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An inexact stochastic optimization model for multi-conflict regional water resources allocation in the south-to-north water benefited area

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ABSTRACT

The cross-regional water diversion project has been widely applied as an important way to relieve water pressure. Study about the tradeoff between multiple regions and multiple water use sectors has caused widespread concern. In this study, an inexact two-stage water resources management model for multi-region water resources planning with a large-scale water diversion project has been developed. The water sources in 11 districts, including independent water sources and public water sources diverted from the project, are considered in the optimization model. Water supply cost and recourse cost are analyzed in the objective function. Based on interval-parameter programming and two-stage stochastic programming, uncertainties in the water resources system are described by both interval values and probability distributions. The result indicates that the water diversion project would greatly change the composition of the water resource system and settle the uneven distribution of regional water resources to achieve district-optimal allocation of water resources. In general, the proposed method can help decision-makers to formulate water management strategies for rational utilization of all kinds of water resources in different regions.

Key words | inexact two-stage stochastic programming, uncertainty, water diversion project, water resources management

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INTRODUCTION

Shortage in the availability of freshwater relative to water demand is the focus of global attention at present (Gain & Giupponi 2015). Various factors, including population growth, economic development, land use change, and environmental degradation, can affect the changes in water demand (Sophocleous 2004). In parallel, climate change is considered as one of the main driving forces to affect the temporal and spatial variability of water availability (Bates *et al.* 2008; Stocker *et al.* 2013). To deal with the uneven distribution of water resources in space, the cross-regional water diversion project, as an important way to relieve water pressure, has been widely applied. Meanwhile, the interaction among these factors further aggravates the shortage of water resources, and can lead to various uncertainties existing in a number of system components and their inter-relationships within a water resources system. These uncertainties would decrease the robustness of the balance between water supply and demand obtained by conventional optimization methods (Xie *et al.* 2013). Therefore, effective water resources planning under uncertainty within a general regional water supply management framework is desired.

Many inexact optimization methods have previously been developed to deal with uncertainties in water resources

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management problems (Sawyer & Lin 1998; Bender & Simonovic 2000; Carter 2005; Castelletti *et al.* 2008; Han *et al.* 2011). For example, Woodward & Shaw (2008) employed robust control as a way to deal with the reduction of uncertainty to a unique probability distribution. Chang *et al.* (1996a, 1996b) developed a grey fuzzy multi-objective linear programming method, for the evaluation of sustainable management strategies for systems analysis. Luo *et al.* (2007) presented an interval stochastic dynamic programming model to deal with the problem of water resources system planning with capacity expansion in an uncertain environment. Ryu *et al.* (2012) used a system dynamics approach to identify long-term aquifer behavior in response to uncertain future hydrological variability with recharge and discharge dynamics within the aquifer system being coded in an environmental modeling framework. Subagadis *et al.* (2016) proposed a new fuzzy-stochastic multiple criteria decision-making approach for water resources management in which a variety of criteria in terms of economic, environmental, and social dimensions were identified and considered.

Among them, two-stage stochastic programming (TSP) is effective in dealing with problems where uncertainties can be expressed as probabilistic distributions. In a TSP model, the first-stage decision is made before the realization of random variables, and the second-stage decision is then made after the random events have happened. TSP with recourse was initially introduced by Beale (1955), and has been widely applied to water resources management during the past decades. For example, Wang & Adams (1986) proposed a two-stage optimization framework for planning reservoir operations, where the hydrologic uncertainty and seasonality of reservoir inflows were modeled as periodic Markov processes. Karuppiah & Grossmann (2008) optimized a superstructure that incorporated all feasible design alternatives for wastewater treatment, recycle and reuse, with a multi-scenario nonconvex mixed integer nonlinear programming model, which is a deterministic equivalent of a TSP model with recourse. In fact, in many real-world problems, the information quality of parameters is often not satisfactory enough to be presented as probabilistic distribution. Even if such distributions are available, reflecting them in large-scale optimization models can be extremely challenging (Huang & Loucks 2000).

Interval-parameter programming (IPP) is an alternative for handling uncertainties in the model's left- and/or right-hand sides, as well as those that cannot be quantified as membership or distribution functions (Huang 1996). Thus, one potential approach for better reflecting uncertainties is to incorporate IPP within a TSP framework when interval numbers are also used as uncertain inputs. Huang & Loucks (2000) proposed an inexact two-stage stochastic programming (ITSP) approach for water resources management. ITSP can not only tackle uncertainties expressed as probabilistic distributions and intervals, but also analyze a variety of policy scenarios that are associated with different levels of economic penalties when the promised policy targets are violated. Xie *et al.* (2013) developed an inexact two-stage water resources management model for multi-regional water resources planning in the Nansihu Lake Basin, China. Zhang & Li (2014) proposed an inexact two-stage water resources allocation model for supporting sustainable development and management of water resources in Sanjiang Plain, China. Li *et al.* (2016) developed an interval-fuzzy two-stage stochastic quadratic programming model to determine the plans for water allocation with maximum benefits. However, few studies have been found in developing an inexact TSP model for a large-scale water diversion project to dispatch multiple water sources for multi-user water supply within a multi-region area.

Therefore, this study aims to develop an inexact two-stage water resources management model for multi-region water resources planning with a large-scale water diversion project in Henan province, China. This model considers multiple water sources and users. It incorporates IPP and TSP into the optimization framework. It can not only present uncertainties as both probability distributions and interval values, but also permit in-depth analysis of various policy scenarios when the promised policy targets are violated. The impact of large water diversion works on regional water resources management planning will be explored. At the same time, the difference between each district, such as industrial structure, technological level, and geographic position, will be reflected in the result. Finally, the modeling results can be useful for supporting the adjustment or justification of the existing water allocation schemes within a complicated water resources system under uncertainty.

