spatial coordinate of the firefly $i$'s position; and $x_{j,d} =$ $d$-th dimension of the spatial coordinate of the firefly $j$'s position.
Equation (16) determines the movement of firefly $i$ towards firefly $j$, the latter being more attractive (that is, brighter):

$$x_{\text{new}i} = x_i + \beta_0 e^{-\gamma |\beta_0|} \left( \frac{x_j}{C_0} - x_i \right) + \alpha \left( \text{rand} - 0.5 \right)$$  \hspace{1cm} (16)

in which $x_{\text{new}i}$ and $x_i$ are the new position and current position of firefly $i$ with the lower brightness, respectively; $x_j$ is the position of firefly $j$ with more brightness; $\alpha$ is a randomized parameter; and rand is a random value in the range $[0, 1]$. In Equation (16) the second and the third terms on its right-hand side play the role of modification of movement and random movement, respectively.

According to Yang’s (2009) recommendation, the value of $\gamma$ is theoretically in the range $[0, \infty]$. However, it practically varies from 0.1 to 10. Furthermore, Yang (2009) pointed out that for most implementations the value of $\beta_0$ can be considered equal to 1. Moreover, according to Yang (2009), the range of values of $\alpha$ is $[0, 1]$.

Figure 2 summarizes the steps of the MOFA. The process starts with the definition of the objective function and the parameters values’ determination. The initial population of fireflies is randomly generated (Garousi-Nejad et al., 2016a, 2016b) and the objective function (evaluation of brightness) is determined for all the fireflies. A random weight vector...
is generated (with the sum of its components equal to 1) in accordance with Yang’s (2013) recommendation with which to compare each pair of fireflies in terms of their brightness. The best non-dominated fireflies are selected based on ranking and crowding distance criteria and are passed to the next iteration. The approximated PF is reported as the final solution at the end of the number of algorithmic iterations specified as the stopping criteria.

**Extended multi-objective developed firefly algorithm**

Multi-objective optimal operation of multi-reservoir systems is complex and multi-dimensional. This type of optimization requires reformulation (or extension) of the parent evolutionary algorithms to cope with specialized conditions and solve efficiently the targeted multi-objective, multi-reservoir, operation problems. The authors propose an extended MOFA, namely MODFA, that is particularly suitable for multi-objective, multi-reservoir, operation problems. First, the steps of the extended FA (namely, DAF as proposed by Garousi-Nejad et al. 2016b) are summarized, followed by the description of MODFA applying the NSGA-II approach.

- **Step 1**: The second and the third terms on the right-hand side of Equation (16) play the roles of modification of position and of random movement, respectively. Yang (2009) suggested that the values of $\beta_0$ and $\gamma$ be set equal to 1 and in the range $[0, \infty)$, respectively. However, Garousi-Nejad et al. (2016b) proposed that these two parameters introduce a trade-off. In other words, the proper values of $\beta_0$ and $\gamma$ are such that they cause the value of $\beta(y, y_i)$ in Equation (14) to vary between 0 and 1. Suitable values for $\beta_0$ and $\gamma$ are found in Table 1 of the work by Garousi-Nejad et al. (2016b).

- **Step 2**: This step increases the conditions under which solutions are modified. It is seen in the flowchart of the MOFA shown in Figure 2 that if firefly $i$ is not worse than firefly $j$ in terms of attractiveness (i.e., objective function value), no modification is effected on firefly $i$’s position. The MODFA, however, modifies firefly $i$’s position through a random walk even if firefly $i$ is not worse than firefly $j$ in terms of attractiveness. This step produces more new positions of fireflies in each iteration.

- **Step 3**: MOFA implements a random walk by means of the third term on the right-hand side of Equation (16) in which a value in the range $[0, 1]$ is applied on all decision variables of a firefly. This type of random walk is not suitable in reservoir operation problems because the decision variables are mostly large values. This calls for random walks that are larger than the range $[0, 1]$. Thus, contrary to MOFA in which the same random walk is effected on all decision variables, the MODFA first categorizes the variables randomly into groups (specified by the user) and each group is assigned a separate range. More details can be found in Garousi-Nejad et al. (2016b).

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**Table 1** Data for three reservoirs and their power plants

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Karoun 4</th>
<th>Kherasan 1</th>
<th>Karoun 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purpose of dam</td>
<td>Hydropower generation</td>
<td>Hydropower generation</td>
<td>Hydropower generation</td>
</tr>
<tr>
<td>Maximum reservoir level (masl)</td>
<td>1.025</td>
<td>1.013</td>
<td>840</td>
</tr>
<tr>
<td>Minimum reservoir level (masl)</td>
<td>990</td>
<td>1.000</td>
<td>800</td>
</tr>
<tr>
<td>Maximum reservoir volume ($10^6$ m$^3$)</td>
<td>2,019</td>
<td>352.55</td>
<td>2,252.58</td>
</tr>
<tr>
<td>Minimum reservoir volume ($10^6$ m$^3$)</td>
<td>1,144.29</td>
<td>262.68</td>
<td>1,110.12</td>
</tr>
<tr>
<td>Active reservoir volume ($10^6$ m$^3$)</td>
<td>748.71</td>
<td>69.87</td>
<td>1,142.66</td>
</tr>
<tr>
<td>Maximum reservoir release ($10^6$ m$^3$)</td>
<td>450</td>
<td>400</td>
<td>1,000</td>
</tr>
<tr>
<td>Power plant capacity ($10^6$ W)</td>
<td>1,000</td>
<td>584</td>
<td>2,000</td>
</tr>
<tr>
<td>Performance coefficient (%)</td>
<td>20</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Power plant efficiency (%)</td>
<td>88</td>
<td>93</td>
<td>92</td>
</tr>
</tbody>
</table>
et al. (2016b). Consequently, small and large random values are produced. It is noted that in the MODFA large values are damped and reduced during the execution of the algorithm. This modification in algorithmic Step 3 creates more solutions in the decision space in the first iterations.

- Step 4: According to Yang (2013), \( \psi_j \) defined in Equation (15) is not limited to the Euclidean distance. In fact, any mathematical statement that can effectively characterize the quantities of interests in the optimization problems can be used as the distance depending on the type of the problem of interest. The term \( e^{-\gamma \delta} \) becomes nearly zero due to the large values of decision variables in reservoir problems (regardless of the value of \( \gamma \)), and it causes the effect of the modification term to vanish. Therefore, Equation (17) is defined instead as the distance between fireflies in the MODFA:

\[
\psi_j = |OF_j - OF_i|
\]

in which \( OF_i = \) and \( OF_j \) the values of objective functions of firefly \( i \) and firefly \( j \), respectively.

- Step 5: The MODFA selects the last sorted solutions of each iteration that are transferred as the best solutions to the next iteration. In other words, solutions that are the same in terms of the decision variables’ values (and not in terms of the same objective functions’ values) are chosen once. Unlike the MOFA, this prevents quick attenuation.

- Step 6: There are also specific recommendations for selecting MODFA parameters in solving reservoir operation problems. The value of \( \beta_0 \) in MODFA is the same as recommended by Yang (2009) in the MOFA. The MODFA uses a range of \( \gamma \) equal to \([1, 5]\). The reason for choosing this range is that these values do not cause the value of \( \beta(\psi_j) \) to be greater than 1. Therefore, according to algorithmic Step 1, the modification term is not combined with the random walk. Garousi-Nejad et al. (2016b) provide a detail analysis of the proper values of \( \gamma \). The complete flowchart of the MODFA is depicted in Figure 3, in which the reformulated (or extended) steps are shown with light gray rectangular shapes and other steps are the same as those of the MOFA.

Pareto performance criteria

It is common practice in multi-objective optimization problems to compare the performance of different multi-objective algorithms. The task of performing accurate performance comparison between different multi-objective algorithms is achieved with several criteria. The criteria chosen in the present study are spacing (\( \delta \)) and hypervolume (\( HV \)). The reason for choosing \( \delta \) is that it can measure the extent of the OP. However, it cannot evaluate the distribution of the points of the OP. Therefore, \( HV \) is also used. The reason for choosing \( HV \) is that this performance criterion has generated increasing interest in recent years (Knowles et al. 2003; Zitzler & Künzli 2004; Emmerich et al. 2005; Beume et al. 2007; Zitzler et al. 2008; Bader & Zitzler 2011). Its success and popularity are due to the fact that it simultaneously accounts for proximity and diversity and is strictly Pareto compliant (Cao et al. 2015). The following addresses the formulation of \( \delta \) and \( HV \).