METHODOLOGY

An inexact two-stage stochastic model can be written as follows:

$$\text{Minimize } f^{\pm} = C_{T_1}^{\pm} X^{\pm} + \sum_{h=1}^{v} P_h D_{T_2}^{\pm} Y^{\pm} \tag{1a}$$

Subject to:

$$A_r^{\pm} X^{\pm} \le B_r^{\pm},\, r = 1, 2, \ldots, m_1 \tag{1b}$$

$$A_t^{\pm} X^{\pm} + A_t^{'\pm} Y^{\pm} \ge \omega_h^{\pm},\, t = 1, 2, \ldots, m_2;\, h = 1, 2, \ldots, v \tag{1c}$$

$$x_j^{\pm} \ge 0,\, x_j^{\pm} \in X^{\pm},\, j = 1, 2, \ldots, n_1 \tag{1d}$$

$$y_{jh}^{\pm} \ge 0,\, y_{jh}^{\pm} \in Y^{\pm},\, j = 1, 2, \ldots, n_2;\, h = 1, 2, \ldots, v \tag{1e}$$

where $\omega_h^{\pm}$, $A_r^{\pm} \in \{R^{\pm}\}^{m_1 \times n_1}$, $A_t^{\pm} \in \{R^{\pm}\}^{m_2 \times n_2}$, $B_r^{\pm} \in \{R^{\pm}\}^{m_1 \times 1}$, $C_{T_1}^{\pm} \in \{R^{\pm}\}^{1 \times n_1}$, $D_{T_2}^{\pm} \in \{R^{\pm}\}^{1 \times n_2}$, $X^{\pm} \in \{R^{\pm}\}^{n_1 \times 1}$, $Y^{\pm} \in \{R^{\pm}\}^{n_2 \times 1}$, $\{R^{\pm}\}$ represent the model parameters or decision variables represented by the interval number. P_h represents the probability of the random variable $\omega_h^{\pm}$ under h probability level, and $\sum_{h=1}^{v} P_h = 1$.

Based on an interactive algorithm (Huang & Loucks 2000), model 1 can be transformed into two deterministic sub-models, respectively corresponding to the upper and lower bounds desired objective function to obtain a stable interval solution. Since the objective is to minimize the net system cost, the sub-model corresponding to lower-bound objective function value (f^-) is first desired, where the lower bounds of cost coefficients and energy demands will correspond to that. Thus, we have the model as follows:

$$\text{Minimize } f^{-} = \sum_{j=1}^{k_1} c_j^- x_j^- + \sum_{j=k_1+1}^{n_1} c_j^- x_j^+ + \sum_{j=1}^{k_2} \sum_{h=1}^{v} p_{jh} d_j^- y_{jh}^- + \sum_{j=k_2+1}^{n_2} \sum_{h=1}^{v} p_{jh} d_j^- y_{jh}^+ \tag{2a}$$

Subject to:

$$\sum_{j=1}^{k_1} |a_{rj}|^+ sign\left(a_{rj}^+\right) x_j^- + \sum_{j=k_1+1}^{n_1} |a_{rj}|^- sign\left(a_{rj}^-\right) x_j^+ \le b_r^+,\, \forall r \tag{2b}$$

$$\sum_{j=1}^{k_1} |a_{tj}|^+ sign\left(a_{tj}^+\right) x_j^- + \sum_{j=k_1+1}^{n_1} |a_{tj}|^- sign\left(a_{tj}^-\right) x_j^+ + \sum_{j=1}^{k_2} \left|a'_{tj}\right|^+ sign\left(a_{tj}^{\prime +}\right) y_{jh}^- + \sum_{j=k_2+1}^{n_2} \left|a'_{tj}\right|^- sign\left(a_{tj}^{\prime -}\right) y_{jh}^+ \ge \omega_h^-,\, \forall t, h \tag{2c}$$

$$x_j^- \ge 0,\, j = 1, 2, \ldots, k_1 \tag{2d}$$

$$x_j^+ \ge 0,\, j = k_1 + 1, k_1 + 2, \ldots, n_1 \tag{2e}$$

$$y_{jh}^- \ge 0,\, \forall h;\, j = 1, 2, \ldots, k_2 \tag{2f}$$

$$y_{jh}^+ \ge 0,\, \forall h;\, j = k_2 + 1, k_2 + 2, \ldots, n_2 \tag{2g}$$

where $c_j^{\pm}$, $d_j^{\pm}$, $a_{rj}^{\pm}$ are parameters or the coefficients of decision variables. $x_j^{\pm}, j = 1, 2, \ldots k_1$ is the positive coefficient of interval variables in objective function. $x_j^{\pm}, j = k_1 + 1, k_1 + 2, \ldots n_1$ is the negative coefficient of interval variables in objective function. $y_{jh}^{\pm}, j = 1, 2, \ldots k_2$; $h = 1, 2, \ldots, v$ is the positive coefficient of random variables in objective function. $y_{jh}^{\pm}, j = k_2 + 1, k_2 + 2, \ldots k_2$; $h = 1, 2, \ldots, v$ is the negative coefficient of random variables in objective function.

Consequently, the sub-model corresponding to the upper bound of the objective function value can be formulated as follows:

$$\text{Minimize } f^{+} = \sum_{j=1}^{k_1} c_j^+ x_j^+ + \sum_{j=k_1+1}^{n_1} c_j^+ x_j^- + \sum_{j=1}^{k_2} \sum_{h=1}^{v} p_{jh} d_j^+ y_{jh}^+ + \sum_{j=k_2+1}^{n_2} \sum_{h=1}^{v} p_{jh} d_j^+ y_{jh}^- \tag{3a}$$

Subject to:

$$\sum_{j=1}^{k_1} |a_{rj}|^- sign\left(a_{rj}^-\right) x_j^+ + \sum_{j=k_1+1}^{n_1} |a_{rj}|^+ sign\left(a_{rj}^+\right) x_j^- \le b_r^-,\, \forall r \tag{3b}$$

$$\sum_{j=1}^{k_1} |a_{tj}|^- sign\left(a_{tj}^-\right) x_j^+ + \sum_{j=k_1+1}^{n_1} |a_{tj}|^+ sign\left(a_{tj}^+\right) x_j^- + \sum_{j=1}^{k_2} \left|a'_{tj}\right|^- sign\left(a'^-_{tj}\right) y_{jh}^+ + \sum_{j=k_2+1}^{n_2} \left|a'_{tj}\right|^+ sign\left(a'^+_{tj}\right) y_{jh}^- \geq \omega_h^+, \forall t, h \tag{3c}$$

$$x_j^+ \geq x_{j\text{opt}}^-, j = 1, 2, \ldots, k_1 \tag{3d}$$

$$0 \leq x_j^- \leq x_{j\text{opt}}^+, j = k_1 + 1, k_1 + 2, \ldots, n_1 \tag{3e}$$

$$y_{jh}^+ \geq y_{jh\text{opt}}^-, \forall h; j = 1, 2, \ldots, k_2 \tag{3f}$$

$$0 \leq y_{jh}^- \leq y_{jh\text{opt}}^+, \forall h; j = k_2 + 1, k_2 + 2, \ldots, n_2 \tag{3g}$$

Solutions of $x_{j\text{opt}}^+(\text{j} = 1, 2, \ldots, \text{k1})$, $x_{j\,\text{opt}}^-(\text{j} = \text{k1} + 1, \text{k1} + 2, \ldots, \text{n1})$, $y_{ls\,\text{opt}}^+(\text{j} = 1, 2, \ldots, \text{k2})$, and $y_{ls\,\text{opt}}^-(\text{j} = \text{k2} + 1, \text{k2} + 2, \ldots, \text{n2})$ can be obtained through sub-model (3). Through integrating solutions of sub-models (2) and (3), the interval solution for model (1) can be obtained:

$$f_{opt}^{\pm} = [f_{opt}^-, f_{opt}^+] \tag{4a}$$

$$x_{j\,opt}^{\pm} = [x_{j\,opt}^-, x_{j\,opt}^+] \tag{4b}$$

$$y_{ls\,opt}^{\pm} = [y_{ls\,opt}^-, y_{ls\,opt}^+] \tag{4c}$$

CASE STUDY

Overview of the case study

China's south-to-north water diversion project, the world's largest, is designed to take water from the country's longest river, the Yangtze, through eastern, middle, and western routes to feed dry areas in the north, including the capital city Beijing. The mid-route project of the south-to-north water diversion started with the construction in 2003. The main canal of the mid-route is 1,432 kilometers in length. As of September 2014, a total of 210.18 billion yuan had been invested in the first phase project of the mid-route. The first phase project will see a massive 9.5 billion cubic meters of water per year pumped through canals and pipes from the Danjiangkou reservoir in central China's Hubei province to the northern provinces of Henan and Hebei and to Beijing.

The first region to be passed through, Henan province, has a temperate climate that is humid subtropical to the south of the Yellow River and bordering on humid continental to the north. It has a distinct seasonal climate characterized by hot, humid summers due to the East Asian monsoon, and generally cool to cold, windy, dry winters that reflect the influence of the vast Siberian anticyclone. Most of the annual rainfall occurs during the summer. Henan province, as the first region to go through, has the largest water consumption plan. It would account for 40% of the total water transfer quantity. As shown in Figure 1, 11 districts would use the water from the water diversion project. In the study area, the main line of the project would be 730 kilometers in length, and affect the daily life of 20 million people.

The water sources of these 11 districts include independent sources of water and public sources of water diverted from the project. With the shortage of independent sources of water being increasing seriously, public sources of water would be the main target of contention between different administrative departments to make up the shortage of water resources. Moreover, the different industrial structures and different plans for national economic and social development in each region would lead to complicating the issue. Four water users are considered in this study, including the agricultural, municipal, industrial, and environmental sectors. Three planning periods are considered in this study and each planning horizon lasts for five years. As shown in Table 1, the water consumption between each district has a significant difference. In districts such as Nanyang city, Zhoukou city, etc. (agriculture as the main industry) most water resources have been used for agriculture. On the contrary, in Pingdingshan city, which is an industrial city, industrial water takes up a large proportion. Zhengzhou, as the provincial capital, has the most developed tertiary industry.

Due to the temporal variation of the water resources system, an obvious stochastic characteristic can be found

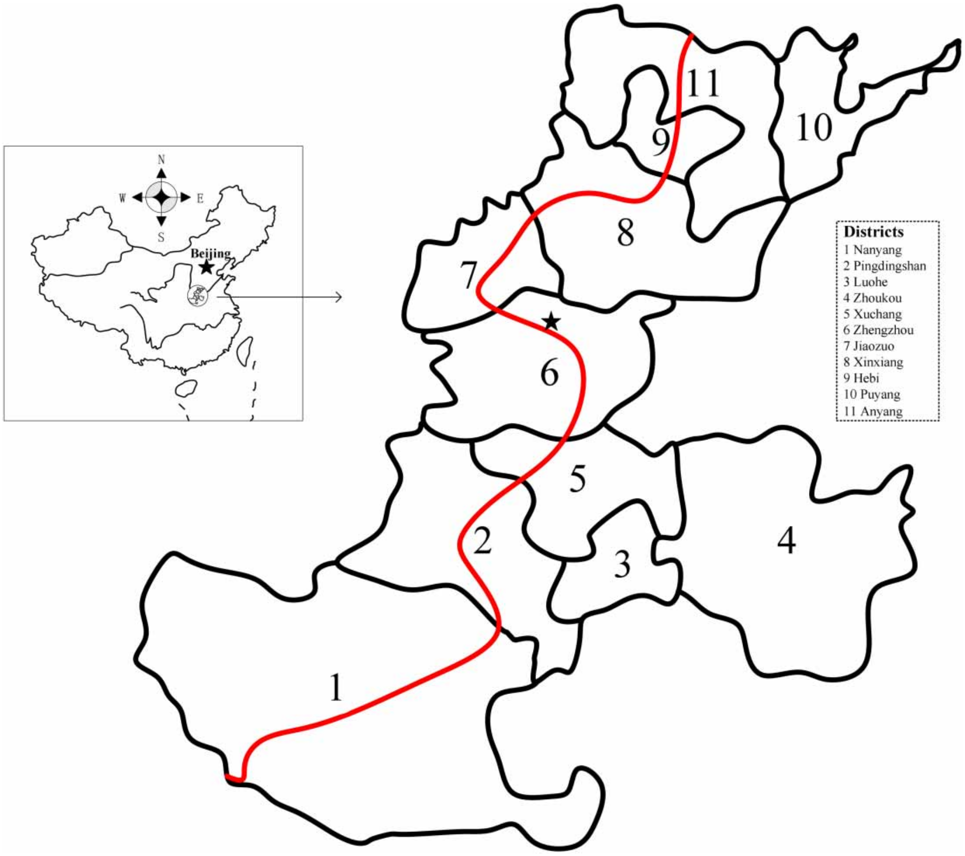

Figure 1 | Geographical position and districts of the study area.

regarding the amount of available water resources. This would change the optimal water allocation schemes within the study area. In addition, many influencing factors and their inter-relationships within the water resources system are uncertain in nature. Such complexity would lead to a number of challenging questions, such as: (a) how to balance development issues in the whole study area and different districts; (b) how much water from the water diversion project should be allocated to each water user sector within each district under different levels of available water amount; (c) how to adapt/adjust the pre-regulated policies to minimize the risk of system disruption under uncertain system conditions. The interval parameter TSP is thus considered to be suitable for tackling this type of water management problem. The following regional water resource management system is used to demonstrate the

Table 1 | Water consumption of each district before planning period (10^6 m^3/year)

	District										
User	**k = 1**	**k = 2**	**k = 3**	**k = 4**	**k = 5**	**k = 6**	**k = 7**	**k = 8**	**k = 9**	**k = 10**	**k = 11**
Agriculture	1,222	265	176	1,025	268	484	697	1,118	328	837	980
Industry	624	631	75	273	261	561	379	267	67	285	201
Municipality	326	131	65	288	138	538	113	22	59	159	181
Environment	97	25	20	18	22	202	26	28	19	43	50

applicability of the ITSP model, as shown in Figure 2. In this study, the uncertainty information included in the water resource management systems was mainly divided into two parts. The first is the uncertainty of the data, especially with a long time and space span. The interval parameter programming was selected to handle this problem. Meanwhile, the uncertainty caused by the promised policy targets in the future was handled by the TSP. The water diversion project as an important source of water resource, along with independent water and reuse water, supports the development of agriculture, industry, municipal, and environment protection.