Spacing (\( \delta \))

Schott (1995) introduced \( \delta \), which is a relative distance measure between consecutive solutions of OP, as follows (Deb et al. 2002):

\[
\delta = \sqrt{\frac{1}{|OP|} \sum_{g=1}^{|OP|} (ds_g - \bar{ds}_g)^2}
\]

in which \(|OP| = \) the total number of OP points; \( g = \) the numerator of the OP points; \( ds_g = \) the shortest (lowest) computed distance of point \( g \) from other OP points; and \( \bar{ds}_g = \) the mean value of \( ds_g \). The terms \( ds_g \) and \( \bar{ds}_g \) are calculated by Equations (19) and (20), respectively:

\[
d_{sg} = \min_{gg=1}^{OP} \sum_{m=1}^{Nof} |f_{m_{OP}}(g_{gg}) - f_{m_{OP}}(g_{g})| \quad gg \in OP \text{ and } gg \neq g
\]

\[
\bar{ds}_g = \frac{\sum_{g=1}^{|OP|} ds_g}{|OP|}
\]

in which \( gg = \) the numerator of the OP points for which \( gg \neq g \); \( m = \) the numerator of the objective functions; \( nOF = \) the total number of objective functions; \( f_{m_{OP}} = \) the
objective function \( m \) associated with the solution (point) \( g \) of OP; and \( f_{m,op}^{(g)} \) = the objective function \( m \) associated with the solution (point) \( gg \) of OP. \( \delta \) measures the standard deviations of different \( d_{Sg} \). The value of \( \delta \) is lower if the OP points are distributed uniformly. In other words, the lower \( \delta \), the better the performance of the algorithm. Thus, the calculated OP is preferable.

Hyper volume

This criterion calculates the volume in the objective space covered by points (members) of the OPs and a reference point \( (W_{Ref}) \). The \( HV \) takes into account the closeness of the OP solutions to the PF set as well as the diversity of solutions across the OP. In the case of the two objective functions shown in Figure 4 \( HV \) is calculated with Equation (21):

\[
HV = \sum_{g=1}^{[OP]-1} ((W_{Ref}^{(2)} - f_2(g+1)) \times (f_1(g+1) - f_1(g)))
\]

in which \( W_{Ref}^{(2)} \) = the value of the second objective function at a reference point; \( f_2(g+1) \) = the value of the second objective function at point \( g + 1 \); and \( f_1(g) \) = the
value of the first objective function at point $g$. The performance of a MODFA algorithm improves with increasing $HV$.

## CASE STUDY

The MODFA was applied to a three-reservoir system that included the Karoun 4, Khersan 1, and Karoun 3 reservoirs in Karoun basin in Iran. Figure 5 depicts the schematic of these three reservoirs. Table 1 shows data for the three reservoirs and the corresponding power plants. A diagram of the monthly inflow volume along with monthly evaporation depth for a 10-year period (1957–1966) is depicted in Figure 6, according to which the maximum inflow to the Karoun 4, Khersan 1, and Karoun 3 are 3,010.8, 1,694.3, and 450.8 $10^6$ m$^3$ per month, respectively (Jahandideh-Tehrani et al. 2003).

Second-order relationships between area and storage (A-S), and water elevation and storage (H-S), and discharge and the elevation of the tailwater (DisRe-TR) for the Karoun 4, Khersan 1, and Karoun 3 reservoirs are used.

Three main approaches that can be used to satisfy the constraints in optimization problems are (1) applying penalty functions, (2) eliminate infeasible solutions and generate new ones, and (3) replace the value of the violated variables with other values. Penalty functions are introduced in typical, constrained, evolutionary optimization problems to penalize constraint violations (see the constraints in Equations (11)–(13)). Equations (22)–(25) present the penalty functions used in this study:

$$P_r(t) = \begin{cases} 0 & \text{if } S_r(t + 1) > S_{\text{max}}(t) \\ \frac{S_{\text{min}}(t) - S_r(t + 1)}{S_{\text{min}}(t)} & \text{Otherwise} \end{cases}$$

in which $P_r(t) =$ penalty for violation of the minimum allowable storage of reservoir $r$ during period $t$; $P_r(t) =$ penalty for violation of the minimum allowable power generation of reservoir $r$ during period $t$; $P_r(t) =$ penalty for the violation of the maximum allowable release of reservoir $r$ during period $t$; and $P_r(t) =$ penalty for the violation of the minimum allowable release of reservoir $r$ during period $t$.

Besides penalty functions, this study applies Equations (26) and (27) that represent the replacement approach to deal with constraints (Choi & Kim 2005). The replacement approach replaces the values of the violated variables with other values. Equations (26) and (27) replace $S_r(t + 1)$ and $P_r(t)$ with $S_{\text{max}}(t)$ and $P_{\text{PCR}}$, respectively, whenever $S_{\text{max}}(t)$ and $P_{\text{PCR}}$ are violated by $S_r(t + 1)$ and $P_r(t)$, respectively:

$$\text{if } S_r(t + 1) > S_{\text{max}}(t) \rightarrow \{ S_r(t + 1) = S_{\text{max}}(t) \}$$

$$\text{if } P_r(t) > P_{\text{PCR}} \rightarrow \{ P_r(t) = \text{Rate} \times P_{\text{PCR}} \}$$

The NLP method did not find a global optimal solution of the two-objective three-reservoir problem introduced in this study due to the complexities and non-linearity of the objective functions, non-linearity of the constraints, multiplicity of simulation relations, and high computational burden. The reservoir operation problem was solved as two separate single-objective problems (considering each objective separately), and was solved as a two-objective problem with the MOGA, MOFA, and MODFA.
RESULTS AND DISCUSSION

Two single-objective problems (one for each of the two considered objective functions) were solved with the DFA to evaluate the calculated OPs. The results from single-objective problems provide the beginning and end points (that is, the two extremal points) of OPs that are valuable in assessing whether or not a calculated OP is correct. In other words, the more the solutions of the calculated OPs are uniformly distributed between the solutions from single-objective results, the better the performance of the algorithm is. Hence, this section is divided into three sub-sections. The first sub-section provides the results of the single-objective three-reservoir operation considering \( OFR_{HP} \) as the objective function. The final sub-section reports the results of the optimal two-objective three-reservoir operation problem.

**The optimal single-objective three-reservoir operation problem considering \( OFR_{HP} \) as the objective function**

The DFA was implemented in this problem with 100 fireflies, 15,000 iterations, 10 groups for categorizing the decision variables, \( \beta_0 = 1 \) and \( \gamma = 3 \). The values of these parameters were chosen after a primary trial and error process. Table 2 lists the values of \( OFR_{HP} \) and the corresponding \( OFV_{HP} \) calculated after 1,500,000 functional evaluations using the obtained decision variables in the formulation of \( OFV_{HP} \) for different values of \( Rate \). It is found that as the value of \( Rate \) decreases the values of \( OFR_{HP} \) and \( OFV_{HP} \)
Figure 6 | Diagram of monthly inflow and monthly evaporation depth for a 10-year period (1957–1966) of (a) Karoun-4, (b) Khersan 1, and (c) Karoun 3 reservoirs.
increase and decrease, respectively. In other words, the reduction of Rate improves the desirability of the two objective functions. It is noted that the values of OFR$_{HP}$ and OFV$_{HP}$ reported in Table 2 are not desirable when Rate = 1. In this instance, the reliability and the vulnerability of the system are less than 1% and almost 60%, respectively. When Rate = 0.5 it is seen that OFR$_{HP}$ and OFV$_{HP}$ are approximately 63 and 66 times greater and lower, respectively, than their values associated with the condition of Rate = 1, respectively. From Table 2, considering Rate = 0.5, the point (0.6333, 0.3959) is used as the right boundary of the OP that is calculated from the two-objective run. It is worth mentioning that this point (together with another point in the next sub-section) is used for calculating the point designated as W$_{ref}$ in Figure 4. W$_{ref}$ is applied with the performance criteria for evaluating the generated OP.

### The optimal single-objective three-reservoir operation problem considering OFV$_{HP}$ as the objective function

The parameters and required initial inputs of this section are the same as those of the previous section. Table 3 lists the values of OFV$_{HP}$ and the corresponding OFR$_{HP}$ calculated after 1,500,000 functional evaluations using the obtained decision variables in the formulation of OFR$_{HP}$ for different values of Rate. It follows from Table 3 that as the value of Rate decreases the values of OFV$_{HP}$ and OFR$_{HP}$ decrease and increase, respectively, which means the reduction of Rate improves the desirability of the two objective functions. According to Table 3, the reservoir system has unsatisfactory values for both objective functions when Rate = 1. The value of Rate = 0.5 produces OFV$_{HP}$ and OFR$_{HP}$ that are approximately 5% and 30%, respectively. From Table 3, considering Rate = 0.5, the point (0.2917, 0.0494) is used as the left boundary of the OP which is calculated from the two-objective run. This point is the second point (as stated at the end of the previous sub-section) that is used for calculating the point entitled as W$_{ref}$ in Figure 4.