Data collection

Table 2 presents the amount of available water resources (represented by an interval number) during each planning period from the water diversion project. As a large cross-regional water diversion project, the gross amount of water resource is more affected by the national planning and policy, which would be very different from natural rivers. On this basis, the relevant parameters have been formulated based on the water resources utilization planning in Henan province and some related references (Xie *et al.* 2013; Li *et al.* 2015). In the table, scenario h represents the inflow levels (low, medium, and high level) which represent the different policy scenarios in the planning period. Policy scenarios in the future would affect the optimal targets of water allocation from the water diversion project. If the water demands of each water user sector in each district are satisfied, it will result in benefits to the regional development. On the other hand, when the water supply is not delivered, either additional water must be obtained from higher-priced alternatives or the water demand must be curtailed by reduced production, which will lead to a reduced net system benefit (Huang & Loucks 2000; Maqsood *et al.* 2005). Specifically, the scenarios of low and high level determine the lower bound and upper bound of the optimal targets of water allocation from the water diversion project. The extra cost would be generated when the optimal target has gaps with each policy scenario, and the total expected system benefit could be further affected. Different policy scenarios and the expectancy value of extra costs caused by different amounts of available water resources would affect the final results.

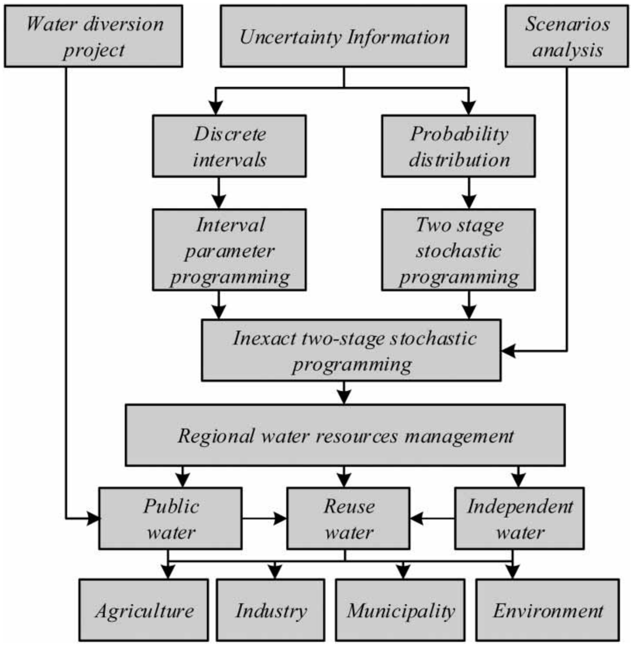

Figure 2 | General framework of the ITSP model.

Table 2 | Amount of available water resources from water diversion project (10^6 m^3/year)

		Period		
Scenario (h)	**Probability**	**t = 1**	**t = 2**	**t = 3**
Low	0.2	[2,415, 2,657]	[2,755, 3,031]	[3,125, 3,438]
Medium	0.6	[2,855, 3,141]	[3,295, 3,625]	[3,615, 3,977]
High	0.2	[3,325, 3,658]	[3,595, 3,955]	[3,840, 4,224]

Meanwhile, the different distance between water consumption districts and Danjiangkou reservoir would result in the price of water changing from 0.18 yuan to 0.58 yuan (HNSY 2014, 2015).

Model development

A 15-year planning horizon has been considered in this study, with the first planning period being from 2016 to 2020, the second from 2021 to 2025, and the last period from 2026 to 2030. It is desired to efficiently allocate water from the public and independent sources of water to four water user sectors within 11 districts. The issue can be formulated as an inexact two-stage water management model.

The objective function is to maximize the expected value of system benefit, which includes (a) the benefit from water supply to different water users, (b) the penalties caused by water shortage, (c) cost of water supply, (d) cost of reused water treatment, and (e) cost of wastewater treatment.

Thus, we have:

$$\max f^{\pm} = (1) - (2) - (3) - (4) - (5) \tag{5a}$$

$$(1) = \sum_{i=1}^{4}\sum_{t=1}^{3}\sum_{k=1}^{11} L_t\left(IW_{itk}^{\pm} + PW_{itk}^{\pm} + RW_{itk}^{\pm}\right) \cdot BW_{itk}^{\pm} \tag{5b}$$

$$(2) = \sum_{i=1}^{4}\sum_{t=1}^{3}\sum_{h=1}^{3}\sum_{k=1}^{11} L_t \cdot p_h \cdot QW_{ithk}^{\pm} \cdot PQW_{itk}^{\pm} \tag{5c}$$

$$(3) = \sum_{i=1}^{4}\sum_{t=1}^{3}\sum_{k=1}^{11} L_t \cdot \left(PW_{itk}^{\pm} - \sum_{h=1}^{3} p_h \cdot QW_{ithk}^{\pm}\right) \cdot CPW_{itk}^{\pm} + \sum_{i=1}^{4}\sum_{t=1}^{3}\sum_{k=1}^{11} L_t \cdot IW_{itk}^{\pm} \cdot CIW_{itk}^{\pm} \tag{5d}$$

$$(4) = \sum_{i=1}^{4}\sum_{t=1}^{3}\sum_{k=1}^{11} L_t \cdot RW_{itk}^{\pm} \cdot CRW_{itk}^{\pm} \tag{5e}$$

$$(5) = \sum_{i=1}^{4}\sum_{t=1}^{3}\sum_{k=1}^{11} L_t \cdot \left(IW_{itk}^{\pm} + PW_{itk}^{\pm} + RW_{itk}^{\pm} - \sum_{h=1}^{3} p_h \cdot QW_{ithk}^{\pm}\right) \cdot \gamma_{itk}^{\pm} \cdot CP_{itk}^{\pm} \tag{5f}$$