### The optimal two-objective three-reservoir operation problem

After determining the solutions of the single-objective problems, the two-objective three-reservoir system was solved with the MOGA, MOFA, and MODFA considering different values of Rate (i.e., with 1, 0.75, and 0.5). Even though other values could have been considered, these values were deemed suitable for reservoir hydropower operation. Reliabilities equal to 100%, 75%, and 50% are applied in hydropower generation in Iran. The latter values are representative of high (Rate = 1), medium (Rate = 0.75), and low (Rate = 0.5) performance thresholds.

The number of parents’ population in MOGA and the number of fireflies’ population in MOFA and MODFA were set equal to 100. Moreover, the number of iterations for all three methods was equal to 2,000. In MOGA the type of selection, crossover function, mutation function, crossover probability, and mutation probability were chosen as the roulette wheel, two-point, uniform, 0.7, and 4.0, respectively. MOFA was implemented with $\beta_0 = 1$, $\gamma = 10$, and $\alpha = 1$. MODFA was implemented with ten groups for categorizing the decision variables, $\beta_0 = 1$, and $\gamma = 3$.

Figure 7 depicts the OPs obtained with the MOGA, MOFA, and MODFA for different values of Rate (1, 0.75, and 0.5) and the results of the single-objective runs as shown with Celtic cross and asterisk symbols related to the conditions in which OFR$_{HP}$ (right boundary) and OFV$_{HP}$ (left boundary) are the objective functions,
It is evident in Figure 7(a) that the results of the MOGA, MOFA, and MODFA are identical when \( \text{Rate} = 1 \). Results corresponding to OPs with \( \text{Rate} = 0.75 \) are illustrated in Figure 7(b), where it is seen that the distribution of OPs obtained with the MOGA, MOFA, and MODFA along the right boundary and the left boundary are more suitable than those shown in Figure 7(a). Clearly, the OPs obtained with the MODFA dominate those obtained with MOGA and MOFA. It is evident that the performance of the MODFA is better than those of the MOGA and MODFA. It is seen in Figure 7(b) that the distance between the first and the last points of MODFA’s OP in Figure 7(b) is too short in terms of \( \text{OFR}_{HP} \). In other words, the OPs’ points vary in a small range (approximately from 0.08 to 0.12). To overcome this issue, the results considering \( \text{Rate} \) equal to 0.5 are presented in Figure 7(c), where it is seen that the
MODFA’s OP points dominate the MOGA and MOFA’s OPs and are well distributed in the range 0.53 to 0.64. It can be seen that the first and the last points of MODFA’s OP are mapped onto the single-objective results. The range and the values of $OFV_{HP}$ in Figure 7(c) are suitable. The reason for this is that the reservoir system experiences a few failure periods in this condition.

The results of the MOGA, MOFA, and MODFA are presented in Table 4, from which it follows that as the value of Rate decreases, the value of HV improves for each method. This means that the performance of the OP improves. The value of $\delta$ increases and worsens when Rate decreases. A key realization is that it is not proper to compare the performance of the algorithms only based on the values of Pareto performance criteria. The OP shown in Figure 7(c) dominates the other OPs, and the value of HV measures closeness and diversity, whereas $\delta$ only measures the spread of solutions. Therefore, it can be asserted that the performance of the MODFA in solving the multi-objective multi-reservoir problem is superior when Rate is equal to 0.5.

Table 5 lists the results for three selected points (solutions) of the obtained OPs illustrated in Figure 7 for a better comparison of the performances of the MOGA, MOFA, and MODFA. The selected points are the first point (from left to right), a middle point (specified with a thick bordered rectangle in Figure 7), and the last point (from left to right) of each OP. According to Table 5, the number of OPs’ points increases for all methods as the value of Rate decreases. Moreover, the MODFA solutions are better that those of MOGA and MOFA for each value of Rate. Also, it can be concluded that the values of $OFR_{HP}$ and $OFV_{HP}$ associated with the three selected points become more favorable for MODFA when Rate = 0.5. In fact, in this case, the distinct number of OPs’ points of MODFA is seven (more than in the OPs of MOGA and MOFA), and, also, the range between the first and the last point of the MODFA’s OP is larger than those of the OPs of MOGA and MOFA when Rate = 0.5.