where $f^{\pm}$ is the net expected system benefit (million RMB¥); $IW_{itk}^{\pm}$ is the water supply targets ($10^6\,\mathrm{m}^3$/year) from the independent source of water in district k during planning period t for industry i, where I = 1, 2, 3, and 4 for the agricultural, industrial, municipal and environmental sectors, respectively; $PW_{itk}^{\pm}$ is the water supply targets ($10^6\,\mathrm{m}^3$/year) from the water diversion project in district k during planning period t for industry i; $RW_{itk}^{\pm}$ is the water supply targets ($10^6\,\mathrm{m}^3$/year) from reuse water in district k during planning period t for industry i; $BW_{itk}^{\pm}$ is the unit benefit of water supply during period t (million RMB￥/$10^6\,\mathrm{m}^3$); L_t is the length of each planning period (five years); $QW_{ithk}^{\pm}$ is the water shortages ($10^6\,\mathrm{m}^3$/year) during period t (when the amount of public water source is corresponding to inflow level h) for the industry i in district k, where h = 1, 2, and 3 for low, medium, and high level; $PQW_{itk}^{\pm}$ is the unit reduction of net benefit during period t (million RMB¥/$10^6\,\mathrm{m}^3$) when the water target is not delivered for the industry i in district k; p_h is the probability of the occurrence of flow level h; $CPW_{itk}^{\pm}$ is the unit cost of water supply (million RMB¥/$10^6\,\mathrm{m}^3$) from the water diversion project during planning period t for industry i; $CIW_{itk}^{\pm}$ is the unit cost of water supply (million RMB¥/$10^6\,\mathrm{m}^3$) from the independent source of water during planning period t for industry i; $CRW_{itk}^{\pm}$ is the unit cost of water supply (million RMB¥/$10^6\,\mathrm{m}^3$) from reused water during planning period t for industry i; $\gamma_{itk}^{\pm}$ is the discharge of wastewater per unit consumption of water during planning period t for industry i; $CP_{itk}^{\pm}$ is the unit cost of wastewater treatment in district k during planning period t for industry i (million RMB¥/$10^6\,\mathrm{m}^3$).

Subject to:

1. Available water quantity constraints

The total water quantity supplied to different sectors should not be more than the available water resource in different districts:

$$\sum_{i=1}^{4}\sum_{k=1}^{11}\left(PW_{itk}^{\pm} - QW_{ithk}^{\pm}\right) \le TPW_{th}^{\pm} \quad \forall t, h \tag{5g}$$

$$\sum_{i=1}^{4} IW_{itk}^{\pm} \le TIW_{tk}^{\pm} \quad \forall t, k \tag{5h}$$

$$QW_{ithk}^{\pm} \le PW_{itk}^{\pm} \quad \forall i, t, h, k \tag{5i}$$

2. Water demand constraints

The total water coming from different sources should not be less than regional minimum water consumption:

$$IW_{itk}^{\pm} + PW_{itk}^{\pm} + RW_{itk}^{\pm} - QW_{ithk}^{\pm} \ge DW_{itk}^{\pm} \tag{5j}$$

3. Wastewater treatment capacity constraints

For regional water resources management system, environmental requirement should be considered as an important constraint, and the wastewater treatment facility should be set up to treat pollutants:

$$\sum_{i=1}^{4}\left(IW_{itk}^{\pm}+PW_{itk}^{\pm}+RW_{itk}^{\pm}-QW_{ithk}^{\pm}\right)\cdot\gamma_{itk}^{\pm}\leq TWW_{tk}^{\pm}\quad\forall t,k,h \tag{5k}$$

4. Water reuse capacity constraints

Reuse water mainly comes from industrial and municipal sectors, and it would be in effect complementary to regional water consumption. In each district, the water supply targets from recycled water should not be greater than the maximum supply capacity of reused water:

$$\sum_{i=1}^{4}RW_{itk}^{\pm}\leq\sum_{i=1}^{4}\left(IW_{itk}^{\pm}+PW_{itk}^{\pm}+RW_{itk}^{\pm}-QW_{ithk}^{\pm}\right)\cdot\beta_{itk}^{\pm},\ \forall t,k,h \tag{5l}$$

$$\sum_{i=1}^{4}\left(IW_{itk}^{\pm}+PW_{itk}^{\pm}+RW_{itk}^{\pm}-QW_{ithk}^{\pm}\right)\cdot\beta_{itk}^{\pm}\leq TRW_{tk}^{\pm},\quad\forall t,k,h \tag{5m}$$

5. Technical constraints

$$IW_{itk}^{\pm},PW_{itk}^{\pm},RW_{itk}^{\pm},BW_{itk}^{\pm},p_h,QW_{ithk}^{\pm},PQW_{itk}^{\pm},CPW_{itk}^{\pm},IW_{itk}^{\pm},CIW_{itk}^{\pm},CRW_{itk}^{\pm},\gamma_{itk}^{\pm},CP_{itk}^{\pm}\geq 0 \tag{5n}$$

$$TPW_{th}^{\pm},TIW_{tk}^{\pm},DW_{itk}^{\pm},TWW_{tk}^{\pm},\beta_{itk}^{\pm},TRW_{tk}^{\pm}\geq 0 \tag{5o}$$

where $TPW_{th}^{\pm}$ is the water resources available for public water coming from the water diversion project under inflow level h in period t (10^6 m^3/year); $TIW_{tk}^{\pm}$ is the water resources available for independent source of water in district k during planning period t (10^6 m^3/year); $DW_{itk}^{\pm}$ is the water resource demand in district k during planning period t for industry i (10^6 m^3/year); $TWW_{tk}^{\pm}$ is the capacity for wastewater treatment during period t in district k (10^6 m^3/year); $\beta_{itk}^{\pm}$ is the recycle ratio of water consumption in district k during planning period t for industry i; $TRW_{tk}^{\pm}$ is the capacity for water recycle treatment in district k during planning period t (10^6 m^3/year).

RESULTS ANALYSIS AND DISCUSSION

The objective of the ITSP model is to ensure the expected system benefit maximized under the circumstances that the quantity of water resources coming from water diversion project is uncertain. Between the associated economic implications and the predefined water resources using policies, an effective linkage would be provided through the solutions. Moreover, the solutions contain a combination of deterministic, interval, and distributional information, and can thus facilitate the reflection for different forms of uncertainties (Li *et al.* 2006). The interval solutions can help managers obtain multiple decision alternatives, as well as provide the basis for further analysis about water resources utilization.

Table 3 presents the optimal targets of water allocation from the water diversion project for different water use sectors in the 11 districts. It can be found that the optimal targets would approach their upper bounds. When PW_{itk} approach their upper bounds, a relative high benefit would be obtained if the demand of water resources is satisfied; a low penalty may have to be paid when the amount of promised water resource is delivered. Conversely, when PW_{itk} reach their lower bounds, the system would have a high cost and a higher risk with the target which has been breached. For different water use sectors, water resources coming from the water diversion project would mainly be distributed to industrial and municipal sectors. For example, in planning period 1, the target of water allocation for Xinxiang city was 75.0, 110.3, 125.0, and 75.0×10^6 m^3/year for agricultural, industrial, municipal, and environmental sectors, respectively. Meanwhile, the different economic activities, total population, and water environment in each district would lead to different targets. For example, in Zhengzhou city, as the core region in the study area, the total water resources from the water diversion project would be 700.0, 870.0, and 710.0×10^6 m^3/year in the three periods, respectively. In comparison, the total water resources from the water diversion project for Zhoukou city would be 150.0, 150.0, and 150.0×10^6 m^3/year in the three periods.

Figure 3 shows the schemes of water allocation for each water user sector in Pingdingshan city, Zhoukou city, Zhengzhou city, and Xinxiang city from the independent

Table 3 | Optimal targets of water allocation from the water diversion project (10^6 m^3/year)

		District										
User	**Period**	**k = 1**	**k = 2**	**k = 3**	**k = 4**	**k = 5**	**k = 6**	**k = 7**	**k = 8**	**k = 9**	**k = 10**	**k = 11**
Agriculture	t = 1	100	50	25	25	50	200	50	75	50	50	75
	t = 2	120	75	25	25	75	250	75	100	75	75	100
	t = 3	140	100	25	25	100	300	100	125	100	100	125
Industry	t = 1	200	100	50	50	100	200	100	110	76	100	125
	t = 2	250	150	50	50	150	250	73	41	86	150	145
	t = 3	300	200	50	50	200	75	200	98	50	200	180
Municipality	t = 1	200	100	50	50	100	200	100	125	100	100	125
	t = 2	250	150	50	50	150	250	150	145	150	150	145
	t = 3	300	200	50	50	183	300	200	180	50	200	90
Environment	t = 1	100	50	25	25	50	100	50	75	50	50	75
	t = 2	120	75	25	25	75	120	39	39	19	75	25
	t = 3	140	46	25	25	29	35	25	31	25	100	31

source of water during the planning periods. The representative districts presented in Figure 3 have different characteristics and trends of economy, in that Pingdingshan city is a typical industrial city, Zhoukou city is a typical agricultural city, Zhengzhou city as the provincial capital has the largest population, and Xinxiang maintained the

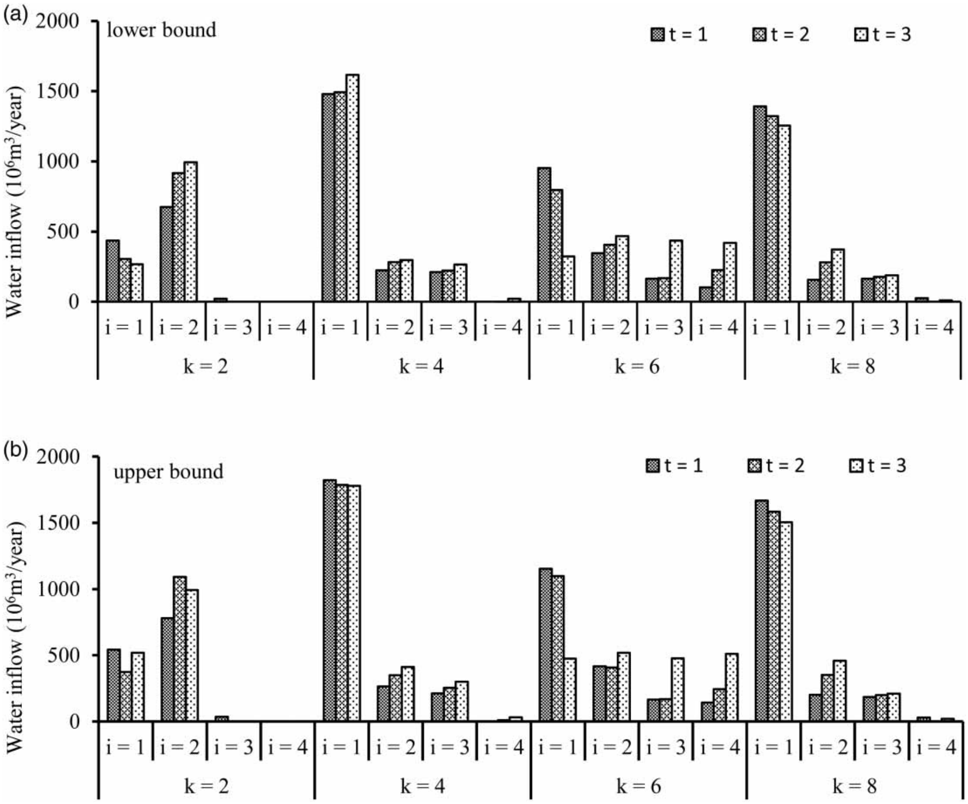

Figure 3 | Water allocation schemes for different water user sectors.

development between agriculture and industry in a balanced way. All these differences would result in various schemes of water allocation. For example, in each planning period, the amount of agriculture water in Zhoukou city would be [1,478.8, 1,822.5], [1,491.6, 1,788.4], and [1,615.7, 1,779.8] $\times 10^6$ m^3/year. In contrast, the distribution amount of agriculture water in Pingdingshan city would be [435.5, 541.9], [304.3, 372.3], and [266.4, 518.4] $\times 10^6$ m^3/year. Meanwhile, in different districts, the water allocation schemes for the municipal sector show significant differences. Unlike the other three districts, the municipal water use in Pingdingshan city comes almost entirely from the water diversion project and in each planning period it would be [20.8, 35.2], 0, and 0×10^6 m^3/year. Water use for environmental conservation is the primary condition for maintaining environmental health. As an important link between the allocation and management of water resources, the water allocation schemes for the environmental sector require extra attention. In Zhengzhou city, the water allocation scheme from the independent source of water to the environmental sector would be [102.0, 142.4], [226.7, 243.6], and [419.5, 510.4] $\times 10^6$ m^3/year. Nevertheless, the water diversion project would be the main source of environmental water in the other three districts.

Figure 4 shows the upper and lower bounds of the objective function of the optimization model n (i.e., net system benefits). In each planning period, the net expected system benefits have an upward trend. However, significant differences in the development situation of each district are presented in each planning period. Because of the differences in economic development and industrial structure,