After analyzing the OPs’ performance, the results of reservoir releases, reservoir storages, hydropower generations, and power plant spills associated with the selected points of the MODFA’s OP (specified with a thick bordered rectangle in Figure 7(c)) are shown in Figure 8 for the three reservoirs for the condition in which Rate = 0.5. It is evident from Figure 8 that the release and the storage results are within the minimum and the maximum allowable values.
Thus, no violation exists in the final results. Also, there are more periods generating PPC in Khersan 1 than in Karroun 4 and Karoun 3. That is why in Figure 8(d) the power plant spills are significant in Khersan 1 but not observed in Karoun 4 and Karoun 3.

Due to population growth, scarce water resources, and the increasing pressure on water in Iran, the optimal operation of the reservoirs (which are the most important structures to store surface water) is imperative. Optimal operation leads to maximizing the efficiency of water use. However, frequently, the operator of a reservoir makes a decision about the releases according to a table adjusted for a standard linear operation policy. This study implemented the MODFA as a reliable and efficient method for solving the complex reservoir optimization problem to provide optimal operation strategies for the three-reservoir system (Karoun 4, Khersan 1, Karoun 3) in Iran. The capability of achieving near-optimal solutions, high-speed convergence rate, and higher reliability highlighted the modified version of firefly algorithm as a serious competitor compared to existing algorithms. Using the optimal strategies to operate the reservoirs leads to decreasing the water shortage of this three-reservoir system in Iran.

LIMITATIONS AND POTENTIAL AREAS FOR FUTURE RESEARCH

It is worth mentioning that one of the significant uncertainties in solving optimal reservoir operation problems using evolutionary or meta-heuristic algorithms is related to the values of the parameters that are used in these algorithms. Even though DFA and MODFA feature fewer parameters than FA, MOFA, GA, and MOGA, there still remain some parameters whose values have uncertainties. Our suggestion for future study is to develop algorithms with fewer or even without parameters. Another uncertainty is related to the fact that in non-linear problems, such as multi-reservoir system with the purpose of hydropower generation, the global solution cannot be obtained. Evolutionary and meta-heuristics algorithms only reach the nearest plausible solution. Therefore, there is always a concern about whether the final result is the optimal solution or not. Even though
this uncertainty exists, using these methods constitutes the most plausible approach to find the nearest optimal solution to the global solution.

MODFA is the multi-objective extension of DFA (proposed by Garousi-Nejad et al. 2016a, 2016b). DFA has been successfully tested with continuous and discrete problems. This study applied the MODFA to a three-reservoir system not previously considered by Garousi-Nejad et al. (2016a, 2016b). Future applications of the MODFA to other reservoir systems most likely will require definitions and calibration of parameters that may differ from those introduced in this work.

**CONCLUDING REMARKS**

Evolutionary and meta-heuristic optimization algorithms are recognized as approaches that provide efficient solutions to challenging multi-objective, multi-reservoir operation problems. The aim of this study was to develop an effective solution to the problem of the operation of a real multi-objective multi-reservoir system with the purpose of hydropower generation. First, the FA was reformulated to tackle this type of reservoir operation problem, leading to the extended FA (DFA). Afterwards, the MODFA was introduced using the NSII principle by reformulating the MOFA. The results of the two-objective, three-reservoir, operation problem was provided in the form of OPs which include non-dominated solutions. Our results showed that the OP of the MODFA dominates those of the MOGA and MOFA. Also, the MODFA produced solutions with more uniform distribution and closer to the values of the single-objective problem than those calculated with MOGA and MOFA.

The performance of the MODFA was examined with different values of performance thresholds. This evaluation revealed that the desirability of the obtained OP increases with decreasing value of the performance threshold. Our results show that the performance of the MODFA in producing a proper OP is best when the value of Rate is equal to 0.5 rather than 1 or 0.75. Since no single performance criterion can entirely evaluate all the capabilities of solution algorithms, herein two Pareto performance criteria were used, δ and HV. These values were applied to evaluate the performance of the calculated OPs with different algorithms. The computed HV values indicated that HV improves as the value of Rate decreases.

The calculated monthly reservoir releases, storages, hydropower generations, and power plant spills established that the Khersan 1 reservoir exhibits more periods during which PPC is produced. Consequently, only this reservoir has power plant spills. From our results it can be asserted that the MODFA is capable of solving the two-objective multi-reservoir hydropower operation problem providing better OPs in terms of obtaining non-dominated solutions compared with the MOGA and MOFA. Thus, MODFA was shown to be an effective method for solving complex multi-objective, multi-reservoir, operation problems.

**REFERENCES**


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