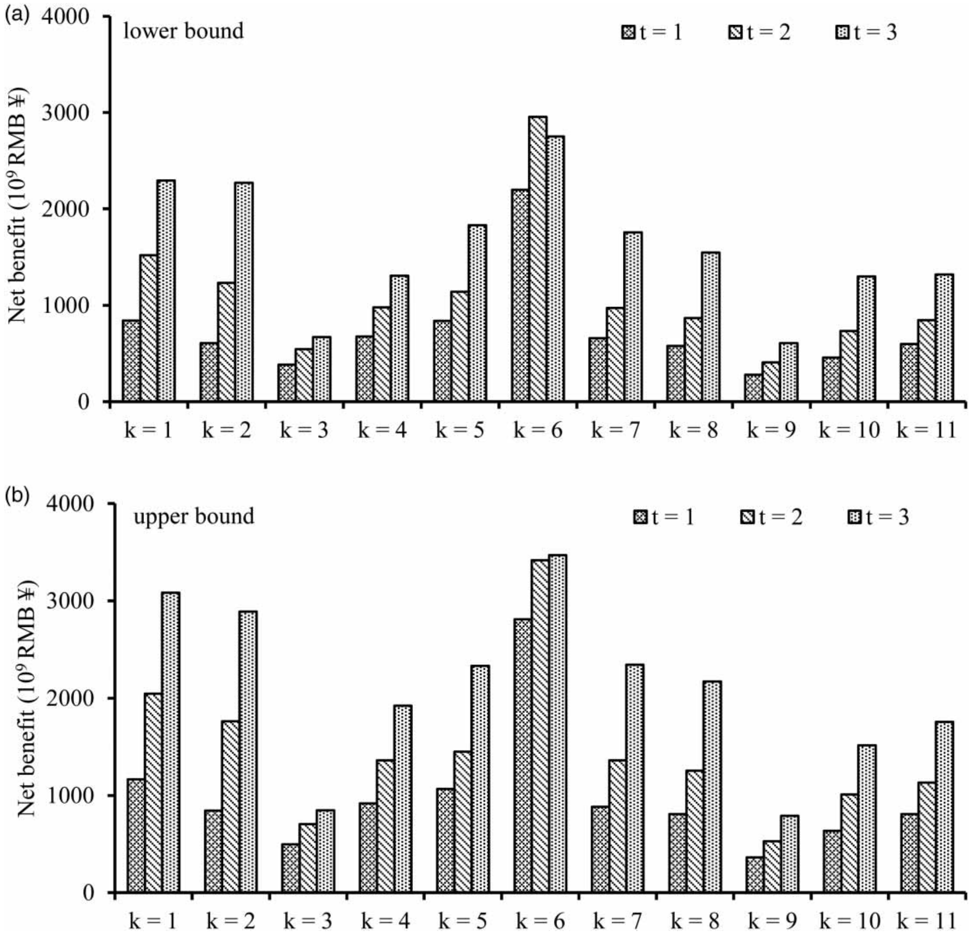

Figure 4 | Net system benefits for each district during the three planning periods.

the development potential of economy in industrial cities would be tremendous in future development. In comparison, the agriculture city would be developing at a slow speed. For example, in Zhoukou city, as an agriculture city, the net expected system benefit would be [675.0, 918.0], [977.2, 1,359.8], and [1,306.7, 1,921.4] × 10^9 RMB ¥ in each planning period. In Pingdingshan city, as an industrial city, the net expected system benefit would be [606.4, 844.6], [1,231.4, 1,761.2], and [2,269.3, 2,890.5] × 10^9 RMB ¥ in each planning period. As the provincial capital, Zhengzhou city would have the largest expected system benefit, and it would be [2,198.1, 2,809.2], [2,956.5, 3,416.6], and [2,750.7, 3,467.6] × 10^9 RMB ¥. However, the decline of agricultural output and the slight increase in industrial output lead to the stabilization of the net expected system benefit.

Figure 5 shows the cost of water supply coming from each water source in the three planning periods. In different districts, considerable changes of water supply cost would take place in a proportion of water charges. In general, the water diversion project would mitigate the problem of water shortage in the study areas. The cost of the water resources coming from the diversion project would claim a significant proportion of the total cost. In the meantime, the reused water treatment cost would be gradually increased. For example, during planning period 1, the cost of water supply from each water source would be [8.7, 9.6], [2.6, 2.7], and [4.0, 5.5] × 10^9 RMB ¥ in Pingdingshan city district. In addition, during the next two planning periods, the cost would increase to [11.8, 13.3], [3.6, 3.7], [7.0, 9.4] × 10^9 RMB ¥ and [12.7, 13.3], [4.3, 4.5], [9.9, 11.3] × 10^9 RMB ¥.

Figure 6 shows the cost of recourse for each district during the planning periods. In general, the net benefit and the recourse cost would increase from period 1 to period 3. This indicates that a high system benefit could be

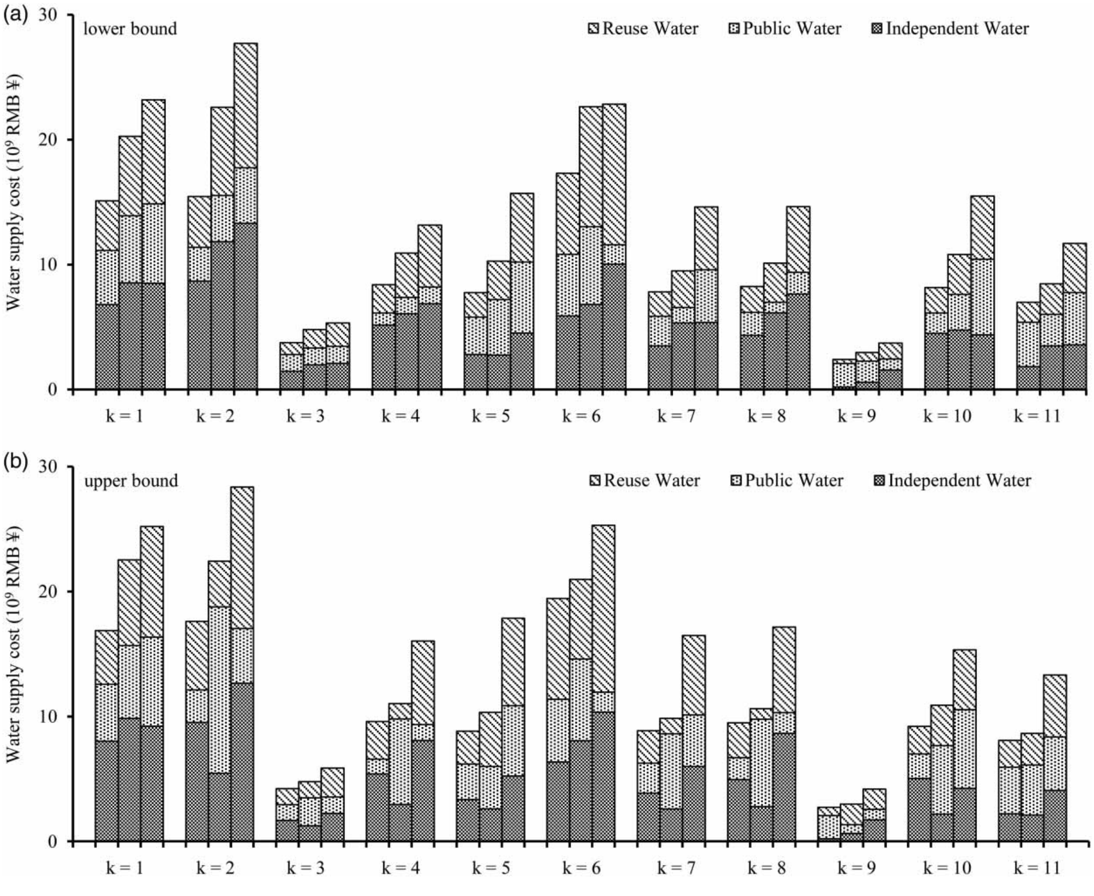

Figure 5 | Cost of water supply coming from each water source.

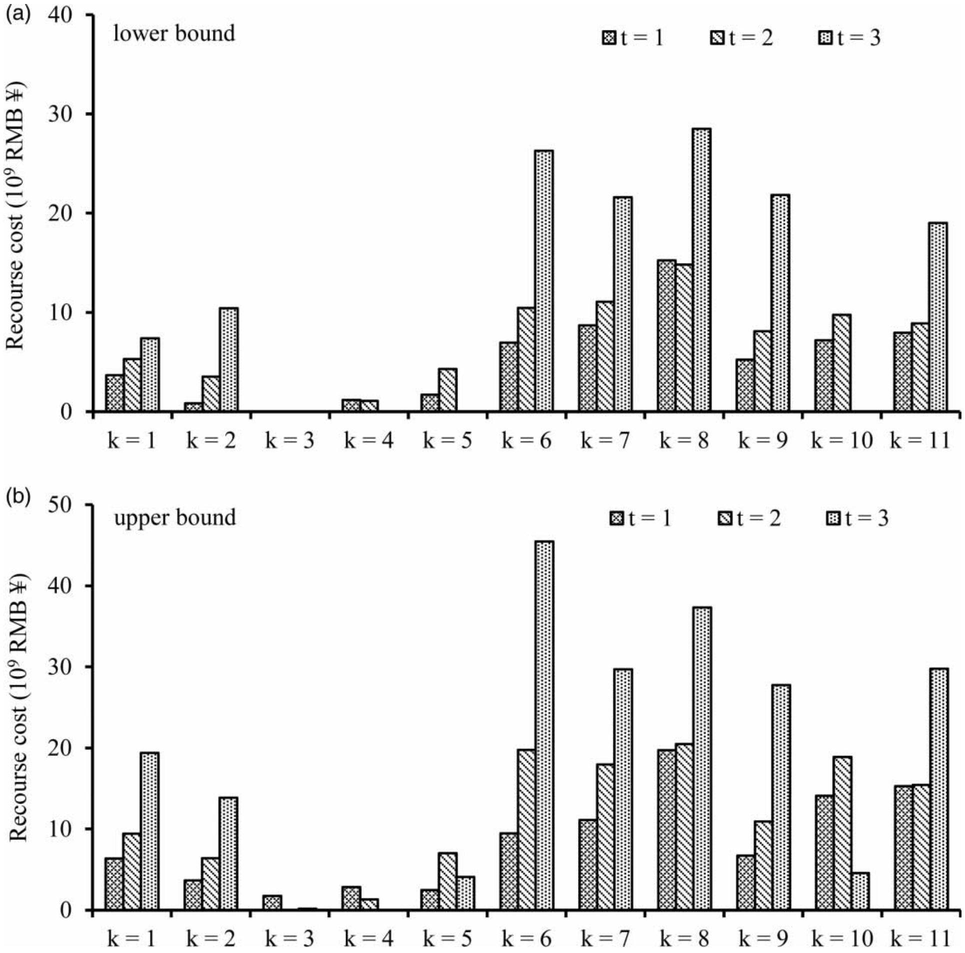

Figure 6 | Recourse cost for each district during the three planning periods.

obtained if the water demands are satisfied, but a high recourse cost might have to be paid when the promised water allocation target was not delivered. For example, in Zhengzhou city, the recourse cost would be [6.97, 9.44], [10.45, 19.75], and [26.30, 45.45] $\times 10^9$ RMB ¥ from period 1 to 3, respectively. Moreover, in Luohe city, the recourse cost would be [0, 1.71], 0, and [0, 0.15] $\times 10^9$ RMB ¥ from period 1 to 3, respectively. In Zhoukou city, the recourse cost would be [1.17, 2.80], [1.17, 1.29], and 0×10^9 RMB ¥ from period 1 to 3, respectively. This indicates that a lower economic growth target would lead to a lower optimal target of water allocation from the water diversion project, and then further lead to a lower recourse cost.

In the study area, the water resources system lacks a unified water resources management mechanism and supervisory measure. The water diversion project would greatly change the composition of the water resources system. In addition to the traditional source of water resources (i.e., independent source of water and reuse water), public water resources from the water diversion project would play an important role in mitigating water shortage. In this study, the inexact two-stage stochastic water resources management model can not only help water resources managers to formulate water management strategies for rational utilization of all kinds of water resources in this region, but also effectively reflect the complexities in the study area among multi-regions, multi-users, and multi-sources. In addition, the results demonstrate that inter-basin water transferred in an effective manner can settle the uneven distribution of regional water resources to achieve the district-optimal allocation of water resources.

CONCLUSIONS

In this study, an ITSP model was developed for water resources management. This method is based on IPP and TSP. It allows uncertainties to be presented as both probability distributions and interval values incorporated within a general optimization framework. The model can provide an effective linkage between conflicting economic benefits and the associated penalties attributed to the violation of predefined policies. The developed model was applied to a water resources allocation case study with a large water diversion project in Henan province, China. A number of scenarios corresponding to different water inflow levels were examined and some important conclusions were obtained.

Cross-regional water diversion projects will become the best solution in cases of water scarcity. Intense competition will take place among different district. The differences between each district, such as industrial structure, technological level, and geographic position, will have an influence on water resources planning. More specifically, the water coming from cross-regional water diversion projects will give priority to the prosperous regions. As for different water use sectors, industrial water and municipal water are superior, and agricultural water will mainly be supplied by an independent source of water. The cost of the water resources coming from the diversion project will claim a significant proportion of the total cost share. In the meantime, reused water treatment cost will be gradually increased. The schemes of water resources allocation have been obtained, and the results are valuable for supporting the adjustment or justification of the existing water allocation schemes within a complicated water resources system under uncertainty.

The proposed method could help decision-makers to formulate water management strategies for rational utilization of all kinds of water resources in different regions. However, the present study has its own limitations. For example, in large-scale water resources management problems, minimum cost or maximum net benefit considered as the only objective would generate the neglect of the risk of model feasibility and reliability. In addition, the probabilities of the occurrence of flow level exist within both public water supplies and independent source of water. The study under a situation with dual probability distribution should be considered. Although ecological water consumption has been considered, an acknowledged calculation method about ecological function (pollutant purification, landscape function, etc.) which can be returned to system benefit has not been determined. Finally, study of how the different conditions and water quality from the water conveyance project will affect regional allocation of water resources should be recommended.

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